National College of Business Administration & Economics Lahore

GROUP ACCEPTANCE SAMPLING PLANS FOR SOME CONTINUOUS DISTRIBUTIONS

BY

MUHAMMAD ASLAM

DOCTOR OF PHILOSOPHY
IN
APPLIED STATISTICS

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NATIONAL COLLEGE OF BUSINESS ADMINISTRATION & ECONOMICS

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A dissertation submitted to
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<u>Dissertation Committee:</u>

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DECLARATION

This is to certify that this research work has not been submitted for obtaining similar degree from any other university / college in Pakistan as well as abroad.

MUHAMMAD ASLAM
April 10, 2010

DEDICATED
TO

My Parents

&

My Wife

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RESEARCH COMPLETION CERTIFICATE

Certified that the research work contained in this thesis entitled **"Group Acceptance Sampling Plans for Some Continuous Distributions"** has been carried out and completed by **"Muhammad Aslam"** under my supervision during his Ph.D. in Applied Statistics program.

(Prof. Dr. Munir Ahmad)
Supervisor

SUMMARY

Acceptance sampling plans are used to accept or reject the submitted product on the basis of random sample taken from the lot. This sampling scheme is used to test the item one by one. In practice, there are testers accommodating multiple items are available. In this situation, group acceptance sampling is used more efficiently. The basic purpose of group acceptance sampling plans is that it can reduce the cost and the time of the experiment than the ordinary acceptance sampling plans. Therefore, the objectives of this study is to propose the group acceptance sampling plans based on truncated life tests are discussed from two approaches. In the first approach, a group acceptance sampling plan from a truncated life test is designed when the lifetime of an item follows either an inverse Rayleigh distribution or a log-logistic distribution, in which a multiple number of items as a group can be tested simultaneously. The minimum number of groups required for a given group size and the acceptance number is determined when the consumer's risk and the test termination time are specified. The operating characteristic values according to various quality levels are found and the minimum ratios of the true average life to the specified life at the specified producer's risk are obtained. Some comparisons are made between the results for the two distributions. In the second approach, a group acceptance sampling plan for a truncated life test is proposed when a multiple number of items as a group can be tested simultaneously assuming that the lifetime of a product follows the Weibull distribution or gamma distribution with known shape parameter. The design parameters such as the number of groups and the acceptance number are determined by satisfying the producer's and the consumer's risks at the specified quality levels, while the termination time and the number of testers are specified. The results are explained with examples in both approaches.

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ACRONYMS

ASN	Average sample number
OC	Operating characteristic
AQL	Acceptable quality level
LQL	Limiting quality level
GASP	Group acceptance sampling plan

NOTATIONS

a	Experiment time multiplier to the specified life
c_1	Acceptance number of the proposed plan
c_2	Rejection number of the proposed plan
g	Total number of groups
$L(p)$	Lot acceptance probability
$1-\beta$	Consumer's confidence level
r	Group size
t_0	Experiment time
α	Producer's risk
$1-\alpha$	Producer's confidence
β	Consumer's risk
λ_w	Scale parameter of the Weibull distribution
μ	Unknown mean life
μ_0	Specified life
λ_I	Scale parameter of the inverse Rayleigh distribution
λ_l	Scale parameter of the log-logistic distribution
m_l	Shape parameter of the log-logistic distribution
μ/μ_0	Ratio of true average life to specified life
$\Gamma(.)$	Gamma function
m_w	Shape parameter of the Weibull distribution
r_1	The mean ratio at the consumer's risk
r_2	The mean ratio at the producer's risk
p_2	Failure probability corresponding to the producer's risk
p_1	Failure probability corresponding to the consumer's risk
n	Sample size
λ_g	Scale parameter of the gamma distribution
m_g	Shape parameter of the gamma distribution

λ_{gb}	Scale parameter of the generalized Birnbaum-Saunders distribution
m_{gb}	Shape parameter of the generalized Birnbaum-Saunders distribution
λ_r	Scale parameter of the generalized Rayleigh distribution
m_r	Shape parameter of the generalized Rayleigh distribution
λ_b	Scale parameter of the Birnbaum-Saunders distribution
m_b	Shape parameter of the Birnbaum-Saunders distribution
λ_p	Scale parameter of the Pareto distribution
m_p	Shape parameter of the Pareto distribution
RGS	Repetitive group sampling
Pdf	Probability density function
PP	Probability plot
cdf	Cumulative distribution function
VGS	Variable group sampling
BS	Birnbaum-Saunders
QQ	Quantile-quantile

CHAPTER 1

BACK GROUND

1.1 INTRODUCTION

Acceptance sampling is an important field of statistical quality control to accept or reject products under inspection. This field was popularized by Dodge and Roming. The acceptance sampling plan first applied in the US Military for testing the bullets during World War II. For example, if every bullet is tested in advance, no bullet is available for shipment. On the other hand if no bullet is tested, then disaster might occur in the battle field at the crucial time. Dodge stated that a sample is randomly taken from a lot and the fate of the products depends on the information obtained from this sample. This process is known as acceptance sampling or lot acceptance sampling. So the acceptance sampling is used for possible acceptance or rejection of the products but not for estimating the quality of the lot.

Schilling (1989) says, "An individual sampling plan has much the effect of a lone sniper, while the sampling plan scheme can provide a fusillade in the battle for quality improvement." Balakrishnan et al. (2007) stated, "Quality is now not only an option or aim of companies, but a necessity for businesses in a global market. Thus, the quality has become a differentiation tool between competitive enterprises. Two important tools for ensuring quality are the statistical quality control and the acceptance sampling." Acceptance sampling plan is a `middle path' between hundred percent inspections and no inspection at all.

1.2 PURPOSE OF ACCEPTANCE SAMPLING

The following two factors are essential to consider an acceptance sampling for testing the quality of a lot of products.

a) Cost and Time

The producers are very careful about the quality of their products from the raw material to final products so that they do not face any difficulty when a consumer comes to buy it. At this stage, it is not possible to check the lifetime of each and every component (100% inspection) taken from the lot for possible acceptance or rejection which definitely leads to the high cost and wastage of time. Cost and time are very important factors that motivate the experimenter to use an acceptance sampling plan scheme. Therefore, an acceptance sampling plan is designed to reach the final decision about the submitted product and to minimize these factors.

b) Destructive Items

If the products under study are for example, electronic components such as energy saver bulbs and an experimenter is interested to see the average life of these bulbs, for this experiment, it is not possible to put all the bulbs in a lot on test and wait for the number of failures. The only way is to pick up few bulbs and put them on test and, on the basis of information so obtained, decide about the average life of energy saver product. Hence for the acceptance or rejection of these destructive products, acceptance sampling is a necessary tool.

1.3 CLASSIFICATION OF ACCEPTANCE SAMPLING

There are two major classifications of acceptance sampling plans (a) by attributes and (b) by variable. These are briefly discussed below:

1.3.1 Attribute Acceptance Sampling

An attribute acceptance sampling has many applications in variety of ways. For example, it is used to check the incoming parts to satisfy certain conditions before they are assembled. The finished products need to satisfy the consumer's specifications. This sampling has three design parameters say (N,n,c), i,e. lot size, N, sample size, n, and the acceptance number, c. The plan is implemented as: select n items from N with the acceptance number, c. If the number of defective items is larger than c, then reject the lot, otherwise accept it.

The quality level, p, indicates the fraction of defective items. A quality level p =0% means that all the units in the lot are good and p =100% means that all the items in the lot are defective/bad. If p =0% or p =100%, there is no need of acceptance sampling plan. But, if p is between 0% and 100%, the acceptance sampling plan is used to accept or reject the entire product on the basis of sample information. With this sampling, there arise two types of risks which are discussed in Section 1.4.

1.3.2 Variable Acceptance Sampling

Producers claim about the average life of the product is tested through an acceptance sampling scheme. Hence, variable acceptance sampling plans are developed to accept or reject a submitted lot of products on the basis of inspected measurable quality characteristic; and sample taken from that lot. To accept or reject the claim of the producer about the products, it is necessary to specify the probability distribution of variable quality characteristic under inspection. In this situation, the normal distribution is used for the final decision. If the failure time of the product under consideration does not follow the normal distribution, the decision constructed on this basis would be misleading. Therefore, variable sampling is applied for the measurement data when the use of the normal distribution is justified to quality characteristics. The main advantage of variable sampling is that the same (as in attribute sampling scheme) operating characteristics (OC) curve can be obtained for smaller sample sizes. A second advantage of this sampling plan is that the measurement data provide more information than the attribute data. There are some disadvantages of the variable sampling plan over the attribute sampling plan. The use of variable sampling plan depends on the assumption that the quality characteristic of the items follows the normal distribution which does not always fulfill in practice. Collani (1990) criticized that attribute sampling can not be used if one is interested in the fraction non-conforming in incoming batches. Seidel (1997) proved that the attribute sampling is always optimal. Even though variable sampling is more economic than the attribute sampling, yet the attribute sampling is widely used in practice because it is easy to use and does not rely on the assumption of the normality. It can be used if lifetime or failure time of a product follows the Weibull distribution or gamma distribution.

1.4 PRODUCER AND CONSUMER RISKS

As mentioned earlier, acceptance sampling scheme is used to reach the final decision of the lot on the basis of information of a few items taken at random from an infinite size lot. Hence, two risks are associated with a sample. For example, if a good product is rejected on the basis of sample information, it will be a loss to producer and if a bad lot is accepted, it will be a loss to consumer. Therefore, producer's and consumer's risks are associated with acceptance sampling scheme. The probability of rejecting a good lot is called the producer's risk and is denoted by α. On the other hand, the probability of accepting the bad lot is called the consumer's risk and is denoted by β. The correct decision, then, is the acceptance of a good lot or rejection of a bad lot on the basis of a sample. The purpose of the acceptance sampling plan is to find the design parameters which satisfy both risks.

1.5 DETERMINATION OF SAMPLE SIZE

The purpose of this study is to design a sampling plan, which is useful to save time and cost of the experimenters (producer and consumer). This goal is achieved if we find a minimum/optimal sample size, n, that satisfies either both risks or only the consumer's risk. A sample size n that satisfies either both risks or only the consumer's risk is said to have a minimum sample size. In the group acceptance sampling, we are interested to find the minimum number of groups so that the producer's and consumer's risks are satisfied. In this case, we select a random sample of size n and distribute the r (number of testers) items to each of groups.

1.6 TYPES OF ACCEPTANCE SAMPLING

The two major types are (a) single acceptance sampling and (b) double acceptance sampling. Next, we discuss these two types of sampling based on truncated life test.

1.6.1 Single Acceptance Sampling Plan

The single acceptance sampling is the combination of sample size, acceptance number/action number, and termination ratio. To test the quality of a product, the null hypothesis is formulated that $\mu \geq \mu_0$ and the alternative hypothesis $\mu < \mu_0$, where μ is the true average or median life of a product and μ_0 is the specified average or median life of a product. These hypotheses are tested through truncated life tests in acceptance sampling. In a single sampling, a sample size n is selected and put on test. The fate of the product depends on the information of this single sample. The life test is implemented as follows:

A random sample is selected and put on the tests. An experimenter runs this experiment for a pre-decided experiment time t. The acceptance number (action number) c is fixed for an experiment, which is truncated if more than c failures are recorded before the end of experiment time or the time of experiment is ended, whichever is earlier. A lot is accepted and realized for consumer's use if no more than c failures are observed during this time and we accept $\mu \geq \mu_0$, otherwise we reject it.

1.6.2 Double Acceptance Sampling Plan

As we mentioned earlier, two risks are associated with the acceptance sampling scheme. The double acceptance sampling is used to minimize the producer's risk. This scheme is used when the submitted product is questionable or if the experimenter can not reach the final decision about the product on the basis of first sample taken from it. The double sampling provides another opportunity to accept or reject the submitted lot. Therefore, this sampling scheme is used to minimize the producer's risk as well as the consumer's risk. Duncan (1986) stated that the double acceptance sampling plan is used to reduce the sample size or producer's risk in the area of quality control where the normal distribution is often adopted. Jun et al. (2006) proposed that the single variable and double group sampling plans for the sudden death testing scheme. They obtained the number groups for both schemes and suggested that the group double sampling is used to reduce the number of groups than the single group sampling scheme. A double sampling plan based on truncated life tests is characterized by n_1, n_2, c_1, c_2 and t/μ_0. This plan operates as follows:

Draw a random sample of size n_1 and put it on the test. Accept the lot if there are no more than c_1 failures and reject the lot if more than c_2 failures are noted before the end of experiment time. If the number of failures are between c_1 and c_2 $(c_1 < c_2)$, take another sample of size n_2 and reject the lot if more than c_2 failures are occurred during the experiment time.

1.6.3 Repetitive Group Acceptance Sampling Plan

The repetitive group sampling (RGS) plan is used for the inspection of attribute characteristics. The operation of RGS is similar to that of the sequential sampling plan. Sherman (1965) stated "RGS plan gives the minimum sample size as well as the desired protection. Furthermore, RGS plans are not nearly as efficient as the sequential sampling plans but they are usually more efficient than the single sampling plan. Specifically, this plan gives an intermediate value in sample size efficiency between the single sampling and sequential sampling plans." The RGS plan is used to improve the OC curve when the variable plan possesses an unsatisfactory OC curve similar to that of the zero acceptance number. To increase the discriminating power of this curve, one way is to increase the sample size, but in life test experiment, it is not possible to increase the sample size. An alternative way is to use the

RGS plan for the attribute inspection. Sherman (1965) also pointed out that variable group sampling (VGS) sampling requires minimum inspection of the submitted product in terms of average sample number (ASN) than the attribute sampling. Furthermore, the VGS is simple and less costly than the attribute acceptance sampling plan scheme.

The assumptions of the VGS plan are

(i) Lots are submitted for serially inspection, in order of production from a process that turns out a constant proportion of non-conforming items.

(ii) The consumer has confidence in the supplier and there should be no reason to believe that a particular lot is poorer than the preceding lots.

1.6.4 Sequential Acceptance Sampling Plan

The sequential sampling approach is quite different from the single, double and multiple sampling plans. In this approach, one takes a sequence of sample size from the product under inspection. If one item is selected as sequence, then the sampling is called the item-by-item sequential sampling plan. On the other hand, if more than one item is selected as sequence, then this sampling is called group sequential sampling plan. Acceptance sampling plan is to accept or the reject the product on the basis of the information taken from the lot whereas sequential acceptance sampling is used to minimize the number of items tested when the early results show that the lot clearly meets the specifications or fails to reach the specified standard.

According to Sherman (1965) RGS plans are not nearly as efficient as the sequential sampling plan but they are most efficient than the single sampling plans. Balamurali and Jun (2006) calculated the average sample number of sequential sampling by the approximation given by Schilling (1982). They observed that the RGS is efficient in terms of average sample number than the single and double sampling but not more efficient than the sequential sampling scheme.

1.7 ACCEPTANCE SAMPLING PLAN AND RELIABILITY PLAN

Two approaches (a) acceptance sampling and (b) reliability plan are discussed as follows:

As the basic purpose of the acceptance sampling is to find sample size which satisfies the specified consumer's risk, the sample size obtained in such a way is called the minimum sample size. In this approach, we are interested in finding the minimum sample size for some specified values of acceptance number, termination time and the specified consumer's risk. Assuming that the decision about the submitted lot is classified as to accept the product or reject it and the sample is taken from the infinite lot so that we can use the binomial distribution (see, Stephen 2001) to find the sample size, probability of acceptance and minimum mean ratio. But, in the reliability approach, we are interested in finding the minimum experiment time for some specified producer's risk. The purpose is to find the experiment time or termination ratio for specified values of producer's risk, acceptance number and the sample size. The latter approach is considered to be more economic in terms of cost and time of the experiment than the former approach to reach the final decision about the submitted product. Therefore, we may call it the economic approach.

1.8 APPLICATION OF BINOMIAL DISTRIBUTION

The binominal distribution is widely used to find the design parameters of attribute acceptance sampling plans. According to Stephen (2001), this distribution can be used in the area of acceptance sampling if the following two assumptions are fulfilled:

a) If a lot size (N) is large enough to be considered infinite for example $n/N \le 0.10$
b) If a decision about the lot is classified into two mutually exclusive groups which are accepted or rejected.

The above assumptions strongly support to use the binomial distribution and determine the parameters of an acceptance sampling plan.

1.9 GROUP ACCEPTANCE SAMPLING

In most acceptance sampling plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. It is implicitly assumed in the usual sampling plan that only a single item is put in a tester. Sometimes, however, testers accommodating multiple items are available in practice because time and cost can be saved by testing those items simultaneously. For example, sudden death testing is frequently adopted by using this type of testers (see, Pascual and Meeker, 1998; Vlcek et al. 2003; Jun et al. 2006).

For this type of testers the number of items to be equipped in a tester is given by specification. When designing an acceptance sampling plan with this type of testers, determining the sample size is equivalent to determining the number of groups. If we call items in a tester as group, then we need to determine the number of groups and the acceptance number as a designed parameters. The proposed acceptance sampling plan is called a group acceptance sampling plan.

In an group acceptance sampling plan, the selected sample size n is distributed to g groups and r (group size) items are put on test in each group so that $n = rg$. The r items in a group are tested simultaneously on each different tester for a pre-assigned time. The experiment is truncated if more than the acceptable number c of failures occur in any group during the experiment time.

1.10 LIFETIME DISTRIBUTION

Failure and repair time of electronic components are random and unpredictable in nature. But, the failure time of an electronic component can be modeled using statistical distributions. The probability density function (pdf) can be used in acceptance sampling. Life distribution are chosen for one or more of the following three reasons, for more detail, (see Tobias, 2004).

a) There is a physical/statistical argument that theoretically matches a failure mechanism to a life distribution model.
b) A particular model has previously been used successfully for the same or similar failure mechanism.
c) A convenient model provides a good empirical fit to all the failure data.

Cox and Oakes (1984) stated that the analysis of the probability density function is not always the best thing. They establish the following ways to identify some suitable distribution:

a) The hazard function $h(t)$ or log survival versus t or log (t).
b) The cumulative hazard function $H(t)$ or log survival versus t or log (t).

Thus, whatever the criteria is to choose a model, this model must be logical and possess statistical tests for fitting the data. To test failure data that follow the pattern of a specified distribution with known shape parameter, a well-known probability plot (PP) can be used to study the behavior of the data. Normally, in acceptance sampling plans based on truncated life tests, the cumulative distribution function (cdf) is used to find the parameters of the proposed sampling plan when the lifetime follows a specific distribution. To solve this problem in acceptance sampling plan, we use various non-normal statistical distributions. There are many distributions which have been used in acceptance sampling including inverse Rayleigh, log-logistic, Weibull and gamma ones. A brief background of these distributions is given in the following sections.

1.10.1 The Inverse Rayleigh Distribution

The cdf of an inverse Rayleigh distribution is given by

$$F(t) = \exp\left(-\lambda_I^2 / t^2\right), \; t > 0, \tag{1.1}$$

where λ_I^2 (>0) is the scale parameter. The mean of this distribution is given by

$$\mu = \sqrt{\pi}\lambda_I \,. \tag{1.2}$$

Mukerjee and Saran (1984) studied the failure rate of an inverse Rayleigh distribution. According to them, the failure rate of a single parameter inverse Rayleigh distribution is increasing for $t < 1.069\sqrt{\lambda_I}$ and decreasing for $t > 1.069\sqrt{\lambda_I}$. Voda (1972) studied the properties of the maximum likelihood estimator of λ_I. Mukerjee and Maiti (1997) considered the percentile estimation of the distribution. Rosaiah and Kantam (2005) used the inverse Rayleigh distribution for an acceptance sampling plan based on truncated life test. More recently, Rosaiah et al. (2008) developed the reliability plans under the assumption that the lifetime of a product follows the inverse Rayleigh distribution.

1.10.2 The Log-Logistic Distribution

The log-logistic distribution has been studied by Shah and Dave (1963) and Tadikamalla and Johnson (1982). The cdf of the log-logistic distribution is given by

$$F(t) = \frac{(t/\lambda_l)^{m_l}}{1+(t/\lambda_l)^{m_l}}, \quad t > 0, \tag{1.3}$$

where $m_l\,(>1)$ is the shape parameter and $\lambda_l\,(>0)$ is the scale parameter. The median of this distribution is just λ_l and the mean is given by

$$\mu = \frac{\pi\lambda_l / m_l}{\sin(\pi / m_l)}. \tag{1.4}$$

We particularly consider the case of $m_l = 2$ for establishing tables for this distribution, in such case the mean is obtained by

$$\mu = 1.5708 m_l. \tag{1.5}$$

O'Quigely and Struthers (1982) study the log-logistic distribution in survival analysis. The log-logistic distribution has been considered by Ragab and Green (1984) for the order statistics and by Balakrishnan and Malik (1987) for the linear unbiased estimation of its parameters. Kantam et al. (2001) and Kantam et al. (2006) used this distribution in acceptance sampling for life test.

1.10.3 The Weibull Distribution

The Weibull distribution has many applications in life testing problems. The exponential distribution is a special case of the Weibull distribution. Epstein (1954) proposed acceptance sampling plans assuming that the lifetime of a product follows the exponential distribution. The Weibull distribution has been considered in many studies for designing reliability sampling plans under various types of censoring schemes. See, for example, Fertig and Mann (1980), Schneider (1989), Chen et al. (2004), Balasooriya and Low (2004), and Ng et al. (2004).

In this case, the cdf is given by

$$F(t;\lambda_w, m_w) = 1 - \exp\left(-(t/\lambda_w)^{m_w}\right), \quad t \geq 0, \quad \lambda_w, m_w > 0, \tag{1.6}$$

where m_w is the shape parameter and λ_w is the scale parameter of the Weibull distribution.
The mean life of a Weibull distributed item is given by

$$\mu = (\lambda_w / m_w)\Gamma(1/m_w). \tag{1.7}$$

1.10.4 The Gamma Distribution

The chi-square and exponential distributions are special cases of the gamma distribution and is widely used as model for the life length of materials. Drenick (1960) and Herd (1959) used this model in reliability problems. The order statistics and some applications of this distribution are discussed by Gupta (1960). The cdf of gamma distribution for integer value (see, Gupta and Groll, 1961) of m_g is given by

$$F_T\left(t,\lambda_g\right)=1-\frac{\sum_{j=0}^{m_g-1}\exp\left(-t/\lambda_g\right)\left(t/\lambda_g\right)^j}{j!},\quad t,\lambda_g>0\,,m_g\geq 1, \qquad (1.8)$$

where m_g is the shape parameter and λ_g is the scale parameter of gamma distribution.

The mean life of a gamma distributed product is given by

$$\mu=m_g\lambda_g. \qquad (1.9)$$

1.11 OBJECTIVE OF THE STUDY

In acceptance sampling plans based on a truncated life test, the sample size is usually determined for the specified experiment time and the acceptance number which satisfy the pre-decided consumer's risk only. This approach uses the one point on the OC curve, so it may not always satisfy the producer's risk. Some studies such as Fertig and Mann (1980), Balasooriya et al. (2000), and Balamurali and Jun (2006) used the two point (by considering producer's and consumer's risks) approach for designing variables acceptance sampling plans for controlling fraction nonconforming. However, it has not been adopted for designing attributes group acceptance sampling based on truncated life tests.

Therefore, the objectives of this study is to propose the group acceptance sampling plans based on truncated life tests are discussed from two approaches. In the first approach, a group acceptance sampling plan from a truncated life test is designed when the lifetime of an item follows either an inverse Rayleigh distribution or a log-logistic distribution, in which a multiple number of items as a group can be tested simultaneously. The minimum number of groups required for a given group size and the acceptance number is determined when the consumer's risk and the test termination time are specified. The operating characteristic values according to various quality levels are found and the minimum ratios of the true average life to the specified life at the specified producer's risk are obtained. Some comparisons are made between the results for the two distributions. In the second approach, a group acceptance sampling plan for a truncated life test is proposed when a multiple number of items as a group can be tested simultaneously assuming that the lifetime of a product follows the Weibull distribution or gamma distribution with known shape parameter. The design parameters such as the number of groups and the acceptance number are determined by satisfying the producer's and the consumer's risks at the specified quality levels, while the termination time and the number of testers are specified. The results are explained with examples in both approaches.

CHAPTER 2

LITERATURE REVIEW

2.1 ATTRIBUTE ACCEPTANCE SAMPLING PLAN

Epstein (1954) developed acceptance sampling plans based on truncated life test assuming that the lifetime of an item follows the exponential distribution. He (1954) considered two situations to develop the acceptance sampling plan for this distribution. The first is the replacement case, in which if an item is failed before the end of experiment time, the failure item is replaced by a new item. In non-replacement case, an item is not replaced by a new one. He (1954) presented tables for both cases. Further, a number of examples are given to justify the use of proposed plan in practice.

Goode and Kao (1961) presented tables of sampling plans and reliability testing for truncated life tests under the assumption that the lifetime of a device follows the Weibull distribution. This procedure is used when inspection of items is by attribute. They (1961) included tables of ratio for adopting the Military Standard 105B to life test and reliability applications. The exponential model is included as a special case of the Weibull distribution when the shape parameter is known or assumed to be known when $\lambda = 1$.

Gupta (1962) obtained sampling plans for truncated life tests from the normal and lognormal distribution. His (1962) tables give the minimum values of sample size necessary to ensure a certain mean life. The modification necessary to assure any other quantile (percentile) of the distribution is obtained. The operating characteristic functions of the proposed plans are obtained for a wide range of values of practical interest and graphed in order to facilitate selection of an appropriate plan in a given situation. Producer's risk is discussed. The table for ratio of true average life to specified average life is given to insure that the producer’s risk does not exceed from 5% to 10%. An approximation for minimum sample size is given.

Kantam and Rosaiah (1998) introduced a new pdf known as the half logistic distribution in the field of acceptance sampling plan. They (1998) developed acceptance sampling plans using the half logistic distribution as a model for lifetime random variates when the test is truncated at a pre-assigned time. They (1998) presented the tables for various acceptance and experiment time. The results in tables are explained by examples.

Kantam et al. (2001) considered the problem of acceptance sampling plans when the test is terminated at a pre-assigned experiment time. Assuming that the lifetime follows the log-logistic distribution with known shape value. The minimum sample values, probability of acceptance and minimum ratio of unknown average life to specified average life are presented. The producer’s risk is also discussed. The results are illustrated by an example.

Baklizi (2003) proposed the acceptance sampling plans based on truncated life tests for the Pareto distribution of the second kind when the shape parameter is assumed to be known. The problem is to find minimum sample size, probability of acceptance and minimum ratio of true average life to specified average life. He (2003) developed a sampling plan that satisfies only the consumer’s risk for specified experiment time and acceptance number. The tables are given for different values of shape parameter. The results are explained by an example.

Baklizi and EI Masri (2004) developed acceptance sampling plans assuming that life test is truncated at pre-assigned time. The lifetime of the units are assumed to follow the Birnbaum-Saunders (BS) distribution widely used for the fatigue process. They (2004) constructed tables of probability of acceptance, minimum sample size and minimum ratio of true median life to specified median life. They (2004) found the design parameters values satisfying the consumer’s risk. The producer’s risk is also presented. An illustrative example is given.

Rosaiah and Kantam (2005) used the inverse Rayleigh distribution in the area of acceptance sampling plan and developed the acceptance sampling plans, assuming that the life time a product follows the inverse Rayleigh distribution. They (2005) proved that the software data given by Wood (1996) is well fitted to the inverse Rayleigh distribution and provided tables for operating characteristics, minimum sample and minimum ratios for their proposed plan. An example is given to use the plan for practical purposes.

Tsai and Wu (2006) considered the problem of acceptance sampling plan based on truncated life tests when the lifetime of product follows the generalized Rayleigh distribution for known shape parameter. They (2006) used the cdf of the generalized Rayleigh given by Voda (1976). For different acceptance number, confidence levels, values of ratio of fixed experiment time to specified mean life, and minimum sample sizes ensuring the specified mean life are found. The operating characteristics values and the producer’s risk are also discussed. The tables of minimum ratio of true average life to specified average life and minimum sample size are constructed. Examples are presented to explain the proposed plan.

Rosaiah et al. (2006) considered the problem of acceptance sampling plans based on life test using the generalized log-logistic distribution (exponentiated log-logistics distribution in the lines of the exponentiated Weibull distribution). In this paper, the probability of acceptance is determined for products submitted to the consumer's use and their lifetimes follow the exponentiated log-logistic distribution with known shape. Tables are constructed to support this plan. The results are illustrated by a numerical example.

Balakrishnan et al. (2007) considered the problem of acceptance sampling assuming that the lifetime of a product follows the generalized BS distribution. They (2007) introduced this distribution and justify its application in the field of acceptance sampling plans and presented tables of minimum sample size, probability of acceptance and minimum ratio of true median life to specified median life. They (2007) showed that the software data given by Wood (1996) is better fitted to these data than the inverse Rayleigh distribution and simple Birnbaum-Saunders distribution using probability plot (PP) charts. They (2007) discussed the approximation method also. The values of sample size obtained in their plan is indeed less than the values of the sample size of Gupta and Groll (1961) for the gamma distribution with shape parameter 2, Kantam et al. (2001) for the log-logistic distribution with shape parameter 2 and Baklizi and EI Masri (2004) for the simple BS distribution with shape parameter 1. The results are explained by some examples.

Aslam et al. (2010) developed the acceptance sampling plans for the generalized exponential distribution when the life tests are truncated at pre-assigned time. Table of sample sizes ensure a certain median life is given. It is shown the tables presented can be used for other life percentiles and shape parameter. Examples are provided for illustrative purposes.

2.2 VARIABLE ACCEPTANCE SAMPLING PLAN

Jun et al. (2006) developed variable acceptance sampling plans for Weibull distributed items under sudden death testing. Variable single and double sampling plans are proposed for the lot acceptance of a product whose life is supposed to follow the Weibull distribution with known shape parameter. The design parameters of this plan are found using the two point approach. The number of group is determined independently of the group size and shape parameter. The comparison between single and double sampling is given and results explained by examples.

Balamurali and Jun (2006) introduced the concept of repetitive group sampling for the variable inspection. They (2006) studied the advantages of this scheme over variable single acceptance sampling plans. Extensive tables are provided and the proposed procedure is illustrated by examples.

2.3 RELIABILITY ACCEPTANCE SAMPLING PLAN

Kantam et al. (2006) proposed the acceptance sampling plans in which the items are put on the test to decide upon acceptance or rejection of a submitted lot when the lifetime of a product follows the log-logistic distribution with shape parameter 2. For a given producer's risk, acceptance number and sample size waiting time to terminate the experiment are found. The tables are given for some values of the producer's risk. The results of this economic reliability sampling plan are compared to the sampling plan given by Kantam et al. (2001). They (2006) argued that the proposed test plans would result in saving the time and cost to reach the final decision about the submitted product.

Rosaiah et al. (2007) used the exponentiated log-logistics distribution as a model of lifetime in the area of acceptance sampling plan and developed economic reliability sampling plans based on truncated life tests for the known parameter of exponentiated log-logistic distribution. They (2007) presented tables of termination time for producer's risk at 1% and 5% and proved that the failure time of soft ware data given by Wood (1996) is well fitted to exponentiated log-logistic distribution with shape parameter 2 using Q-Q plots. They (2007) presented examples to show that the reliability approach is more economic to reach the decision about a product than the approach given by Rosaiah et al. (2006).

Aslam and Shahbaz (2007) developed reliability acceptance sampling plans for generalized exponential distributed items for known shape parameter values. The tables of termination time are given for producer's risks at 1% and 5%. The results are compared to plan given by Kantam et al. (2006) for the log-logistic distribution with shape parameter 2. It is proved that the reliability plan is more economic in terms of cost and time to reach the final decision about the product. To illustrate the idea, a numerical example is given.

Rosaiah and Kantam (2008) provided reliability sampling plans based on life tests for the inverse Rayleigh distributed items. The preferability of the proposed plan over plan proposed by Rosaiah and Kantam (2005) is established with respect to experiment time.

Aslam and Kantam (2008) proposed an economic reliability plan for the Birnbaum-Saunders distribution. It is assumed that the shape parameter is known. The tables are constructed for termination ratio for producer's risk at 1% and 5%. The results are compared to acceptance sampling plans given by Baklizi and EI Masri (2004) for shape parameter 1. The results are explained by examples.

Aslam (2008) proposed the reliability acceptance sampling plans based on truncated life tests assuming that the lifetime of a product follows the generalized Rayleigh distribution with known shape parameter. He (2008) discussed the advantages of the proposed plans over the existing plans. Results are explained by two examples.

2.4 DOUBLE ACCEPTANCE SAMPLING PLAN

Aslam (2007) extended the idea of single sampling to double sampling plan based on truncated life tests and used the standard scheme to develop the double sampling plan assuming that the lifetime of a product follows the simple Rayleigh distribution. Tables of probability of acceptance are presented for various values of sample sizes and some specified values of true average life to specified average life.

Aslam and Jun (2010c) developed the double acceptance sampling plans based on truncated life tests assuming that the lifetime of a product follows the generalized log-logistic distribution. The zero and one failure scheme is considered. The minimum samples sizes for the first and the second sample are determined to ensure that the median or mean life is larger than the specified average or median life. The operating characteristics are analyzed according to various ratios of the true median life to the specified life. The minimum such ratios are also obtained so as to lower the producer's risk at the specified level. The results are explained by examples.

Aslam et al. (2009c) developed double acceptance sampling plan based on the truncated assuming that the lifetime of a product follows any distribution. The design parameters such as sample sizes and acceptance numbers for the first and second samples are determined so as to minimize the average sample number subject to satisfying the consumer's and producer's risks at the respectively specified quality levels. The resultant tables can be used regardless of the underlying distribution as long as the reliability requirements are specified at two risks. In addition, gamma and Weibull distributions are particularly considered to report the design parameters according to the quality levels in terms of the mean ratios.

Aslam et al. (2009b) proposed double sampling for the Weibull distribution and considered the zero and one failure test to find the design parameters at the specified consumer's risk and test termination time. The minimum mean ratio is calculated at given level of producer's risk. They (2009b) found that the double sampling can reduce the producer's risk as compared to single sampling plan.

2.5 GROUP ACCEPTANCE SAMPLING PLAN

Aslam and Jun (2009a) designed the group acceptance sampling plans based on life test when the lifetime of products follows either an inverse Rayleigh or a log-logistic distribution, in which a multiple number of items as group can be tested simultaneously in a tester. The minimum number of groups, probability of acceptance and minimum ratio of true average life to specified average life are found and the producer's and consumer's risks are discussed. Comparison of results for both distributions is given by examples.

Aslam and Jun (2009b) proposed group acceptance sampling plans for the Weibull distribution with known shape parameter. The design parameters such as the number of groups and the acceptance number are determined by satisfying producer's and consumer's risks for given termination time, mean ratio and the number of testers. The results are explained by examples.

Aslam et al. (2009a) studied group acceptance sampling plans based on life test assuming that the lifetime follows the gamma distribution with known shape parameters. The design parameters are determined for specified values of termination time, mean ratio and number of testers. The tables are constructed and results are explained by examples.

CHAPTER 3

EARLIER WORK

3.1 INTRODUCTION

In this chapter, some lifetime distributions earlier used by the researchers to develop acceptance sampling plans are presented. Some formulas to find the minimum sample size to ensure (a) true average life to specified average life, (b) the minimum ratio of true average life to specified average life and (c) the probability of acceptance of a lot for attribute single acceptance sampling, variable acceptance sampling, reliability acceptance sampling and double acceptance sampling plans are given. Formulas for group acceptance plans are given in next chapter.

3.2 ATTRIBUTE ACCEPTANCE SAMPLING PLAN

Kantam et al. (2006) used the log-logistic distribution as a lifetime model with the cumulative distribution function (cdf)

$$F(t;\lambda_l)=\left(t/\lambda_l\right)^{m_l}\Big/\left[1+(t/\lambda_l)^{m_l}\right],\ t\geq 0, m_l>1, \lambda_l>0, \tag{3.1}$$

and pdf as

$$f\left(t;\lambda_l\right)=m_l\left(t/\lambda_l\right)^{m_l-1}\Big/\lambda_l\left[1+\left(t/\lambda_l\right)^{m_l}\right]^2, \tag{3.2}$$

where λ_l is a scale parameter and m_l is a shape parameter. The minimum sample size n is determined using the following inequality

$$\sum_{i=0}^{c}\binom{n}{i}p^i\left(1-p\right)^{n-i}\leq\beta, \tag{3.3}$$

where p is the function (1.3) and β is the consumer's risk. Kantam et al. (2006) considered β=0.75, 0.90, 0.95, 0.99. The mean life of log-logistic distribution is

$$\mu=\lambda_l\Gamma\left(1+1/m_l\right)\Gamma\left(1-1/m_l\right).$$

If n is large and p is small, the Poisson distribution can be used to find the minimum sample size such that the following inequality satisfy

$$\sum_{i=0}^{c}\left(e^{-\lambda_l}\mu_l^i / i!\right) \leq \beta. \tag{3.4}$$

The value of ratio of true average life to specified average life μ/μ_0 are found to satisfy the following inequality

$$\sum_{i=0}^{c}\binom{n}{i} p^i (1-p)^{n-i} \geq 1-\alpha, \tag{3.5}$$

where α is producer's risk and $1-\alpha$ is producer's confidence level.
The probability of acceptance of this sampling plan is given by

$$L(p) = \sum_{i=0}^{c}\binom{n}{i} p^i (1-p)^{n-i}. \tag{3.6}$$

Baklizi (2003) considered the Pareto distribution of second kind with the following cdf

$$F(t,\lambda_P,m_P) = 1-\left(1+\frac{t}{\lambda_P}\right)^{-m_P},\ t,m_P,\lambda_P > 0, \tag{3.7}$$

and pdf as

$$f(t,\lambda_P,m_P) = \frac{m_P}{\lambda_P}\left(1+\frac{t}{\lambda_P}\right)^{-(m_P+1)}, \tag{3.8}$$

where m_P is a shape parameter and λ_P is a scale parameter. To find the minimum sample size under the Pareto distribution, Baklizi (2003) used (3.3) when the p is as in (3.7). The inequality (3.5) is used to find the mean minimum ratio μ/μ_0 . The probability of acceptance is calculated using (3.6). Baklizi (2003) constructed tables for m_P=2, 3.

Baklizi and EI Masri (2004) developed the acceptance sampling plan when the lifetime of a product follows the Birnbaum-Saunders distribution with known shape parameter. The cdf of BS distribution is given as

$$F(t,\lambda_b,m_b)=\Phi\left(\frac{1}{m_b}\xi\left(\frac{t}{\lambda_b}\right)\right), t>0, \lambda_b, m_b>0, \tag{3.9}$$

and pdf is

$$f(t,\lambda_b,m_b)=\frac{\exp\left(m_b^{-2}\right)}{2m_b\sqrt{2\pi\lambda_b}}t^{-3/2}\left(t+\lambda_b\right)\times\exp\left(-\frac{1}{2m_b^{2}}\left(\frac{t}{\lambda_b}+\frac{\lambda_b}{t}\right)\right), \tag{3.10}$$

where $\xi\left(\frac{t}{\lambda_b}\right)=\sqrt{\frac{t}{\lambda_b}}-\sqrt{\frac{\lambda_b}{t}}$, m_b is the shape parameter and λ_b is the scale parameter.

The design parameters of this single acceptance sampling plan are (n,c). Baklizi and EI Masri (2004) determined the minimum values of sample size for known m_b=1 by using the (3.3) when the p is as in (3.9). The minimum ratio and probability of acceptance are found by using (3.5) and (3.6) respectively. Baklizi and EI Masri (2004) used the median life λ_b (the median is same as the scale parameter) to develop this acceptance sampling plan for the BS distribution.

Rosaiah and Kantam (2005) assumed that the lifetime of a product follows the inverse Rayleigh distribution with cdf and pdf given respectively as follows:

$$F(t;\lambda_I)=\begin{cases}\exp\left(-\lambda_I^{2}/t^2\right), & t>0\\ 0 & , \quad t\le 0\end{cases} \tag{3.11}$$

$$f(t;\lambda_I)=\begin{cases}\frac{2\lambda_I^{2}}{t^3}\exp\left(\lambda_I^{2}/t^2\right), & t>0\\ 0 & , \quad t\le 0\end{cases} \tag{3.12}$$

where λ_I is the scale parameter of the inverse Rayleigh distribution.

Rosaiah and Kantam (2005) determined the design parameters of the single acceptance samplin plan for the inverse Rayleigh distribution when p in (3.3) is function at (3.11). The inequalities in (3.5) and (3.6) are used to complete the tables of minimum mean ratio and probability of acceptance, respectively. Rosaiah and Kantam (2005) used (3.4) to find the approximate values of n and c. In this approach, they specified the values of c=0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and determined the minimum sample size satisfying (3.4). A real data of software reliability given by Wood (1996) used to explain the results. By using the Q-Q chart, they showed that the inverse Rayleigh distribution is well fitted to above data.

Tasi and Wu (2006) proposed the acceptance sampling plan for the generalized Rayleigh distribution. The cdf and pdf of generalized Rayleigh distribution originally derived by Voda (1976) is given as

$$F_k(t;\lambda_r) = \Gamma\left(I, m_r + 1\right)\left(\frac{t^2}{\lambda_r}\right), \qquad (3.13)$$

where $\Gamma(.)$ is the incomplete gamma function, m_r is the shape parameter and λ_r is the scale parameter of the generalized Rayleigh distribution. If m_r is an integer, then (3.13) can be written as

$$F_{m_r}\left(t;\lambda_r\right) = 1 - \sum_{j=0}^{m_r} \frac{\left(t^2/\lambda_r\right)^j \exp\left(-\lambda_r\right)}{j!}. \qquad (3.14)$$

The mean life of generalized Raleigh distribution is $\mu_r = m_r\sqrt{\lambda_r}$. The equations (3.3), (3.5) and (3.6) are used to find the minimum sample, minimum mean ratio and probability of acceptance for known shape parameter m_r=0.

Balakrishnan et al. (2007) discussed an acceptance sampling plan based on truncated life test for the generalized BS distribution which is a generalization of the work by Baklizi and EI Masri (2004) function. The BS distribution is defined in terms of the normal distribution by means of random variate $T = \lambda_{gb}\left[m_{gb}Z/2 + \sqrt{\left(m_{gb}Z/2\right)^2 + 1}\right]^2$, where $Z = 1/m_{gb}\left(\sqrt{T/\lambda_{gb}} - \sqrt{\lambda_{gb}/T}\right) \sim N(0,1)$, $m_{gb} > 0$ is the shape parameter and $\lambda_{gb} > 0$ is the scale parameter of the distribution. The generalize BS distribution is given by assuming that Z follows the symmetrical distribution in elliptically contoured distribution with location parameter is 0 and scale parameter is 1, i.e., $Z \sim EC(0,1,f)$, where $f(.)$ is the pdf of Z, for more detail, reader can refer to Balakrishnan et al. (2007).

The pdf of generalized BS distribution is given as

$$f_{T(t)} = f\left(a_t\left(m_{gb},\lambda_{gb}\right)\frac{d}{dt}a_t\left(m_{gb},\lambda_{gb}\right),\ t, m_{gb}, \lambda_{gb} > 0, \tag{3.15}$$

where $f(.)$ is the pdf of $Z \sim EC(0,1,f)$,

$$a_t\left(m_{gb},\lambda_{gb}\right) = \sqrt{\frac{t}{\lambda_{gb}}} - \sqrt{\frac{\lambda_{gb}}{t}},$$

and

$$\frac{d}{dt}a_t\left(m_{gb},\lambda_{gb}\right) = \frac{1}{2m_{gb}\lambda_{gb}}\left(\frac{t}{\lambda_{gb}}+1\right)\left(\frac{t}{\lambda_{gb}}\right)^{-\frac{3}{2}}. \tag{3.16}$$

The cdf of T is

$$F_T(t) = P\left(T \le t\right) = F\left(a_t\left(m_{gb},\lambda_{gb}\right)\right), \tag{3.17}$$

where $a_t\left(m_{gb},\lambda_{gb}\right)$ is given by (3.16).

Balakrishnan et al. (2007) constructed the tables of minimum ratio, minimum sample and probability of acceptance for various values of shape parameter. The tables are given when the Poisson distribution is used instead of the binomial distribution.

3.3 VARIABLE ACCEPTANCE SAMPLING PLAN

Jun et al. (2006) utilized variable sampling plans for sudden death assuming the life time of a product to follow the Weibull distribution with known shape parameter. The cdf of Weibull distribution is given by

$$F(t) = 1 - \exp\left(-\left(\lambda_w t\right)^{m_w}\right),\ t \ge 0, \lambda_w, m_w > 0, \tag{3.18}$$

where m_w is the shape parameter and λ_w is the scale parameter of the Weibull distribution.

Jun et al. (2006) further assumed that there is lower specification limit regarding the life time. Thus, the fraction nonconforming or unreliability is given as

$$p = \Pr\{X < L\} = F(L), \tag{3.19}$$

where L is the specification limit.

If p is given, then the corresponding $\lambda_w L$ is obtained from (3.19) through

$$w = (\lambda_w L)^{m_w} = -\ln(1-p), \;\; 0 < p < 1. \tag{3.20}$$

The lot acceptance probability is given as

$$P_a(p) = P_r\left\{v \ge kL^{m_w}/p\right\} = P_r\left\{2\lambda^{m_w}\, rvkL^{m_w}/p\right\}, \tag{3.21}$$

where k is the design parameter of the plan, r is the number of testers and $v = \sum_{i=1}^{g} Y_i^{m_w}$; Y_i, the time to first failure from the ith groups $(i = 1, 2, 3, ..., g)$.

The $2\lambda^{m_w} rv$ quantity follows the chi-square distribution with $2g$ degree of freedom, where g is the number of groups.

The number of groups g satisfying the producer's and consumer's risk are determined by using

$$\frac{w_0}{w_1} = \frac{\ln(1-p_0)}{\ln(1-p_1)} \le \frac{\chi^2_{1-\alpha,2g}}{\chi^2_{\beta,2g}}. \tag{3.22}$$

3.4 RELIABILITY ACCEPTANCE SAMPLING PLAN

Rosaiah and Kantam (2008) proposed the reliability acceptance sampling plan for the inverse Rayleigh distribution. They found the termination time using (3.11) to reach a decision about a product. Two tables of termination time for producer's risks of 0.05 and 0.01 are constructed using (3.5). A real example by Wood (1996) used to show that the reliability approach is more economic than the sampling plan proposed by them.

3.5 DOUBLE ACCEPTANCE SAMPLING PLAN

Aslam et al. (2009c) proposed double sampling plans for general lifetime distributions. They proposed the following plan:

Step 1
Draw the first sample of size n_1 from a lot and put them on test for t_0 units of time.

Step 2
Accept the lot if there are c_1 or smaller number of failures. Reject the lot and terminate the test as soon as more than c_2 failures are observed. If the number of failures is between c_1 and c_2, then draw the second sample of size n_2 from the lot and put them on test for another t_0 units of time.

Step 3
Accept the lot if the total number of failures from the first and the second samples is no greater than c_2. Otherwise, terminate the test and reject the lot.

where n_1 and n_2 are sample sizes of the first and the second sample, respectively, whereas c_1 and c_2 are the acceptance numbers associated with the first and the second sample, respectively. The single sampling plan is a special case of the double sampling plan when $c_1 = c_2$.

Let F be the cdf of the lifetime of a product. Then, the probability that a product fails before time t_0, denoted by p, is obtained by

$$p = F(t_0). \tag{3.23}$$

The probability that the lot is accepted from the two samples under the double sampling plan is given by

$$P_a(p) = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{x=c_1+1}^{c_2} \binom{n_1}{x} p^x (1-p)^{n_1-x} \left[\sum_{i=0}^{c_2-x} \binom{n_2}{i} p^i (1-p)^{n_2-i} \right]. \tag{3.24}$$

Aslam et al. (2009c) determined the design parameters such that the following inequities satisfy

$$P_a(p_1) \geq 1-\alpha, \tag{3.25}$$

and

$$P_a\left(p_2\right) \leq \beta. \qquad (3.26)$$

The average sample number (ASN) for the double sampling plan is calculated by

$$\text{ASN}(p) = n_1 P_1 + \left(n_1 + n_2\right)\left(1 - P_1\right), \qquad (3.27)$$

where P_1 is the probability that the decision was made by the first sample and given by

$$P_1 = 1 - \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i \left(1 - p\right)^{n_1 - i}. \qquad (3.28)$$

Therefore, the parameters for our double sampling plan are determined by solving the following optimization problem:

Minimize $\text{ASN}(p_2)$ (3.29a)

Subject to

$$P_a(p_1) \geq 1 - \alpha, \qquad (3.29b)$$

$$P_a(p_2) \leq \beta, \qquad (3.29c)$$

$$1 \leq n_2 \leq n_1 \text{ and} \qquad (3.29d)$$

$$n_1, n_2 \text{: integers.} \qquad (3.29e)$$

Aslam and Jun (2009c) developed the double acceptance sampling plans based on life test under the assumption that the life time follows the generalized log-logistic distribution with known shape parameter. For the zero and one failure scheme, the lot acceptance probability is given as

$$P_a = \left(1 - p\right)^{n_1}\left[1 + n_1 p\left(1 - p\right)^{n_2 - 1}\right], \qquad (3.30)$$

where p is the function of cdf of any underlying distribution.

The minimum sample sizes n_1 and n_2 ensuring $\mu \geq \mu_0$ at the consumer's confidence level $1 - \beta$ can be found as the solution to the following inequality:

$$(1-p_0)^{n_1}\left[1+n_1 p_0 (1-p_0)^{n_2-1}\right] \le \beta. \tag{3.31}$$

For $c_1 = 0$ and $c_2 = 1$, we have

$$\text{ASN} = n_1 + n_1 n_2 p (1-p)^{n_1-1}. \tag{3.32}$$

At the producer's risk of α, the minimum ratio μ / μ_0 can be obtained by solving

$$P_a \ge 1-\alpha , \tag{3.33}$$

where $1-\alpha$ is the producer's confidence level.

CHAPTER 4

THE GROUP ACCEPTANCE SAMPLING PLANS

4.1 INTRODUCTION

In this chapter, some mathematical formulas are used to find the design parameters of a new developed group acceptance sampling plan under various statistical distributions such as the inverse Rayleigh, log-logistic, the Weibull and gamma distributions based on life test are discussed.

The ordinary sampling plan stated that a lot of product is accepted or rejected on the basis of single sample taken from the lot. Furthermore, this sampling scheme considers only the consumer's risk. The idea of this ordinary sampling to the group acceptance sampling plan is extended. Design parameters for two cases (1) satisfying only the consumer's risk and (2) satisfying both the producer's and consumer's risks are simultaneously determined. To develop the idea of a group sampling plan, a lot of size N is considered. A sample of size $n = r \times g$ is selected from N items where g is the number of groups and r is the number of testers. The r items are distributed to g groups. For this experiment, the lot of product is accepted if no more than c failures are found in each and every group, otherwise the lot is rejected. As the decision about the lot lies into two mutually exclusive categories, that is, reject the lot or accept it on the basis of a sample size taken from an infinite lot size, where all the groups are independent of each other. Then, the probability of acceptance for the lot is given as

$$L(p) = \left[\sum_{i=0}^{c} \binom{r}{i} \; p^{i} \left(1-p\right)^{r-i} \right]^{g}, \qquad (4.1)$$

where p is the function of the cdf of any underlying distribution, g is the number of groups and r is the number of testers.

4.2 PROBABILITY OF FAILURE FOR VARIOUS DISTRIBUTIONS

Suppose that the lifetime of a product follows the inverse Rayleigh distribution. It is convenient to determine the termination time t_0 as a multiple of the specified life μ_0 that is

$$t_0 = a\mu_0, \tag{4.2}$$

where a is a constant and μ_0 is specified mean at t_0.

The scale parameter λ_I of the inverse Rayleigh distribution is

$$\lambda_I = \frac{\mu}{2\sqrt{\pi}}. \tag{4.3}$$

Using (4.2) and (4.3) the cdf of inverse Rayleigh (1.1) can be written as

$$p = \exp\left(\frac{-\mu^2}{\pi a^2 \mu^2{}_0}\right). \tag{4.4}$$

or

$$p = \exp\left(\frac{-1}{\pi a^2}\left(\frac{\mu}{\mu_0}\right)^2\right), \tag{4.5}$$

where μ/μ_0 is a ratio of true mean life of the inverse Rayleigh distribution to the specified average life of a product and p is probability failure of a components under inverse Rayleigh distribution.

The cdf and mean life of the log-logistic distribution are given in (1.3) and (1.4), respectively. The scale parameter λ_l of the log-logistic distribution from (1.4) is

$$\lambda_l = \frac{\mu}{\Gamma(1+1/m_l)\Gamma(1-1/m_l)}, \tag{4.6}$$

where m_l is the shape parameter and λ_l is the scale parameter of the log-logistic distribution.

Under (4.2) and (4.6), (1.3) can be written as follows

$$p = \frac{\left(a\Gamma\left(1+1/m_l\right)\Gamma\left(1-1/m_l\right)\right)^{m_l}}{\left[\left(\mu/\mu_0\right)^2 + \left(a\Gamma\left(1+1/m_l\right)\Gamma\left(1-1/m_l\right)\right)^{m_l}\right]}. \qquad (4.7)$$

where a is the termination ratio.

Alternatively, for m_l =2, (4.7) can be written as

$$p = \frac{\left(1.5708a\right)^2}{\left[\left(\mu/\mu_0\right)^2 + \left(1.5708a\right)^2\right]}. \qquad (4.8)$$

For the Weibull distribution, its scale parameter λ_w can be found from (1.7) as

$$\lambda_w = \frac{\mu m_w}{\Gamma\left(1/m_w\right)}. \qquad (4.9)$$

where m_w is the shape parameter and λ_w is the scale parameter of the Weibull distribution.

Using (4.2) and (4.9) the cdf of the Weibull distribution (1.6) can be written as

$$p = 1 - \exp\left(-\left(t_0/\lambda_w\right)^{m_w}\right), \qquad (4.10)$$

or

$$p = 1 - \exp\left(-a^{m_w}\left(\mu/\mu_0\right)^{-m_w}\left(\Gamma\left(1/m_w\right)/m_w\right)^{m_w}\right), \qquad (4.11)$$

For the gamma distribution, the scale parameter λ_g from (1.9) can be obtained as

$$\lambda_g = \frac{\mu}{m_g}, \qquad (4.12)$$

where m_g is the shape parameter and λ_g is the scale parameter of gamma distribution.

For $m_g = 2$, the cdf given in (1.8) can be extended as

$$p = 1 - \sum_{j=0}^{1}\left[\exp\left(-\frac{t}{\lambda_g}\right)\left(\frac{t}{\lambda_g}\right)^j \Big/ j!\right]. \tag{4.13}$$

(4.13) can be written as

$$p = 1 - e^{-\frac{t}{\lambda_g}}\left[1 + \frac{t}{\lambda_g}\right]. \tag{4.14}$$

Similarly, m_g =3, (1.8) can be written as

$$p = 1 - e^{-\frac{t}{\lambda_g}}\left[1 + \frac{t}{\lambda_g} + \frac{1}{2}\left(\frac{t}{\lambda_g}\right)^2\right]. \tag{4.15}$$

Using (4.2) and (4.12), (4.14) and (4.15) can written respectively, as

$$p = 1 - e^{-\frac{am_g}{\mu/\mu_0}}\left[1 + \frac{am_g}{\mu/\mu_0}\right], \tag{4.16}$$

and

$$p = 1 - e^{-\frac{am_g}{\mu/\mu_0}}\left[1 + \frac{am_g}{\mu/\mu_0} + \frac{1}{2}\left(\frac{am_g}{\mu/\mu_0}\right)^2\right]. \tag{4.17}$$

4.3 PROBABILITY OF ACCEPTANCE FOR VARIOUS DISTRIBUTIONS

As described in (4.1), p is the cdf underlying statistical distribution. Here we give the OC function for the distributions under study.

When the lifetime follows the inverse Rayleigh distribution, by using (4.5), then (4.1) can be written as

$$L(p)=\left[\sum_{i=0}^{c}\binom{r}{i}\left(\exp\left(\frac{-1}{\pi a^2(\mu/\mu_0)^2}\right)\right)^i\left(1-\exp\left(\frac{-1}{\pi a^2(\mu/\mu_0)^2}\right)\right)^{r-i}\right]^g, \tag{4.18}$$

where a is termination ratio and μ/μ_0 is mean ratio.

The OC function (4.1) under (4.7) can be written as

$$L(p)=\left[\sum_{i=0}^{c}\binom{r}{i}\left(\frac{\left(a\Gamma(1+1/m_l)\Gamma(1-1/m_l)\right)^{m_l}}{\left[(\mu/\mu_0)^2+\left(a\Gamma(1+1/m_l)\Gamma(1-1/m_l)\right)^{m_l}\right]}\right)^i\right.$$
$$\left.\times\left(1-\frac{\left(a\Gamma(1+1/m_l)\Gamma(1-1/m_l)\right)^{m_l}}{\left[(\mu/\mu_0)^2+\left(a\Gamma(1+1/m_l)\Gamma(1-1/m_l)\right)^{m_l}\right]}\right)^{r-i}\right]^g, \tag{4.19}$$

where a is termination ratio , μ/μ_0 is mean ratio and m_l is the shape parameter of the log-logistic distribution.

The OC function under (4.11) is

$$L(p)=\left[\sum_{i=0}^{c}\binom{r}{i}\left(1-\exp\left(-a^{m_w}(\mu/\mu_0)^{-m_w}\left(\Gamma(1/m_w)/m_w\right)^{m_w}\right)\right)^i\right.$$
$$\left.\times\left(\left(\exp\left(-a^{m_w}(\mu/\mu_0)^{-m_w}\left(\Gamma(1/m_w)/m_w\right)^{m_w}\right)\right)\right)^{r-i}\right]^g, \tag{4.20}$$

where a is termination ratio , μ/μ_0 is mean ratio and m_w is the shape parameter of the Weibull distribution.

Similarly, the OC function under (4.16) is

$$L(p)=\left[\sum_{i=0}^{c}\binom{r}{i}\left(1-e^{-\frac{am_g}{\mu/\mu_0}}\left[1+\frac{am_g}{\mu/\mu_0}\right]\right)^i\left(e^{-\frac{am_g}{\mu/\mu_0}}\left[1+\frac{am_g}{\mu/\mu_0}\right]\right)^{r-i}\right]^g, \tag{4.21}$$

where a is termination ratio , μ/μ_0 is mean ratio and m_g is the shape parameter of gamma distribution

The OC function under (4.17) is

$$L(p)=\left[\sum_{i=0}^{c}\binom{r}{i}\left(1-e^{-\frac{am_g}{\mu/\mu_0}}\left[1+\frac{am_g}{\mu/\mu_0}+\frac{1}{2}\left(\frac{am_g}{\mu/\mu_0}\right)^2\right]\right)^i\right.$$

$$\left.\times\left(e^{-\frac{am_g}{\mu/\mu_0}}\left[1+\frac{am_g}{\mu/\mu_0}+\frac{1}{2}\left(\frac{am_g}{\mu/\mu_0}\right)^2\right]\right)^{r-i}\right]^g . \quad (4.22)$$

4.4 MINIMUM RATIO FOR INVERSE RAYLEIGH AND LOG-LOGISTIC DISTRIBUTIONS

The minimum ratio of true average life to specified average life for the inverse Rayleigh distribution can be obtained by simplifying (4.5) as follows

$$\ln(p)=\frac{-1}{\pi a^2}\left(\frac{\mu}{\mu_0}\right)^2, \quad (4.23)$$

or

$$\left(\frac{\mu}{\mu_0}\right)=\sqrt{-\pi a^2(\ln p)}, \quad (4.24)$$

where μ/μ_0 is the mean ratio of unknown average life to specified average life and p is the probability of failure of a component before time t_0.

The minimum ratio for the log-logistic from (4.7) can be obtained as

$$\frac{\mu}{\mu_0}=\frac{a\left(\Gamma(1+1/m_l)\Gamma(1-1/m_l)\right)(1-p)^{1/m_l}}{(p)^{1/m_l}}, \quad (4.25)$$

where a is termination ratio and m_l is the shape parameter of log-logistic distribution.

It is important to note that (4.24) and (4.25) are used to develop the group acceptance sampling plan by considering only producer's risk. Also note that p in (4.24) and (4.25) is the probability at which the following inequality is satisfied

$$\left[\sum_{i=0}^{c}\binom{r}{i} \; p^{i}(1-p)^{r-i}\right]^{g} \geq 1-\alpha, \tag{4.26}$$

where α is producer's risk.

The purpose of acceptance sampling plan is to find the minimum group size which satisfy the following inequality

$$\left[\sum_{i=0}^{c}\binom{r}{i} \; p^{i}(1-p)^{r-i}\right]^{g} \leq \beta, \tag{4.27}$$

where β is consumer's risk.

4.5 FORMULATION OF TWO POINT ON OC CURVE

In this section, we give some formulas for the Weibull and gamma distributions, which can be used to find the design parameters simultaneously satisfying the both risks.

The quality level of a product can be expressed in terms of the ratio of its mean lifetime to the specified life, say μ/μ_0. The consumer demands that the lot acceptance probability should be smaller than the specified consumer's risk β at a lower quality level (usually at ratio 1), whereas the producer requires that the lot rejection probability should be smaller than the specified producer's risk α at a higher quality level. When the quality level is expressed by the ratio mentioned earlier, the proposed two-point approach for finding the design parameters is to determine the number of groups and the acceptance number that simultaneously satisfy the following two inequalities for the Weibull distribution with known shape parameters:

$$\left[\sum_{i=0}^{c}\binom{r}{i} \left(1-\exp\left(-a^{m_w}(\mu/\mu_0)^{-m_w}\left(\Gamma(1/m_w)/m_w\right)^{m_w}\right)\right)^{i} \times\left(\left(\exp\left(-a^{m_w}(\mu/\mu_0)^{-m_w}\left(\Gamma(1/m_w)/m_w\right)^{m_w}\right)\right)\right)^{r-i}\right]^{g} \geq 1-\alpha, \tag{4.28}$$

and

$$\left[\sum_{i=0}^{c}\binom{r}{i}\left(1-\exp\left(-a^{m_w}\left(\Gamma(1/m_w)/m_w\right)^{m_w}\right)\right)^{i}\right.$$

$$\left.\times\left(\left(\exp\left(-a^{m_w}\left(\Gamma(1/m_w)/m_w\right)^{m_w}\right)\right)\right)^{r-i}\right]^{g}\leq\beta. \qquad (4.29)$$

where a is termination ratio, g is the number of groups, m_w is the shape parameter of the Weibull distribution and r is the number of testers.

To find the design parameters of the gamma distribution the following two inequalities satisfy at a time.

For γ=2.

$$\left[\sum_{i=0}^{c}\binom{r}{i}\left(1-e^{-\frac{am_g}{\mu/\mu_0}}\left[1+\frac{am_g}{\mu/\mu_0}\right]\right)^{i}\left(e^{-\frac{am_g}{\mu/\mu_0}}\left[1+\frac{am_g}{\mu/\mu_0}\right]\right)^{r-i}\right]^{g}\geq 1-\alpha, \qquad (4.30)$$

and

$$\left[\sum_{i=0}^{c}\binom{r}{i}\left(1-e^{-am_g}\left[1+am_g\right]\right)^{i}\left(e^{-am_g}\left[1+am_g\right]\right)^{r-i}\right]^{g}\leq\beta. \qquad (4.31)$$

For γ=3.

$$\left[\sum_{i=0}^{c}\binom{r}{i}\left(1-e^{-\frac{am_g}{\mu/\mu_0}}\left[1+\frac{am_g}{\mu/\mu_0}+\frac{1}{2}\left(\frac{am_g}{\mu/\mu_0}\right)^{2}\right]\right)^{i}\right.$$

$$\left.\times\left(e^{-\frac{am_g}{\mu/\mu_0}}\left[1+\frac{am_g}{\mu/\mu_0}+\frac{1}{2}\left(\frac{am_g}{\mu/\mu_0}\right)^{2}\right]\right)^{r-i}\right]^{g}\geq 1-\alpha, \qquad (4.32)$$

and

$$\left[\sum_{i=0}^{c}\binom{r}{i}\left(1-e^{-am_g}\left[1+am_g+\frac{1}{2}\left(am_g\right)^{2}\right]\right)^{i}\right.$$

$$\times\left(e^{-am_g}\left[1+am_g+\frac{1}{2}\left(am_g\right)^2\right]\right)^{r-i}\Bigg]^g \leq \beta. \qquad (4.33)$$

where a is termination ratio, g is the number of groups, m_g is the shape parameter of gamma distribution and r is the number of testers.

CHAPTER 5

THE GROUP ACCEPTANCE SAMPLING PLANS DESIGN

5.1 INTRODUCTION

In this chapter, the methodology to find the design parameters of proposed group sampling plans based on truncated life tests is given. The separate procedure to find the design parameters considering a single and two points on the operating characteristics curve is given. For single point on OC curve, a procedure for the log-logistic and inverse Rayleigh distributions is developed. For the two points on OC curve, the Weibull and gamma distributions are used. Tables of minimum number of groups, probabilities of acceptance and minimum mean ratios of the true average life to specified average life are given for the distributions under consideration. A number of examples are given to illustrate the group sampling procedure developed here. The pattern of design parameters are also studied by using graphical approaches.

5.2 THE GROUP SAMPLING PLAN FOR SINGLE POINT ON OC CURVE

We are interested in designing a group sampling plan in order to assure that the mean life of an item in a lot (say μ) is greater than the specified life (say μ_0) under the assumption that the lifetime of an item follows either an inverse Rayleigh distribution or a log-logistic one with known shape parameter. A lot of products or items is considered to be "good" if the true average life is greater than the specified life. The lot is accepted if $\mu \geq \mu_0$ at a certain level of consumer's risk. Otherwise, the lot is rejected.

The following group acceptance sampling plan based on truncated life tests is proposed:

1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot is $n = gr$.
2) Select the acceptance number c for a group and specify the experiment time t_0.
3) Perform the experiment for the g groups simultaneously and record the number of failures for each group.

4) Accept the lot if at most c failures occur in each of all groups by the experiment time.

5) Terminate the experiment as soon as more than c failures occur in any group and reject the lot.

The proposed sampling plan is an extension of the ordinary sampling plan available in literature such as in Kantam et al. (2001) and Rosaiah and Kantam (2005), for which $r = 1$. We are interested in determining the number of groups g required for each of two distributions under study, whereas the various values of the acceptance number c and the termination time t_0 are assumed to be specified.

When we want to determine the parameters of the proposed sampling plan, the consumer's risk is used. Often, the consumer's risk is expressed by the consumer's confidence level. If the confidence level is $1-\beta$, then the consumer's risk is β. The number of groups in the proposed sampling plan is determined so that the consumer's risk does not exceed β. The lot of products is accepted only if there are at most c failures occurred in each of g groups. Thus, the lot acceptance probability is given by

$$L(p) = \left[\sum_{i=0}^{c} \binom{r}{i} \; p^i (1-p)^{r-i} \right]^g , \qquad (4.1)$$

where p is the probability that an item in a group fails before the termination time, r is the number of testers and g is the number of groups. The probability p for the inverse Rayleigh distribution is given by

$$p = \exp\left(-\left(\frac{\lambda_I}{t_0} \right)^2 \right) = \exp\left(-\frac{1}{a^2 \pi} \left(\frac{\mu}{\mu_0} \right)^2 \right), \qquad (4.5)$$

and for the log-logistic distribution with $m_l = 2$ it is calculated by (4.8)

$$p = \frac{(1.5708a)^2}{(\mu/\mu_0)^2 + (1.5708a)^2} . \qquad (4.8)$$

The minimum number of groups required can be determined by considering the consumer's risk when the true mean equals the specified life $(\mu = \mu_0)$ through the following inequality:

$$L(p_0) \le \beta, \tag{5.1}$$

where β is the consumer's risk, $L(p_0)$ is given by (4.1) and p_0 are given by (4.5) and (4.8) for the inverse Rayleigh and log-logistic distributions, respectively. Particularly, for $c=0$ (so-called zero failure test), g can be determined by the minimum integer satisfying the following inequality:

$$g \ge \frac{\ln m_l}{r \ln(1-p_0)}, \tag{5.2}$$

where m_l the shape parameter of the log-logistic distribution and r is the number of testers.

5.2.1 Operating Characteristics

Once the minimum number of groups is obtained, one may be interested to find the probability of acceptance of a lot, when the quality (or reliability) of the product is good enoughly. As mentioned earlier, the product is considered to be good if $\mu > \mu_0$ or $\mu/\mu_0 > 1$. The probability of acceptance is obtained by using the expression given in (4.1).

5.2.2 The Minimum Mean Ratio

Further, the producer may be interested in enhancing the quality level of the product so that the acceptance probability should be greater than a specified level. At the producer's risk α the minimum ratio μ/μ_0 can be obtained by satisfying the inequality

$$\left[\sum_{i=1}^{c} \binom{r}{i} p^i (1-p)^{r-i}\right]^g \ge 1-\alpha, \tag{5.3}$$

where p are given by (4.5) and (4.8) for the inverse Rayleigh and log-logistic distribution, respectively, and g is chosen at the consumer's risk β when $\mu / \mu_0 = 1$.

5.3 THE GROUP SAMPLING PLAN FOR TWO POINTS ON OC CURVE

We are interested in designing a sampling plan in order to assure that the mean life of items in a lot (say μ) is greater than the specified life (say μ_0). We accept the lot if there is enough evidence that $\mu \geq \mu_0$ at certain levels of consumer's and producer's risks, otherwise, reject the lot. Let us propose the following group acceptance sampling plan based on the truncated life test:

1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be $n = gr$.
2) Select the acceptance number (or action limit) c for a group and the experiment time t_0.
3) Perform the experiment for the g groups simultaneously and record the number of failures for each group.
4) Accept the lot if at most c failures occur in each of all groups.
5) Truncate the experiment if more than c failures occur in any group and reject the lot.

We are interested in determining the number of groups g and the action limit c which satisfies both risks at the same time, whereas the group size r, and the termination time t_0 are assumed to be specified. In practice, c=0 is usually adopted in order to reduce the sample size, but in general c can be determined as well.

5.3.1 GASP for the Weibull Distribution

Suppose that the lifetime of an item or a product follows a Weibull distribution, the pdf of this distribution is given as

$$f(t) = \frac{m_w}{\lambda_w}\left(\frac{t}{\lambda_w}\right)^{m_w - 1} e^{-\left(\frac{t}{\lambda_w}\right)^{m_w}}, \quad t \geq 0, \tag{5.4}$$

where m_w is the shape parameter and λ_w is the scale parameter of the Weibull distribution.

Assume that the shape parameter m_w is known such that the cdf is given in (1.6).

Practitioners would probably have some prior knowledge of the model parameters when they select the acceptance sampling plan based on life test.

The shape parameter of the Weibull distribution can be easily estimated by the data of several failure times. Most manufacturers keep the estimated shape parameter for each product. Thus, when the shape parameter is unknown, they can use the estimated value.

Remark. Note that the cdf given in (1.6) depends on the scale parameter λ_w only through t/λ_w. Note also that the mean life of a Weibull distributed item is given by (1.7).

The lot of products is accepted only if there are c or less failures in each of g groups. Thus, the lot acceptance probability is given in (4.1).

For the known value of shape parameter, p can be evaluated when the multiplier a and the ratio μ/μ_0 are specified.

As mentioned earlier in page 33 of the dissertation , consumer demands that the lot acceptance probability should be smaller than the specified consumer's risk β at a lower quality level (usually at ratio 1), whereas the producer requires that the lot rejection probability should be smaller than the specified producer's risk α at a higher quality level. When the quality level is expressed by the mentioned ratio, the proposed two-point approach for finding the design parameters is to determine the number of groups and the acceptance number that satisfies the two inequalities (4.28) and (4.29) simultaneously for the Weibull distribution.

These inequalities can be written as

$$L(p \mid \mu/\mu_0 = r_1) \le \beta, \tag{5.5}$$

and

$$L(p \mid \mu/\mu_0 = r_2) \ge 1-\alpha, \tag{5.6}$$

respectively.
where r_1 is the mean ratio at the consumer's risk and r_2 is the mean ratio at the producer's risk

5.3.2 GASP for the Gamma Distribution

For the gamma distribution, inequalities (4.30)-(4.33) can be written as follows

$$L(p \mid \mu/\mu_0 = r_1) \le \beta, \tag{5.7}$$

and

$$L(p \mid \mu/\mu_0 = r_2) \ge 1-\alpha, \tag{5.8}$$

respectively, where r_1 is the mean ratio at the consumer's risk and r_2 is the mean ratio at the producer's risk. Let p_1 and p_2 be the failure probabilities corresponding to the consumer's and producer's risks, respectively. Then, the minimum number of groups and action number required can be determined by considering the consumer's and producer's risks at the same time through the inequalities (4.28) and (4.29) for the Weibull distribution and (4.30)-(4.33) for the gamma distribution and in general written as

$$L(p_1) = \left[\sum_{i=0}^{c} \binom{r}{i} p_1^i (1-p_1)^{r-i} \right]^g \leq \beta, \tag{5.9}$$

and

$$L(p_2) = \left[\sum_{i=0}^{c} \binom{r}{i} p_2^i (1-p_2)^{r-i} \right]^g \geq 1-\alpha. \tag{5.10}$$

The values of g and c are found using a search by increasing integers, which can be easily implemented in Excel sheets. Particularly for c=0, g can be determined by the minimum integer satisfying the following inequality:

$$\frac{-\ln\beta}{rb\left(a/r_1\right)^{m_w}} \leq g \leq \frac{-\ln(1-\alpha)}{rb\left(a/r_2\right)^{m_w}}, \tag{5.11}$$

where $b = \left(\Gamma\left(1/m_w\right)/m_w\right)^{m_w}$, r_1 is the mean ratio at the consumer's risk and r_2 is the mean ratio at the producer's risk, β is the consumer's risk, $1-\alpha$ is producer's confidence level, r is the number of testers, m_w is the shape parameter of the Weibull distribution and a is termination ratio.

5.4 GASP with Single Point on OC Curve

Since p_0 for the inverse Rayleigh distribution in (4.5) is smaller than that for the log-logistic distribution for $a \leq 0.825$, the number of groups required is larger under the inverse Rayleigh distribution than under the log-logistic one when the test time is shorter than $0.825\mu_0$ and the converse is true when the test time is longer than $0.825\mu_0$.

Table 1 and 2 show the minimum number of groups required for the proposed sampling plan according to various values of confidence level (β =0.25, 0.10, 0.05, 0.01), group size (r), acceptance number (c) and the test termination time multiplier (a =0.7, 0.8, 1.0, 1.2, 1.5, 2.0) under the inverse Rayleigh and log-logistic distributions with m_l =2, respectively.

It can be seen from Tables 1 and 2 that the number of groups required for the inverse Rayleigh distribution is quite similar to that for the log-logistic distribution, although the former is sometimes smaller than the latter when the test time is shorter than the specified average life.

Once the minimum number of groups is obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is good enough. As mentioned earlier, the product is considered to be good if $\mu > \mu_0$ or $\mu/\mu_0 > 1$. The probabilities of acceptance based on (4.1) for various mean lifetimes (μ / μ_0= 2, 4, 6, 8, 10, 12) under the plan parameters chosen before are reported in Tables 3 and 4 for the inverse Rayleigh and log-logistic distributions, respectively. Here again, the shape parameter in a log-logistic distribution is assumed as m_l =2.

We see from Tables 3 and 4 that OC values increase more quickly under the inverse Rayleigh distribution than under the log-logistic distribution as the quality increases. For example, when β=0.10, r=6, c=2 and a=0.7, the number of groups required is g=2 for both distributions. However, the OC value goes to 1.0 when the true mean becomes four times, the specified average life under the inverse Rayleigh distribution, whereas it requires almost ten times under the log-logistic distribution.

Furthermore, the producer may be interested in enhancing the quality level of the product so that the acceptance probability should be greater than a specified level. At the producer's risk α, the minimum ratio μ / μ_0 can be obtained by satisfying the inequality given in (5.3).

Tables 5 and 6 show the minimum ratio of μ / μ_0 for the inverse Rayleigh and log-logistic distributions at the producer's risk of α =0.05 under the plan parameters chosen before, respectively.

It can be also seen from Tables 5-6 that the effect of improving the quality on the lot acceptance probability is quicker for the inverse Rayleigh case than for the log-logistic one. For example, when β =0.10, r=4, g=1, c=0 and a=0.7, the manufacturer requires to increase the true mean 2.60 times the specified life under the inverse Rayleigh distribution in order to keep the producer's risk at 5 percent, whereas it requires to increase the true mean 9.77 times under the log-logistic distribution.

5.5 GASP WITH TWO POINTS ON OC CURVE

Tables 7-9 show the minimum number of groups and the acceptance/action number required for the proposed group sampling plan when the lifetime of a product follows the Weibull distribution. Tables 10-11 show these designs parameters for shape parameter 2 and 3, for the gamma distribution according to various values of the consumer's risk (β=0.25, 0.10, 0.05, 0.01); when the true mean equals to the specified life r_1 and 5 percent of producer's risk; when the true mean is r_2 (=2, 4, 6, 8, 10) times the specified life. Two levels of group size (r=5, 10) and two levels of the test termination time multiplier (a=0.5, 1.0) are considered. We consider three values of the shape parameter of the Weibull distribution: m_w=1 (exponential case) in Table 7, m_w=2 in Table 8 and m_w=3 in Table 9. We also consider the two values of shape parameter of the gamma distribution: m_g=2 in Table 10 and m_g=3 in Table 11. It is important to note that if one wants to determine the minimum sample size from the groups acceptance plan, it can be obtained by $n = r \times g$.

In tables 7-9, the probability of lot acceptance is included when the true mean is r_2 times the specified life, which is greater than 0.95 because the sample size increases with an increment of r. Note that for the exponential distribution, there exists no combination of acceptance number and group size for some cases. It is seen from these tables that a reasonable number of groups is required to meet the producer's risk at a higher quality level but a quite large number of groups are needed to meet the producer's risk at a lower quality level such as the mean ratio of 2. When comparing the results for m_w=2 and m_w=3, the numbers of groups required are not much different if the test time is large enough. It is observed that the number of groups tends to decrease as m_w increases, r increases or a increases. However, the trend is not monotonic since it depends on the acceptance number as well. Note that the reader can get EXCEL program from the author upon request.

5.6 COMPARISON OF GROUP SAMPLING

In this section, we present some graphs to compare number of groups of proposed acceptance sampling plan for both approaches.

Figure 1 shows that for the values of number of testers $r=7$ and acceptance number $c=5$ and $\beta=0.25$ the log-logistic distribution provides a less number of groups than the inverse Rayleigh one. This figure shows that as the experiment time (ratio) increases, we need a less number of groups to be tested. The inverse Rayleigh distribution provides a large number of groups than the log-logistic one when we want to run an experiment for 700 hours. These numbers of groups are very close for both distributions if the experiment ratio is greater than 1.2.

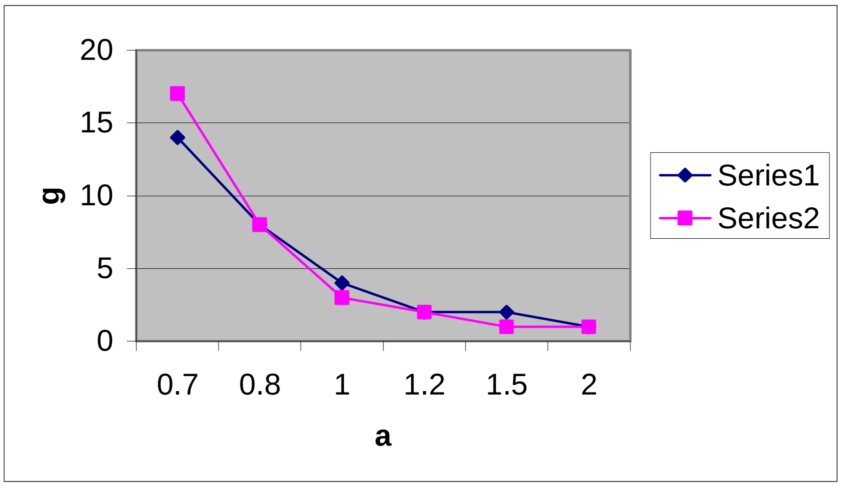

Figure 1: Number of groups for the log-logistics (serie 1) and inverse Rayleigh (serie 2) distributions for single point on OC curve.

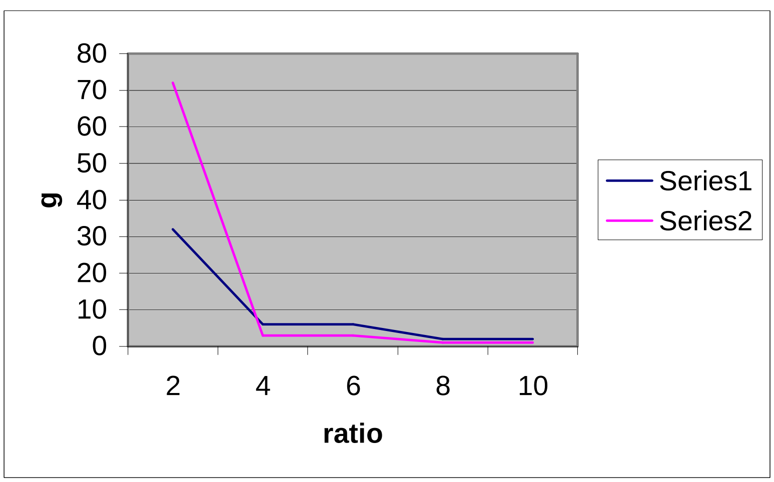

Figure 2: Number of groups for the Weibull (serie 1) and gamma (serie 2) distributions for two points on OC curve.

The graph in figure 2 is drawn by considering the values of β=0.25, $r = 5$ and $a = 0.5$ from Tables 10 and 11. It is clear from Figure 2 that as the quality level of the product increases, the number of groups for the Weibull and gamma distributions with shape parameter equal to 2 decreases. Also, for a ratio equal to 2, the gamma distribution gives a large number of groups than the Weibull distribution. But a careful examination of this figure shows that the numbers of groups from the gamma distribution are less than those from the Weibull one for a ratio greater than 2.

CHAPTER 6

EXAMPLES

The use of proposed group acceptance sampling plans is illustrated based on truncated life tests in this chapter. Tables are described and the results are explained by some numerical examples for GASP with single and two points on OC curve.

6.1 GASP with Single Point on OC Curve

We illustrate our method in an example. Suppose bulb manufacturers would like to know if the mean life of their product is greater than the specified average life, μ_0=1000 hours. Suppose that they want to run an experiment by 700 hours using testers equipped with 12 items each. It is assumed that c=5 and β=0.01. This leads to the termination multiplier a=0.700 and from Table 1 the minimum group required is g=5 for the inverse Rayleigh distribution and g=4 for the log-logistic distribution. If the underlying distribution is the inverse Rayleigh one, then we draw a random sample of size 60 items and allocate 12 items to 5 groups to put on test for 700 hours. Suppose now that we observed only one failure from group 1 before the termination time and three failures from group 2, two failures from group 3, no failures from group 4 and six failures from group 5. Then, we reject the lot and declare that a bulb product in this lot has the mean life smaller than 1000 hours at the consumer's risk of 1 percent. On the other hand, if the underlying distribution is the log-logistic distribution, then we have to draw 48 items and allocate them into 4 groups to put on test. If the numbers of failures from group 1 to group 4 are same as the above, then we accept the lot and declare that a bulb product in this lot has the mean life at least 1000 hours at the consumer's risk of 1 percent.

Suppose that the lifetime of a product under consideration is known to follow the inverse Rayleigh or the log-logistic distribution with $m_l = 2$. Also it is required to demonstrate that through the proposed sampling plan, the mean life of the product under consideration is at least 1000 hours at consumer's risk of 5 percent. We want to run this experiment 700 hours using the tester to be equipped with seven items each. When the acceptance number is c=2, we accept the lot if at most two failures occur before 700 hours in each of two groups. We truncate the experiment as soon as the 3rd failure occurs before the

700^{th} hours. The minimum number of groups for the inverse Rayleigh and log-logistic distributions can be found as g=2 from Table 1 and 2. This means that a total of 14 products are needed and that 7 items are allocated at each of 2 testers. For this proposed sampling plan under the two distributions, the operating characteristics can be seen from Tables 3 and 4 as follows:

μ / μ_0	1	2	4	6	8	10	12
Inverse Rayleigh	0.0368	0.9772	1.0000	1.0000	1.0000	1.0000	1.0000
Log-logistic	0.0245	0.6283	0.9805	0.9978	0.9996	0.9999	1.0000

This shows that, if the true average life is 4 times of 1000 hours, the producer's risk is almost zero for the inverse Rayleigh distribution, whereas for the log-logistic distribution it requires almost 10 times to have zero risk. It can also be seen that the OC values for the inverse Rayleigh distribution are more rapidly increasing as the quality increases than for the log-logistic distribution. If we need the ratio corresponding to the producer's risk of 0.05, we can obtain it from Table 3. For example, when r=7, g=2, c=2, a=0.700, the ratios μ / μ_0 for the inverse Rayleigh and log-logistic distributions are 1.88 and 3.29, respectively.

6.2 GASP WITH TWO POINTS ON OC CURVE

Suppose for example that the lifetime of a product under consideration is known to follow a Weibull distribution with shape parameter of 2. Suppose that it is desired to design a group sampling plan to assure that the mean life is greater than 1,000 hours through the experiment to be completed by 1,000 hours, using testers equipped with 5 products each. It is assumed that the consumer's risk is 25 percent when the true mean is 1,000 hours and the producer's risk is 5 percent when the true mean is 2,000 hours. Since m_w=2, β=0.25, r=5, a=1.0 and $r_2 = 2$, the minimum number of groups and acceptance number can be found as g=5 and c=3 from Table 2. This means that a total of 25 products are needed and that 5 items are allocated at each of 5 testers. We accept the lots if no more than 3 failures occur before 1000 hours in each of 5 groups. For this proposed sampling plan under the Weibull distribution, the number of groups decreases and the operating characteristics values increase as follows when the true mean life increases, which was summarized from Table 8.

Probability of acceptance for above plan

$\mu / \mu_0 = r_2$	2	4	6	8	10
g	5	1	1	1	1
c	3	1	1	0	0
$L(p_2)$	0.9785	0.9792	0.9955	0.9985	0.9994

Let us consider another example. Suppose a bearing manufacturer would like to know if the mean life of their ball bearings is greater than the specified life, μ_0=5,000 cycles, say. The lifetime of a bearing follows a Weibull distribution with m_w=3. Suppose that they want to run an experiment 2,500 cycles by using testers equipped with 5 items each. It is assumed that the consumer's risk is 10 percent when the true mean is 5,000 cycles and the producer's risk is 5 percent when the true mean is 20,000 cycles. This leads to β=0.1, r=5, a=0.5 and $r_2 = 4$. Thus, from Table 9 the minimum group number required is g=6 and the acceptance number is c=0. We draw a random sample of size 30 and allocate 5 bearings to each of 6 groups for the experiment with 2,500 cycles. We accept the lot and declare that a ball bearing in this lot has a mean life greater than 5,000 cycles when there are no failures observed from all 6 groups.

In tables 10-11, note that, as the ratio r_2 increases, the number of groups and the acceptance numbers decrease at the same time. We need a smaller number of groups and the acceptance number if the termination ratio increases at a fixed group size. For example, from Table 10, if r=5 and a changes from 0.5 to 1.0, the required values of design parameters of GASP have been changed from g=72, c= 3 to g=19, c=4 when r_2=2. It is also noted that when r_2=2 we find some high values of g and c at some conditions, and we cannot find them to satisfy the conditions given in (5.9) to (5.10) in some cases. It is observed that the number of groups tends to decrease as m_g increases or a increases. However, the trend is not monotonic since it depends on the acceptance number as well. The probability of acceptance for the lot at the mean ratio corresponding to the producer's risk is also given in Table 10 and Table 11.

Suppose that the lifetime of a product follows the gamma distribution with shape parameter of 2. It is desired to design a GASP to test that the mean life is greater than 1,000 hours and experimenter wants to run an experiment for 500 hours using testers equipped with 5 products each. The producer's risk is 5% when the true mean is 4,000 hours and the consumer's risk is 25% when the true mean is 1,000 hours. Since m_g =2, β =0.25, r =5, a =0.5 and $r_2 = 4$, the minimum number of groups and acceptance number can be found as g =3 and c =1 from Table 10. This indicates that a total of 15 products are needed and that 3 products are allocated at each of 5 testers. We accept the lot if no more than 1 failure occurs before 500 hours in each of 3 groups. For this proposed sampling plan, the probability of acceptance is 0.9802 when the true mean is 4,000 hours.

CHAPTER 7

CONCLUSION AND FUTURE RESEARCH

We have proposed an attribute group sampling plan, which can be utilized when a multi-item tester is adopted for a life test. The two-point approach has been adopted for determining the plan parameters such as the number of groups and the acceptance number. The group sampling plan would be beneficial in terms of test time and cost because a group of items can be simultaneously tested. Furthermore, a group acceptance sampling plan from a truncated life test is proposed and the number of groups has been determined for the inverse Rayleigh and log-logistic distributions when the consumer's risk and the other plan parameters are specified. It can be observed that the number of groups required is larger for the inverse Rayleigh than for the log-logistic distribution when the test time is shorter than the specified life. However, the operating characteristics for the inverse Rayleigh distribution is more desirable for the log-logistic distribution in a sense that the former increases more rapidly than the latter, as the quality improves. The major two parameters in an acceptance sampling plan based on a truncated life test are the sample size and the test termination time. However, most studies have been focused on determining the sample size while the test termination time is assumed to be specified. Obviously, the termination time can be determined by a similar approach when the sample size is specified.

7.1 FUTURE RESEARCH

1. A further study is needed to propose a more efficient or economic group sampling plan in terms of the sample size required or the test cost.

2. The study can also be extended to develop the group sampling plan considering only consumer's risk using many other distributions.

3. It may be interesting to simultaneously study the determination of the parameters g and c as a future research using some other distributions.

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APPENDIX

Table 1:
Number of groups required for the proposed plan for the inverse Rayleigh distribution

β	*r*	*c*	*a*					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	1	1	1	1	1	1
	3	1	2	2	1	1	1	1
	4	2	4	3	2	1	1	1
	5	3	6	4	2	2	1	1
	6	4	10	5	3	2	1	1
	7	5	17	8	3	2	1	1
0.10	4	0	1	1	1	1	1	1
	5	1	2	1	1	1	1	1
	6	2	2	2	1	1	1	1
	7	3	3	2	1	1	1	1
	8	4	5	3	2	1	1	1
	9	5	7	4	2	1	1	1
0.05	5	0	1	1	1	1	1	1
	6	1	2	1	1	1	1	1
	7	2	2	2	1	1	1	1
	8	3	3	2	1	1	1	1
	9	4	4	3	2	1	1	1
	10	5	6	3	2	1	1	1
0.01	7	0	1	1	1	1	1	1
	8	1	1	1	1	1	1	1
	9	2	2	2	1	1	1	1
	10	3	3	2	1	1	1	1
	11	4	4	2	2	1	1	1
	12	5	5	3	2	1	1	1

Table 2:
Number of groups required for the proposed plan for the log-logistic distribution when $m_l = 2$

β	r	c	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	1	1	1	1	1	1
	3	1	2	1	1	1	1	1
	4	2	3	3	2	1	1	1
	5	3	5	4	2	2	1	1
	6	4	8	5	3	2	1	1
	7	5	14	8	4	2	2	1
0.10	4	0	1	1	1	1	1	1
	5	1	2	1	1	1	1	1
	6	2	2	2	1	1	1	1
	7	3	3	2	2	1	1	1
	8	4	4	3	2	1	1	1
	9	5	6	4	2	2	1	1
0.05	5	0	1	1	1	1	1	1
	6	1	2	1	1	1	1	1
	7	2	2	2	1	1	1	1
	8	3	3	2	1	1	1	1
	9	4	4	3	2	1	1	1
	10	5	5	3	2	1	1	1
0.01	7	0	1	1	1	1	1	1
	8	1	2	1	1	1	1	1
	9	2	2	1	1	1	1	1
	10	3	3	2	1	1	1	1
	11	4	3	2	1	1	1	1
	12	5	4	3	2	1	1	1

Table 3:
Operating characteristics values of the group sampling plan with c=2 for inverse Rayleigh distribution

β	μ / μ_0								
	r	*g*	*a*	2	4	6	8	10	12
0.25	4	4	0.7	0.9938	1.0000	1.0000	1.0000	1.0000	1.0000
	4	3	0.8	0.9727	1.0000	1.0000	1.0000	1.0000	1.0000
	4	2	1.0	0.8662	1.0000	1.0000	1.0000	1.0000	1.0000
	4	1	1.2	0.8055	0.9999	1.0000	1.0000	1.0000	1.0000
	4	1	1.5	0.5795	0.9959	1.0000	1.0000	1.0000	1.0000
	4	1	2.0	0.3004	0.9307	0.9993	1.0000	1.0000	1.0000
0.10	6	2	0.7	0.9862	1.0000	1.0000	1.0000	1.0000	1.0000
	6	2	0.8	0.9272	1.0000	1.0000	1.0000	1.0000	1.0000
	6	1	1.0	0.7806	1.0000	1.0000	1.0000	1.0000	1.0000
	6	1	1.2	0.5173	0.9995	1.0000	1.0000	1.0000	1.0000
	6	1	1.5	0.2266	0.9824	1.0000	1.0000	1.0000	1.0000
	6	1	2.0	0.0508	0.7806	0.9968	1.0000	1.0000	1.0000
0.05	7	2	0.7	0.9772	1.0000	1.0000	1.0000	1.0000	1.0000
	7	2	0.8	0.8866	1.0000	1.0000	1.0000	1.0000	1.0000
	7	1	1.0	0.6921	1.0000	1.0000	1.0000	1.0000	1.0000
	7	1	1.2	0.3918	0.9992	1.0000	1.0000	1.0000	1.0000
	7	1	1.5	0.1308	0.9715	1.0000	1.0000	1.0000	1.0000
	7	1	2.0	0.0189	0.6921	0.9946	1.0000	1.0000	1.0000
0.01	9	2	0.7	0.9514	1.0000	1.0000	1.0000	1.0000	1.0000
	9	2	0.8	0.7854	1.0000	1.0000	1.0000	1.0000	1.0000
	9	1	1.0	0.5173	1.0000	1.0000	1.0000	1.0000	1.0000
	9	1	1.2	0.2081	0.9982	1.0000	1.0000	1.0000	1.0000
	9	1	1.5	0.0394	0.9416	1.0000	1.0000	1.0000	1.0000
	9	1	2.0	0.0023	0.5173	0.9880	1.0000	1.0000	1.0000

Table 4:
Operating characteristics values of the group sampling plan with c=2 for the log-logistic distribution when $m_l = 2$

β	μ / μ_0								
	r	g	a	2	4	6	8	10	12
0.25	4	3	0.7	0.8812	0.9961	0.9996	0.9999	1.0000	1.0000
	4	3	0.8	0.8007	0.9919	0.9991	0.9998	1.0000	1.0000
	4	2	1.0	0.7081	0.9829	0.9980	0.9996	0.9999	1.0000
	4	1	1.2	0.7306	0.9793	0.9973	0.9994	0.9998	0.9999
	4	1	1.5	0.5570	0.9449	0.9914	0.9981	0.9994	0.9998
	4	1	2.0	0.3279	0.8415	0.9666	0.9914	0.9973	0.9990
0.10	6	2	0.7	0.7359	0.9882	0.9987	0.9998	0.9999	1.0000
	6	2	0.8	0.6008	0.9766	0.9973	0.9995	0.9999	1.0000
	6	1	1.0	0.5827	0.9652	0.9954	0.9991	0.9997	0.9999
	6	1	1.2	0.4008	0.9223	0.9882	0.9974	0.9993	0.9997
	6	1	1.5	0.2062	0.8184	0.9652	0.9915	0.9974	0.9991
	6	1	2.0	0.0617	0.5827	0.8816	0.9652	0.9882	0.9954
0.05	7	2	0.7	0.6283	0.9805	0.9978	0.9996	0.9999	1.0000
	7	2	0.8	0.4695	0.9619	0.9954	0.9991	0.9997	0.9999
	7	1	1.0	0.4608	0.9450	0.9924	0.9984	0.9995	0.9998
	7	1	1.2	0.2780	0.8820	0.9808	0.9957	0.9987	0.9995
	7	1	1.5	0.1156	0.7405	0.9450	0.9861	0.9957	0.9984
	7	1	2.0	0.0243	0.4608	0.8250	0.9450	0.9808	0.9924
0.01	9	2	0.7	0.4235	0.9582	0.9950	0.9990	0.9997	0.9999
	9	1	0.8	0.5088	0.9597	0.9949	0.9989	0.9997	0.9999
	9	1	1.0	0.2682	0.8923	0.9835	0.9964	0.9989	0.9996
	9	1	1.2	0.1226	0.7853	0.9597	0.9904	0.9971	0.9989
	9	1	1.5	0.0328	0.5795	0.8923	0.9704	0.9904	0.9964
	9	1	2.0	0.0328	0.5795	0.8923	0.9704	0.9904	0.9964

Table 5:
Minimum ratio of true average life to specified life for a producer's risk of 5% for the inverse Rayleigh distribution

β	c	a						
		r	0.700	0.800	1.0	1.20	1.50	2.0
0.25	0	2	2.38	2.72	3.40	4.08	5.10	6.80
	1	3	1.90	2.17	2.51	3.01	3.76	5.02
	2	4	1.70	1.89	2.26	2.51	3.14	4.19
	3	5	1.54	1.70	1.99	2.38	2.75	3.67
	4	6	1.43	1.56	1.87	2.15	2.48	3.31
	5	7	1.36	1.48	1.71	1.97	2.28	3.05
0.10	0	4	2.60	2.97	3.71	4.45	5.57	7.42
	1	5	2.12	2.27	2.84	3.41	4.26	5.68
	2	6	1.81	2.07	2.43	2.92	3.64	4.86
	3	7	1.67	1.84	2.17	2.60	3.25	4.33
	4	8	1.57	1.74	2.10	2.37	2.96	3.95
	5	9	1.49	1.64	1.92	2.19	2.74	3.66
0.05	0	5	2.66	3.04	3.80	4.55	5.69	7.59
	1	6	2.19	2.36	2.95	3.54	4.43	5.91
	2	7	1.88	2.15	2.54	3.05	3.81	5.08
	3	8	1.74	1.92	2.27	2.73	3.41	4.55
	4	9	1.62	1.82	2.20	2.50	3.12	4.17
	5	10	1.54	1.69	2.04	2.32	2.90	3.87
0.01	0	7	2.75	3.15	3.93	4.72	5.90	7.86
	1	8	2.17	2.49	3.11	3.73	4.66	6.21
	2	9	1.99	2.28	2.70	3.24	4.05	5.41
	3	10	1.85	2.05	2.44	2.93	3.66	4.88
	4	11	1.74	1.89	2.36	2.70	3.38	4.50
	5	12	1.64	1.81	2.21	2.52	3.15	4.20

Table 6:
Minimum ratio of true average life to specified life for a producer's risk of 5% for the log-logistic distribution

β	*c*	*a*						
		r	0.700	0.800	1.0	1.20	1.50	2.0
0.25	0	2	6.82	7.80	9.75	11.69	14.62	19.49
	1	3	3.38	3.18	3.98	4.77	5.96	7.95
	2	4	2.44	2.78	3.18	3.29	4.11	5.48
	3	5	1.88	2.25	2.49	2.99	3.27	4.36
	4	6	1.80	1.92	2.25	2.51	2.78	3.71
	5	7	1.67	1.77	2.03	2.20	2.75	3.28
0.10	0	4	9.77	11.17	13.96	16.75	20.94	27.92
	1	5	4.61	4.37	5.46	6.55	8.19	10.92
	2	6	2.98	3.41	3.70	4.44	5.55	7.40
	3	7	2.48	2.63	3.29	3.50	4.37	5.83
	4	8	2.13	2.34	2.75	2.96	3.70	4.93
	5	9	1.93	2.10	2.35	2.82	3.25	4.33
0.05	0	5	10.83	12.38	15.47	18.57	23.21	30.95
	1	6	5.12	4.88	6.10	7.32	9.15	12.20
	2	7	3.29	3.76	4.09	4.90	6.13	8.17
	3	8	2.72	2.89	3.21	3.86	4.82	6.43
	4	9	2.34	2.59	3.01	3.25	4.07	5.42
	5	10	2.06	2.22	2.62	2.86	3.57	4.76
0.01	0	7	12.83	14.66	18.33	21.99	27.49	36.66
	1	8	6.00	5.70	7.13	8.55	10.69	14.25
	2	9	3.84	3.82	4.77	5.73	7.16	9.55
	3	10	3.15	3.36	3.74	4.49	5.61	7.48
	4	11	3.15	3.36	3.74	4.49	5.61	7.48
	5	12	2.32	2.56	3.03	3.31	4.13	5.51

Table 7:
Minimum number of groups and acceptance number for the Weibull distribution when $m_w = 1$

β	$\mu/\mu_0 = r_2$	r=5						r=10					
		a=0.5			a=1.0			a=0.5			a=1.0		
		g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	-	-	-	-	-	-	27	6	0.9568	20	8	0.9712
	4	17	3	0.9854	3	3	0.9707	2	3	0.9559	1	4	0.9511
	6	4	2	0.9820	3	3	0.9927	1	2	0.9600	1	4	0.9891
	8	4	2	0.9919	2	2	0.9732	1	2	0.9807	1	3	0.9777
	10	2	1	0.9574	2	2	0.9852	1	2	0.9893	1	3	0.9892
0.10	2	-	-	-	-	-	-	209	7	0.9658	32	8	0.9543
	4	27	3	0.9769	5	3	0.9517	6	4	0.9799	2	5	0.9787
	6	7	2	0.9688	5	3	0.9879	3	3	0.9827	2	4	0.9783
	8	7	2	0.9859	2	2	0.9732	2	2	0.9617	1	3	0.9777
	10	7	2	0.9925	2	2	0.9852	2	2	0.9786	1	3	0.9892
0.05	2	-	-	-	-	-	-	272	7	0.9557	225	9	0.9802
	4	35	3	0.9702	29	4	0.9848	7	4	0.9766	3	5	0.9683
	6	9	2	0.9601	6	3	0.9855	4	3	0.9770	2	4	0.9783
	8	9	2	0.9819	3	2	0.9600	2	2	0.9617	1	3	0.9777
	10	9	2	0.9904	3	2	0.9779	2	2	0.9786	1	3	0.9892
0.01	2	-	-	-	-	-	-	3156	8	0.9685	451	9	0.9607
	4	54	3	0.9944	44	4	0.9770	11	4	0.9634	4	5	0.9579
	6	54	3	0.9897	10	3	0.9759	6	3	0.9658	3	4	0.9676
	8	13	2	0.9740	10	3	0.9914	6	3	0.9874	2	3	0.9559
	10	13	2	0.9861	4	2	0.9706	3	2	0.9681	2	3	0.9786

Remark. The cells with hyphens (-) indicate that g and c cannot be found to satisfy the conditions.

Table 8:
Minimum number of groups and acceptance number for the Weibull distribution when $m_w = 2$

β	$\mu/\mu_0 = r_2$	r=5						r=10					
		a=0.5			a=1.0			a=0.5			a=1.0		
		g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	32	2	0.9878	5	3	0.9785	16	3	0.9913	2	4	0.9594
	4	6	1	0.9913	1	1	0.9792	2	1	0.9875	1	2	0.9898
	6	6	1	0.9982	1	1	0.9955	2	1	0.9974	1	1	0.9813
	8	2	0	0.9698	1	1	0.9985	1	0	0.9698	1	1	0.9937
	10	2	0	0.9806	1	1	0.9994	1	0	0.9806	1	1	0.9974
0.10	2	531	3	0.9866	9	3	0.9617	26	3	0.9775	2	4	0.9594
	4	10	1	0.9856	2	1	0.9588	3	1	0.9813	1	2	0.9898
	6	10	1	0.9971	2	1	0.9911	3	1	0.9961	1	1	0.9813
	8	3	0	0.9550	2	1	0.9971	3	1	0.9988	1	1	0.9937
	10	3	0	0.9710	1	0	0.9615	2	0	0.9615	1	1	0.9974
0.05	2	591	3	0.9851	11	3	0.9534	34	3	0.9707	5	5	0.9827
	4	13	1	0.9813	2	1	0.9588	4	1	0.9751	1	2	0.9898
	6	13	1	0.9962	2	1	0.9911	4	1	0.9948	1	1	0.9813
	8	13	1	0.9988	2	1	0.9971	4	1	0.9983	1	1	0.9937
	10	4	0	0.9615	1	0	0.9615	2	0	0.9615	1	1	0.9974
0.01	2	1063	3	0.9734	95	4	0.9830	52	3	0.9555	7	5	0.9758
	4	19	1	0.9728	6	2	0.9939	6	1	0.9629	2	2	0.9796
	6	19	1	0.9945	3	1	0.9867	6	1	0.9923	1	1	0.9813
	8	19	1	0.9982	3	1	0.9957	6	1	0.9995	1	1	0.9937
	10	5	0	0.9521	3	1	0.9982	6	1	0.9990	1	1	0.9974

Table 9:
Minimum number of groups and acceptance number for the Weibull distribution $m_w = 3$

β	$\mu/\mu_0 = r_2$	r=5						r=10					
		a=0.5			a=1.0			a=0.5			a=1.0		
		g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	23	1	0.9728	2	2	0.9892	6	1	0.9692	1	2	0.9530
	4	4	0	0.9726	1	1	0.9988	2	0	0.9726	1	1	0.9948
	6	4	0	0.9918	1	1	0.9999	2	0	0.9918	1	0	0.9676
	8	4	0	0.9965	1	1	1	2	0	0.9965	1	0	0.9862
	10	4	0	0.9982	1	1	1	2	0	0.9982	1	0	0.9929
0.10	2	37	1	0.9566	4	2	0.9785	10	1	0.9493	1	2	0.9530
	4	6	0	0.9591	2	1	0.9976	3	0	0.9591	1	1	0.9948
	6	6	0	0.9877	1	0	0.9837	3	0	0.9877	1	0	0.9676
	8	6	0	0.9848	1	0	0.9931	3	0	0.9948	1	0	0.9862
	10	6	0	0.9973	1	0	0.9964	3	0	0.9973	1	0	0.9929
0.05	2	48	1	0.9441	5	2	0.9732	63	2	0.9904	1	2	0.9530
	4	7	0	0.9525	2	1	0.9976	13	1	0.9989	1	1	0.9948
	6	7	0	0.9857	1	0	0.9837	4	0	0.9837	1	0	0.9676
	8	7	0	0.9939	1	0	0.9931	4	0	0.9931	1	0	0.9862
	10	7	0	0.9969	1	0	0.9964	4	0	0.9964	1	0	0.9929
0.01	2	256	2	0.9966	7	2	0.9627	96	2	0.9854	3	3	0.9784
	4	74	1	0.9986	3	1	0.9964	20	1	0.9983	1	1	0.9948
	6	11	0	0.9776	2	0	0.9676	6	0	0.9756	1	0	0.9676
	8	11	0	0.9905	2	0	0.9862	6	0	0.9896	1	0	0.9862
	10	11	0	0.9951	2	0	0.9929	6	0	0.9947	1	0	0.9929

Table 10:
Minimum number of groups and acceptance number for the gamma distribution when $m_g = 2$

β	$\mu/\mu_0 = r_2$	r=5						r=10					
		a=0.5			a=1.0			a=0.5			a=1.0		
		g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	72	3	0.9781	19	4	0.9758	5	3	0.9563	4	6	0.9804
	4	3	1	0.9802	2	2	0.9873	1	1	0.9726	1	3	0.9911
	6	3	1	0.9955	1	1	0.9818	1	1	0.9935	1	2	0.9916
	8	1	0	0.9646	1	1	0.9933	1	1	0.9978	1	1	0.9726
	10	1	0	0.9768	1	1	0.9970	1	0	0.9542	1	1	0.9874
0.10	2	119	3	0.9641	30	4	0.9621	23	4	0.9768	6	6	0.9708
	4	5	1	0.9672	3	2	0.9810	4	2	0.9923	1	3	0.9911
	6	5	1	0.9925	1	1	0.9818	2	1	0.9870	1	2	0.9916
	8	5	1	0.9975	1	1	0.9933	2	1	0.9955	1	1	0.9726
	10	2	0	0.9542	1	1	0.9970	1	0	0.9542	1	1	0.9874
0.05	2	155	3	0.9535	*	*	*	30	4	0.9698	7	6	0.9660
	4	6	1	0.9607	3	2	0.9810	5	2	0.9903	2	3	0.9823
	6	6	1	0.9910	2	1	0.9639	2	1	0.9870	1	2	0.9916
	8	6	1	0.9969	2	1	0.9867	2	1	0.9955	1	1	0.9726
	10	2	0	0.9542	2	1	0.9970	1	0	0.9542	1	1	0.9874
0.01	2	-	-	-	*	*	*	46	4	0.9541	27	7	0.9832
	4	37	2	0.9934	5	2	0.9685	7	2	0.9865	2	3	0.9823
	6	10	1	0.9850	2	1	0.9639	3	1	0.9806	2	2	0.9832
	8	10	1	0.9949	2	1	0.9867	3	1	0.9933	1	1	0.9726
	10	10	1	0.9978	2	1	0.9941	3	1	0.9971	1	1	0.9874

Remark. The cells with hyphens (-) indicate that *g* and *c* are found to be large. The cells with hyphens (*) indicates that *g* and *c* can not satisfy the conditions.

Table 11:
Minimum number of groups and acceptance number for the gamma distribution m_g =3

β	μ/μ₀ $=r_2$	r=5						r=10					
		a=0.5			a=1.0			a=0.5			a=1.0		
		g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$	g	c	$L(p_2)$
0.25	2	27	2	0.9833	4	3	0.9776	4	2	0.9745	1	4	0.9727
	4	5	1	0.9978	1	1	0.9849	2	1	0.9962	1	2	0.9936
	6	2	0	0.9786	1	1	0.9980	1	0	0.9786	1	1	0.9914
	8	2	0	0.9905	1	0	0.9672	1	0	0.9905	1	1	0.9981
	10	2	0	0.9950	1	0	0.9821	1	0	0.9950	1	0	0.9646
0.10	2	44	2	0.9729	7	3	0.9611	7	2	0.9558	3	5	0.9850
	4	9	1	0.9961	2	1	0.9700	3	1	0.9942	1	2	0.9936
	6	3	0	0.9681	2	1	0.9960	2	0	0.9576	1	1	0.9914
	8	3	0	0.9858	1	0	0.9672	2	0	0.9811	1	1	0.9981
	10	3	0	0.9925	1	0	0.9821	2	0	0.9900	1	0	0.9646
0.05	2	57	2	0.9650	9	3	0.9502	27	3	0.9875	4	5	0.9800
	4	11	1	0.9952	2	1	0.9700	4	1	0.9923	1	2	0.9936
	6	3	0	0.9681	2	1	0.9960	2	0	0.9576	1	1	0.9914
	8	3	0	0.9858	1	0	0.9672	2	0	0.9811	1	1	0.9981
	10	3	0	0.9925	1	0	0.9821	2	0	0.9900	1	0	0.9646
0.01	2	-	-	-	70	4	0.9823	42	3	0.9807	6	5	0.9702
	4	17	1	0.9926	3	1	0.9553	6	1	0.9885	2	2	0.9872
	6	17	1	0.9992	3	1	0.9940	6	1	0.9988	1	1	0.9914
	8	5	0	0.9764	3	1	0.9987	3	0	0.9717	1	1	0.9981
	10	5	0	0.9975	2	0	0.9646	3	0	0.9850	1	0	0.9646

Remark. The cells with hyphens (-) indicate that g and c are found to be large.