National College of Business Administration & Economics Lahore

A FAMILY OF ESTIMATORS FOR TWO-PHASE SAMPLING USING MULTI- AUXILIARY ATTRIBUTES

BY

INAM-UL-HAQ

**DOCTOR OF PHILOSOPHY
IN
APPLIED STATISTICS**

December, 2009

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Dissertation Committee:

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(Chairman)

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(Member)

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(Member)

____
Rector
National College of Business Administration & Economics

DECLARATION

This is to certify that this research work has not been submitted for obtaining similar degree from any other university / college.

INAM-UL-HAQ
December 10, 2009

DEDICATED

TO

My Parents,

My Spouse

And

My Daughters Emman & Laiba

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RESEARCH COMPLETION CERTIFICATE

Certified that the research work contained in this thesis entitled **"A FAMILY OF ESTIMATORS FOR TWO-PHASE SAMPLING USING MULTI-AUXILIARY ATTRIBUTES"** has been carried out and completed by **"Inam-ul-Haq"** under my supervision during his Ph. D**.** Applied Statistics programmed.

(Dr. Muhammad Hanif)
Supervisor

SUMMARY

After Introduction, use of supplementary information in context to full, partial, and no information along with notation is discussed in Chapter 1. Review of literature has been given in Chapter 2. In Chapter 3 some existing estimators, which utilizes auxiliary information, has been described.

Chapter 4 include the generalized families of estimators for full, partial and no information cases; using "k" auxiliary attributes along with improved form of generalized families of estimators. Chapter 5 includes generalized ratio and product estimators using "k" auxiliary attributes along with improved generalized ratio and product estimators. In Chapter 6 generalized family of estimators proposed for full, partial and no information cases, and improved form of proposed generalized family of estimators using idea of shrinkage for two auxiliary attributes is given .In chapter 7 some new shrinkage regression type estimators for single and two auxiliary attributes, using full, partial and no information cases are suggested. The numerical comparison of suggested and existing estimators is given in chapter 8.

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CHAPTER 1
INTRODUCTION

1.1 ROLE OF SAMPLING

The sampling methods considered here are applied for purpose of enumerative statistics and relate to actual finite populations. Most survey work involves sampling from finite populations. There are two parts to any sampling strategy. First there is the selection procedure, the manner in which sampling units are to be selected from a finite population. Second there is the estimation procedure, which prescribes how inferences are to be made from sample to population. These inferences may be either enumerative or analytical. Enumerative inference seeks only to describe the finite population under study whereas analytical inference attempts to explain the underlying distributional and functional characteristics of a population. Enumerative inference typically concerns with the estimation of some parameters of a population such as means, totals, proportion and ratios.

Analytical inference consists of specifying appropriate model that adequately describes the sample. It is customary to distinguish between enumerative and analytical inference in terms of complexity of the population characteristics. Estimating a mean, total etc., is regarded as an enumerative problem whereas estimating a regression or correlation coefficient as an analytical one, but if the mean in question is a parameter of a simple explanatory model, the inference is enumerative. For enumerative inference, a quite different probability structure is used. It depends on the manner in which the sample is selected. This is the classical finite population sampling inference developed by Neyman (1934), who based his results on Bowley (1926).

Systematic interest in the use of sampling theory appeared towards the end of the second last century when Kiaer (1895, 99) used the representative method for collecting data independently of the census. In (1901) he also demonstrated empirically that stratification could provide good estimates of finite population totals and means. On the recommendation of Kiaer (1895), International Statistical Institute (1903) recommended the adoption of stratified sampling with proportional allocation as an acceptable method of data collection. The primary difference in the earlier uses of the sampling methods lay in the selection of the sample; it was thought that a purposive selection based on sample's knowledge of the population with regard to closely correlated characters is the best way of getting a sample that could be considered representative of the population.

Neyman (1934)'s paper was the revolutionary paper and in that paper he made it clear that random selection had its basis on a sound scientific theory, which definitely gave to sample survey the character of an objective research tool and made it possible to predict the validity of the survey estimates. Actually in this sense it can be said to mark the beginning of a new era.

1.2 USE OF AUXILIARY VARIABLE AND ATTRIBUTE

Use of auxiliary information to increase the efficiency of estimators for population characteristics is an integral part of several estimators. Auxiliary information is often used to improve the efficiency of estimators while using ratio or regression method of estimation in survey sampling.

The history of using auxiliary information in survey sampling is as old as history of the survey sampling. The work of Neyman (1938) may be referred to as the initial works where auxiliary information has been used. Cochran (1940) used auxiliary information in single phase sampling to develop the ratio estimator for estimation of population mean. Hansen and Hurwitz (1943) also suggested the use of auxiliary information in selecting the sample with varying probabilities. Triphati (1970) and Das (1988) used the auxiliary information in four ways as (i) the values of one or more auxiliary variable may be known for all units of population (ii) the values of one or more auxiliary variable may be known only for some units of population (iii) the values of the parameter are not known for auxiliary variables but their estimated values are known (iv) the values of one or more population parameter of auxiliary variable are known.

In most of the surveys the auxiliary information is always available. Various research works during last 70 years suggest that every form of the auxiliary information should always be used in developing sampling strategies. Samiuddin and Hanif (2007) introduced the following approach of using auxiliary variable.

- Full information case: Information for all auxiliary variables is available for all population units.
- No information case: Information for all auxiliary variables is available for sample.
- Partial information case: Information for some auxiliary variables is available for all population units.

The use of auxiliary information has been studied by various authors in various forms to improve the efficiency of their constructed estimators. Some note able references are; [Cochran (1942), Quenouille (1956), Robson (1957), Olkin (1958),Mickey (1959),Nieto De Pascual (1961), Murthy (1964), Searls (1964), Tin (1965), Sastry (1965), Singh (1967), Rao and Mudholkar (1967), Rao and

Perieria (1968), Rao(1969), Srivastava (1967, 70, 71, 80), Chand (1975), Srivenkataramana and Srinath (1976), Kulkarni (1978), Sahai (1979), Vos (1980),Sahai and Ray (1980), Das and Tripathi (1980), Kiregyra (1980, 84), Triphati (1970, 80, 87), Sisodia and Dwivedi (1981), Srivastava and Jhajj (1981, 83).Kaur (1983), Das(1988), Prasad (1989), Lui (1990), Srivastava et al. (1990), Naik and Gupta (1991), Sahoo and Sahoo (1993, 94), Singh and Upadhyaya (1995), Biradar and Singh (1997), Ahmed (1998), Singh (2001), Roy (2003), Singh et al. (2004), Dubey (2006), Samiuddin and Hanif (2006,2007), Ahmad (2007), Hanif et al. (2009a, 2009b), Ahmad et al. (2009a, 2009b,2009c), etc.]

Use of quantitative auxiliary variable is common practice to improve efficiency of estimators. Niak and Gupta (1996) used auxiliary attribute instead of quantitative auxiliary variable to construct ratio, product and regression estimator. Jhajj et al. (2006) introduced a family of estimators by using known information on one auxiliary attribute. Shabbir and Gupta (2007) modified Jhajj et al. (2006) estimator and proposed an improved ratio cum regression type estimator using idea of Roy (2003).

1.3- APPLICATION OF STUDY IN REAL LIFE

In this section some of the real life examples are given as,
(1)- It is reported about some particular part of City that infant of this area are under weighted at birth. A researcher wants to estimate the mean weight of child at birth then main variable of this study will be weight of infant at birth and auxiliary variables will (i) Mother's proper check up during pregnancy (ii) Use of proper medicine during pregnancy (iii) Age of mother at birth (iv) Use of proper food during pregnancy etc.

(2)- The government wants to estimate the mean yield of cotton per acre to know whether this year's average yield is improved than previous year or not then main variable will be per acre yield of cotton from different sources, and auxiliary variables will be (i) Fertility of land (ii) Use of recommended seeds (iii) Area of land (iv) proper control of cotton's insects, etc.

(3)- It is reported about some particular part of City that women of this area are weak and they fell into complexities of pregnancy, a researcher wants to estimate the mean amount of hemoglobin in their blood then main variable of this study will be amount of hemoglobin in the blood of women who can become mother in future, and auxiliary variables will (i) Use of folic acid (ii) Use of proper foods that contain iron (iii) Age of mother at birth (iv) Function of liver etc.

It is worth noting that we can convert quantitative auxiliary variable into an auxiliary attribute, for example height is quantitative auxiliary variable but height of person can be categorized as (i) less than or equal to 60 inches (ii) more than 60 inches.

1.4 OBJECTIVES OF STUDY

The objectives of thesis are;

- Improvement in the Jhajj et al. (2006) estimator and produce new improved estimators.
- To improve the Shabbir and Gupta (2007) estimator for single and two auxiliary attributes.
- To generalize Jhajj et al. (2006) family of estimators using "k" auxiliary attributes as well as to produce some new families of estimators and shrinkage families of estimators.
- To construct generalized ratio and product estimators as well as shrinkage ratio and product estimators using "k" auxiliary attributes.
- To conduct a numerical study for comparison of newly constructed and existing estimators as well as comparison of full, partial and no information cases.

1.5 NOTATIONS AND ASSUMPTIONS

Following notations has been used in this thesis.

1. N: Size of Population, n: Size of sample, $\bar{y}$ is sample mean, and $\theta = \left(\frac{1}{n} - \frac{1}{N}\right)$.

2. $$\tau_{ij} = \begin{cases} 1, & \text{if ith unit of population possess attribute } \tau_j. \\ 0, & \text{otherwise.} \qquad \text{"for } j = 1, 2..., k\text{".} \end{cases}$$

3. $A_j = \sum_{i=1}^{N} \tau_{ij}$: Total number of units in population possessing attribute, τ_j for $j = 1, 2..., k$.

   $a_j = \sum_{i=1}^{n} \tau_{ij}$: Total number of units in sample possessing attribute τ_j, $j = 1, 2......., k$.

4. $\bar{\tau}_j = \frac{A_j}{N} = P_j$, $\hat{\bar{\tau}}_j = \frac{a_j}{n} = p_j$ for $j = 1, 2......., k$.

5. $S^2_{\tau_j} = \frac{N}{N-1} P_j \left(1 - P_j\right)$, $S_{y\tau_j} = \frac{1}{N-1}\left[\sum_{i=1}^{N} Y_i \tau_{ij} - NP_j \bar{Y}\right]$

for $j = 1, 2......., k$.

6. $\rho_{Pb_j} = \frac{C_{y\tau_j}}{C_y C_{\tau_j}}$, point bi-serial correlation coefficient (for $j = 1, 2......., k$.)

7. $\bar{e}_{\tau_j} = p_j - P_j$ for $j = 1, 2......., k$. (1.5.1)

$$v_j = \frac{p_j}{P_j},\ v_j - 1 = \frac{p_j - P_j}{P_j} = \frac{1}{P_j}\bar{e}_{\tau_j},$$

$$\text{So } v_j = 1 + \frac{\bar{e}_{\tau_j}}{P_j} = 1 + \frac{\bar{e}_{\tau_{j(1)}}}{P_j} = 1 + \frac{\bar{e}_{\tau_{j(2)}}}{P_j} \quad (1.5.2)$$

for $j = 1, 2......., k$.(for two phase we can select by our choice)

8. $\bar{e}_y = \bar{y} - \bar{Y}$. (1.5.3)

9. $E\left(\bar{e}_y\right) = E\left(\bar{e}_{\tau_j}\right) = 0$ for $j = 1, 2......., k$.

$$E\left(\bar{e}_y^2\right) = \theta \bar{Y}^2 C_y^2, \quad (1.5.4)$$

$$E\left(\bar{e}^2_{\tau_j}\right) = \theta P_j^2 C^2_{\tau_j}, \text{ for } j = 1, 2......., k. \quad (1.5.5)$$

10. $E\left(\bar{e}_y \bar{e}_{\tau_j}\right) = \theta \bar{Y} C_y P_j C_{\tau_j} \rho_{Pbj}$ for $j = 1, 2......., k$. (1.5.6)

$$E\left(\bar{e}_{\tau_j} \bar{e}_{\tau_\psi}\right) = \theta P_j P_\psi C_{\tau_j} C_{\tau_\psi} Q_{j\psi} \quad \& \quad j \neq \psi, \quad (1.5.7)$$

Where $Q_{j\psi}$ is the coefficient of association between jth and ψ th auxiliary attribute;

11. n_1 : Size of first phase sample, n_2 : Size of second phase sample. $\overline{y}_2$ is mean of second phase sample.

$$\theta_1 = \left(\frac{1}{n_1} - \frac{1}{N}\right), \theta_2 = \left(\frac{1}{n_2} - \frac{1}{N}\right), \quad \theta_3 = \theta_2 - \theta_1$$

12. $p_{j(1)}$ = Proportion of units possessing attribute τ_j in first phase sample of size n_1.

$p_{j(2)}$ = Proportion of units possessing attribute τ_j in second phase sample of size n_2.

13. $$\overline{e}_{y_2} = \overline{y}_2 - \overline{Y}, \tag{1.5.8}$$

$$\overline{e}_{\tau_{j(1)}} = p_{j(1)} - P_j, \tag{1.5.9}$$

$$\overline{e}_{\tau_{j(2)}} = p_{j(2)} - P_j. \tag{1.5.10}$$

$$v_{jd} = \frac{p_{j(2)}}{P_{j(1)}}, \quad v_{jd} - 1 = \frac{p_{j(2)} - p_{j(1)}}{p_{j(1)}},$$

$$\text{or } v_{jd} - 1 = \frac{\overline{e}_{\tau_{j(2)}} - \overline{e}_{\tau_{j(1)}}}{P_j + \overline{e}_{\tau_{j(1)}}} = \frac{\left(\overline{e}_{\tau_{j(2)}} - \overline{e}_{\tau_{j(1)}}\right)}{P_j}\left(1 + \frac{\overline{e}_{\tau_{j(1)}}}{P_j}\right)^{-1}$$

Here we shall take $\left(v_{jd} - 1\right)$ to term of order 1/n as

$$v_{jd} - 1 \approx \frac{\left(\overline{e}_{\tau_{j(2)}} - \overline{e}_{\tau_{j(1)}}\right)}{P_j} \text{ for } j = 1, 2......., k. \tag{1.5.11}$$

14. $$E\left(\overline{e}_{\tau_{j(1)}}\right) = E\left(\overline{e}_{\tau_{j(2)}}\right) = E\left(\overline{e}_{y_2}\right) = 0,$$

$$E\left(\overline{e}^2_{y_2}\right) = \theta_2 \overline{Y}^2 C_y^2, \tag{1.5.12}$$

$$\left.\begin{cases} E\left(\overline{e}_{\tau_{j(1)}} - \overline{e}_{\tau_{j(2)}}\right)^2 = \theta_3 P_j^2 C_{\tau_j}^2 \\ E\left(\overline{e}^2_{\tau_{j(1)}}\right) = \theta_1 P_j^2 C_{\tau_j}^2 \\ \& E\left(\overline{e}^2_{\tau_{j(2)}}\right) = \theta_2 P_j^2 C_{\tau_j}^2 \end{cases}\right\}, \quad \text{for } j = 1, 2,........, k. \tag{1.5.13}$$

15. $$\left\{\begin{array}{l} E\left[\bar{e}_{y_2}\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)\right]=\theta_3\bar{Y}C_yP_jC_{\tau_j}\rho_{Pbj} \\ E\left(\bar{e}_{y_2}\bar{e}_{\tau_{j(1)}}\right)=\theta_1\bar{Y}C_yP_jC_{\tau_j}\rho_{Pbj} \\ E\left(\bar{e}_{y_2}\bar{e}_{\tau_{j(2)}}\right)=\theta_2\bar{Y}C_yP_jC_{\tau_j}\rho_{Pbj} \end{array}\right\} \qquad (1.5.14)$$

16. $$\left\{\begin{array}{l} E\left[\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)\left(\bar{e}_{\tau_{\psi(2)}}-\bar{e}_{\tau_{\psi(1)}}\right)\right]=\theta_3P_jP_\psi C_{\tau_j}C_{\tau_\psi}Q_{j\psi} \\ E\left[\bar{e}_{\tau_{j(2)}}\left(\bar{e}_{\tau_{\psi(2)}}-\bar{e}_{\tau_{\psi(1)}}\right)\right]=\theta_3P_jP_\psi C_{\tau_j}C_{\tau_\psi}Q_{j\psi} \\ E\left[\bar{e}_{\tau_{j(1)}}\left(\bar{e}_{\tau_{\psi(2)}}-\bar{e}_{\tau_{\psi(1)}}\right)\right]=0 \qquad \& \ j\neq\psi \end{array}\right\} \qquad (1.5.15)$$

17. (i) $\underline{\alpha}_{(k\times1)}=\left[\alpha_j\right]_{(k\times1)}$ (ii) $\underline{v}=\left[v_j\right]_{(k\times1)}$ (iii) $\underline{v}_d=\left[v_{jd}\right]_{k\times1}$

(iv) $\underline{\varphi}=\left[\underline{v}-1\right]_{k\times1}=\left[\dfrac{\bar{e}_{\tau_j}}{P_j}\right]_{k\times1}$ (v) $\underline{\varphi}e_y=\left[\dfrac{e_y\bar{e}_{\tau_j}}{P_j}\right]_{k\times1}$

(vi) $\underline{\varphi}\underline{\varphi}'=\left[\dfrac{\bar{e}_{\tau_j}\bar{e}_{\tau_\psi}}{P_jP_\psi}\right]_{k\times k}\ \forall\ j=\psi=1,2...,k$

18. $E\left(\underline{\varphi}\underline{\varphi}'\right)=\theta\left[\Phi_\tau\right]_{k\times k}$ and $E\left(\underline{\varphi}\bar{e}_y\right)=\theta\left[\phi_{y\tau}\right]_{k\times1}$ (1.5.16)

Where $\left[\Phi_\tau\right]_{k\times k}=\left[C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right]_{k\times k}$ and $Q_{jj}=1,\forall\ j=1,2,...,k$

$\left[\underline{\phi}_{y\tau}\right]_{k\times1}=\left[\bar{Y}C_yC_{\tau_j}\rho_{Pb_j}\right],\ \forall\ j=1,2,...,k$

19. (i) $\underline{\varphi}_1=\left[\underline{v}_1-1\right]_{m\times1}=\left[\dfrac{\bar{e}_{\tau_{j(1)}}}{P_j}\right]_{m\times1}$ and $\underline{v}_1=\left[v_j\right]_{(m\times1)}\ \forall\ j=1,2,...,m$

(ii) $\underline{\varphi}_2 = [\underline{v}_2 - 1]_{(k-m)\times 1} = \left[\dfrac{\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)}{P_j}\right]_{(k-m)\times 1}$ and $\underline{v}_2 = \left[v_j\right]_{(k-m)\times 1}$

$\forall\, j = m+1, m+2, \ldots, k$

(iii) $\underline{\varphi}_1 e_{y_2} = \left[\dfrac{e_{y_2}\bar{e}_{\tau_{j(1)}}}{P_j}\right]_{m\times 1}$ (iv) $\underline{\varphi}_2 e_{y_2} = \left[\dfrac{\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)e_{y_2}}{P_j}\right]_{(k-m)\times 1}$

(v) $\underline{\varphi}_1\underline{\varphi}_1^{/} = \left[\dfrac{\bar{e}_{\tau_{j(1)}}\bar{e}_{\tau_{\psi(1)}}}{P_j P_\psi}\right]_{m\times m}$ (vi) $\underline{\varphi}_2\underline{\varphi}_1^{/} = \left[\dfrac{\bar{e}_{\tau_{j(1)}}\left(\bar{e}_{\tau_{\psi(2)}} - \bar{e}_{\tau_{\psi(1)}}\right)}{P_j P_\psi}\right]_{m\times(k-m)}$

(vii) $\underline{\varphi}_2\underline{\varphi}_2^{/} = \left[\dfrac{\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\left(\bar{e}_{\tau_{\psi(2)}} - \bar{e}_{\tau_{\psi(1)}}\right)}{P_j P_\psi}\right]_{(k-m)\times(k-m)}$

(ix) $\underline{\varphi}_d = [\underline{v}_d - 1]_{k\times 1} = \left[\dfrac{\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)}{P_j}\right]_{k\times 1}$ $\forall\, j = 1, 2, \ldots, k$

(x) $\underline{\varphi}_d e_{y_2} = \left[\dfrac{\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)e_{y_2}}{P_j}\right]_{k\times 1}$

(xi) $\underline{\varphi}_d\underline{\varphi}_d^{/} = \left[\dfrac{\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\left(\bar{e}_{\tau_{\psi(2)}} - \bar{e}_{\tau_{\psi(1)}}\right)}{P_j P_\psi}\right]_{k\times k}$

(xii) $\underline{\alpha}_1 = \left[\alpha_j\right]_{(m\times 1)}$ $\forall\, j = 1, 2, \ldots, m$

and $\underline{\alpha}_2 = \left[\alpha_j\right]_{(k-m)\times 1}$ $\forall\, j = m+1, m+2, \ldots, k$

20. $E\left(\underline{\varphi}_d\,\underline{\varphi}_d^{/}\right) = (\theta_2 - \theta_1)[\Phi_\tau]_{k\times k}$, $E\left(\underline{\varphi}_d\bar{e}_{y_2}\right) = (\theta_2 - \theta_1)\left[\underline{\phi}_{y\tau}\right]_{k\times 1}$ (1.5.17)

$E\left(\underline{\varphi}_1\bar{e}_{y_2}\right) = \theta_1\left[\underline{\phi}_{y\tau_1}\right]_{m\times 1}$ $E\left(\underline{\varphi}_2\bar{e}_{y_2}\right) = (\theta_2 - \theta_1)\left[\underline{\phi}_{y\tau_2}\right]_{(k-m)\times 1}$ (1.5.18)

$E\left(\underline{\varphi}_1\,\underline{\varphi}_1^{/}\right) = \theta_1\left[\Phi_{\tau_1}\right]_{m\times m}$ $E\left(\underline{\varphi}_2\,\underline{\varphi}_2^{/}\right) = (\theta_2 - \theta_1)\left[\Phi_{\tau_2}\right]_{(k-m)\times(k-m)}$ (1.5.19)

$E\left(\underline{\varphi}_2\,\underline{\varphi}_1^{/}\right) = E\left(\underline{\varphi}_1\,\underline{\varphi}_2^{/}\right) = 0$ (1.5.20) where $\underline{\phi}_{y\tau_1} = \left[\bar{Y} C_y C_{\tau_j} \rho_{Pb_j}\right]_{m\times 1} \ \forall\, j = 1, 2, ..., m.$

$\underline{\phi}_{y\tau_2} = \left[\bar{Y} C_y C_{\tau_j} \rho_{Pb_j}\right]_{(k-m)\times 1} \ \forall\, j = m+1, m+2, ..., k.$

$\left[\Phi_{\tau_1}\right]_{m\times m} = \left[C_{\tau_j} C_{\tau_\psi} Q_{j\psi}\right]_{m\times m}$ and $Q_{jj} = 1, \forall\, j = 1, 2, ..., m$

$\left[\Phi_{\tau_2}\right]_{(k-m)\times(k-m)} = \left[C_{\tau_j} C_{\tau_\psi} Q_{j\psi}\right]_{(k-m)\times(k-m)}$
and $Q_{jj} = 1, \forall\, j = m+1, m+2, ..., k$

21. $\underline{\phi}'_{y\tau} \Phi_{\tau}^{-1} \underline{\phi}_{y\tau} = \left(\rho^2_{y.\tau_1\tau_2...\tau_k}\right) \bar{Y}^2 C_y^2,$ (1.5.21)

$$\left\{\begin{matrix} \underline{\phi}'_{y\tau_1} \Phi_{\tau_1}^{-1} \underline{\phi}_{y\tau_1} = \left(\rho^2_{y.\tau_1\tau_2...\tau_m}\right) \bar{Y}^2 C_y^2 \\ \underline{\phi}'_{y\tau_2} \Phi_{\tau_2}^{-1} \underline{\phi}_{y\tau_2} = \left(\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}\right) \bar{Y}^2 C_y^2 \end{matrix}\right\}, \qquad (1.5.22)$$

Where $\rho^2_{y.\tau_1\tau_2...\tau_k}$ is squared multiple bi-serial correlation coefficient.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION TO TWO-PHASE SAMPLING

Several sampling strategies heavily depend on the use of auxiliary information. When such information is not available, one could consider taking a large preliminary sample in which only auxiliary variables are observed. Later on a relatively smaller sample could be considered for studying main variable. This idea along with idea of stratified random sampling was first given by Neyman (1938). Neyman (1938) gave the concept of two-phase sampling as,

> *"A more accurate estimate of the original character may be obtained for the same total expenditure by arranging the sampling of population in two steps. The first step is to secure data, for the second character only, from a relatively large random sample of the population in order to obtain an accurate estimate of the distribution of this character.*
>
> *The second step is to divide this sample, as in stratified sampling into classes or strata according to the value of the second character and to draw at random from each of the strata, a small sample for the costly intensive interviewing necessary to secure data regarding the first character.*
>
> *An estimate of the first character based on these samples may be more accurate than based on an equally expensive sample drawn at random without stratification. The question is to determine for a given expenditure, the sizes of the initial sample and the subsequent samples which yield the most accurate estimate of the first character".*

Two-phase sampling is cost effective. It is well known that two-phase (or double) sampling is of significant use in practice when the population parameter(s) (say, population mean $\bar{X}$) of the auxiliary variable x is not known. Consider the problem of estimating population mean $\bar{Y}$ of a study variable Y from a finite population of N units. When information on one or more auxiliary variable say, X and Z which are correlated with the variable Y are available or can be obtained, ratio and regression estimator can be used to improve the efficiency. Two-phase sampling is used to obtain information about X and Z cheaply from a bigger sample at first phase and relatively small samples (sub-sample of first phase sample) are drawn at the second phase and information for Y can be obtained from that sample.

In the past various researchers used two-phase sampling to develop different estimators. Sukhatme (1962) used two-phase sampling scheme to develop a general ratio-type estimator. Mohanty (1967) provided an expression to estimate the population mean of study variable by combining the ratio and regression method.

Chand (1975) developed two chain ratio-type estimators for finite population mean by using two-auxiliary variables. Kiregyera (1980) proposed a chain ratio-to-regression type estimator to investigate relative efficiency of sample mean and other estimators. Kiregyera (1984) developed two regression type estimators which are regression-in-regression and ratio-in-regression estimators. Mukherjee et al. (1987) proposed three estimators and extended their results to case when multi-auxiliary information is available.

Sahoo and Sahoo (1993) suggested a regression type estimator by using an additional auxiliary variable for two-phase sampling when population mean of main auxiliary variable was unknown. Sahoo and Sahoo (1993) class of estimators covered a large number of estimators.

Singh and Upadhyaya (1995) considered a generalized estimator to estimate the populations mean using two auxiliary variables in two-phase sampling. Ahmed (1998) suggested a few corrections for the estimators proposed by Mukherjee et al. (1987). Roy (2003) proposed a regression type estimator to estimate population mean. Roy (2003) estimator was more efficient than estimators proposed by Mohanty (1967), Chand (1975), Kiregyera (1980, 84) and Sahoo et al. (1993).

Singh and Espejo (2007) suggested a class of ratio-product estimators in two-phase sampling with its properties and identified asymptotically optimum estimators from proposed class of estimators. Singh and Espejo (2007) also investigated the conditions for the proposed estimator to be more efficient than the two-phase sampling ratio, product and mean per unit estimator.

Samiuddin and Hanif (2007) proposed ratio and regression estimation procedures to estimate the population mean in two-phase sampling using idea of partial and no information cases . Samiuddin and Hanif (2007) estimators were more efficient than estimator proposed by Roy (2003) under some particular situations.

Some more note able references in area of two-phase sampling are,
Srivastava (1970), Rao (1973), Singh et al. (1989), Singh, (1987, 93), Upadhyaya et al. (1992), Sahoo et al. (1993), Armstrong and St-Jean (1993), Sahoo and Sahoo (1994), Ahmad et al. (1994), Singh and Biradar (1994), Dorfman (1994), Prasad et al. (1996), Singh et al. (1996), Hidirogou and Sarndal (1998), Tracy and Singh (1999a, 1999b), Singh (2001), Upadhyaya and Singh (2001), Singh et al. (2001), Singh and Singh (2001), Radhey et al. (2002), Chandra and Singh (2003), Diana and Tommasi (2003, 04), Singh et al. (2004), Pradhan (2005), Samiuddin and Hanif (2007),Singh et al. (2006), Upadhyaya et al. (2006), Diana et al. (2007), Tahir (2008), Fuller and Legg (2008), Kamal et al. (2009), Hanif et al. (2009a, 2009b),Ahmad et al. (2009a, 2009b, 2009c).

2.2- SHRINKAGE ESTIMATOR

The shrinkage estimator is a statistical tradeoff between the bias and the estimation error. In Shrinkage estimation a naive or raw estimator is improved by combining it with other information. A certain parameter is introduced with any conventional estimator to have its shrinkage version and value for this parameter can be obtained as that minimizing the mean square of the new estimator. One general result of shrinkage estimation is that many standard estimators can be improved in term of mean square error by shrinking their mean square error towards zero.
For the first time the shrinkage estimator appeared in Stein (1956), where it was shown that “shrinking” sample means of multivariate normal distribution to an appropriate common constant improves estimation accuracy. In general, the shrinkage estimate is obtained via a “shrinkage” of sample unbiased estimate towards some biased target with lower estimation error. Shahbaz and Hanif (2009) proposed an easy way to determine mean square error for shrinkage version of any estimator.

References in area of shrinkage estimation are,
James and Stein (1961), Stien (1962), Theil (1963), Thompson (1968), Metha and Srinivasan (1971), Pandey (1979), Copas (1983, 97), Stigler (1990), George (1991), Krishnamoorthy (1992), Sarkar (1994), Kubokawa (1994), Ahmed and Rohatgi (1996), Khan and Saleh (1997)

CHAPTER 3

SOME PREVIOUS ESTIMATORS USING SINGLE AUXILIARY ATTRIBUTES

3.1 INTRODUCTION

In this chapter some existing estimators, which utilizes auxiliary information, has been described.

3.2 ESTIMATORS BY NAIK AND GUPTA (1996)

Naik and Gupta (1996) proposed ratio, product and regression estimators for single phase sampling.
Ratio estimator proposed by Naik and Gupta (1996) is:

$$t_{1(1)} = \bar{y}\frac{P_1}{p_1}. \tag{3.2.1}$$

The mean square error of $t_{1(1)}$ is:

$$MSE\left(t_{1(1)}\right) = \theta\bar{Y}^2\left[C_y^2 + C_{\tau_1}^2 - 2\rho_{Pb_1}C_yC_{\tau_1}\right]. \tag{3.2.2}$$

Product estimator proposed by Naik and Gupta (1996) is:

$$t_{2(1)} = \bar{y}\frac{p_1}{P_1}. \tag{3.2.3}$$

The mean square error of $t_{2(1)}$ is:

$$MSE\left(t_{2(1)}\right) = \theta\bar{Y}^2\left[C_y^2 + C_{\tau_1}^2 + 2\rho_{Pb_1}C_yC_{\tau_1}\right]. \tag{3.2.4}$$

Regression estimator proposed by Naik and Gupta (1996) is:

$$t_{3(1)} = \bar{y} + b\left(p_1 - P_1\right). \tag{3.2.5}$$

The mean square error of $t_{3(1)}$ is:

$$MSE\left(t_{3(1)}\right) = \theta\left(1 - \rho_{Pb_1}^2\right)\bar{Y}^2C_y^2. \tag{3.2.6}$$

3.3 FAMILY OF ESTIMATORS BY JHAJJ et al. (2006)

Jhajj et al. (2006) has proposed the following family of estimators, using information of single auxiliary attribute, for single-phase sampling.

$$\hat{T}_{4(1)} = g_\omega\left(\bar{y}, v_1\right), \text{ for } v_1 = \frac{p_1}{P_1} > 0; \tag{3.3.1}$$

where $g_\omega\left(\bar{y}, v_1\right)$ is a parametric function of $\bar{y}$ and v_1 such that

$g_\omega(\bar{Y},1)=\bar{Y},$ for all $\bar{Y}$ and satisfy following conditions.

i) What ever sample be chosen, the point $(\bar{y},v_1)$ assumes value in a bounded closed convex subset R_2 of two-dimensional real space containing the point $(\bar{Y},1)$.

ii) The function $g_\omega(\bar{y},v_1)$ is continuous and bounded in R_2.

iii) The first and second order partial derivatives of $g_\omega(\bar{y},v_1)$ exist and are continuous as well as bounded in R_2.

Some functions given by Jhajj et al. (2006) that can be used as members of family given in (3.3.1) are;

i) $g_\omega(\bar{y},v_1)=\bar{y}\,v_1{}^{\alpha},$

ii) $g_\omega(\bar{y},v_1)=\bar{y}\,e^{\alpha(v_1-1)},$

iii) $g_\omega(\bar{y},v_1)=\bar{y}+\alpha(v_1-1),$

Some more functions that can be members of the above mentioned family are;

iv) $g_\omega(\bar{y},v_1)=\bar{y}\;v_1{}^{\alpha/2},$

v) $g_\omega(\bar{y},v_1)=\bar{y}\;v_1{}^{\alpha}e^{\alpha(v_1-1)},$

vi) $g_\omega(\bar{y},v_1)=\bar{y}\left[k\,v_1{}^{\alpha}+(1-k)\,e^{\alpha(v_1-1)}\right],$

where $0<k<1$, and α is unknown parameter.

The mean square error of the family given in (3.3.1) to the terms of order $1/n$ is:

$$MSE\left(\hat{T}_{4(1)}\right)\approx\theta\left(1-\rho^2_{Pb_1}\right)\bar{Y}^2C_y^2. \tag{3.3.2}$$

Jhajj et al. (2006) proposed following family of estimators, using information of single auxiliary attribute, for two-phase sampling.

$$\hat{T}_{5(2)}=g_\omega(\bar{y}_2,v_{1d}), \tag{3.3.3}$$

where $v_{1d}=\dfrac{p_{1(2)}}{p_{1(1)}}$, and $g_\omega(\bar{Y},1)=\bar{Y}.$

Following are some member estimators of the family given in (3.3.3).

i) $g_{\omega}(\bar{y}_2, v_{1d}) = \bar{y}_2 (v_{1d})^{\alpha}$,

ii) $g_{\omega}(\bar{y}_2, v_{1d}) = \bar{y}_2 + \alpha(v_{1d} - 1)$,

iii) $g_{\omega}(\bar{y}_2, v_{1d}) = \bar{y}_2 + e^{\alpha(v_{1d}-1)}$,

iv) $g_{\omega}(\bar{y}_2, v_{1d}) = \bar{y}_2 + e^{\alpha(v_{1d}-1)} v_{1d}^{\alpha}$,

where α is unknown parameter. The mean square error of each estimator to the terms of order $1/n$ of this family (3.3.3) is:

$$MSE(\hat{T}_{5(2)}) \approx (\theta_2 - \theta_3 \rho_{Pb_1}^2) \bar{Y}^2 C_y^2. \quad (3.3.4)$$

3. 4 ESTIMATORS BY SHABBIR AND GUPTA (2007)

Shabbir and Gupta (2007) proposed following estimator for single phase sampling,

$$t_{6(1)} = [d_1 \bar{y} + d_2 (P_1 - p_1)] \frac{P_1}{p_1}, \text{ for } p_1 > 0 \quad (3.4.1)$$

where d_1 and d_2 are unknown parameters.

The mean square error of $t_{6(1)}$ to the terms of order $1/n^2$ is:

$$MSE(t_{6(1)}) \approx \frac{\theta(1 - \rho_{Pb_1}^2) \bar{Y}^2 C_y^2}{1 + \theta(1 - \rho_{Pb_1}^2) C_y^2}. \quad (3.4.2)$$

Shabbir and Gupta (2007) proposed following estimator for two phase sampling,

$$t_{7(2)} = \left[W_1 \bar{y}_2 + W_2 (p_{1(1)} - p_{1(2)}) \right] \frac{p_{1(1)}}{p_{1(2)}}, \text{ for } p_{1(2)} > 0 \quad (3.4.3)$$

where W_1 and W_2 are unknown quantities to be determined.

The mean square error of $t_{7(2)}$ to the terms of order $1/n^2$ is:

$$MSE(t_{7(2)}) \approx \frac{(\theta_2 - \theta_3 \rho_{Pb_1}^2) \bar{Y}^2 C_y^2}{1 + (\theta_2 - \theta_3 \rho_{Pb_1}^2) C_y^2}. \quad (3.4.4)$$

3.5- GENERAL SHRINKAGE ESTIMATOR BY SHAHBAZ AND HANIF (2009)

In this section a general shrinkage estimator proposed by Shahbaz and Hanif (2009), is discussed. The shrinkage estimator alongside its mean square error is given in the following theorem.

THEOREM: If $\hat{t}$ is any available estimator of parameter "T" then shrinkage estimator of $\hat{t}$ is given as,

$$\hat{t}_s = \frac{\hat{t}}{1 + T^{-2}MSE(\hat{t})}. \tag{3.5.1}$$

The minimum mean square error of $\hat{t}_s$ is,

$$MSE\left(\hat{t}_s\right) = \frac{MSE(\hat{t})}{1 + T^{-2}MSE(\hat{t})}. \tag{3.5.2}$$

CHAPTER 4
GENERALIZATION OF A FAMILY OF ESTIMATORS FOR FULL PARTIAL AND NO INFORMATION CASES USING MULTI- AUXILIARY ATTRIBUTES

4.1 INTRODUCTION

In this chapter a generalized family of estimators based on the information of "k" auxiliary attributes has been proposed. Three different cases have been discussed that include the full, partial and no information cases. Shrinkage version of proposed generalized family of estimators for same cases is also discussed.

4.2- A GENERALIZED FAMILY OF ESTIMATORS FOR SINGLE-PHASE SAMPLING

A generalized family of estimator by using "k" auxiliary attributes for full information case is:

$$\hat{T}_{8(1)} = \left[g_{\omega}\left(\bar{y}, v_1, v_2, ..., v_k\right) \right]; \tag{4.2.1}$$

where $v_j = p_j / P_j, v_j > 0$ and p_j is the sample proportion of jth attribute. Also $g_{\omega}\left(\bar{y}, v_1, v_2, ..., v_k\right)$ is the parametric function such that $g_{\omega}\left(\bar{Y}, 1, 1, ..., 1\right) = \bar{Y}$ and the point $\left(\bar{y}, v_1, v_2, ..., v_k\right)$ are to be in a bounded set in R_k containing a point$\left(\bar{Y}, 1, 1, ..., 1\right)$.Some proposed estimators under above condition are:

(i) $$t_{8(1)} = \bar{y} + \sum_{j=1}^{k} \alpha_j \left(v_j - 1\right), \tag{4.2.2}$$

(ii) $$t_{9(1)} = \bar{y} e^{\sum_{j=1}^{k} \alpha_j \left(v_j - 1\right)}, \tag{4.2.3}$$

(iii) $$t_{10(1)} = \bar{y}\left(v_1\right)^{\alpha_1}\left(v_2\right)^{\alpha_2} ... \left(v_k\right)^{\alpha_k}, \tag{4.2.4}$$

4.2.1- Mean Square Error of $t_{8(1)}$

Consider the estimator defined in (4.2.2),

$$t_{8(1)} = \bar{y} + \sum_{j=1}^{k} \alpha_j \left(v_j - 1\right). \tag{4.2.2}$$

Changing (4.2.2) in vector notation,

$$t_{8(1)} = \bar{y} + \underline{\alpha}^{/}\left(\underline{v} - \underline{1}\right) = \bar{y} + \underline{\alpha}^{/}\underline{\varphi}; \tag{4.2.5}$$

where $\underline{\alpha}^{/}, \underline{\varphi}, \left(\underline{v} - \underline{1}\right)$ are defined in section (1.5).

Using (1.5.3) in (4.2.5) and on simplification:

$$\left(t_{8(1)} - \bar{Y}\right) = \bar{e}_y + \underline{\alpha}^{/}\underline{\varphi}.$$

The mean square error of $t_{8(1)}$ is:

$$MSE\left(t_{8(1)}\right) = E\left[\bar{e}_y^2 + \underline{\alpha}^{/}\underline{\varphi}\underline{\varphi}^{/}\underline{\alpha} + 2\underline{\alpha}^{/}\underline{\varphi}\bar{e}_y\right]. \tag{4.2.6}$$

Using (1.5.4) and (1.5.16) in (4.2.6) and on simplification the mean square error of $t_{8(1)}$ is:

$$MSE\left(t_{8(1)}\right) = \theta\left[\bar{Y}^2 C_y^2 + \underline{\alpha}^{/}\Phi_\tau\underline{\alpha} + 2\underline{\alpha}^{/}\underline{\phi}_{y\tau}\right]. \tag{4.2.7}$$

Differentiating (4.2.7) with respect to $\underline{\alpha}$ and equating the derivative to zero,

$$2\Phi_\tau\underline{\alpha} + 2\underline{\phi}_{y\tau} = 0.$$

On simplification:

$$\underline{\alpha} = -\Phi_\tau^{-1}\underline{\phi}_{y\tau}. \tag{4.2.8}$$

Using (4.2.8) in (4.2.7) and on simplification:

$$MSE\left(t_{8(1)}\right) = \theta\left[\bar{Y}^2 C_y^2 + \underline{\phi}_{y\tau}^{/}\Phi_\tau^{-1}\Phi_\tau\Phi_\tau^{-1}\underline{\phi}_{y\tau} - 2\underline{\phi}_{y\tau}^{/}\Phi_\tau^{-1}\underline{\phi}_{y\tau}\right] \text{ Or}$$

$$MSE\left(t_{8(1)}\right) = \theta\left[\bar{Y}^2 C_y^2 + \underline{\phi}_{y\tau}^{/}\Phi_\tau^{-1}\underline{\phi}_{y\tau} - 2\underline{\phi}_{y\tau}^{/}\Phi_\tau^{-1}\underline{\phi}_{y\tau}\right]. \tag{4.2.9}$$

Using (1.5.21) in (4.2.9),

$$MSE\left(t_{8(1)}\right) = \theta\left[\bar{Y}^2 C_y^2 + \left(\rho_{y.\tau_1\tau_2\ldots\tau_k}^2\right)\bar{Y}^2 C_y^2 - 2\left(\rho_{y.\tau_1\tau_2\ldots\tau_k}^2\right)\bar{Y}^2 C_y^2\right].$$

On simplification:

$$MSE\left(t_{8(1)}\right) = \left[\theta\left(1 - \rho_{y.\tau_1\tau_2\ldots\tau_k}^2\right)\right]\bar{Y}^2 C_y^2. \tag{4.2.10}$$

4.2.2- Mean Square Error of $t_{9(1)}$

Consider the estimator defined in (4.2.3),

$$t_{9(1)} = \bar{y}e^{\sum_{j=1}^{k}\alpha_j\left(v_j - 1\right)}. \tag{4.2.3}$$

Expanding (4.2.3) and retaining linear terms only:

$$t_{9(1)} \approx \bar{y}\left(1 + \sum_{j=1}^{k}\alpha_j\left(v_j - 1\right)\right). \tag{4.2.11}$$

Changing (4.2.11) into vector notation,

$$t_{9(1)} \approx \bar{y}\left(1 + \underline{\alpha}^{/}\left(\underline{v} - 1\right)\right) = \bar{y}\left(1 + \underline{\alpha}^{/}\underline{\varphi}\right). \tag{4.2.12}$$

Using (1.5.3) in (4.2.12) and on simplification:

$\left(t_{9(1)} - \bar{Y}\right) \approx \bar{e}_y + \bar{Y}\underline{\alpha}'\underline{\varphi}.$

The mean square error of $t_{9(1)}$ is:

$$\text{MSE}\left(t_{9(1)}\right) \approx E\left[\bar{e}_y^2 + \bar{Y}^2\underline{\alpha}'\underline{\varphi}\underline{\varphi}'\underline{\alpha} + 2\bar{Y}\underline{\alpha}'\underline{\varphi}\bar{e}_y\right]. \tag{4.2.13}$$

Using (1.5.4) and (1.5.16) in (4.2.13) and on simplification the mean square error of $t_{9(1)}$ is:

$$\text{MSE}\left(t_{9(1)}\right) \approx \theta\left[\bar{Y}^2C_y^2 + \bar{Y}^2\underline{\alpha}'\Phi_\tau\underline{\alpha} + 2\bar{Y}\underline{\alpha}'\underline{\phi}_{y\tau}\right]. \tag{4.2.14}$$

Differentiating (4.2.14) with respect to $\underline{\alpha}$ and equating the derivative to zero,

$2\bar{Y}^2\Phi_\tau\underline{\alpha} + 2\bar{Y}\underline{\phi}_{y\tau} = 0.$

On simplification:

$$\underline{\alpha} = -\frac{1}{\bar{Y}}\Phi_\tau^{-1}\underline{\phi}_{y\tau}. \tag{4.2.15}$$

Using (4.2.15) in (4.2.14) and on simplification:

$$\text{MSE}\left(t_{9(1)}\right) = \theta\left(1 - \rho^2_{y.\tau_1\tau_2\ldots\tau_k}\right)\bar{Y}^2C_y^2. \tag{4.2.16}$$

4.2.3- Mean Square Error of $t_{10(1)}$

Consider the estimator defined in (4.2.4),

$$t_{10(1)} = \bar{y}\left(v_1\right)^{\alpha_1}\left(v_2\right)^{\alpha_2}\ldots\left(v_k\right)^{\alpha_k}. \tag{4.2.4}$$

Expanding (4.2.4) and retaining linear terms only and on simplification:

$t_{10(1)} \approx \bar{y}\left(1 + \sum_{j=1}^{k}\alpha_j\left(v_j - 1\right)\right).$

This is (4.2.11); so

$$\text{MSE}\left(t_{10(1)}\right) = \text{MSE}\left(t_{9(1)}\right) \approx \left[\theta\left(1 - \rho^2_{y.\tau_1\tau_2\ldots\tau_k}\right)\right]\bar{Y}^2C_y^2. \tag{4.2.17}$$

The minimized mean square error of each of three considered estimators of proposed general family $\hat{T}_{8(1)}$ is:

$$\text{MSE}\left(\hat{T}_{8(1)}\right) \approx \theta\left(1 - \rho^2_{y.\tau_1\tau_2\ldots\tau_k}\right)\bar{Y}^2C_y^2. \tag{4.2.18}$$

Expression (4.2.18) is not general mean square error expression for general family defined in (4.2.1) but taken as common mean square error for considered estimators. The proposed general family of estimators $\hat{T}_{8(1)}$ is very large any function which satisfies given conditions is member of proposed family. Similar are remarks for upcoming families of estimators.

4.3- A GENERALIZED FAMILY OF ESTIMATORS FOR TWO-PHASE SAMPLING

In this section some generalized families of estimators for two phase sampling have been proposed.

4.3.1 - A Generalized Family of Estimators for Partial Information Case

A generalized family of estimators for partial information case by using "k" auxiliary attributes, with "m" known and "(k-m)"unknown, is:

$$\hat{T}_{11(2)} = g_{\omega}\left(\bar{y}_2, v_1, v_2 .....v_m, v_{(m+1)}, v_{(m+2)},.....v_k\right), \qquad (4.3.1)$$

where $v_j = p_{j(1)} / P_j ,(j = 1,2...m); v_j > 0$

and $v_j = p_{j(2)} / p_{j(1)}$, for $(j = m+1, m+2...k)$.

The function $g_{\omega}\left(\bar{y}_2, v_1, v_2 .....v_m, v_{(m+1)}, v_{(m+2)},.....v_k\right)$ satisfies all the conditions stated for (4.2.1). Some proposed estimators under above condition are:

(i) $$t_{11(2)} = \bar{y}_2 + \sum_{j=1}^{m} \alpha_j\left(v_j - 1\right) + \sum_{j=m+1}^{k} \alpha_j\left(v_j - 1\right), \qquad (4.3.2)$$

(ii) $$t_{12(2)} = \bar{y}_2 e^{\sum_{j=1}^{m}\alpha_j\left(v_j-1\right)+\sum_{j=m+1}^{k}\alpha_j\left(v_j-1\right)}, \qquad (4.3.3)$$

(iii) $$t_{13(2)} = \bar{y}_2 \left(v_1\right)^{\alpha_1}\left(v_2\right)^{\alpha_2}...\left(v_m\right)^{\alpha_m}\left(v_{(m+1)}\right)^{\alpha_{m+1}}\left(v_{(m+2)}\right)^{\alpha_{m+2}}...\left(v_k\right)^{\alpha_k}, \qquad (4.3.4)$$

4.3.1.1 Mean Square Error of $t_{11(2)}$

Consider the estimator defined in (4.3.2),

$$t_{11(2)} = \bar{y}_2 + \sum_{j=1}^{m} \alpha_j\left(v_j - 1\right) + \sum_{j=m+1}^{k} \alpha_j\left(v_j - 1\right). \qquad (4.3.2)$$

Changing (4.3.2) into vector notation,

$$t_{11(2)} = \bar{y}_2 + \underline{\alpha}_1^{/}\left(\underline{v}_1 - 1\right) + \underline{\alpha}_2^{/}\left(\underline{v}_2 - 1\right) ; \qquad (4.3.5)$$

where $\underline{\alpha}_1^{/}, \underline{\alpha}_2^{/}$,$\left(\underline{v}_1 - \underline{1}\right),\left(\underline{v}_2 - \underline{1}\right)$ are defined in section (1.5).

Using (1.5.8) in (4.3.5) and on simplification:

$$\left(t_{11(2)} - \bar{Y}\right) = \bar{e}_{y_2} + \underline{\alpha}_1^{/}\underline{\varphi}_1 + \underline{\alpha}_2^{/}\underline{\varphi}_2;$$

where $\underline{\varphi}_1, \underline{\varphi}_2$ are defined in section (1.5).

The mean square error of $t_{11(2)}$ is:

$$\text{MSE}\left(t_{11(2)}\right)=E\begin{bmatrix}\bar{e}_{y_2}^2+\underline{\alpha}_1^{/}\underline{\varphi}_1\underline{\varphi}_1^{/}\underline{\alpha}_1+\underline{\alpha}_2^{/}\underline{\varphi}_2\underline{\varphi}_2^{/}\underline{\alpha}_2+2\underline{\alpha}_1^{/}\underline{\varphi}_1\bar{e}_{y_2}\\+2\underline{\alpha}_2^{/}\underline{\varphi}_2\bar{e}_{y_2}+\underline{\alpha}_1^{/}\underline{\varphi}_1\underline{\varphi}_2^{/}\underline{\alpha}_2+\underline{\alpha}_2^{/}\underline{\varphi}_2\underline{\varphi}_1^{/}\underline{\alpha}_1\end{bmatrix}. \quad (4.3.6)$$

Using (1.5.12), (1.5.18), (1.5.19) and (1.5.20) in (4.3.6) and on simplification the mean square error of $t_{11(2)}$ is:

$$\text{MSE}\left(t_{11(2)}\right)=\theta_2\bar{Y}^2C_y^2+\theta_1\left\{\underline{\alpha}_1^{/}\Phi_{\tau_1}\underline{\alpha}_1+2\underline{\alpha}_1^{/}\underline{\phi}_{y\tau_1}\right\}+\left(\theta_2-\theta_1\right)\left\{\underline{\alpha}_2^{/}\Phi_{\tau_2}\underline{\alpha}_2+2\underline{\alpha}_2^{/}\underline{\phi}_{y\tau_2}\right\}. \quad (4.3.7)$$

Differentiating (4.3.7) with respect to $\underline{\alpha}_1$ and $\underline{\alpha}_2$ and equating the derivative to zero,

$2\Phi_{\tau_1}\underline{\alpha}_1+2\underline{\phi}_{y\tau_1}=0$ and $2\Phi_{\tau_2}\underline{\alpha}_2+2\underline{\phi}_{y\tau_2}=0.$

From above two equations $\underline{\alpha}_1$ and $\underline{\alpha}_2$ are:

$$\underline{\alpha}_1=-\Phi_{\tau_1}^{-1}\underline{\phi}_{y\tau_1}, \quad (4.3.8)$$

$$\underline{\alpha}_2=-\Phi_{\tau_2}^{-1}\underline{\phi}_{y\tau_2}. \quad (4.3.9)$$

Using (4.3.8) and (4.3.9) in (4.3.7):

$$\text{MSE}\left(t_{11(2)}\right)=\begin{bmatrix}\theta_2\bar{Y}^2C_y^2+\theta_1\left\{\underline{\phi}_{y\tau_1}^{/}\Phi_{\tau_1}^{-1}\Phi_{\tau_1}\Phi_{\tau_1}^{-1}\underline{\phi}_{y\tau_1}-2\underline{\phi}_{y\tau_1}^{/}\Phi_{\tau_1}^{-1}\underline{\phi}_{y\tau_1}\right\}\\+\left(\theta_2-\theta_1\right)\left\{\underline{\phi}_{y\tau_2}^{/}\Phi_{\tau_2}^{-1}\Phi_{\tau_2}\Phi_{\tau_2}^{-1}\underline{\phi}_{y\tau_2}-2\underline{\phi}_{y\tau_2}^{/}\Phi_{\tau_2}^{-1}\underline{\phi}_{y\tau_2}\right\}\end{bmatrix}. \text{ Or}$$

$$\text{MSE}\left(t_{11(2)}\right)=\begin{bmatrix}\theta_2\bar{Y}^2C_y^2+\theta_1\left\{\underline{\phi}_{y\tau_1}^{/}\Phi_{\tau_1}^{-1}\underline{\phi}_{y\tau_1}-2\underline{\phi}_{y\tau_1}^{/}\Phi_{\tau_1}^{-1}\underline{\phi}_{y\tau_1}\right\}\\+\left(\theta_2-\theta_1\right)\left\{\underline{\phi}_{y\tau_2}^{/}\Phi_{\tau_2}^{-1}\underline{\phi}_{y\tau_2}-2\underline{\phi}_{y\tau_2}^{/}\Phi_{\tau_2}^{-1}\underline{\phi}_{y\tau_2}\right\}\end{bmatrix}. \quad (4.3.10)$$

Using (1.5.22) in (4.3.10) the mean square error of $t_{11(2)}$ is:

$$\text{MSE}\left(t_{11(2)}\right)=\begin{bmatrix}\theta_2\bar{Y}^2C_y^2+\theta_1\left\{\left(\rho_{y.\tau_1\tau_2...\tau_m}^2\right)\bar{Y}^2C_y^2-2\left(\rho_{y.\tau_1\tau_2...\tau_m}^2\right)\bar{Y}^2C_y^2\right\}\\+\left(\theta_2-\theta_1\right)\left\{\left(\rho_{y.\tau_{m+1}\tau_{m+2}...\tau_k}^2\right)\bar{Y}^2C_y^2-2\left(\rho_{y.\tau_{m+1}\tau_{m+2}...\tau_k}^2\right)\bar{Y}^2C_y^2\right\}\end{bmatrix}.$$

On simplification:

$$\text{MSE}\left(t_{11(2)}\right)=\left[\theta_2\left\{1-\rho_{y.\tau_{m+1}\tau_{m+2}...\tau_k}^2\right\}+\theta_1\left\{\rho_{y.\tau_{m+1}\tau_{m+2}...\tau_k}^2-\rho_{y.\tau_1\tau_2...\tau_m}^2\right\}\right]\bar{Y}^2C_y^2. \quad (4.3.11)$$

4.3.1.2- Mean Square Error of $t_{12(2)}$

Consider the estimator defined in (4.3.3),

$$t_{12(2)}=\bar{y}_2e^{\sum_{j=1}^{m}\alpha_j\left(v_j-1\right)+\sum_{j=m+1}^{k}\alpha_j\left(v_j-1\right)}. \quad (4.3.3)$$

Expanding (4.3.3) and retaining linear terms only and on simplification:

$$t_{12(2)} \approx \bar{y}_2\left(1+\sum_{j=1}^{m}\alpha_j\left(v_j-1\right)+\sum_{j=m+1}^{k}\alpha_j\left(v_{jd}-1\right)\right). \quad (4.3.12)$$

Changing (4.3.12) into vector notation,

$$t_{12(2)} \approx \bar{y}_2\left(1+\underline{\alpha}_1\left(\underline{v}_1-1\right)+\underline{\alpha}_2\left(\underline{v}_2-1\right)\right)=\bar{y}_2\left(1+\underline{\alpha}_1^{/}\underline{\varphi}_1+\underline{\alpha}_2^{/}\underline{\varphi}_2\right). \quad (4.3.13)$$

Using (1.5.8) in (4.3.13) and on simplification:

$$\left(t_{12(2)}-\bar{Y}\right)=\bar{e}_{y_2}+\bar{Y}\underline{\alpha}_1^{/}\underline{\varphi}_1+\bar{Y}\underline{\alpha}_2^{/}\underline{\varphi}_2.$$

The mean square error of $t_{12(2)}$ is:

$$MSE\left(t_{12(2)}\right)\approx E\begin{bmatrix}\bar{e}_{y_2}^2+\bar{Y}^2\underline{\alpha}_1^{/}\underline{\varphi}_1\underline{\varphi}_1^{/}\underline{\alpha}_1+\bar{Y}^2\underline{\alpha}_2^{/}\underline{\varphi}_2\underline{\varphi}_2^{/}\underline{\alpha}_2+2\bar{Y}\underline{\alpha}_1^{/}\underline{\varphi}_1\bar{e}_{y_2}\\ +2\bar{Y}\underline{\alpha}_2^{/}\underline{\varphi}_2\bar{e}_{y_2}+\bar{Y}^2\underline{\alpha}_1^{/}\underline{\varphi}_1\underline{\varphi}_2^{/}\underline{\alpha}_2+\bar{Y}^2\underline{\alpha}_2^{/}\underline{\varphi}_2\underline{\varphi}_1^{/}\underline{\alpha}_1\end{bmatrix}. \quad (4.3.14)$$

Using (1.5.12), (1.5.18), (1.5.19) and (1.5.20) in (4.3.14) and on simplification the mean square error of $t_{12(2)}$ is:

$$MSE\left(t_{12(2)}\right)\approx\theta_2\bar{Y}^2C_y^2+\theta_1\left\{\bar{Y}^2\underline{\alpha}_1^{/}\Phi_{\tau_1}\underline{\alpha}_1+2\bar{Y}\underline{\alpha}_1^{/}\underline{\phi}_{y\tau_1}\right\}$$
$$+\left(\theta_2-\theta_1\right)\left\{\bar{Y}^2\underline{\alpha}_2^{/}\Phi_{\tau_2}\underline{\alpha}_2+2\bar{Y}\underline{\alpha}_2^{/}\underline{\phi}_{y\tau_2}\right\}. \quad (4.3.15)$$

Differentiating (4.3.15) with respect to $\underline{\alpha}_1$ and $\underline{\alpha}_2$ and equating the derivative to zero:

$$2\bar{Y}^2\Phi_{\tau_1}\underline{\alpha}_1+2\bar{Y}\underline{\phi}_{y\tau_1}=0,\ \text{and}\ 2\bar{Y}^2\Phi_{\tau_2}\underline{\alpha}_2+2\bar{Y}\underline{\phi}_{y\tau_2}=0.$$

From above two equations $\underline{\alpha}_1$ and $\underline{\alpha}_2$ are:

$$\underline{\alpha}_1=\frac{-1}{\bar{Y}}\Phi_{\tau_1}^{-1}\underline{\phi}_{y\tau_1}. \quad (4.3.16)$$

$$\underline{\alpha}_2=\frac{-1}{\bar{Y}}\Phi_{\tau_2}^{-1}\underline{\phi}_{y\tau_2}. \quad (4.3.17)$$

Using (4.3.16) and (4.3.17) in (4.3.15) and on simplification:

$$MSE\left(t_{12(2)}\right)=\left[\theta_2\left\{1-\rho_{y.\tau_{m+1}\tau_{m+2}\ldots\tau_k}^2\right\}+\theta_1\left\{\rho_{y.\tau_{m+1}\tau_{m+2}\ldots\tau_k}^2-\rho_{y.\tau_1\tau_2\ldots\tau_m}^2\right\}\right]\bar{Y}^2C_y^2. \quad (4.3.18)$$

4.3.1.3- Mean Square Error of $t_{13(2)}$

Consider the estimator defined in (4.3.4),

$$t_{13(2)}=\bar{y}_2\left(v_1\right)^{\alpha_1}\left(v_2\right)^{\alpha_2}\ldots\left(v_m\right)^{\alpha_m}\left(v_{(m+1)}\right)^{\alpha_{m+1}}\left(v_{(m+2)}\right)^{\alpha_{m+2}}\ldots\left(v_k\right)^{\alpha_k}. \quad (4.3.4)$$

Expanding (4.3.4) and retaining linear terms only and on simplification:

$$t_{13(2)} \approx \bar{y}_2\left(1+\sum_{j=1}^{m}\alpha_j\left(v_j-1\right)+\sum_{j=m+1}^{k}\alpha_j\left(v_{jd}-1\right)\right).$$

This is (4.3.13); so

$$\text{MSE}\left(t_{13(2)}\right)=\text{MSE}\left(t_{12(2)}\right)\approx$$
$$\left[\theta_2\left\{1-\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}\right\}+\theta_1\left\{\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}-\rho^2_{y.\tau_1\tau_2...\tau_m}\right\}\right]\bar{Y}^2C^2_y. \quad (4.3.19)$$

The minimized mean square error of each of three considered estimators of proposed general family $\hat{T}_{11(2)}$ is:

$$\text{MSE}\left(\hat{T}_{11(2)}\right)\approx\left[\theta_2\left\{1-\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}\right\}+\theta_1\left\{\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}-\rho^2_{y.\tau_1\tau_2...\tau_m}\right\}\right]\bar{Y}^2C^2_y. \quad (4.3.20)$$

4.3.2- A Generalized Family of Estimators for No Information Case

A generalized family of estimators by using "k" auxiliary attributes for no information case is:

$$\hat{T}_{14(2)}=g_{\omega}\left(\bar{y}_2,v_{1d},v_{2d},...,v_{kd}\right); \quad (4.3.21)$$

where $v_{jd}=p_{j(2)}/p_{j(1)}$ and $v_{jd}>0$. The function $g_{\omega}\left(\bar{y}_2,v_{1d},v_{2d},...,v_{kd}\right)$ satisfies all the conditions stated for (4.2.1). Some proposed estimators are:

(i) $$t_{14(2)}=\bar{y}_2+\sum_{j=1}^{k}\alpha_j\left(v_{jd}-1\right), \quad (4.3.22)$$

(ii) $$t_{15(2)}=\bar{y}_2e^{\sum_{j=1}^{k}\alpha_j\left(v_{jd}-1\right)}, \quad (4.3.23)$$

(iii) $$t_{16(2)}=\bar{y}\left(v_{1d}\right)^{\alpha_1}\left(v_{2d}\right)^{\alpha_2}...\left(v_{kd}\right)^{\alpha_k}, \quad (4.3.24)$$

4.3.2.1- Mean Square Error of $t_{14(2)}$

Consider the estimator defined in (4.3.22),

$$t_{14(2)}=\bar{y}_2+\sum_{j=1}^{k}\alpha_j\left(v_{jd}-1\right). \quad (4.3.22)$$

Changing (4.3.22) in vector notation,

$$t_{14(2)}=\bar{y}_2+\underline{\alpha}'\left(\underline{v}_d-\underline{1}\right)=\bar{y}_2+\underline{\alpha}'\underline{\varphi}_d; \quad (4.3.25)$$

where $\underline{\varphi}_d$,$\left(\underline{v}_d-\underline{1}\right)$ are defined in section (1.5).

Using (1.5.8) in (4.3.25) and on simplification:

$$\left(t_{14(2)}-\bar{Y}\right)=\bar{e}_{y_2}+\underline{\alpha}'\underline{\varphi}_d.$$

The mean square error of $t_{14(2)}$ is:

$$MSE\left(t_{14(2)}\right)=E\left[\bar{e}_{y_2}^2+\underline{\alpha}^{/}\underline{\varphi}_d\underline{\varphi}_d^{/}\underline{\alpha}+2\underline{\alpha}^{/}\underline{\varphi}_d\bar{e}_{y_2}\right]. \tag{4.3.26}$$

Using (1.5.12) and (1.5.17) in (4.3.26) and on simplification the mean square error of $t_{14(2)}$ is:

$$MSE\left(t_{14(2)}\right)=\theta_2\bar{Y}^2C_y^2+\left(\theta_2-\theta_1\right)\left\{\underline{\alpha}^{/}\Phi_\tau\underline{\alpha}+2\underline{\alpha}^{/}\underline{\phi}_{y\tau}\right\}. \tag{4.3.27}$$

The optimum value of $\underline{\alpha}$ is same as derived for full information case in (4.2.8).Using (4.2.8) in (4.3.27):

$$MSE\left(t_{14(2)}\right)=\left[\theta_2\bar{Y}^2C_y^2+\left(\theta_2-\theta_1\right)\left\{\underline{\phi}_{y\tau}^{/}\Phi_\tau^{-1}\Phi_\tau\Phi_\tau^{-1}\underline{\phi}_{y\tau}-2\underline{\phi}_{y\tau}^{/}\Phi_\tau^{-1}\underline{\phi}_{y\tau}\right\}\right].\text{Or}$$

$$MSE\left(t_{14(2)}\right)=\left[\theta_2\bar{Y}^2C_y^2+\left(\theta_2-\theta_1\right)\left\{\underline{\phi}_{y\tau}^{/}\Phi_\tau^{-1}\underline{\phi}_{y\tau}-2\underline{\phi}_{y\tau}^{/}\Phi_\tau^{-1}\underline{\phi}_{y\tau}\right\}\right]. \tag{4.3.28}$$

Using (1.5.21) in (4.3.28) and on simplification the mean square error of $t_{14(2)}$ is:

$$MSE\left(t_{14(2)}\right)=\left\{\theta_2\left(1-\rho_{y.\tau_1\tau_2\ldots\tau_k}^2\right)+\theta_1\rho_{y.\tau_1\tau_2\ldots\tau_k}^2\right\}\bar{Y}^2C_y^2. \tag{4.3.29}$$

4.3.2.2- Mean Square Error of $t_{15(2)}$

Consider the estimator defined in (4.3.23),

$$t_{15(2)}=\bar{y}_2 e^{\sum_{j=1}^{k}\alpha_j\left(v_{jd}-1\right)}. \tag{4.3.23}$$

Expanding (4.3.23) and retaining linear terms only:

$$t_{15(2)}\approx\bar{y}_2\left(1+\sum_{j=1}^{k}\alpha_j\left(v_{jd}-1\right)\right). \tag{4.3.30}$$

Changing (4.3.30) into vector notation,

$$t_{15(2)}\approx\bar{y}_2\left(1+\underline{\alpha}^{/}\left(\underline{v}_d-1\right)\right)=\bar{y}_2\left(1+\underline{\alpha}^{/}\underline{\varphi}_d\right). \tag{4.3.31}$$

Using (1.5.8) in (4.3.31) and on simplification:

$$\left(t_{15(2)}-\bar{Y}\right)\approx\bar{e}_{y_2}+\bar{Y}\underline{\alpha}^{/}\underline{\varphi}_d.$$

The mean square error of $t_{15(2)}$ is:

$$MSE\left(t_{15(2)}\right)\approx E\left[\bar{e}_{y_2}^2+\bar{Y}^2\underline{\alpha}^{/}\underline{\varphi}_d\underline{\varphi}_d^{/}\underline{\alpha}+2\bar{Y}\underline{\alpha}^{/}\underline{\varphi}_d\bar{e}_{y_2}\right]. \tag{4.3.32}$$

Using (1.5.12) and (1.5.17) in (4.3.32) and on simplification:

$$MSE\left(t_{15(2)}\right)\approx\left[\theta_2\bar{Y}^2C_y^2+\left(\theta_2-\theta_1\right)\left\{\bar{Y}^2\underline{\alpha}^{/}\Phi_\tau\underline{\alpha}+2\bar{Y}\underline{\alpha}^{/}\underline{\phi}_{y\tau}\right\}\right]. \tag{4.3.33}$$

The optimum value of $\underline{\alpha}$ is same as derived for full information case in (4.2.15).Using (4.2.15) in (4.3.33) and on simplification the mean square error of $t_{15(2)}$ is:

$$\text{MSE}\left(t_{15(2)}\right) \approx \left\{\theta_2\left(1-\rho^2_{y.\tau_1\tau_2...\tau_k}\right)+\theta_1\rho^2_{y.\tau_1\tau_2...\tau_k}\right\}\bar{Y}^2C_y^2. \tag{4.3.34}$$

4.3.2.3- Mean Square Error of $t_{16(2)}$

Consider the estimator defined in (4.3.24),

$$t_{16(2)} = \bar{y}_2\left(v_{1d}\right)^{\alpha_1}\left(v_{2d}\right)^{\alpha_2}...\left(v_{kd}\right)^{\alpha_k}. \tag{4.3.24}$$

Expanding (4.3.24) and retaining linear terms only:

$$t_{16(2)} \approx \bar{y}_2\left(1+\sum_{j=1}^{k}\alpha_j\left(v_{jd}-1\right)\right).$$

This is (4.3.30); so

$$\text{MSE}\left(t_{16(2)}\right) = \text{MSE}\left(t_{15(2)}\right) \approx \left\{\theta_2\left(1-\rho^2_{y.\tau_1\tau_2...\tau_k}\right)+\theta_1\rho^2_{y.\tau_1\tau_2...\tau_k}\right\}\bar{Y}^2C_y^2. \tag{4.3.35}$$

The minimized mean square error of each of three considered estimators of proposed general family $\hat{T}_{14(2)}$ is:

$$\text{MSE}\left(\hat{T}_{14(2)}\right) \approx \left\{\theta_2\left(1-\rho^2_{y.\tau_1\tau_2...\tau_k}\right)+\theta_1\rho^2_{y.\tau_1\tau_2...\tau_k}\right\}\bar{Y}^2C_y^2. \tag{4.3.36}$$

4.4- A GENERALIZED SHRINKAGE FAMILY OF ESTIMATORS FOR FULL PARTIAL AND NO INFORMATION CASES

In this section the shrinkage version of proposed generalized family of estimators for full, partial and no information cases is discussed. Shahbaz and Hanif (2009) approach given in (3.5.2) is used to find mean square error of shrinkage version.

4.4.1- A Generalized Shrinkage Family of Estimators for Single-Phase Sampling

Shrinkage version of generalized family of estimators $\hat{T}_{8(1)}$ is:

$$\hat{T}_{17(1)} = \frac{\hat{T}_{8(1)}}{1+\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{8(1)}\right)}, \tag{4.4.1}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2),the mean square error of each of considered estimator of proposed general family $\hat{T}_{17(1)}$ is:

$$\text{MSE}\left(\hat{T}_{17(1)}\right) = \frac{\text{MSE}\left(\hat{T}_{8(1)}\right)}{1+\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{8(1)}\right)} = \frac{\theta\left(1-\rho^2_{y.\tau_1\tau_2...\tau_k}\right)\bar{Y}^2C_y^2}{1+\theta\left(1-\rho^2_{y.\tau_1\tau_2...\tau_k}\right)C_y^2}, \tag{4.4.2}$$

It can be easily shown that generalized shrinkage family of estimators $\hat{T}_{17(1)}$ is more efficient than generalized family of estimators $\hat{T}_{8(1)}$. Similar are remarks for partial and no information cases.

4.4.2- A Generalized Shrinkage Family of Estimators for Two-Phase Sampling for Partial Information Case

Shrinkage version of generalized family of estimators $\hat{T}_{11(2)}$ is:

$$\hat{T}_{18(2)} = \frac{\hat{T}_{11(2)}}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{11(2)}\right)}, \tag{4.4.3}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of each of considered estimator of proposed general family $\hat{T}_{18(2)}$ is:

$$MSE\left(\hat{T}_{18(1)}\right) = \frac{MSE\left(\hat{T}_{11(2)}\right)}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{11(2)}\right)} =$$

$$\frac{\left[\theta_2\left\{1-\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}\right\}+\theta_1\left\{\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}-\rho^2_{y.\tau_1\tau_2...\tau_m}\right\}\right]\bar{Y}^2C^2_{y.}}{1+\left[\theta_2\left\{1-\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}\right\}+\theta_1\left\{\rho^2_{y.\tau_{m+1}\tau_{m+2}...\tau_k}-\rho^2_{y.\tau_1\tau_2...\tau_m}\right\}\right]C^2_{y.}} \tag{4.4.4}$$

4.4.3- A Generalized Shrinkage Family of Estimators for Two-Phase Sampling for No Information Case

Shrinkage version of generalized family of estimators $\hat{T}_{14(2)}$ is:

$$\hat{T}_{19(2)} = \frac{\hat{T}_{14(2)}}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{14(2)}\right)}, \tag{4.4.5}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of each of considered estimator of proposed general family $\hat{T}_{19(2)}$ is:

$$MSE\left(\hat{T}_{19(2)}\right) = \frac{MSE\left(\hat{T}_{14(2)}\right)}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{14(2)}\right)}$$

$$= \frac{\left\{\theta_2\left(1-\rho^2_{y.\tau_1\tau_2...\tau_k}\right)+\theta_1\rho^2_{y.\tau_1\tau_2...\tau_k}\right\}\bar{Y}^2C^2_y}{1+\left\{\theta_2\left(1-\rho^2_{y.\tau_1\tau_2...\tau_k}\right)+\theta_1\rho^2_{y.\tau_1\tau_2...\tau_k}\right\}C^2_y}. \tag{4.4.6}$$

CHAPTER 5
GENERALIZATION OF RATIO AND PRODUCT ESTIMATOR FOR FULL PARTIAL AND NO INFORMATION CASES USING MULTI - AUXILIARY ATTRIBUTES

5.1 INTRODUCTION

In this chapter the generalized ratio and product estimators for full information case (single-phase sampling), partial and no information cases (two-phase sampling) using "k" auxiliary attributes has been proposed. Shrinkage version of newly generalized ratio and product estimators has also been discussed.

5.2- GENERALIZED RATIO ESTIMATOR FOR FULL PARTIAL AND NO INFORMATION CASES

In this section the generalized ratio estimator for full, partial and no information cases is proposed. The mean square error of generalized ratio estimator for same cases is also derived. It is also shown that generalized ratio estimator for full and no information cases is special case of corresponding generalized families of estimators.

5.2.1- Generalized Ratio Estimator for Single-Phase Sampling Using "k" Auxiliary Attributes

Generalized ratio estimator by using "k" auxiliary attributes for full information case is:

$$t_{20(1)} = \bar{y}\left(\frac{P_1}{p_1}\right)\left(\frac{P_2}{p_2}\right).....\left(\frac{P_k}{p_k}\right). \tag{5.2.1}$$

Using (1.5.1) and (1.5.3) in (5.2.1) and on simplification:

$$\left(t_{20(1)} - \bar{Y}\right) \approx \left(\bar{e}_y - \sum_{j=1}^{k}\frac{\bar{Y}}{P_j}\bar{e}_{\tau_j}\right).$$

The mean square error of $t_{20(1)}$ is:

$$MSE\left(t_{20(1)}\right) \approx E\left(\bar{e}_y - \sum_{j=1}^{k}\frac{\bar{Y}}{P_j}\bar{e}_{\tau_j}\right)^2. \tag{5.2.2}$$

Expanding R.H.S. of (5.2.2):

$$\mathrm{MSE}\left(t_{20(1)}\right)\approx E\left(\bar{e}_y^2+\sum_{j=1}^{k}\frac{\bar{Y}^2\bar{e}_{\tau_j}^2}{P_j^2}-2\sum_{j=1}^{k}\frac{\bar{Y}\bar{e}_y\bar{e}_{\tau_j}}{P_j}+2\sum_{j\neq\psi=1}^{k}\frac{\bar{Y}}{P_j}\frac{\bar{Y}}{P_\psi}\bar{e}_{\tau_j}\bar{e}_{\tau_\psi}\right). \quad (5.2.3)$$

Using (1.5.4), (1.5.5), (1.5.6) and (1.5.7) in (5.2.3) and on simplification the mean square error of $t_{20(1)}$ is:

$$\mathrm{MSE}\left(t_{20(1)}\right)\approx\theta\bar{Y}^2\left[C_y^2+\sum_{j=1}^{k}C_{\tau_j}^2-2\sum_{j=1}^{k}C_yC_{\tau_j}\rho_{Pb_j}+2\sum_{j\neq\psi=1}^{k}C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right]. \quad (5.2.4)$$

It can be easily shown that generalized ratio estimator $t_{20(1)}$ given in (5.2.1) is special case of the estimator $t_{10(1)}$ given in (4.2.4) for $\alpha_1=\alpha_2=\alpha_3...=\alpha_k=-1$. Similarly mean square error of $t_{20(1)}$ given in (5.2.4) can easily be obtained by putting $\underline{\alpha}=\left[-\underline{1}\right]_{k\times1}$ in (4.2.14).Therefore generalized ratio estimator $t_{20(1)}$ is special case of the estimators $t_{10(1)}$.

5.2.2 Generalized Ratio Estimator for Two-Phase Sampling for Partial Information Case Using "k" Auxiliary Attributes With m Known and (k-m) Unknown (m<k)

Generalized ratio estimator by using "k" auxiliary attributes for partial information case is:

$$t_{21(2)}=\bar{y}_2\left(\frac{P_1}{p_{1(2)}}\right)\left(\frac{P_2}{p_{2(2)}}\right).....\left(\frac{P_m}{p_{m(2)}}\right)\left(\frac{p_{(m+1)(1)}}{p_{(m+1)(2)}}\right)......\left(\frac{p_{k(1)}}{p_{k(2)}}\right). \quad (5.2.5)$$

Using (1.5.8) and (1.5.9) and (1.5.10) in (5.2.5) and on simplification:

$$\left(t_{21(2)}-\bar{Y}\right)\approx\left(\bar{e}_{y_2}-\sum_{j=1}^{m}\frac{\bar{Y}}{P_j}\bar{e}_{\tau_{j(2)}}-\sum_{j=m+1}^{k}\frac{\bar{Y}}{P_j}\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)\right).$$

The mean square error of $t_{21(2)}$ is:

$$\mathrm{MSE}\left(t_{21(2)}\right)\approx E\left(\bar{e}_{y_2}-\sum_{j=1}^{m}\frac{\bar{Y}}{P_j}\bar{e}_{\tau_{j(2)}}-\sum_{j=m+1}^{k}\frac{\bar{Y}}{P_j}\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)\right)^2. \quad (5.2.6)$$

Expanding R.H.S. of (5.2.6):

$$\mathrm{MSE}\left(t_{21(2)}\right) \approx E\left(\begin{array}{l} \bar{e}_{y_2}^2 + \sum_{j=1}^{m} \frac{\bar{Y}^2 \bar{e}_{\tau_{j(2)}}^2}{P_j^2} + \sum_{j=m+1}^{k} \frac{\bar{Y}^2}{P_j^2}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)^2 \\ -2\sum_{j=1}^{m} \frac{\bar{Y}\bar{e}_{y_2}\bar{e}_{\tau_{j(2)}}}{P_j} - 2\sum_{j=m+1}^{k} \frac{\bar{Y}\bar{e}_{y_2}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)}{P_j} \\ +2\sum_{j\neq\psi=1}^{m} \frac{\bar{Y}}{P_j}\frac{\bar{Y}}{P_\psi}\bar{e}_{\tau_{j(2)}}\bar{e}_{\tau_{\psi(2)}} + 2\sum_{\substack{1\leq j\leq m \\ \psi\geq m+1}}^{m} \frac{\bar{Y}}{P_j}\frac{\bar{Y}}{P_\psi}\bar{e}_{\tau_{j(2)}}\left(\bar{e}_{\tau_{\psi(2)}} - \bar{e}_{\tau_{\psi(1)}}\right) \\ +2\sum_{j\neq\psi=m+1}^{k} \frac{\bar{Y}}{P_j}\frac{\bar{Y}}{P_\psi}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\left(\bar{e}_{\tau_{\psi(2)}} - \bar{e}_{\tau_{\psi(1)}}\right) \end{array}\right). \qquad (5.2.7)$$

Using (1.5.12) (1.5.13) (1.5.14) and (1.5.15) in (5.2.7) and on simplification the mean square error of $t_{21(2)}$ is:

$$\mathrm{MSE}\left(t_{21(2)}\right) \approx \bar{Y}^2\left[\begin{array}{l} \theta_2\left\{C_y^2 + \sum_{j=1}^{m} C_{\tau_j}^2 - 2\sum_{j=1}^{m} C_y C_{\tau_j}\rho_{Pb_j} + 2\sum_{j\neq\psi=1}^{m} C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right\} \\ +\theta_3\left\{\begin{array}{l} \sum_{j=m+1}^{k} C_{\tau_j}^2 - 2\sum_{j=m+1}^{k} C_y C_{\tau_j}\rho_{Pb_j} \\ +2\sum_{\substack{1\leq j\leq m \\ \psi\geq m+1}}^{k} C_{\tau_j}C_{\tau_\psi}Q_{j\psi} + 2\sum_{j\neq\psi=m+1}^{k} C_{\tau_j}C_{\tau_\psi}Q_{j\psi} \end{array}\right\} \end{array}\right]. \qquad (5.2.8)$$

If P_j for $j =, m+1, m+2, \ldots k$ were also known then $\theta_2 = \theta_3 = \theta$ for this $\mathrm{MSE}\left(t_{21(2)}\right) \approx \theta\bar{Y}^2\left[C_y^2 + \sum_{j=1}^{k} C_{\tau_j}^2 - 2\sum_{j=1}^{k} C_y C_{\tau_j}\rho_{Pb_j} + 2\sum_{j\neq\psi=1}^{k} C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right]$.

This is mean square error of full information case given in (5.2.4).

5.2.3 Generalized Ratio Estimator for Two-Phase Sampling for No Information Case Using "k" Auxiliary Attributes

Generalized ratio estimator by using "k" auxiliary attributes for no information case is:

$$t_{22(2)} = \bar{y}_2 \left(\frac{p_{1(1)}}{p_{1(2)}}\right)\left(\frac{p_{2(1)}}{p_{2(2)}}\right)\ldots\ldots\left(\frac{p_{k(1)}}{p_{k(2)}}\right). \tag{5.2.9}$$

Using (1.5.8), (1.5.9) and (1.5.10) in (5.2.9) and on simplification:

$$\left(t_{22(2)} - \bar{Y}\right) \approx \left(\bar{e}_{y_2} - \sum_{j=1}^{k} \frac{\bar{Y}}{P_j}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\right).$$

The mean square error of $t_{22(2)}$ is:

$$MSE\left(t_{22(2)}\right) \approx E\left(\bar{e}_{y_2} - \sum_{j=1}^{k} \frac{\bar{Y}}{P_j}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\right)^2. \tag{5.2.10}$$

Expanding R.H.S. of (5.2.10):

$$MSE\left(t_{22(2)}\right) \approx E\left(\begin{array}{l} \bar{e}_{y_2}^2 + \sum\limits_{j=1}^{k} \dfrac{\bar{Y}^2\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)^2}{P_j^2} - 2\sum\limits_{j=1}^{k} \dfrac{\bar{Y}\bar{e}_{y_2}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)}{P_j} \\ +2\sum\limits_{j\neq\psi=1}^{k} \dfrac{\bar{Y}}{P_j}\dfrac{\bar{Y}}{P_\psi}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\left(\bar{e}_{\tau_{\psi(2)}} - \bar{e}_{\tau_{\psi(1)}}\right) \end{array}\right).$$

(5.2.11)

Using (1.5.12) (1.5.13) (1.5.14) and (1.5.15) in (5.2.11) and on simplification the mean square error of $t_{22(2)}$ is:

$$MSE\left(t_{22(2)}\right) \approx \bar{Y}^2\left[\theta_2 C_y^2 + \theta_3\left\{\sum_{j=1}^{k} C_{\tau_j}^2 - 2\sum_{j=1}^{k} C_y C_{\tau_j}\rho_{Pb_j} + 2\sum_{j\neq\psi=1}^{k} C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right\}\right].$$

(5.2.12)

It can be easily shown that generalized ratio estimator $t_{22(2)}$ given in (5.2.9) is special case of the estimator $t_{16(2)}$ given in (4.3.24) for $\alpha_1 = \alpha_2 = \alpha_3 ... = \alpha_k = -1$. Similarly mean square error of $t_{22(2)}$ given in (5.2.12) can easily be obtained by putting $\underline{\alpha} = [-\underline{1}]_{k\times 1}$ and in (4.3.33).Therefore generalized ratio estimator $t_{22(2)}$ is special case of the estimator $t_{16(2)}$.

If all P_j were known then $\theta_2 = \theta_3 = \theta$ for this

$$MSE\left(t_{28(2)}\right) \approx \theta\bar{Y}^2\left[C_y^2 + \sum_{j=1}^{k} C_{\tau_j}^2 - 2\sum_{j=1}^{k} C_y C_{\tau_j}\rho_{Pb_j} + 2\sum_{j\neq\psi=1}^{k} C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right].$$

This is mean square error of full information case given in (5.2.4).

5.3 GENERALIZED SHRINKAGE RATIO ESTIMATOR FOR FULL PARTIAL AND NO INFORMATION CASES

In this section the shrinkage generalized ratio estimator full, partial and no information cases has been proposed. The mean square error of generalized shrinkage ratio estimator for same cases using “k” auxiliary attributes has also been derived by using Shahbaz and Hanif (2009) approach given in (3.5.2).

5.3.1 Generalized Shrinkage Ratio Estimator for Single-Phase Sampling Using “k” Auxiliary Attributes

Generalized shrinkage ratio estimator by using “k” auxiliary attributes for full information case is:

$$t_{23(1)} = \frac{t_{20(1)}}{1+\bar{Y}^{-2}MSE\left(t_{20(1)}\right)}, \tag{5.3.1}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{23(1)}$ is:

$$MSE\left(t_{23(1)}\right) = \frac{MSE\left(t_{20(1)}\right)}{1+\bar{Y}^{-2}MSE\left(t_{20(1)}\right)}. \tag{5.3.2}$$

5.3.2 Generalized Shrinkage Ratio Estimator for Two-Phase Sampling for Partial Information Case Using “k” Auxiliary Attributes With m Known and (k-m) Unknown (m<k)

Generalized shrinkage ratio estimator by using “k” auxiliary attributes for partial information case is:

$$t_{24(2)} = \frac{t_{21(2)}}{1+\bar{Y}^{-2}MSE\left(t_{21(2)}\right)}, \tag{5.3.3}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{24(2)}$ is:

$$MSE\left(t_{24(2)}\right) = \frac{MSE\left(t_{21(2)}\right)}{1+\bar{Y}^{-2}MSE\left(t_{21(2)}\right)}. \tag{5.3.4}$$

5.3.3 Generalized Shrinkage Ratio Estimator for Two-Phase Sampling for No Information Case Using “k” Auxiliary Attributes

Generalized shrinkage ratio estimator by using “k” auxiliary attributes for no information case is:

$$t_{25(2)}=\frac{t_{22(1)}}{1+\bar{Y}^{-2}MSE\left(t_{22(1)}\right)}, \qquad (5.3.5)$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{25(2)}$ is:

$$MSE\left(t_{25(2)}\right)=\frac{MSE\left(t_{22(2)}\right)}{1+\bar{Y}^{-2}MSE\left(t_{22(2)}\right)}. \qquad (5.3.6)$$

It can be easily observed that $MSE\left(t_{23(1)}\right)<MSE\left(t_{20(1)}\right)$, $MSE\left(t_{24(2)}\right)<MSE\left(t_{21(2)}\right)$ and $MSE\left(t_{25(2)}\right)<MSE\left(t_{22(2)}\right)$,because generalized shrinkage ratio estimators are more efficient than conventional generalized ratio estimators.

5.4 GENERALIZED PRODUCT ESTIMATOR FOR FULL PARTIAL AND NO INFORMATION CASES

In this section the generalized product estimator for full, partial and no information cases has been proposed. The mean square error of generalized product estimator for same cases has also been derived. It is also shown that generalized product estimator for full and no information cases are special case of corresponding generalized families of estimators.

5.4.1 Generalized Product Estimator for Single-Phase Sampling Using "k" Auxiliary Attributes

Generalized product estimator by using "k" auxiliary attributes for full information case is:

$$t_{26(1)}=\bar{y}\left(\frac{p_1}{P_1}\right)\left(\frac{p_2}{P_2}\right).....\left(\frac{p_k}{P_k}\right). \qquad (5.4.1)$$

Using (1.5.1) and (1.5.3) in (5.4.1) and on simplification:

$$\left(t_{26(1)}-\bar{Y}\right)\approx\left(\bar{e}_y+\sum_{j=1}^{k}\frac{\bar{Y}}{P_j}\bar{e}_{\tau_j}\right).$$

The mean square error of $t_{26(1)}$ is:

$$MSE\left(t_{26(1)}\right)\approx E\left(\bar{e}_y+\sum_{j=1}^{k}\frac{\bar{Y}}{P_j}\bar{e}_{\tau_j}\right)^2. \qquad (5.4.2)$$

Expanding R.H.S. of (5.4.2):

$$MSE\left(t_{26(1)}\right) \approx E\left(\bar{e}_y^2 + \sum_{j=1}^{k} \frac{\bar{Y}^2 \bar{e}_{\tau_j}^2}{P_j^2} + 2\sum_{j=1}^{k} \frac{\bar{Y}\bar{e}_y \bar{e}_{\tau_j}}{P_j} + 2\sum_{j \neq \psi = 1}^{k} \frac{\bar{Y}}{P_j}\frac{\bar{Y}}{P_\psi} \bar{e}_{\tau_j} \bar{e}_{\tau_\psi}\right). \quad (5.4.3)$$

Using (1.5.4) (1.5.5) (1.5.6) and (1.5.7) in (5.4.3) and on simplification the mean square error of $t_{26(1)}$ is:

$$MSE\left(t_{26(1)}\right) \approx \theta \bar{Y}^2 \left[C_y^2 + \sum_{j=1}^{k} C_{\tau_j}^2 + 2\sum_{j=1}^{k} C_y C_{\tau_j} \rho_{Pb_j} + 2\sum_{j \neq \psi = 1}^{k} C_{\tau_j} C_{\tau_\psi} Q_{j\psi}\right]. \quad (5.4.4)$$

It can be easily shown that generalized product estimator $t_{26(1)}$ given in (5.4.1) is special case of the estimator $t_{10(1)}$ given in (4.2.4) for $\alpha_1 = \alpha_2 = \alpha_3 ... = \alpha_k = 1$. Similarly mean square error of $t_{26(1)}$ given in (5.4.4) can easily be obtained by putting $\underline{\alpha} = \left[\underline{1}\right]_{k \times 1}$ and in (4.2.14).Therefore generalized product estimator $t_{26(1)}$ is special case of the estimator $t_{10(1)}$.

5.4.2 Generalized Product Estimator for Two-Phase Sampling for Partial Information Case Using "k" Auxiliary Attributes With m Known and (k-m) Unknown (m<k)

Generalized product estimator by using "k" auxiliary attributes for partial information case is:

$$t_{27(2)} = \bar{y}_2 \left(\frac{p_{1(2)}}{P_1}\right)\left(\frac{p_{2(2)}}{P_2}\right) ..... \left(\frac{p_{m(2)}}{P_m}\right)\left(\frac{p_{(m+1)(2)}}{p_{(m+1)(1)}}\right) ...... \left(\frac{p_{k(2)}}{p_{k(1)}}\right). \quad (5.4.5)$$

Using (1.5.8), (1.5.9) and (1.5.10) in (5.4.5) and on simplification:

$$\left(t_{27(2)} - \bar{Y}\right) \approx \left(\bar{e}_{y_2} + \sum_{j=1}^{m} \frac{\bar{Y}}{P_j} \bar{e}_{\tau_{j(2)}} + \sum_{j=m+1}^{k} \frac{\bar{Y}}{P_j}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\right).$$

The mean square error of $t_{27(2)}$ is:

$$MSE\left(t_{27(2)}\right) \approx E\left(\bar{e}_{y_2} + \sum_{j=1}^{m} \frac{\bar{Y}}{P_j} \bar{e}_{\tau_{j(2)}} + \sum_{j=m+1}^{k} \frac{\bar{Y}}{P_j}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\right)^2. \quad (5.4.6)$$

Expanding R.H.S. of (5.4.6):

$$
MSE\left(t_{27(2)}\right)\approx E\left(\begin{array}{l}
\bar{e}_{y_2}^2+\sum_{j=1}^{m}\frac{\bar{Y}^2\bar{e}_{\tau_{j(2)}}^2}{P_j^2}+\sum_{j=m+1}^{k}\frac{\bar{Y}^2}{P_j^2}\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)^2\\
+2\sum_{j=1}^{m}\frac{\bar{Y}\bar{e}_{y_2}\bar{e}_{\tau_{j(2)}}}{P_j}+2\sum_{j=m+1}^{k}\frac{\bar{Y}\bar{e}_{y_2}\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)}{P_j}\\
+2\sum_{j\neq\psi=1}^{m}\frac{\bar{Y}}{P_j}\frac{\bar{Y}}{P_\psi}\bar{e}_{\tau_{j(2)}}\bar{e}_{\tau_{\psi(2)}}+2\sum_{\substack{1\le j\le m\\ \psi\ge m+1}}^{m}\frac{\bar{Y}}{P_j}\frac{\bar{Y}}{P_\psi}\bar{e}_{\tau_{j(2)}}\left(\bar{e}_{\tau_{\psi(2)}}-\bar{e}_{\tau_{\psi(1)}}\right)\\
+2\sum_{j\neq\psi=m+1}^{k}\frac{\bar{Y}}{P_j}\frac{\bar{Y}}{P_\psi}\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)\left(\bar{e}_{\tau_{\psi(2)}}-\bar{e}_{\tau_{\psi(1)}}\right)
\end{array}\right).
$$

(5.4.7)

Using (1.5.12) (1.5.13) (1.5.14) and (1.5.15) in (5.4.7) and on simplification the mean square error of $t_{27(2)}$ is:

$$
MSE\left(t_{27(2)}\right)\approx\bar{Y}^2\left[\begin{array}{l}
\theta_2\left\{C_y^2+\sum_{j=1}^{m}C_{\tau_j}^2+2\sum_{j=1}^{m}C_yC_{\tau_j}\rho_{Pb_j}+2\sum_{j\neq\psi=1}^{m}C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right\}\\
+\theta_3\left\{\begin{array}{l}
\sum_{j=m+1}^{k}C_{\tau_j}^2+2\sum_{j=m+1}^{k}C_yC_{\tau_j}\rho_{Pb_j}\\
+2\sum_{\substack{1\le j\le m\\ \psi\ge m+1}}^{k}C_{\tau_j}C_{\tau_\psi}Q_{j\psi}+2\sum_{j\neq\psi=m+1}^{k}C_{\tau_j}C_{\tau_\psi}Q_{j\psi}
\end{array}\right\}
\end{array}\right].
$$

(5.4.8)

If P_j for $(j=,m+1,m+2,...k)$ were also known then $\theta_2=\theta_3=\theta$ for this

$$
MSE\left(t_{27(2)}\right)\approx\theta\bar{Y}^2\left[C_y^2+\sum_{j=1}^{k}C_{\tau_j}^2+2\sum_{j=1}^{k}C_yC_{\tau_j}\rho_{Pb_j}+2\sum_{j\neq\psi=1}^{k}C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right].
$$

This is mean square error of full information case given in (5.4.4).

5.4.3 Generalized Product Estimator for Two-Phase Sampling for No Information Case Using "k" Auxiliary Attributes

Generalized product estimator by using "k" auxiliary attributes for no information case is:

$$t_{28(2)} = \bar{y}_2\left(\frac{p_{1(2)}}{p_{1(1)}}\right)\left(\frac{p_{2(2)}}{p_{2(1)}}\right)......\left(\frac{p_{k(2)}}{p_{k(1)}}\right). \tag{5.4.9}$$

Using (1.5.8), (1.5.9) and (1.5.10) in (5.4.9) and on simplification:

$$\left(t_{28(2)} - \bar{Y}\right) \approx \left(\bar{e}_{y_2} + \sum_{j=1}^{k}\frac{\bar{Y}}{P_j}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\right).$$

The mean square error of $t_{28(2)}$ is:

$$MSE\left(t_{28(2)}\right) \approx E\left(\bar{e}_{y_2} + \sum_{j=1}^{k}\frac{\bar{Y}}{P_j}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\right)^2. \tag{5.4.10}$$

Expanding R.H.S. of (5.4.10):

$$MSE\left(t_{28(2)}\right) \approx E\left(\begin{array}{l}\bar{e}_{y_2}^2 + \sum\limits_{j=1}^{k}\dfrac{\bar{Y}^2\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)^2}{P_j^2} + 2\sum\limits_{j=1}^{k}\dfrac{\bar{Y}\bar{e}_{y_2}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)}{P_j} \\ +2\sum\limits_{j\neq\psi=1}^{k}\dfrac{\bar{Y}}{P_j}\dfrac{\bar{Y}}{P_\psi}\left(\bar{e}_{\tau_{j(2)}} - \bar{e}_{\tau_{j(1)}}\right)\left(\bar{e}_{\tau_{\psi(2)}} - \bar{e}_{\tau_{\psi(1)}}\right)\end{array}\right). \tag{5.4.11}$$

Using (1.5.12) (1.5.13) (1.5.14) and (1.5.15) in (5.4.11) and on simplification the mean square error of $t_{28(2)}$ is:

$$MSE\left(t_{28(2)}\right) \approx \bar{Y}^2\left[\theta_2 C_y^2 + \theta_3\left\{\sum_{j=1}^{k}C_{\tau_j}^2 + 2\sum_{j=1}^{k}C_y C_{\tau_j}\rho_{Pb_j} + 2\sum_{j\neq\psi=1}^{k}C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right\}\right]. \tag{5.4.12}$$

It can be easily shown that generalized product estimator $t_{28(2)}$ given in (5.4.9) is special case of the estimator $t_{16(2)}$ given in (4.3.24) for $\alpha_1 = \alpha_2 = \alpha_3 ... = \alpha_k = 1$. Similarly mean square error of $t_{28(2)}$ given in (5.4.12) can easily be obtained by putting $\underline{\alpha} = \left[\underline{1}\right]_{k\times 1}$ and in (4.3.33).Therefore generalized ratio estimator $t_{28(2)}$ is special case of the estimator $t_{16(2)}$.

If all P_j were known then $\theta_2 = \theta_3 = \theta$ for this

$$MSE\left(t_{28(2)}\right) \approx \theta\bar{Y}^2\left[C_y^2 + \sum_{j=1}^{k}C_{\tau_j}^2 + 2\sum_{j=1}^{k}C_y C_{\tau_j}\rho_{Pb_j} + 2\sum_{j\neq\psi=1}^{k}C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right].$$

This is mean square error of full information case given in (5.4.4)

5.5 GENERALIZED SHRINKAGE PRODUCT ESTIMATOR FOR FULL PARTIAL AND NO INFORMATION CASES

In this section the shrinkage version of generalized product estimator for full, partial and no information cases has been proposed. The mean square error of generalized shrinkage product estimator for same cases using "k" auxiliary attributes has also been derived by using Shahbaz and Hanif (2009) approach given in (3.5.2).

5.5.1 Generalized Shrinkage Product Estimator for Single-Phase Sampling Using "k" Auxiliary Attributes

Generalized shrinkage product estimator by using "k" auxiliary attributes for full information case is:

$$t_{29(1)} = \frac{t_{26(1)}}{1+\bar{Y}^{-2}MSE\left(t_{26(1)}\right)}, \tag{5.5.1}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{29(1)}$ is:

$$MSE\left(t_{29(1)}\right) = \frac{MSE\left(t_{26(1)}\right)}{1+\bar{Y}^{-2}MSE\left(t_{26(1)}\right)}. \tag{5.5.2}$$

5.5.2 Generalized Shrinkage Product Estimator for Two-Phase Sampling for Partial Information Case Using "k" Auxiliary Attributes With m Known and (k-m) Unknown (m<k)

Generalized shrinkage product estimator by using "k" auxiliary attributes for partial information case is:

$$t_{30(2)} = \frac{t_{27(2)}}{1+\bar{Y}^{-2}MSE\left(t_{27(2)}\right)}, \tag{5.5.3}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{30(2)}$ is:

$$MSE\left(t_{30(2)}\right) = \frac{MSE\left(t_{27(2)}\right)}{1+\bar{Y}^{-2}MSE\left(t_{27(2)}\right)}. \tag{5.5.4}$$

5.5.3 Generalized Shrinkage Product Estimator for Two-Phase Sampling for No Case Information Using "k" Auxiliary Attributes

Generalized shrinkage product estimator by using "k" auxiliary attributes for no information case is:

$$t_{31(2)} = \frac{t_{28(2)}}{1 + \bar{Y}^{-2} MSE\left(t_{28(2)}\right)}, \tag{5.5.5}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{31(2)}$ is:

$$MSE\left(t_{31(2)}\right) = \frac{MSE\left(t_{28(2)}\right)}{1 + \bar{Y}^{-2} MSE\left(t_{28(2)}\right)}. \tag{5.5.6}$$

It can be easily observed that $MSE\left(t_{29(1)}\right) < MSE\left(t_{26(1)}\right)$, $MSE\left(t_{30(2)}\right) < MSE\left(t_{27(2)}\right)$ and $MSE\left(t_{31(2)}\right) < MSE\left(t_{28(2)}\right)$, because generalized shrinkage product estimators are more efficient than conventional generalized product estimators.

CHAPTER 6

A FAMILY OF ESTIMATORS FOR FULL PARTIAL AND NO INFORMATION CASES USING TWO – AUXILIARY ATTRIBUTES

6.1 INTRODUCTION

In this chapter generalization of the estimators given by Jhajj et al. (2006) for full, partial and no information cases using two-auxiliary attributes, have been proposed. The behaviors of bias for various estimators of generalized family of estimators have also been studied. The shrinkage families of estimators for these cases have also been discussed.

6.2- A FAMILY OF ESTIMATORS FOR FULL PARTIAL AND NO INFORMATION CASES USING TWO AUXILIARY ATTRIBUTES

In this section a general family of estimators has been suggested for full, partial and no information cases. Bias and mean square error have also been derived.

6.2.1- A Family of Estimators for Single-Phase Sampling

General family of estimators for full information case is:

$$\hat{T}_{32(1)} = g_{\omega}\left(\bar{y}, v_1, v_2\right), \tag{6.2.1}$$

where $v_1 = \dfrac{p_1}{P_1}$, $v_2 = \dfrac{p_2}{P_2}$, $v_1 > 0, v_2 > 0$, p_1 and p_2 are sample proportions possessing attributes τ_1 and τ_2 respectively, $g_{\omega}\left(\bar{y}, v_1, v_2\right)$ is the parametric function such that $g_{\omega}\left(\bar{Y},1,1\right) = \bar{Y}$, and satisfying the point $\left(\bar{y}, v_1, v_2\right)$ to be in a bounded set in R_3 containing a point $\left(\bar{Y},1,1\right)$.Some proposed estimators under above condition are:

i) $$t_{32(1)} = \bar{y}\, v_1^{\alpha_1} v_2^{\alpha_2}, \tag{6.2.2}$$

ii) $$t_{33(1)} = \bar{y}\, e^{\alpha_1\left(v_1-1\right)+\alpha_2\left(v_2-1\right)}, \tag{6.2.3}$$

iii) $t_{34(1)} = \bar{y}\left(v_1 e^{(v_1-1)}\right)^{\alpha_1}\left(v_2 e^{(v_2-1)}\right)^{\alpha_2}$, (6.2.4)

iv) $t_{35(1)} = \bar{y} v_1^{\alpha_1} e^{\alpha_2(v_2-1)}$, (6.2.5)

v) $t_{36(1)} = \frac{\bar{y}}{2}\left[v_1^{\alpha_1} v_2^{\alpha_2} + e^{\alpha_1(v_1-1)+\alpha_2(v_2-1)}\right]$, (6.2.6)

vi) $t_{37(1)} = \bar{y} + \alpha_1\left(v_1^{\alpha_3} - 1\right) + \alpha_2\left(v_2^{\alpha_4} - 1\right)$, (6.2.7)

vii) $t_{38(1)} = \bar{y} + \alpha_1\left(v_1^{\alpha_3} - 1\right) + \alpha_2\left(v_2 - 1\right)$, (6.2.8)

viii) $t_{39(1)} = \frac{\bar{y}}{k_1 + k_2}\left[k_1 v_1^{\frac{\alpha_1}{2}} + k_2 e^{\alpha_2(v_2-1)}\right]$, (6.2.9)

ix) $t_{40(1)} = \bar{y}\left[k e^{\alpha_1(v_1-1)} + (1-k) e^{\alpha_2(v_2-1)}\right]$, (6.2.10)

x) $t_{41(1)} = \bar{y} + \alpha_1\left(v_1 - 1\right) + \alpha_2\left(v_2 - 1\right)$, (6.2.11)

where k_1 and k_2 are positive numbers and $0 < k < 1$.

6.2.1.1- Bias and Mean Square Error Expression for $t_{32(1)}$

Consider the estimator defined in (6.2.2),

$$t_{32(1)} = \bar{y} v_1^{\alpha_1} v_2^{\alpha_2}. \quad (6.2.2)$$

Using (1.5.2) and (1.5.3) in (6.2.2) and on simplification:

$$t_{32(1)} = \left(\bar{Y} + \bar{e}_y\right)\left(1 + \frac{\bar{e}_{\tau_1}}{P_1}\right)^{\alpha_1}\left(1 + \frac{\bar{e}_{\tau_2}}{P_2}\right)^{\alpha_2}. \quad (6.2.12)$$

Expanding (6.2.12) up to second degree of approximation and on simplification:

$$\left(t_{32(1)} - \bar{Y}\right) \approx \left[\bar{e}_y + \frac{\alpha_1 \bar{Y}}{P_1}\bar{e}_{\tau_1} + \frac{\alpha_2 \bar{Y}}{P_2}\bar{e}_{\tau_2} + \frac{\alpha_1}{P_1}\bar{e}_y \bar{e}_{\tau_1} + \frac{\alpha_2}{P_2}\bar{e}_y \bar{e}_{\tau_2} + \frac{\alpha_1 \alpha_2 \bar{Y}}{P_1 P_2}\bar{e}_{\tau_1}\bar{e}_{\tau_2}\right]. \quad (6.2.13)$$

Taking expectation on both sides of (6.2.13):

$$\text{Bias}\left(t_{32(1)}\right) \approx E\left(t_{32(1)} - \bar{Y}\right) \approx$$

$$E\left[\bar{e}_y + \frac{\alpha_1 \bar{Y}}{P_1}\bar{e}_{\tau_1} + \frac{\alpha_2 \bar{Y}}{P_2}\bar{e}_{\tau_2} + \frac{\alpha_1}{P_1}\bar{e}_y \bar{e}_{\tau_1} + \frac{\alpha_2}{P_2}\bar{e}_y \bar{e}_{\tau_2} + \frac{\alpha_1 \alpha_2 \bar{Y}}{P_1 P_2}\bar{e}_{\tau_1}\bar{e}_{\tau_2}\right]. \tag{6.2.14}$$

Using (1.5.6) and (1.5.7) in (6.2.14):

$$\text{Bias}\left(t_{32(1)}\right) \approx \theta\left[\alpha_1 \bar{Y} C_y C_{\tau_1} \rho_{Pb_1} + \alpha_2 \bar{Y} C_y C_{\tau_1} \rho_{Pb_2} + \alpha_1 \alpha_2 \bar{Y} C_{\tau_1} C_{\tau_2} Q_{12}\right]. \tag{6.2.15}$$

The mean square error of $t_{32(1)}$ is:

$$\text{MSE}\left(t_{32(1)}\right) \approx E\left[\bar{e}_y + \frac{\alpha_1 \bar{Y}}{P_1}\bar{e}_{\tau_1} + \frac{\alpha_2 \bar{Y}}{P_2}\bar{e}_{\tau_2}\right]^2. \tag{6.2.16}$$

Expanding the R.H.S. of (6.2.16):

$$\text{MSE}\left(t_{32(1)}\right) \approx E\left[\begin{array}{l} \bar{e}_y^2 + \dfrac{\alpha_1^2 \bar{Y}^2}{P_1^2}\bar{e}_{\tau_1}^2 + \dfrac{\alpha_2^2 \bar{Y}^2}{P_2^2}\bar{e}_{\tau_2}^2 + 2\dfrac{\alpha_1 \bar{Y}}{P_1}\bar{e}_y \bar{e}_{\tau_1} \\ +2\dfrac{\alpha_2 \bar{Y}}{P_2}\bar{e}_y \bar{e}_{\tau_2} + 2\dfrac{\alpha_1 \alpha_2 \bar{Y}^2}{P_1 P_2}\bar{e}_{\tau_1}\bar{e}_{\tau_2} \end{array}\right]. \tag{6.2.17}$$

Using (1.5.4), (1.5.5), (1.5.6) and (1.5.7) in (6.2.17) and on simplification the mean square error of $t_{32(1)}$ is:

$$\text{MSE}\left(t_{32(1)}\right) \approx \theta\left[\begin{array}{l} \bar{Y}^2 C_y^2 + \alpha_1^2 \bar{Y}^2 C_{\tau_1}^2 + \alpha_2^2 \bar{Y}^2 C_{\tau_2}^2 + 2\alpha_1 \bar{Y}^2 C_y C_{\tau_1} \rho_{Pb_1} \\ +2\alpha_2 \bar{Y}^2 C_y C_{\tau_1} \rho_{Pb_2} + 2\alpha_1 \alpha_2 \bar{Y}^2 C_{\tau_1} C_{\tau_2} Q_{12} \end{array}\right]. \tag{6.2.18}$$

For optimum value of α_1 and α_2, differentiate (6.2.18) with respect to α_1 and α_2, respectively and on simplification:

$$\alpha_1 = \frac{-C_y\left(\rho_{Pb_1} - \rho_{Pb_2} Q_{12}\right)}{C_{\tau_1}\left(1 - Q_{12}^2\right)}, \tag{6.2.19}$$

$$\alpha_2 = \frac{-C_y\left(\rho_{Pb_2} - \rho_{Pb_1} Q_{12}\right)}{C_{\tau_2}\left(1 - Q_{12}^2\right)}. \tag{6.2.20}$$

Using (6.2.19), (6.2.20) in (6.2.18) and on simplification the mean square error of $t_{32(1)}$ is:

$$\text{MSE}\left(t_{32(1)}\right) \approx \theta\left[1 - \frac{\rho_{Pb_1}^2 + \rho_{Pb_2}^2 - 2Q_{12}\rho_{Pb_1}\rho_{Pb_2}}{\left(1 - Q_{12}^2\right)}\right]\bar{Y}^2 C_y^2.$$

Or

$$\text{MSE}\left(t_{32(1)}\right) \approx \theta\left(1 - \rho_{y.\tau_1\tau_2}^2\right)\bar{Y}^2 C_y^2. \tag{6.2.21}$$

Using (6.2.19), (6.2.20) in (6.2.15) and on simplification:

$$\text{Bias}\left(t_{32(1)}\right) \approx \frac{-\theta\bar{Y}^2 C_y^2}{\bar{Y}\left(1-Q_{12}^2\right)^2}\left[\rho_{Pb_1}^2 + \rho_{Pb_2}^2 + \rho_{Pb_1}\rho_{Pb_2}Q_{12}^3 - 3\rho_{Pb_1}\rho_{Pb_2}Q_{12}\right] \quad (6.2.22)$$

Likewise deriving bias and mean square error of other proposed estimators of general family:

(i) $$\text{Bias}\left(t_{33(1)}\right) \approx \frac{-\theta\bar{Y}^2 C_y^2}{\bar{Y}\left(1-Q_{12}^2\right)}\left[\rho_{Pb_1}^2 + \rho_{Pb_2}^2 - 2\rho_{Pb_1}\rho_{Pb_2}Q_{12}\right]. \quad (6.2.23)$$

(ii) $$\text{Bias}\left(t_{34(1)}\right) \approx \frac{-\theta\bar{Y}^2 C_y^2}{4\bar{Y}\left(1-Q_{12}^2\right)^2}\begin{bmatrix}3\rho_{Pb_1}^2 + 3\rho_{Pb_2}^2 - \rho_{Pb_1}^2 Q_{12}^2 - \rho_{Pb_2}^2 Q_{12}^2 \\ -8\rho_{Pb_1}\rho_{Pb_2}Q_{12} + 4\rho_{Pb_1}\rho_{Pb_2}Q_{12}^3\end{bmatrix}. \quad (6.2.24)$$

(iii) $$\text{Bias}\left(t_{35(1)}\right) = \text{Bias}\left(t_{32(1)}\right). \quad (6.2.25)$$

(iv) $$\text{Bias}\left(t_{36(1)}\right) \approx \frac{-\theta\bar{Y}^2 C_y^2}{2\bar{Y}\left(1-Q_{12}^2\right)^2}\begin{bmatrix}2\rho_{Pb_1}^2 + 2\rho_{Pb_2}^2 - \rho_{Pb_1}^2 Q_{12}^2 - \rho_{Pb_2}^2 Q_{12}^2 \\ -5\rho_{Pb_1}\rho_{Pb_2}Q_{12} + 3\rho_{Pb_1}\rho_{Pb_2}Q_{12}^2\end{bmatrix}. \quad (6.2.26)$$

(v) $$\text{Bias}\left(t_{37(1)}\right) \approx \text{Zero}. \quad (6.2.27)$$

(vi) $$\text{Bias}\left(t_{38(1)}\right) \approx \text{Zero}. \quad (6.2.28)$$

(vii) $$\text{Bias}\left(t_{39(1)}\right) \approx \text{Bias}\left(t_{33(1)}\right). \quad (6.2.29)$$

(viii) $$\text{Bias}\left(t_{40(1)}\right) \approx \text{Bias}\left(t_{33(1)}\right). \quad (6.2.30)$$

(ix) $$\text{Bias}\left(t_{41(1)}\right) = \text{Zero}. \quad (6.2.31)$$

Where as the minimized mean square error of each of ten proposed estimators of general family $\hat{T}_{32(1)}$ is:

$$\text{MSE}\left(\hat{T}_{32(1)}\right) \approx \theta\left(1-\rho_{y.\tau_1\tau_2}^2\right)\bar{Y}^2 C_y^2. \quad (6.2.32)$$

Expression (6.2.32) is not general mean square error expression for general family given in (6.2.1) but taken as common mean square error for considered estimators. The bias is same for a few estimators but differ in mostly proposed estimators. We will have same remarks in all of upcoming families of estimators. It can be easily observed that $\text{MSE}\left(\bar{y}\right) - \text{MSE}\left(\hat{T}_{32(1)}\right) \geq 0$ and $\text{MSE}\left(\hat{T}_{4(1)}\right) - \text{MSE}\left(\hat{T}_{32(1)}\right) \geq 0$. Hence $\hat{T}_{32(1)}$ is more efficient than $\bar{y}$. Also $\hat{T}_{32(1)}$ is more efficient than $\hat{T}_{4(1)}$ under the condition that $\rho_{y.\tau_1\tau_2}^2 > \rho_{pb_1}^2$.

The proposed family of estimators $\hat{T}_{32(1)}$ is very large any function which satisfies given conditions is member of proposed family. Similarly all the general families developed below are very large and have same behaviors like general family $\hat{T}_{32(1)}$.

6.2.2- A Family of Estimators for Two-Phase Sampling for Partial Information Case

General family of estimators for partial information case is:

$$\hat{T}_{42(2)} = g_{\omega}\left(\bar{y}_2, v_1, v_{2d}\right); \quad (6.2.33)$$

where $v_1 = \frac{p_{1(2)}}{P_1}$, $v_{2d} = \frac{p_{2(2)}}{p_{2(1)}}$, $v_1 > 0$, $v_{2d} > 0$, where $g_{\omega}\left(\bar{y}_2, v_1, v_{2d}\right)$ is parametric function such that $g_{\omega}\left(\bar{Y},1,1\right) = \bar{Y}$, and satisfying conditions mentioned for family defined in (6.2.1). Some proposed estimators under above condition are:

i) $$t_{42(2)} = \bar{y}_2 \, v_1^{\lambda_1} v_{2d}^{\lambda_2}, \quad (6.2.34)$$

ii) $$t_{43(2)} = \bar{y}_2 \, e^{\lambda_1(v_1-1)+\lambda_2(v_{2d}-1)}, \quad (6.2.35)$$

iii) $$t_{44(2)} = \bar{y}_2\left(v_1 e^{(v_1-1)}\right)^{\lambda_1}\left(v_{2d} e^{(v_{2d}-1)}\right)^{\lambda_2}, \quad (6.2.36)$$

iv) $$t_{45(2)} = \bar{y}_2 v_1^{\lambda_1} e^{\lambda_2(v_{2d}-1)}, \quad (6.2.37)$$

v) $$t_{46(2)} = \frac{\bar{y}_2}{2}\left[v_1^{\lambda_1} v_{2d}^{\lambda_2} + e^{\lambda_1(v_1-1)+\lambda_2(v_{2d}-1)}\right], \quad (6.2.38)$$

vi) $$t_{47(2)} = \bar{y}_2 + \lambda_1\left(v_1^{\alpha_3} - 1\right) + \lambda_2\left(v_{2d}^{\alpha_4} - 1\right), \quad (6.2.39)$$

vii) $$t_{48(2)} = \bar{y}_2 + \lambda_1\left(v_1^{\alpha_3} - 1\right) + \lambda_2\left(v_{2d} - 1\right), \quad (6.2.40)$$

viii) $$t_{49(2)} = \frac{\bar{y}_2}{k_1 + k_2}\left[k_1 v_1^{\frac{\lambda_1}{2}} + k_2 e^{\lambda_2(v_{2d}-1)}\right], \quad (6.2.41)$$

ix) $$t_{50(2)} = \bar{y}_2\left[k e^{\lambda_1(v_1-1)} + (1-k) e^{\lambda_2(v_{2d}-1)}\right], \quad (6.2.42)$$

x) $t_{51(2)} = \bar{y}_2 + \lambda_1 \left(v_1 - 1 \right) + \lambda_2 \left(v_{2d} - 1 \right),$ (6.2.43)

where k_1 and k_2 are positive numbers and $0 < k < 1$.

6.2.2.1- Bias and Mean Square Error Expression for $t_{42(2)}$

Consider the estimator of the family defined in (6.2.34),

$$t_{42(2)} = \bar{y}_2 \, v_1^{\lambda_1} v_{2d}^{\lambda_2}. \quad (6.2.34)$$

Using (1.5.2) (1.5.8) and (1.5.11) in (6.2.34) and on simplification:

$$t_{42(2)} \approx \left(\bar{Y} + \bar{e}_{y_2} \right) \left(1 + \frac{\bar{e}_{\tau_{1(2)}}}{P_1} \right)^{\lambda_1} \left(1 + \frac{\left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right)}{P_2} \right)^{\lambda_2}. \quad (6.2.44)$$

Expanding (6.2.44) up to second degree of approximation and on simplification:

$$\mathrm{Bias}\left(t_{42(2)} \right) \approx E \left[\begin{array}{l} \bar{e}_{y_2} + \dfrac{\lambda_1 \bar{Y}}{P_1} \bar{e}_{\tau_{1(2)}} + \dfrac{\lambda_2 \bar{Y} \left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right)}{P_2} \\ + \dfrac{\lambda_1}{P_1} \bar{e}_{y_2} \bar{e}_{\tau_{1(2)}} + \dfrac{\lambda_2 \left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right) \bar{e}_{y_2}}{P_2} + \dfrac{\lambda_1 \lambda_2 \bar{Y} \bar{e}_{\tau_{1(2)}} \left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right)}{P_1 P_2} \end{array} \right].$$

(6.2.45)

Using (1.5.14) and (1.5.15) in (6.2.45):

$$\mathrm{Bias}\left(t_{42(2)} \right) \approx \left[\lambda_1 \theta_2 \bar{Y} C_y C_{\tau_1} \rho_{Pb_1} + \theta_3 \left\{ \lambda_2 \bar{Y} C_y C_{\tau_1} \rho_{Pb_2} + \lambda_1 \lambda_2 \bar{Y} C_{\tau_1} C_{\tau_2} Q_{12} \right\} \right].$$

(6.2.46)

The mean square error of $t_{42(2)}$ is:

$$\mathrm{MSE}\left(t_{42(2)} \right) \approx E \left[\bar{e}_{y_2} + \frac{\lambda_1 \bar{Y}}{P_1} \bar{e}_{\tau_{1(2)}} + \frac{\lambda_2 \bar{Y} \left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right)}{P_2} \right]^2. \quad (6.2.47)$$

Expanding the R.H.S. of (6.2.47):

$$\mathrm{MSE}\left(t_{42(2)} \right) \approx E \left[\begin{array}{l} \bar{e}_{y_2}^2 + \dfrac{\lambda_1^2 \bar{Y}^2}{P_1^2} \bar{e}_{\tau_{1(2)}}^2 + \dfrac{\lambda_2^2 \bar{Y}^2}{P_2^2} \left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right)^2 + 2 \dfrac{\lambda_1 \bar{Y}}{P_1} \bar{e}_{y_2} \bar{e}_{\tau_{1(2)}} \\ + 2 \dfrac{\lambda_2 \bar{Y}}{P_2} \bar{e}_{y_2} \left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right) + 2 \dfrac{\lambda_1 \lambda_2 \bar{Y}^2}{P_1 P_2} \bar{e}_{\tau_{1(2)}} \left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right) \end{array} \right].$$

(6.2.48)

Using (1.5.12), (1.5.13) (1.5.14) and (1.5.15) in (6.2.48) and on simplification:

$$\mathrm{MSE}\left(t_{42(2)}\right) \approx \begin{bmatrix} \theta_2\left\{\bar{Y}^2C_y^2 + \lambda_1^2\bar{Y}^2C_{\tau_1}^2 + 2\lambda_1\bar{Y}^2C_yC_{\tau_1}\rho_{Pb_1}\right\} + \\ \theta_3\left\{\lambda_2^2\bar{Y}^2C_{\tau_2}^2 + 2\lambda_2\bar{Y}^2C_yC_{\tau_2}\rho_{Pb_2} + 2\lambda_1\lambda_2\bar{Y}^2C_{\tau_1}C_{\tau_2}Q_{12}\right\}\end{bmatrix}. \quad (6.2.49)$$

For optimum value of λ_1 and λ_2, differentiate (6.2.49) with respect to λ_1 and λ_2 respectively and on simplification:

$$\lambda_1 = \frac{-C_y\left(\theta_2\rho_{Pb_1} - \rho_{Pb_2}\theta_3Q_{12}\right)}{C_{\tau_1}\left(\theta_2 - \theta_3Q_{12}^2\right)}, \quad (6.2.50)$$

$$\lambda_2 = \frac{-\theta_2C_y\left(\rho_{Pb_2} - \rho_{Pb_1}Q_{12}\right)}{C_{\tau_2}\left(\theta_2 - \theta_3Q_{12}^2\right)}. \quad (6.2.51)$$

Using (6.2.50), (6.2.51) in (6.2.49) and on simplification:

$$\mathrm{MSE}\left(t_{42(2)}\right) \approx \theta_2\left[1 - \frac{\theta_2\rho_{Pb_1}^2 + \theta_3\rho_{Pb_2}^2 - 2\theta_3Q_{12}\rho_{Pb_1}\rho_{Pb_2}}{\left(\theta_2 - \theta_3Q_{12}^2\right)}\right]\bar{Y}^2C_y^2. \quad (6.2.52)$$

If P_2 was also known then $\theta_2 = \theta_3 = \theta$, for this expression (6.2.52) is:

$$\mathrm{MSE}\left(t_{42(2)}\right) \approx \theta\left(1 - \rho_{y.\tau_1\tau_2}^2\right)\bar{Y}^2C_y^2.$$

This is corresponding mean square error for full information case given in (6.2.21).

Using (6.2.50), (6.2.51) in (6.2.46) and on simplification:

$$\mathrm{Bias}\left(t_{42(2)}\right) \approx \frac{-\theta_2\bar{Y}^2C_y^2}{\bar{Y}\left(\theta_2 - \theta_3Q_{12}^2\right)^2}\left[\theta_2^2\rho_{Pb_1}^2 + \theta_2\theta_3\rho_{Pb_2}^2 + \theta_3^2\rho_{Pb_1}\rho_{Pb_2}Q_{12}^3 - 3\theta_2\theta_3\rho_{Pb_1}\rho_{Pb_2}Q_{12}\right]. \quad (6.2.53)$$

If P_2 was also known then $\theta_2 = \theta_3 = \theta$, for this expression (6.2.53) is:

$$\mathrm{Bias}\left(t_{42(2)}\right) \approx \frac{-\theta\bar{Y}^2C_y^2}{\bar{Y}\left(1 - Q_{12}^2\right)^2}\left[\rho_{Pb_1}^2 + \rho_{Pb_2}^2 + \rho_{Pb_1}\rho_{Pb_2}Q_{12}^3 - 3\rho_{Pb_1}\rho_{Pb_2}Q_{12}\right].$$

This is corresponding bias for full information case given in (6.2.22).

Likewise deriving bias and mean square error of other proposed estimators of general family:

$$\text{(i)}\ \mathrm{Bias}\left(t_{43(2)}\right) \approx \frac{-\theta_2\bar{Y}^2C_y^2}{\bar{Y}\left(\theta_2 - \theta_3Q_{12}^2\right)}\left[\theta_2\rho_{Pb_1}^2 + \theta_3\rho_{Pb_2}^2 - 2\theta_3\rho_{Pb_1}\rho_{Pb_2}Q_{12}\right]. \quad (6.2.54)$$

(ii) $\text{Bias}\left(t_{44(2)}\right) \approx \frac{-\theta_2 \bar{Y}^2 C_y^2}{4\bar{Y}\left(1-Q_{12}^2\right)^2} \begin{bmatrix} 3\theta_2^2\rho_{Pb_1}^2 + 3\theta_2\theta_3\rho_{Pb_2}^2 - \theta_2\theta_3\rho_{Pb_1}^2 Q_{12}^2 \\ -\theta_3^2\rho_{Pb_2}^2 Q_{12}^2 - \theta_2\theta_3 8\rho_{Pb_1}\rho_{Pb_2} Q_{12} \\ +4\theta_3^2\rho_{Pb_1}\rho_{Pb_2} Q_{12}^3 \end{bmatrix}.$ (6.2.55)

(iii) $\text{Bias}\left(t_{45(2)}\right) = \text{Bias}\left(t_{42(2)}\right).$ (6.2.56)

(iv) $\text{Bias}\left(t_{46(2)}\right) \approx \frac{-\theta_2 \bar{Y}^2 C_y^2}{2\bar{Y}\left(\theta_2 - \theta_3 Q_{12}^2\right)^2} \begin{bmatrix} 2\theta_2^2\rho_{Pb_1}^2 + 2\theta_2\theta_3\rho_{Pb_2}^2 - \theta_3^2\rho_{Pb_2}^2 Q_{12}^2 \\ -\theta_2\theta_3\rho_{Pb_1}^2 Q_{12}^2 - 5\theta_2\theta_3\rho_{Pb_1}\rho_{Pb_2} Q_{12} \\ +3\theta_2^2\rho_{Pb_1}\rho_{Pb_2} Q_{12}^2 \end{bmatrix}.$

(6.2.57)

(v) $\text{Bias}\left(t_{47(2)}\right) \approx \text{Zero}.$ (6.2.58)

(vi) $\text{Bias}\left(t_{48(2)}\right) \approx \text{Zero}.$ (6.2.59)

(vii) $\text{Bias}\left(t_{49(2)}\right) \approx \text{Bias}\left(t_{43(2)}\right).$ (6.2.60)

(viii) $\text{Bias}\left(t_{50(2)}\right) \approx \text{Bias}\left(t_{43(2)}\right).$ (6.2.61)

(ix) $\text{Bias}\left(t_{51(2)}\right) = \text{Zero}.$ (6.2.62)

Where as the minimized mean square error of each of ten of proposed estimators of general family $\hat{T}_{42(2)}$ is:

$$\text{MSE}\left(\hat{T}_{42(2)}\right) \approx \theta_2 \left[1 - \frac{\theta_2\rho_{Pb_1}^2 + \theta_3\rho_{Pb_2}^2 - 2\theta_3 Q_{12}\rho_{Pb_1}\rho_{Pb_2}}{\left(\theta_2 - \theta_3 Q_{12}^2\right)}\right] \bar{Y}^2 C_y^2. \qquad (6.2.63)$$

If P_2 was also known then $\theta_2 = \theta_3 = \theta$, for this expression (6.2.63) is:

$$\text{MSE}\left(\hat{T}_{42(2)}\right) \approx \theta\left(1 - \rho_{y.\tau_1\tau_2}^2\right) \bar{Y}^2 C_y^2.$$

This is corresponding mean square error for full information case given in (6.2.32).

It is interested to note that another family of estimators was developed for partial information case as,

$$\hat{T}_{52(2)} = g_\omega\left(\bar{y}_2, v_1, v_{2d}\right), \qquad (6.2.64)$$

where $v_1 = \frac{p_{1(1)}}{P_1}$, $v_{2d} = \frac{p_{2(2)}}{p_{2(1)}}$, $v_1 > 0$, $v_{2d} > 0$, where $g_\omega\left(\bar{y}_2, v_1, v_{2d}\right)$ is parametric function such that $g_\omega\left(\bar{Y},1,1\right) = \bar{Y}$, and satisfying condition mentioned in section (6.2.1). The same the ten estimators of the family given in (6.2.64) were considered. The common mean square error of each of ten estimators to the term of order 1/n is:

$$MSE\left(\hat{T}_{52(2)}\right) \approx \left\{\theta_2\left(1-\rho^2_{y.\tau_2}\right) + \theta_3\left(\rho^2_{y.\tau_2} - \rho^2_{y.\tau_1}\right)\right\}\bar{Y}^2C_y^2 \; . \quad (6.2.65)$$

Where $\rho^2_{y.\tau_1} = \rho^2_{pb_1}$ and $\rho^2_{y.\tau_2} = \rho^2_{pb_2}$.

6.2.3-A Family of Estimators for Two-Phase Sampling for No Information Case

General family of estimators for partial information case is:

$$\hat{T}_{53(2)} = g_\omega\left(\bar{y}_2, v_{1d}, v_{2d}\right); \quad (6.2.66)$$

where $v_{1d} = \frac{p_{1(2)}}{p_{1(1)}}$, $v_{2d} = \frac{p_{2(2)}}{p_{2(1)}}$, $v_{1d} > 0$, $v_{2d} > 0$, where $g_\omega\left(\bar{y}_2, v_{1d}, v_{2d}\right)$ is parametric function such that $g_\omega\left(\bar{Y},1,1\right) = \bar{Y}$, and satisfying conditions mentioned for family defined in (6.2.1). Some proposed estimators under above condition are:

i) $t_{53(2)} = \bar{y}_2\, v_{1d}^{\alpha_1} v_{2d}^{\alpha_2}$, (6.2.67)

ii) $t_{54(2)} = \bar{y}_2\, e^{\alpha_1(v_{1d}-1)+\alpha_2(v_{2d}-1)}$, (6.2.68)

iii) $t_{55(2)} = \bar{y}_2\left(v_{1d}e^{(v_{1d}-1)}\right)^{\alpha_1}\left(v_{2d}e^{(v_{2d}-1)}\right)^{\alpha_2}$, (6.2.69)

iv) $t_{56(2)} = \bar{y}_2 v_{1d}^{\alpha_1}\, e^{\alpha_2(v_{2d}-1)}$, (6.2.70)

v) $t_{57(2)} = \frac{\bar{y}_2}{2}\left[v_{1d}^{\alpha_1} v_{2d}^{\alpha_2} + e^{\alpha_1(v_{1d}-1)+\alpha_2(v_{2d}-1)}\right]$, (6.2.71)

vi) $t_{58(2)} = \bar{y}_2 + \alpha_1\left(v_{1d}^{\alpha_3} - 1\right) + \alpha_2\left(v_{2d}^{\alpha_4} - 1\right)$, (6.2.72)

vii) $t_{59(2)} = \bar{y}_2 + \alpha_1\left(v_{1d}^{\alpha_3} - 1\right) + \alpha_2\left(v_{2d} - 1\right)$, (6.2.73)

viii) $t_{60(2)} = \frac{\bar{y}_2}{k_1 + k_2}\left[k_1 v_{1d}^{\frac{\alpha_1}{2}} + k_2 e^{\alpha_2 (v_{2d} - 1)} \right],$ (6.2.74)

ix) $t_{61(2)} = \bar{y}_2 \left[k e^{\alpha_1 (v_{1d} - 1)} + (1-k) e^{\alpha_2 (v_{2d} - 1)} \right],$ (6.2.75)

x) $t_{62(2)} = \bar{y}_2 + \alpha_1 (v_{1d} - 1) + \alpha_2 (v_{2d} - 1),$ (6.2.76)

where k_1 and k_2 are positive numbers and $0 < k < 1$.

6.2.3.1- Bias and Mean Square Error Expression for $t_{53(2)}$

Consider the estimator defined in (6.2.67),

$$t_{53(2)} = \bar{y}_2 \, v_{1d}^{\alpha_1} v_{2d}^{\alpha_2}. \tag{6.2.67}$$

Using (1.5.8) and (1.5.11) in (6.2.67) and on simplification:

$$t_{53(2)} \approx \left(\bar{Y} + \bar{e}_{y_2} \right) \left(1 + \frac{\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}} \right)}{P_1} \right)^{\alpha_1} \left(1 + \frac{\left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right)}{P_2} \right)^{\alpha_2}. \tag{6.2.77}$$

Expanding (6.2.77) up to second degree of approximation and on simplification:

$$\text{Bias}\left(t_{53(2)}\right) \approx E\left[\begin{array}{l} \bar{e}_{y_2} + \frac{\alpha_1 \bar{Y}\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}} \right)}{P_1} + \frac{\alpha_2 \bar{Y}\left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right)}{P_2} + \frac{\alpha_1}{P_1} \bar{e}_{y_2} \left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}} \right) \\ + \frac{\alpha_2 \left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right) \bar{e}_{y_2}}{P_2} + \frac{\alpha_1 \alpha_2 \bar{Y}}{P_1 P_2} \left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}} \right)\left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right) \end{array} \right]. \tag{6.2.78}$$

Using (1.5.14) and (1.5.15) in (6.2.78):

$$\text{Bias}\left(t_{53(2)}\right) \approx \theta_3 \left[\alpha_1 \bar{Y} C_y C_{\tau_1} \rho_{Pb_1} + \alpha_2 \bar{Y} C_y C_{\tau_1} \rho_{Pb_2} + \alpha_1 \alpha_2 \bar{Y} C_{\tau_1} C_{\tau_2} Q_{12} \right]. \tag{6.2.79}$$

The mean square error of $t_{53(2)}$ is:

$$\text{MSE}\left(t_{53(2)}\right) \approx E\left[\bar{e}_{y_2} + \frac{\alpha_1 \bar{Y}\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}} \right)}{P_1} + \frac{\alpha_2 \bar{Y}\left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}} \right)}{P_2} \right]^2. \tag{6.2.80}$$

Expanding the R.H.S. of (6.2.80) we get,

$$\text{MSE}\left(t_{53(2)}\right) \approx E\begin{bmatrix} \bar{e}_{y_2}^2 + \dfrac{\alpha_1^2\bar{Y}^2\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}}\right)^2}{P_1^2} + \dfrac{\alpha_2^2\bar{Y}^2}{P_2^2}\left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}}\right)^2 \\ +2\dfrac{\alpha_1\bar{Y}}{P_1}\bar{e}_{y_2}\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}}\right) + 2\dfrac{\alpha_2\bar{Y}}{P_2}\bar{e}_{y_2}\left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}}\right) \\ +2\dfrac{\alpha_1\alpha_2\bar{Y}^2}{P_1P_2}\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}}\right)\left(\bar{e}_{\tau_{2(2)}} - \bar{e}_{\tau_{2(1)}}\right) \end{bmatrix}. \quad (6.2.81)$$

Using (1.5.12), (1.5.13) (1.5.14) and (1.5.15) in (6.2.81) and on simplification:

$$\text{MSE}\left(t_{53(2)}\right) \approx \left[\theta_2\bar{Y}^2C_y^2 + \theta_3\begin{Bmatrix} \alpha_1^2\bar{Y}^2C_{\tau_1}^2 + \alpha_2^2\bar{Y}^2C_{\tau_2}^2 + 2\alpha_1\bar{Y}^2C_yC_{\tau_1}\rho_{Pb_1} \\ +2\alpha_2\bar{Y}^2C_yC_{\tau_2}\rho_{Pb_2} + 2\alpha_1\alpha_2\bar{Y}^2C_{\tau_1}C_{\tau_2}Q_{12} \end{Bmatrix}\right].$$

(6.2.82)

The optimum value of α_1 and α_2 are same as derived in (6.2.19) and (6.2.20) for full information case. Using (6.2.19) and (6.2.20) in (6.2.82) and on simplification the mean square error of $t_{53(2)}$ is:

$$\text{MSE}\left(t_{53(2)}\right) \approx \left[\theta_2\left(1-\rho_{y.\tau_1\tau_2}^2\right) + \theta_1\rho_{y.\tau_1\tau_2}^2\right]\bar{Y}^2C_y^2. \quad (6.2.83)$$

Using (6.2.19) and (6.2.20) in (6.2.79) and on simplification:

$$\text{Bias}\left(t_{53(2)}\right) \approx \frac{-\theta_3\bar{Y}^2C_y^2}{\bar{Y}\left(1-Q_{12}^2\right)^2}\left[\rho_{Pb_1}^2 + \rho_{Pb_2}^2 + \rho_{Pb_1}\rho_{Pb_2}Q_{12}^3 - 3\rho_{Pb_1}\rho_{Pb_2}Q_{12}\right]. \quad (6.2.84)$$

If both P_1 and P_2 were known then $\theta_3 = \theta$, for this expression (6.2.84) is:

$$\text{Bias}\left(t_{53(2)}\right) \approx \frac{-\theta\bar{Y}^2C_y^2}{\bar{Y}\left(1-Q_{12}^2\right)^2}\left[\rho_{Pb_1}^2 + \rho_{Pb_2}^2 + \rho_{Pb_1}\rho_{Pb_2}Q_{12}^3 - 3\rho_{Pb_1}\rho_{Pb_2}Q_{12}\right]$$

This is corresponding bias for full information case given in (6.2.22).

Likewise deriving bias and mean square error of other proposed estimators of general family:

$$\text{(i) Bias}\left(t_{54(2)}\right) \approx \frac{-\theta_3\bar{Y}^2C_y^2}{\bar{Y}\left(1-Q_{12}^2\right)}\left[\rho_{Pb_1}^2 + \rho_{Pb_2}^2 - 2\rho_{Pb_1}\rho_{Pb_2}Q_{12}\right]. \quad (6.2.85)$$

$$\text{(ii) Bias}\left(t_{55(2)}\right) \approx \frac{-\theta_3\bar{Y}^2C_y^2}{4\bar{Y}\left(1-Q_{12}^2\right)^2}\begin{bmatrix} 3\rho_{Pb_1}^2 + 3\rho_{Pb_2}^2 - \rho_{Pb_1}^2Q_{12}^2 - \rho_{Pb_2}^2Q_{12}^2 \\ -8\rho_{Pb_1}\rho_{Pb_2}Q_{12} + 4\rho_{Pb_1}\rho_{Pb_2}Q_{12}^3 \end{bmatrix}. \quad (6.2.86)$$

$$\text{(iii) Bias}\left(t_{56(2)}\right) = \text{Bias}\left(t_{53(2)}\right). \quad (6.2.87)$$

(iv) $\text{Bias}\left(t_{57(2)}\right) \approx \frac{-\theta_3 \bar{Y}^2 C_y^2}{2\bar{Y}\left(1-Q_{12}^2\right)^2}\begin{bmatrix} 2\rho_{Pb_1}^2 + 2\rho_{Pb_2}^2 - \rho_{Pb_1}^2 Q_{12}^2 - \rho_{Pb_2}^2 Q_{12}^2 \\ -5\rho_{Pb_1}\rho_{Pb_2}Q_{12} + 3\rho_{Pb_1}\rho_{Pb_2}Q_{12}^2 \end{bmatrix}$. (6.2.88)

(v) $\text{Bias}\left(t_{58(2)}\right) \approx \text{Zero}$. (6.2.89)

(vi) $\text{Bias}\left(t_{59(2)}\right) \approx \text{Zero}$. (6.2.90)

(vii) $\text{Bias}\left(t_{60(2)}\right) \approx \text{Bias}\left(t_{54(2)}\right)$. (6.2.91)

(viii) $\text{Bias}\left(t_{61(2)}\right) \approx \text{Bias}\left(t_{54(2)}\right)$. (6.2.92)

(ix) $\text{Bias}\left(t_{62(2)}\right) = \text{Zero}$. (6.2.93)

Where as the minimized mean square error for each of ten of proposed estimators of general family $\hat{T}_{53(2)}$ is:

$$\text{MSE}\left(\hat{T}_{53(2)}\right) \approx \left[\theta_2\left(1-\rho_{y.\tau_1\tau_2}^2\right) + \theta_1\rho_{y.\tau_1\tau_2}^2\right]\bar{Y}^2 C_y^2. \quad (6.2.94)$$

It can be easily checked that

$\text{MSE}\left(\hat{T}_{5(2)}\right) - \text{MSE}\left(\hat{T}_{53(2)}\right) \geq 0$. So $\hat{T}_{53(2)}$ is more efficient than $\hat{T}_{5(2)}$ under the condition that $\rho_{y.\tau_1\tau_2}^2 > \rho_{pb_1}^2$.

.

If both P_1 and P_2 were known then $\theta_2 = \theta \,\&\, \theta_1 = 0$, for this expression (6.2.94) is:

$$\text{MSE}\left(\hat{T}_{53(2)}\right) \approx \theta\left(1-\rho_{y.\tau_1\tau_2}^2\right)\bar{Y}^2 C_y^2 .$$

This is corresponding mean square error for full information case given in (6.2.32).

6.3- THEOREM ON GENERALIZED BIAS OF SHRINKAGE ESTIMATORS

Statement: Let "T" be generalized family of estimators for population mean $\bar{Y}$ using "k" auxiliary attributes, for any case (full, partial or no information case), such that generalized mean square error of each estimator is "$\text{MSE}(T) = \sigma_T^2$".Let t be a particular estimator of above mentioned family of estimator, such that "$\text{Bias}(t) = B_t$".Then for any shrinkage estimator "$t^* = \eta t$" of t, we have $\text{Bias}(t^*) = \frac{B_t - \bar{Y}^{-1}\sigma_T^2}{1+\bar{Y}^{-2}\sigma_T^2}$.

Proof:

We have $t = (\bar{y}, \xi_1, \xi_2, ..., \xi_k)$, (6.3.1) where $\xi_1, \xi_2, ..., \xi_k$ are auxiliary attributes.

After some mathematical expansion to term of order n^{-2} we have,

$$\text{Bias}(t) = E\left(t - \bar{Y}\right) \approx$$

$$E\left[\text{Terms of order "}n^{-1}\text{"} + \text{Terms of order "}n^{-2}\text{"}\right] = B_t. \quad (6.3.2)$$

Such that $E\left[\text{Terms of order "}n^{-1}\text{"}\right] = 0$.

Let $t^* = \eta t = \eta(\bar{y}, \xi_1, \xi_2, ..., \xi_k)$, (6.3.3)

After some mathematical expansion to term of order n^{-2} we have,

$\text{Bias}(t^*) \approx E\left[\left(\eta - 1\right)\bar{Y} + \eta\left\{\text{Terms of order "}n^{-1}\text{"} + \text{Terms of order "}n^{-2}\text{"}\right\}\right]$ or

$$\text{Bias}(t^*) \approx \left(\eta - 1\right)\bar{Y} + \eta B_t. \quad (6.3.4)$$

The mean square error of t^* will be,

$\text{MSE}(t^*) \approx E\left[\left(\eta - 1\right)\bar{Y} + \eta\left\{\text{Terms of order "}n^{-1}\text{"}\right\}\right]^2$. Or

$$\text{MSE}(t^*) \approx E\left[\left(\eta - 1\right)^2 \bar{Y} + \eta^2\left\{\text{Terms of order "}n^{-1}\text{"}\right\}^2 + 2\eta\left(\eta - 1\right)\bar{Y}\left\{\text{Terms of order "}n^{-1}\text{"}\right\}\right].$$

On simplification we get,

$$\text{MSE}(t^*) \approx \left(\eta - 1\right)^2 \bar{Y} + \eta^2 \sigma_T^2. \quad (6.3.5)$$

For optimum value of η we differentiate (6.4.5) with respect to η and on simplification we get,

$$\eta = \frac{1}{1 + \bar{Y}^{-2}\sigma_T^2}. \quad (6.3.6)$$

Using (6.4.6) in (6.4.4) and on simplification we get,

$$\text{Bias}(t^*) = \frac{B_t - \bar{Y}^{-1}\sigma_T^2}{1 + \bar{Y}^{-2}\sigma_T^2}. \quad (6.3.7)$$

If t is unbiased estimator of $\bar{Y}$ then $B_t = 0$, for which $\text{Bias}(t^*) = \frac{-\bar{Y}^{-1}\sigma_T^2}{1 + \bar{Y}^{-2}\sigma_T^2}$,

this shows shrinkage estimator of unbiased conventional estimator is biased.

6.4- A SHRINKAGE FAMILY OF ESTIMATORS FOR FULL PARTIAL AND NO INFORMATION CASES USING TWO AUXILIARY ATTRIBUTES

In this section a general shrinkage family of estimators has been suggested for full, partial and no information cases. A general expression for bias and mean square error has also been given.

6.4.1-A Shrinkage Family of Estimators for Single-Phase Sampling

Shrinkage version of general family of estimators $\hat{T}_{32(1)}$ is:

$$\hat{T}_{63(1)} = \frac{\hat{T}_{32(1)}}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{32(1)}\right)}, \tag{6.4.1}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error for each estimator of proposed general Shrinkage family $\hat{T}_{63(1)}$ is:

$$MSE\left(\hat{T}_{63(1)}\right) = \frac{MSE\left(\hat{T}_{32(1)}\right)}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{32(1)}\right)}. \tag{6.4.2}$$

If $B_{\hat{T}_{32(1)}}$ is bias for any estimator of conventional family of estimators $\hat{T}_{32(1)}$, then bias for corresponding estimator of Shrinkage family $\hat{T}_{63(1)}$ is: [using (6.3.7)],

$$Bias\left(\hat{T}_{63(1)}\right) = \frac{B_{\hat{T}_{32(1)}} - \bar{Y}^{-2}MSE\left(\hat{T}_{32(1)}\right)}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{32(1)}\right)}. \tag{6.4.3}$$

6.4.2- A Shrinkage Family of Estimators for Two-Phase Sampling for Partial Information Case

Shrinkage version of general family of estimators $\hat{T}_{42(2)}$ is:

$$\hat{T}_{64(2)} = \frac{\hat{T}_{42(2)}}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{42(2)}\right)}, \tag{6.4.4}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error for each estimator of proposed general Shrinkage family $\hat{T}_{64(2)}$ is:

$$MSE\left(\hat{T}_{64(2)}\right) = \frac{MSE\left(\hat{T}_{42(2)}\right)}{1+\bar{Y}^{-2}MSE\left(\hat{T}_{42(2)}\right)}. \tag{6.4.5}$$

If $B_{\hat{T}_{42(2)}}$ is bias for any estimator of conventional family of estimators $\hat{T}_{42(2)}$, then bias for corresponding estimator of Shrinkage family $\hat{T}_{64(2)}$ is: [using (6.3.7)],

$$\text{Bias}\left(\hat{T}_{64(2)}\right)=\frac{B_{\hat{T}_{42(2)}}-\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{42(2)}\right)}{1+\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{42(2)}\right)}. \tag{6.4.6}$$

The shrinkage version of general family of estimators $\hat{T}_{52(2)}$ is:

$$\hat{T}_{65(2)}=\frac{\hat{T}_{52(2)}}{1+\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{52(2)}\right)}, \tag{6.4.7}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error for each estimator of proposed general Shrinkage family $\hat{T}_{65(2)}$ is:

$$\text{MSE}\left(\hat{T}_{65(2)}\right)=\frac{\text{MSE}\left(\hat{T}_{52(2)}\right)}{1+\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{52(2)}\right)}. \tag{6.4.8}$$

6.4.3 A Shrinkage Family of Estimators for Two-Phase Sampling for No Information Case

Shrinkage version of general family of estimators $\hat{T}_{53(2)}$ is:

$$\hat{T}_{66(2)}=\frac{\hat{T}_{53(2)}}{1+\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{53(2)}\right)}, \tag{6.4.9}$$

using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error for each estimator of proposed general family $\hat{T}_{66(2)}$ is:

$$\text{MSE}\left(\hat{T}_{66(2)}\right)=\frac{\text{MSE}\left(\hat{T}_{53(2)}\right)}{1+\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{53(2)}\right)}. \tag{6.4.10}$$

If $B_{\hat{T}_{53(2)}}$ is bias for any estimator of conventional family of estimators $\hat{T}_{53(2)}$, then bias for corresponding estimator of Shrinkage family $\hat{T}_{66(2)}$ is: [using (6.3.7)],

$$\text{Bias}\left(\hat{T}_{66(2)}\right)=\frac{B_{\hat{T}_{53(2)}}-\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{53(2)}\right)}{1+\bar{Y}^{-2}\text{MSE}\left(\hat{T}_{53(2)}\right)}. \tag{6.4.11}$$

CHAPTER 7

SHRINKAGE REGRESSION TYPE ESTIMATORS FOR SINGLE AND TWO PHASE SAMPLING USING SINGLE AND TWO AUXILIARY- ATTRIBUTES

7.1 INTRODUCTION

In this chapter some new shrinkage regression type estimators for single and two phase sampling using single auxiliary attribute have been proposed. The new shrinkage regression type estimators may be considered as an alternate to Shabbir and Gupta (2007) estimator. This new approach has an advantage over Shabbir and Gupta (2007) estimators as it is defined for any value of sample proportion p_1 and mean square error of proposed estimator is not approximated to certain term like Shabbir and Gupta (2007) because new shrinkage regression type estimators do not contain any ratio. These new shrinkage regression type estimators have also been proposed for full partial and no information cases using two auxiliary attributes.

7.2- A SHRINKAGE REGRESSION TYPE ESTIMATOR USING SINGLE AUXILIARY ATTRIBUTE

In this section a shrinkage regression type estimator for full and no information cases, using single auxiliary attribute is proposed.

7.2.1- A Shrinkage Regression Type Estimator for Single-Phase Sampling for Full Information Case

A shrinkage regression type estimator for full information case is:

$$t_{67(1)} = d_0\left[\bar{y} - d_1\left(p_1 - P_1\right)\right] = d_0 t^*_{67(1)}, \tag{7.2.1}$$

or $$t_{67(1)} = \frac{t^*_{67(1)}}{1 + \bar{Y}^{-2}MSE\left(t^*_{67(1)}\right)}$$

where $$d_0 = \frac{1}{1 + \bar{Y}^{-2}MSE\left(t^*_{67(1)}\right)}$$ and

$t^*_{67(1)} = \bar{y} - d_1\left(p_1 - P_1\right)$, d_1 is unknown parameter to be determined later.

Using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{67(1)}$ is:

$$MSE\left(t_{67(1)}\right)=\frac{MSE\left(t^*_{67(1)}\right)}{1+\bar{Y}^{-2}MSE\left(t^*_{67(1)}\right)} \tag{7.2.2}$$

The mean square error of $t^*_{67(1)}$ is:

$$MSE\left(t^*_{67(1)}\right)=E\left(t^*_{67(1)}-\bar{Y}\right)^2=E\left(\bar{e}_y-d_1\bar{e}_{\tau_1}\right)^2. \tag{7.2.3}$$

Expanding the R.H.S. of equation (7.2.3):

$$MSE\left(t^*_{67(1)}\right)=E\left[\bar{e}_y^2+d_1^2\bar{e}_{\tau_1}^2-2d_1\bar{e}_y\bar{e}_{\tau_1}\right]. \tag{7.2.4}$$

Using (1.5.4), (1.5.5), (1.5.6) and (1.5.7) in (7.2.4) and on simplification

$$MSE\left(t^*_{67(1)}\right)=\theta\left(\bar{Y}^2C_y^2+d_1^2P_1^2C_{\tau_1}^2-2d_1\bar{Y}P_1C_yC_{\tau_1}\rho_{Pb_1}\right).$$

(7.2.5)

Differentiating (7.2.5) w.r.t. d_1 and equating the derivative to zero:

$$d_1=\frac{\bar{Y}C_y}{P_1C_{\tau_1}}\rho_{Pb_1}. \tag{7.2.6}$$

Using (7.2.6) in (7.2.5) and on simplification the mean square error of $t^*_{67(1)}$ is:

$$MSE\left(t^*_{67(1)}\right)=\theta\left(1-\rho_{Pb_1}^2\right)\bar{Y}^2C_y^2$$

(7.2.7)

Using (7.2.7) in (7.2.2):

$$MSE\left(t_{67(1)}\right)=\frac{\theta\left(1-\rho_{Pb_1}^2\right)\bar{Y}^2C_y^2}{1+\theta\left(1-\rho_{Pb_1}^2\right)C_y^2}. \tag{7.2.8}$$

This is not approximate like Shabbir and Gupta (2007) but identical to the mean square error of their suggested estimator given in (3.4.1).

7.2.2-A Shrinkage Regression Type Estimator for Two-Phase Sampling for No Information Case

A shrinkage regression type estimator for no information case is:

$$t_{68(2)}=\phi_0\left[\bar{y}_2-\phi_1\left(p_{1(2)}-p_{1(1)}\right)\right]=\phi_0t^*_{68(2)}, \tag{7.2.9}$$

or $$t_{68(2)}=\frac{t^*_{68(2)}}{1+\bar{Y}^{-2}MSE\left(t^*_{68(2)}\right)}$$

where $\phi_0 = \dfrac{1}{1+\bar{Y}^{-2}MSE\left(t^*_{68(2)}\right)}$ and $t^*_{68(2)} = \bar{y}_2 - \phi_1\left(p_{1(2)} - p_{1(1)}\right)$, ϕ_1 is unknown parameter to be determined later.

Using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{68(2)}$ is:

$$MSE\left(t_{68(2)}\right) = \frac{MSE\left(t^*_{68(2)}\right)}{1+\bar{Y}^{-2}MSE\left(t^*_{68(2)}\right)} \tag{7.2.10}$$

The mean square error of $t^*_{68(2)}$ is:

$$MSE\left(t^*_{68(2)}\right) = E\left\{\bar{e}_{y_2} - \phi_1\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}}\right)\right\}^2. \tag{7.2.11}$$

Expanding the R.H.S. of equation (7.2.11) and on simplification:

$$MSE\left(t^*_{68(2)}\right) = E\left\{\bar{e}^2_{y_2} + \phi_1^2\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}}\right)^2 - 2\phi_1\bar{e}_{y_2}\left(\bar{e}_{\tau_{1(2)}} - \bar{e}_{\tau_{1(1)}}\right)\right\}. \tag{7.2.12}$$

Using (1.5.12), (1.5.13) and (1.5.14), (1.5.15) in (7.2.12) and on simplification:

$$MSE\left(t^*_{68(2)}\right) = \left\{\theta_2\bar{Y}^2C_y^2 + \theta_3\left(\phi_1^2P_1^2C_{\tau_1}^2 - 2\phi_1\bar{Y}P_1C_yC_{\tau_1}\rho_{Pb_1}\right)\right\}. \tag{7.2.13}$$

Differentiating (7.2.13) w.r.t. ϕ_1 and equating the derivative to zero

$$\phi_1 = \frac{\bar{Y}C_y}{P_1C_{\tau_1}}\rho_{Pb_1}. \tag{7.2.14}$$

Using (7.2.14) in (7.2.13) and on simplification the mean square error of $t^*_{68(2)}$ is:

$$MSE\left(t^*_{68(2)}\right) = \left(\theta_2 - \theta_3\rho^2_{Pb_1}\right)\bar{Y}^2C_y^2$$

(7.2.15)

Using (7.2.15) in (7.2.10):

$$MSE\left(t_{68(2)}\right) = \frac{\left[\theta_2 - \theta_3\rho^2_{Pb_1}\right]\bar{Y}^2C_y^2}{1+\left[\theta_2 - \theta_3\rho^2_{Pb_1}\right]C_y^2}. \tag{7.2.16}$$

This is not approximate like Shabbir and Gupta (2007) but identical to the mean square error of their suggested estimator given in (3.4.3).

7.3- A SHRINKAGE REGRESSION TYPE ESTIMATOR USING TWO AUXILIARY ATTRIBUTES

In this section a new shrinkage regression type estimator for full, partial and no information cases, using two auxiliary attributes is proposed.

7.3.1 - A Shrinkage Regression Type Estimator for Single-Phase Sampling

A shrinkage regression type estimator for full information case, using two auxiliary attributes is:

$$t_{69(1)} = d_0\left[\bar{y} - d_1\left(p_1 - P_1\right) - d_2\left(p_2 - P_2\right)\right] = d_0 t^*_{69(1)}, \tag{7.3.1}$$

or $$t_{69(1)} = \frac{t^*_{69(1)}}{1+\bar{Y}^{-2}MSE\left(t^*_{69(1)}\right)}$$

where $d_0 = \dfrac{1}{1+\bar{Y}^{-2}MSE\left(t^*_{69(1)}\right)}$ and

$t^*_{69(1)} = \bar{y} - d_1\left(p_1 - P_1\right) - d_2\left(p_2 - P_2\right)$,

d_1, d_2 are unknown parameters to be determined later.

Using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{69(1)}$ is:

$$MSE\left(t_{69(1)}\right) = \frac{MSE\left(t^*_{69(1)}\right)}{1+\bar{Y}^{-2}MSE\left(t^*_{69(1)}\right)} \tag{7.3.2}$$

The mean square error of $t^*_{69(1)}$ is:

$$MSE\left(t^*_{69(1)}\right) = E\left\{\bar{e}_y - \sum_{j=1}^{2} d_j \bar{e}_{\tau_j}\right\}^2 . \tag{7.3.3}$$

Expanding the R.H.S. of (7.3.3):

$$MSE\left(t^*_{69(1)}\right) = E\left\{\bar{e}_y^2 + \sum_{j=1}^{2} d_j^2 \bar{e}_{\tau_j}^2 - 2\sum_{j=1}^{2} d_j \bar{e}_y \bar{e}_{\tau_j} + 2\sum_{j\neq\psi=1}^{2} d_j d_\psi \bar{e}_{\tau_j}\bar{e}_{\tau_\psi}\right\}. \tag{7.3.4}$$

Using (1.5.4), (1.5.5) (1.5.6) and (1.5.7) in (7.3.4) and on simplification:

$$MSE\left(t^*_{69(1)}\right) = \theta\left\{\bar{Y}^2 C_y^2 + \sum_{j=1}^{2} d_j^2 P_j^2 C_{\tau_j}^2 - 2\sum_{j=1}^{2} d_j \bar{Y} P_j C_y C_{\tau_j}\rho_{Pb_j} + 2\sum_{j\neq\psi=1}^{2} d_j d_\psi P_j P_\psi C_{\tau_j} C_{\tau_\psi} Q_{i\psi}\right\}. \tag{7.3.5}$$

Differentiating (7.3.5) with respect to d_1 and d_2 and equating the derivative to zero:

$$\sum_{j=1}^{2} d_j P_j C_{\tau_j} Q_{1j} = \bar{Y} C_y \rho_{Pb_1}. \quad (7.3.6)$$

$$\sum_{j=1}^{2} d_j P_j C_{\tau_j} Q_{2j} = \bar{Y} C_y \rho_{Pb_2}. \quad (7.3.7)$$

Solving (7.3.6) and (7.3.7)

$$d_1 = \frac{\bar{Y} C_y \left(\rho_{pb_1} - Q_{12}\rho_{pb_2}\right)}{P_1 C_{\tau_1}\left(1 - Q_{12}^2\right)}, \quad (7.3.8)$$

$$\text{and } d_2 = \frac{\bar{Y} C_y \left(\rho_{Pb_2} - Q_{12}\rho_{Pb_1}\right)}{P_2 C_{\tau_2}\left(1 - Q_{12}^2\right)}. \quad (7.3.9)$$

Using (7.3.6) and (7.3.7) in (7.3.5) the mean square error of $t^*_{69(1)}$ is:

$$MSE(t^*_{69(1)}) = \theta\left(1 - \rho^2_{y.\tau_1\tau_2}\right)\bar{Y}^2 C_y^2 \quad (7.3.10)$$

Using (7.3.10) in (7.3.2):

$$MSE(t_{69(1)}) = \frac{\theta\left(1 - \rho^2_{y.\tau_1\tau_2}\right)\bar{Y}^2 C_y^2}{1 + \theta\left(1 - \rho^2_{y.\tau_1\tau_2}\right) C_y^2}. \quad (7.3.11)$$

It can be easily checked that $MSE\left(t_{6(1)}\right) - MSE\left(t_{69(1)}\right) \geq 0$.

Hence shrinkage regression type estimator $t_{69(1)}$ is more efficient than Shabbir and Gupta (2007) estimator $t_{6(1)}$ under the condition that $\rho^2_{y.\tau_1\tau_2} > \rho^2_{pb_1}$.

7.3.2 - A Shrinkage Regression Type Estimator for Two-Phase Sampling for Partial Information Case

A shrinkage regression type estimator for partial information case, using two auxiliary attributes is:

$$t_{70(2)} = \gamma_0\left[\bar{y}_2 - \gamma_1\left(p_{1(2)} - P_1\right) - \gamma_2\left(p_{2(2)} - p_{2(1)}\right)\right] = \gamma_0 t^*_{70(2)}, \quad (7.3.12)$$

or $$t_{70(2)} = \frac{t^*_{70(2)}}{1 + \bar{Y}^{-2} MSE\left(t^*_{70(2)}\right)}$$

where $\gamma_0 = \frac{1}{1 + \bar{Y}^{-2} MSE\left(t^*_{70(2)}\right)}$ and

$$t^*_{70(2)} = \bar{y}_2 - \gamma_1\left(p_{1(2)} - P_1\right) - \gamma_2\left(p_{2(2)} - p_{2(1)}\right),$$

γ_1, γ_2 are unknown parameters to be determined later.
Using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{70(2)}$ is:

$$MSE\left(t_{70(2)}\right)=\frac{MSE\left(t^*_{70(2)}\right)}{1+\bar{Y}^{-2}MSE\left(t^*_{70(2)}\right)} \tag{7.3.13}$$

The mean square error of $t^*_{70(2)}$ is:

$$MSE\left(t^*_{70(2)}\right)=\left\{\bar{e}_{y_2}-\gamma_1\bar{e}_{\tau_{1(2)}}-\gamma_2\left(\bar{e}_{\tau_{2(2)}}-\bar{e}_{\tau_{2(1)}}\right)\right\}^2. \tag{7.3.14}$$

Expanding the R.H.S. of (7.3.14):

$$MSE\left(t^*_{70(2)}\right)=E\left\{\begin{array}{l}\bar{e}^2_{y_2}+\gamma_1^2\bar{e}^2_{\tau_{1(2)}}+\gamma_2^2\left(\bar{e}_{\tau_{2(2)}}-\bar{e}_{\tau_{2(1)}}\right)^2-2\gamma_1\bar{e}_{y_2}\bar{e}_{\tau_{1(2)}}\\ -2\gamma_2\bar{e}_{y_2}\left(\bar{e}_{\tau_{2(2)}}-\bar{e}_{\tau_{2(1)}}\right)+2\gamma_1\gamma_2\bar{e}_{\tau_{1(2)}}\left(\bar{e}_{\tau_{2(2)}}-\bar{e}_{\tau_{2(1)}}\right)\end{array}\right\}. \tag{7.3.15}$$

Using (1.5.12), (1.5.13), (1.5.14) and (1.5.15) in (7.3.15) and on simplification

$$MSE\left(t^*_{70(2)}\right)=\left\{\begin{array}{l}\theta_2\bar{Y}^2C_y^2+\gamma_1^2\theta_2P_1^2C_{\tau_1}^2+\gamma_2^2\theta_3P_2^2C_{\tau_2}^2-2\gamma_1\theta_2\bar{Y}P_1C_yC_{\tau_1}\rho_{Pb_1}\\ -2\gamma_2\theta_3\bar{Y}P_2C_yC_{\tau_2}\rho_{Pb_2}+2\gamma_1\gamma_2\theta_3P_1P_2C_{\tau_1}C_{\tau_1}Q_{12}\end{array}\right\}. \tag{7.3.16}$$

Differentiating (7.3.16) w.r.t. γ_1 and γ_2 and on simplification:

$$\gamma_1\theta_2P_1C_{\tau_1}+\gamma_2\theta_3P_1C_{\tau_2}Q_{12}=\theta_2\bar{Y}C_y\rho_{Pb_1}, \tag{7.3.17}$$

and $$\gamma_1\theta_3P_1C_{\tau_1}Q_{21}+\gamma_2\theta_3P_2C_{\tau_2}=\theta_3\bar{Y}C_y\rho_{Pb_2}. \tag{7.3.18}$$

Solving (7.3.17) and (7.3.18):

$$\gamma_1=\frac{\bar{Y}C_y\left(\theta_2\rho_{Pb_1}-\theta_3\rho_{Pb_2}Q_{12}\right)}{P_1C_{\tau_1}\left(\theta_2-\theta_3Q_{12}^2\right)}, \tag{7.3.19}$$

and $$\gamma_2=\frac{\theta_2\bar{Y}C_y\left(\rho_{Pb_2}-Q_{12}\rho_{Pb_1}\right)}{P_2C_{\tau_2}\left(\theta_2-\theta_3Q_{12}^2\right)}. \tag{7.3.20}$$

Using the value of γ_1 and γ_2 in (7.3.16) and on simplification the mean square error of $t^*_{70(2)}$ is:

$$MSE\left(t^*_{70(2)}\right)=\theta_2\left[1-\frac{\theta_2\rho^2_{Pb_1}+\theta_3\rho^2_{Pb_2}-2\theta_3Q_{12}\rho_{Pb_1}\rho_{Pb_2}}{\left(\theta_2-\theta_3Q_{12}^2\right)}\right]\bar{Y}^2C_y^2 \tag{7.3.21}$$

Using (7.3.21) in (7.3.13) and on simplification:

$$\mathrm{MSE}\left(t_{70(2)}\right)=\frac{\theta_2\left[1-\frac{\theta_2\rho^2_{Pb_1}+\theta_3\rho^2_{Pb_2}-2\theta_3 Q_{12}\rho_{Pb_1}\rho_{Pb_2}}{\left(\theta_2-\theta_3 Q^2_{12}\right)}\right]\bar{Y}^2C^2_y}{1+\theta_2\left[1-\frac{\theta_2\rho^2_{Pb_1}+\theta_3\rho^2_{Pb_2}-2\theta_3 Q_{12}\rho_{Pb_1}\rho_{Pb_2}}{\left(\theta_2-\theta_3 Q^2_{12}\right)}\right]C^2_y}. \qquad (7.3.22)$$

If P_2 was also known then $\theta_2=\theta_3=\theta$, for this expression (7.3.22) become,

$$\mathrm{MSE}\left(t_{70(2)}\right)=\frac{\theta\left(1-\rho^2_{y.\tau_1\tau_2}\right)\bar{Y}^2C^2_y}{1+\theta\left(1-\rho^2_{y.\tau_1\tau_2}\right)C^2_y} .$$

This is equivalent to full information case as given in (7.3.11).

7.3.3 -A Shrinkage Regression Type Estimator for Two-Phase Sampling for No Information Case

A shrinkage regression type estimator for no information case, using two auxiliary attributes is:

$$t_{71(2)}=\delta_0\left[\bar{y}_2-\delta_1\left(p_{1(2)}-p_{1(1)}\right)-\delta_2\left(p_{2(2)}-p_{2(1)}\right)\right]=\delta_0 t^*_{71(2)}, \qquad (7.3.23)$$

or $$t_{71(2)}=\frac{t^*_{71(2)}}{1+\bar{Y}^{-2}\mathrm{MSE}\left(t^*_{71(2)}\right)}$$

where $\delta_0=\frac{1}{1+\bar{Y}^{-2}\mathrm{MSE}\left(t^*_{71(2)}\right)}$ and

$t^*_{71(2)}=\bar{y}_2-\delta_1\left(p_{1(2)}-p_{1(1)}\right)-\delta_2\left(p_{2(2)}-p_{2(1)}\right)$,

δ_1, δ_2 are unknown parameters to be determined later.

Using Shahbaz and Hanif (2009) approach given in (3.5.2), the mean square error of $t_{71(2)}$ is:

$$\mathrm{MSE}\left(t_{71(2)}\right)=\frac{\mathrm{MSE}\left(t^*_{71(2)}\right)}{1+\bar{Y}^{-2}\mathrm{MSE}\left(t^*_{71(2)}\right)} \qquad (7.3.24)$$

The mean square error of $t^*_{71(2)}$ is:

$$\mathrm{MSE}\left(t^*_{71(2)}\right)=E\left\{\bar{e}_{y_2}-\sum_{j=1}^{2}\delta_1\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)\right\}^2. \qquad (7.3.25)$$

Expanding the R.H.S. of (7.3.25) we get,

$$MSE\left(t^*_{71(2)}\right)=E\begin{Bmatrix}\bar{e}^2_{y_2}+\sum_{j=1}^{2}\delta_j^2\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)^2-2\sum_{j=1}^{2}\delta_j\bar{e}_y\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)\\ +2\sum_{j\neq\psi=1}^{2}\delta_j\delta_\psi\left(\bar{e}_{\tau_{j(2)}}-\bar{e}_{\tau_{j(1)}}\right)\left(\bar{e}_{\tau_{\psi(2)}}-\bar{e}_{\tau_{\psi(1)}}\right)\end{Bmatrix}. \quad (7.3.26)$$

Using (1.5.12), (1.5.13) (1.5.14) and (1.5.15) in (7.3.26) and on simplification:

$$MSE\left(t^*_{71(2)}\right)=\begin{Bmatrix}\theta_2\bar{Y}^2C_y^2\\ +\theta_3\left\{\sum_{j=1}^{2}\delta_j^2P_j^2C^2_{\tau_j}-2\sum_{j=1}^{2}\delta_j\bar{Y}P_jC_yC_{\tau_j}\rho_{Pb_j}+2\sum_{j\neq\psi=1}^{2}\delta_j\delta_\psi P_jP_\psi C_{\tau_j}C_{\tau_\psi}Q_{j\psi}\right\}\end{Bmatrix}.$$

(7.3.27)

The optimum value of δ_1 and δ_2 are, $\delta_1=d_1$, $\delta_2=d_2$, using the optimum value of δ_1 and δ_2 in (7.3.27) and on simplification the mean square error of $t^*_{71(2)}$ is:

$$MSE(t^*_{71(2)})=\left\{\theta_2\left(1-\rho^2_{y.\tau_1\tau_2}\right)+\theta_1\rho^2_{y.\tau_1\tau_2}\right\}\bar{Y}^2C_y^2 \quad (7.3.28)$$

Using (7.3.28) in (7.3.24) and on simplification:

$$MSE(t_{71(2)})=\frac{\left\{\theta_2\left(1-\rho^2_{y.\tau_1\tau_2}\right)+\theta_1\rho^2_{y.\tau_1\tau_2}\right\}\bar{Y}^2C_y^2}{1+\left\{\theta_2\left(1-\rho^2_{y.\tau_1\tau_2}\right)+\theta_1\rho^2_{y.\tau_1\tau_2}\right\}C_y^2}. \quad (7.3.29)$$

It can be easily checked that $MSE\left(t_{7(2)}\right)-MSE\left(t_{71(2)}\right)\geq 0$.

Hence shrinkage regression type estimator $t_{71(2)}$ is more efficient than Shabbir and Gupta (2007) estimator $t_{7(2)}$ under the condition that $\rho^2_{y.\tau_1\tau_2}>\rho^2_{pb_1}$.

It can be noted that the optimum value of δ_0,δ_1 and δ_2 involve some population parameters, these parameters are assumed to be known for the efficient use of proposed family $t_{71(2)}$ but if values of these parameters are unknown then these can be estimated from the sample.

Srivastava and Jhajj (1983) approach may also be used; say the estimator of proposed family $t_{71(2)}$ will have the same minimum mean square error, if we replace the unknown value of parameters involved in optimum value of δ_0,δ_1 and δ_2 with their consistent estimators. Similar are the remarks for all estimators proposed previously.

If there has been no first phase then $\theta_2=\theta\ \&\ \theta_1=0$, for this expression (7.3.29) become as $MSE\left(t_{71(2)}\right)=\dfrac{\left\{\theta\left(1-\rho^2_{y.\tau_1\tau_2}\right)\right\}\bar{Y}^2C_y^2}{1+\left\{\theta\left(1-\rho^2_{y.\tau_1\tau_2}\right)\right\}C_y^2}$.

This equals to expression (7.3.11) of full information case.

CHAPTER 8
EMPIRICAL STUDY

8.1 INTRODUCTION

In this Chapter numerical studies have been conducted to compare the efficiency of various estimators for single-phase and two-phase sampling. Twelve real populations (given in Appendix) from Government of Pakistan (1998) Ministry of food, agriculture and Livestock were taken. Various population parameters and sample statistics have been calculated. Mean square errors of various estimators are computed and sample estimate of each estimators are obtained. The detail of numerical study can be seen from various Tables.

Table-8.1
Relative Efficiency of Suggested Estimators (Ref) With Respect To Mean Per Unit Estimator ($\bar{y}$) for Single-Phase Sampling

Pop #	$\bar{y}$	$\hat{T}_{4(1)}$	$t_{6(1)}$	$\hat{T}_{32(1)}$ (New Estimator)	$t_{69(1)}$ (New Estimator)
1	100	108.77	120.1	118.20	129.53
2	100	142.25	153.36	149.52	160.63
3	100	142.03	152.92	143.39	154.27
4	100	122.30	134.59	124.56	136.84
5	100	102.49	114.96	126.02	138.34
6	100	111.41	123.65	114.58	126.82
7	100	146.61	155.3	239.88	248.56
8	100	225.5	233.66	268.09	276.28
9	100	125.4	132.8	186.98	194.42
10	100	105.9	137.4	107.13	136.63
11	100	107.07	136.09	107.73	136.96
12	100	101.8	129.2	109.90	137.30

REF = $[M\hat{S}E(\bar{y})/M\hat{S}E(.)]*100$

Relative efficiency of both families (Jhajj et al. (2006) family and proposed family) have been computed with respect to mean per unit estimator, $\bar{y}$ so it can be observe from Table-8.1 that proposed family of estimators $\hat{T}_{32(1)}$ is more efficient than Jhajj et al. (2006) family of estimators, $\hat{T}_{4(1)}$. Similarly it can also be observed from Table-8.1 that proposed estimator, $t_{69(1)}$ is always more efficient than Shabbir and Gupta (2007) estimator, $t_{6(1)}$ when both of these estimators are compared with mean per unit estimator $\bar{y}$.

Table-8.2
Relative Efficiency (Ref) of Suggested Estimators With Respect To Jhajj et al. (2006) Estimator ($\hat{T}_{4(1)}$) for Single-Phase Sampling

Pop #	$\hat{T}_{4(1)}$	$t_{6(1)}$	$\hat{T}_{32(1)}$ (New Estimator)	$t_{69(1)}$ (New Estimator)
1	100	110.41	108.67	119.08
2	100	107.81	105.11	112.92
3	100	107.67	100.96	108.62
4	100	110.06	101.85	111.90
5	100	112.2	122.95	135.1
6	100	110.99	102.85	113.83
7	100	105.92	163.62	169.54
8	100	103.63	118.90	122.53
9	100	105.94	149.12	155.06
10	100	129.74	101.17	130.91
11	100	127.10	100.64	127.92
12	100	126.92	107.95	134.87

REF= $[\hat{MSE}\left(t_{4(1)}\right)/\hat{MSE}(.)]$*100

Relative efficiency of proposed family of estimators, $\hat{T}_{32(1)}$ have been computed with respect to Jhajj et al. (2006) family of estimators, $\hat{T}_{4(1)}$, it can be observed from Table-8.2 that proposed family of estimators, $\hat{T}_{32(1)}$ is more efficient than Jhajj et al. (2006) family of estimators, $\hat{T}_{4(1)}$. Similarly it can also be observed from Table-8.2 that proposed estimator, $t_{69(1)}$ is always more efficient than Shabbir and Gupta (2007) estimator, $t_{6(1)}$ when both of these estimators are compared with Jhajj et al. (2006) family of estimators $\hat{T}_{4(1)}$.

Table-8.3
Ranking of Suggested Estimators for Single-Phase Sampling

Pop #	$\bar{y}$	$\hat{T}_{4(1)}$	$t_{6(1)}$	$\hat{T}_{32(1)}$ **(New Estimator)**	$t_{69(1)}$ **(New Estimator)**
1	5	4	2	3	1
2	5	4	2	3	1
3	5	4	2	3	1
4	5	4	2	3	1
5	5	4	3	2	1
6	5	4	2	3	1
7	5	4	3	2	1
8	5	4	3	2	1
9	5	4	3	2	1
10	5	4	2	3	1
11	5	4	2	3	1
12	5	4	2	3	1
Average Ranks	5	4	2.333	2.667	1

Ranks of various estimators have been given in Table-8.3. Average ranks are given in last row of Table-8.3, it can be observed from Table-8.3 that, the average ranks of new proposed estimator, $t_{69(1)}$ is always 1 which shows that proposed estimator, $t_{69(1)}$ is the most efficient than all existing estimators, in which auxiliary attributes are used. The average rank of proposed family of estimators, $\hat{T}_{32(1)}$ is 2.667 which shows that proposed family of estimators, $\hat{T}_{32(1)}$ is always more efficient than Jhajj et al. (2006) family of estimators $\hat{T}_{4(1)}$.

Table-8.4
Relative Efficiency (Ref) of Suggested Estimators With Respect To Mean Per Unit Estimator ($\bar{y}_2$) for Two-Phase Sampling

Pop #	$\bar{y}_2$	$\hat{T}_{5(2)}$	$t_{7(2)}$	$\hat{T}_{53(2)}$ (New Estimator)	$t_{71(2)}$ (New Estimator)
1	100	104.62	115.94	109.20	120.52
2	100	119.40	130.50	122.12	133.23
3	100	119.31	130.20	119.84	130.72
4	100	111.08	123.37	112.09	124.375
5	100	101.11	113.58	112.73	125.20
6	100	105.93	118.17	107.48	119.72
7	100	115.15	123.83	140.33	149.01
8	100	147.90	156.08	157.47	165.65
9	100	113.36	120.80	137.12	144.36
10	100	103.40	134.90	104.10	135.60
11	100	104.04	396.06	104.43	396.44
12	100	101.05	128.45	105.60	133.00

REF= $[\hat{MSE}\ (\bar{y}_2)/\hat{MSE}\ (.)]$*100

Relative efficiency of both families (Jhajj et al. (2006) family and proposed family) have been computed with respect to mean per unit estimator, $\bar{y}_2$ so it can be observed from Table-8.4 that proposed family of estimators, $\hat{T}_{53(2)}$ is more efficient than Jhajj et al. (2006) family of estimators, $\hat{T}_{5(2)}$ for two-phase sampling. Similarly it can also be observed from Table-8.4 that proposed estimator, $t_{71(2)}$ is always more efficient than Shabbir and Gupta (2007) estimator, $t_{7(2)}$ for two-phase sampling, when both of these estimator are compared with mean per unit estimator $\bar{y}_2$. It can be observed that efficiencies of $t_{7(2)}$ and $t_{71(2)}$ are approximately same for population "3" and "5", this is because $\rho^2_{pb_1}$ and $\rho^2_{y.\tau_1\tau_2}$ approximately same.

Table-8.5
Relative Efficiency (Ref) of Suggested Estimator With Respect To Jhajj et al. (2006) Estimator ($\hat{T}_{5(2)}$) for Two-Phase Sampling

Pop #	$\hat{T}_{5(2)}$	$t_{7(2)}$	$\hat{T}_{53(2)}$ **(New Estimator)**	$t_{71(2)}$ **(New Estimator)**
1	100	108.80	104.38	115.20
2	100	110.97	102.28	111.59
3	100	117.18	100.44	109.56
4	100	108.04	100.91	111.97
5	100	111.04	111.49	123.81
6	100	115.33	101.46	113.01
7	100	107.54	121.87	129.40
8	100	105.53	106.47	112.00
9	100	106.55	120.95	127.51
10	100	130.46	100.67	131.13
11	100	380.66	100.37	128.43
12	100	127.12	104.49	131.61

REF= $[\hat{MSE}\ (t_{6(2)})/\hat{MSE}\ (.)]*100$

Relative efficiency of proposed family of estimators, $\hat{T}_{53(2)}$ have been computed with respect to Jhajj et al. (2006) family of estimators, $\hat{T}_{5(2)}$ so it can be observed from Table-8.5 that proposed family of estimators, $\hat{T}_{53(2)}$ is more efficient than Jhajj et al. (2006) family of estimators, $\hat{T}_{5(2)}$ for two-phase sampling. Similarly it can also be observed from Table-8.5 that proposed estimator, $t_{71(2)}$ is always more efficient than Shabbir and Gupta (2007) estimator, $t_{7(2)}$ when both of these estimators are compared with Jhajj et al. (2006) family of estimators, $\hat{T}_{5(2)}$ for two-phase sampling.

Table-8.6
Ranking of Suggested Estimators for Two-Phase Sampling

Pop #	$\bar{y}_2$	$\hat{T}_{5(2)}$	$t_{7(2)}$	$\hat{T}_{53(2)}$	$t_{71(2)}$
1	5	4	2	3	1
2	5	4	2	3	1
3	5	4	2	3	1
4	5	4	2	3	1
5	5	4	2	3	1
6	5	4	2	3	1
7	5	4	3	2	1
8	5	4	3	2	1
9	5	4	3	2	1
10	5	4	2	3	1
11	5	4	2	3	1
12	5	4	2	3	1
Average Ranks	5	4	2.25	2.75	1

Ranks of various estimators have been given in Table-8.6 for two-phase sampling. Average ranks are given in last row of Table-8.6, it can be observed from Table-8.6 that, the average ranks of new proposed estimator, $t_{71(2)}$ is always 1 which shows that proposed estimator $t_{71(2)}$ is the most efficient than all existing estimators for two-phase sampling, in which auxiliary attributes are used. The average rank of proposed family of estimators, $\hat{T}_{53(2)}$ is 2.5 which shows that proposed family of estimators, $\hat{T}_{53(2)}$ is always more efficient than Jhajj et al. (2006) family of estimators, $\hat{T}_{5(2)}$ also proposed family of estimators, $\hat{T}_{53(2)}$ is as efficient as Shabbir and Gupta (2007) estimator $t_{7(2)}$ for two-phase sampling.

Table-8.7
Comparison of Full, Partial and No Information Cases for Generalized Jhajj et al. (2006) Estimators (Relative Efficiency)

Pop #	Relative efficiency of full & partial information to no information		Relative efficiency of Full information to partial information
	$\hat{T}_{32(1)}$ (Full information)	$\hat{T}_{42(2)}$ (Partial information)	$\hat{T}_{32(1)}$ (Full information)
1	108.2466	103.5194	104.56646
2	122.4347	116.9979	104.64689
3	119.6564	118.7813	100.73673
4	111.129	109.8239	101.18835
5	111.7863	100.9679	110.71469
6	106.6035	104.9007	101.62329
7	170.9397	108.7959	157.11966
8	170.2505	144.9148	117.48314
9	136.3569	111.3599	122.44703
10	102.9238	102.5836	100.3317
11	103.192	102.7445	100.43559
12	104.0688	100.7416	103.30275

Table-8.8
Comparison of Full, Partial and No Information Cases for Generalized New Shrinkage Estimators (Relative Efficiency)

Pop #	Relative efficiency of full & partial information to no information		Relative efficiency of Full information to partial information
	$t_{69(1)}$ (Full information)	$t_{70(2)}$ (Partial information)	$t_{69(1)}$ (Full information)
1	107.4748	103.192	104.1504
2	120.5643	115.5812	104.3114
3	118.0197	117.2217	100.6808
4	110.0295	109.008	100.9371
5	110.6128	100.9286	109.5951
6	105.9285	104.2866	101.5745
7	166.8066	108.2819	154.0484
8	166.7876	142.6934	116.8853
9	134.4831	110.7719	121.4053
10	102.2447	101.9841	100.2555
11	102.4953	102.1453	100.3426
12	103.2306	100.5894	102.6257

8.2 CONCLUSIONS

Proposed estimator $t_{69(1)}$ is recommended to estimate the population mean for full information case as $t_{69(1)}$ outperform all the existing estimators for full information. Similarly $t_{71(2)}$ is recommended to estimate the population mean for no information case as $t_{71(2)}$ outperform all the existing estimators for no information.

It is also recommended that full information should always be preferred if possible, otherwise partial information are the best choice, no information case are recommended when there is no other choice. It can easily observed from table 8.7 and 8.8 that the estimators based on full information case are always more efficient than estimators based on partial and no information. It can also be observed that estimators based on partial information case are always more efficient than estimators based no information

REFERENCES

1. Ahmad, Z. (2007). *Generalized multivariate ratio and regression estimators for multi -phase sampling.* Ph.D. thesis submitted to National College of Business Administration & Economics Lahore 40E-I, Gulberg III, Lahore, Pakistan.

2. Ahmad, Z., Hanif, M. and Ahmad, M. (2009a). Generalized regression-in- regression estimators for two-phase sampling using multi-auxiliary variables. Submitted for publication.

3. Ahmad, Z., Hanif, M. and Ahmad, M. (2009b). Generalized regression-cum-ratio estimators for two-phase sampling using multi-auxiliary variables. *Pak. J. Statist.,* 25(2), 93-106.

4 Ahmad, Z., Hanif, M. and Ahmad, M. (2009c). Generalized multivariate regression estimators for multi-phase sampling using multi-auxiliary variables (submitted for publication).

5. Ahmed, M.S. (1998). A note on regression-type estimators using multiple auxiliary information. *Austral. & New Zealand J. Stat., 43(3), 373-376*

6. Ahmed, M.S., Khan, S.U. and Tripathi, T.P. (1994). Two general classes of chain ratio and product estimators for a finite population mean based on two-phase sampling and multivariate information. *Jour. Stat. Stud.,* 14, 86-99.

7. Ahmed, S.E. and Rohatgi, V.K. (1996).Shrinkage estimation of the proportion in randomized response. *Metrika,* 43, 17-30.

8. Armstrong, J. and St-Jean, H. (1993).Generalized regression estimator for two phase sample of tax records. *Survey Methodology,* 20, 91-105.

9. Biradar, R.S. and Singh, H.P.(1997). A class of estimators for population parameter using supplementary information. *Aligarh, J. Stat.,* 17, 54-71.

10. Bowley, A.L. (1926). Measurements of precision attained in sampling. *Bull. Inst. Internat. Statist,* 22(1), 1-62.

11. Chand, L. (1975). *Some ratio type estimators based on two or more auxiliary variables*, Unpublished Ph.D. thesis, Iowa State University, Ames, Iowa (USA).

12. Chandra, P. and Singh, H.P. (2003). A family of unbiased estimators in two- phases sampling using two auxiliary variables. *Statistics in Transition*, Vol. 6, No.1, 131-141.

13. Cochran, W.G. (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. *J. Agri. Sc.,* 30, 262-275.

14. Cochran, W.G. (1942). Sampling theory when the sampling units are of unequal sizes. *J. Amer. Statist. Assoc.,* 37, 199-212.

15. Copas, J. B. (1997). Using regression models for prediction: shrinkage and regression to the mean, *Statistical Methods in Medical Research,* Vol. 6, No. 2, 167-183

16. Copas, J.B. (1983). Regression, prediction, shrinkage. *J. Roy. Statist. Soc.,* (Ser. B) 45, 311-354.

17. Das, A.K. (1988). *Contribution to the theory of sampling strategies based on auxiliary information*, Ph.D. thesis submitted to B.C.K.V., Mohanpur, Nadia, and West Bengal, India.

18. Das, A.K. and Tripathi, T.P. (1980). Sampling strategies for population mean when the coefficient of variation of an auxiliary character is known. *Sankhya.,* Ser C. 42, 76-86.

19. Diana, G. and Tommasi, C. (2003). On optimal estimation for finite population mean in two-phase sampling. *Statistical Methods and Application,* 12, 41-48.

20. Diana, G. and Tommasi, C. (2004). Optimal use of two auxiliary variables in double sampling. *Statistical Methods and Application,* 13,275-284.

21. Diana, G., Tommasi, C. and Pero, P. (2007). Multi-Phase sampling under cost constraints. *Statistical Methods and Application,* 16,309-319.

22. Dorfman, A.H. (1994). A note on variance estimation for the regression estimator in double sampling. *Jour. Amer. Statist. Assoc.,* 89, 137-140.

23. Dubey, V. (2006). A generalized estimator of population mean using auxiliary information in general sampling design. *Aligarh J.Statist.,* Vol. 26, 57-64.

24. Fuller, W.A. and Legg, J.C. (2008). Two-phase sampling. A Chapter in the *Book Foundation of Survey Sampling,* to be published by J.Wiley.

25. George, E.I. (1991). Shrinkage domination in a multivariate common mean problem. *Ann. Statist.,* 19, 952-960.

26. Government of Pakistan (1998). *Crop Area Production by Districts (1995-96 to 1997-98).* Ministry of Food, Agriculture and Livestock. Food, Agriculture and Livestock Division, Economic Wing, Islamabad.

27. Hanif, M., Ahmad, Z. and Ahmad, M. (2009a). Generalized multivariate ratio estimators using multi-auxiliary variables for multi-phase sampling (submitted for publication).

28 Hanif, M., Hammd, N. and Shahbaz, M. Q. (2009b). A Modifie Regression Type Estimator in Survey Sampling, (submitted for publication).

29. Hansen, M.H. and Hurwitz, W.N. (1943). On the theory of sampling from finite populations. *Ann. Math. Statist.,* 14, 333-362.

30. Hidirogou, M.A. and Sarndal, C.E. (1998). Use of auxiliary information for two-phase sampling. *Survey Methodology,* 24(1), 11-20.

31. James, W. and Stein, C. (1961). Estimation with quadratic loss. *Proc. Fourth Berkeley Symp. Math. Statist. Prob.,* Vol. 1, 361--379. Univ. of California Press.

32. Jhajj, H.S., Sharma, M.K. and Grover, L.K. (2006). A family of estimators of population mean, using information on auxiliary attribute. *Pak. J. Statist.,* 22(1), 43-50.

33. Kamal, A., Shahbaz, M. Q. and Hanif M. (2009). Modified regression-type estimator in two-phase sampling using arbitrary probabilities. *Pak. J. Statist.* 25(2), 141-147.

34. Kaur, P. (1983). Generalized unbiased product estimators. *Pure Appl. Math. Sc.,* 17, 1-2, 67-79.

35. Khan, S. and Saleh, A.K.Md.E. (1997). Shrinkage Pre-test estimator of the intercept parameter for a regression model with multivariate

Student-t errors. *Biometrical Journal,* Vol. 29, 131-147.

36. Kiaer, A.N. (1895). Observations et expériences concernant les dénombrements representatives. *Bulletin de l'institut international de statistique*, 1895, 9, livre 2, 176-183. Session de Berne en 1895. La discussion est dans le livre 1, xciii-xcviii.

37. Kiaer, A.N. (1899). Sur les méthodes représentatives ou typologiques appliquées à la statistique. *Bulletin de l'institut international de statistiques*, 1899, 11, livre 1, 180-185. Session de Saint Petersbourg en 1897.

38. Kiaer, A.N. (1903). Sur les méthodes représentatives ou typologiques. *Bulletin de l'institut international de statistique*, 1903, 13, livre 1, 66-70, discussion 70-78. Session de Budapest en 1901.

39. Kiaer, A.N. (1905). Discours sans titre sur la méthode représentative. *Bulletin de l'institut international de statistiques*, 1905, 14, livre 1, 119-126, discussion 126-134. Session de Berlin en 1903.

40. Kiregyera, B. (1980). A chain ratio type estimator in finite population double sampling using two auxiliary variables. *Metrika,* 27, 217-223.

41. Kiregyera, B. (1984). Regression type estimators using two auxiliary variables and the model of double sampling from finite population. *Metrika,* 31, 215-226.

42. Krishnamoorthy, K. (1992). On shrinkage estimator of a normal common mean vector. *J. Multivariate Anal.,* 40, 109-114.

43. Kubokawa, T. (1994). Double shrinkage estimation of ratio of scale parameters. *Ann. Inst. Statist. Math.,* 46, 95-116.

44. Kulkarni, S.P. (1978). A note on modified ratio estimator using transformation. *J. Ind. Soc. Agri. Statist.,* 30, 2, 125-128.

45. Lui, K.J. (1990). Modified product estimators of finite population mean infinite sampling. *Comm. in Statist. Theory and Methods.,* 19(10), 3700-3807.

46. Metha, J.S. and Srinivasan, R. (1971). Estimation of the mean by shrinkage to a point. *Jour. Amer. Statist. Assoc.,* 66, 86-90.

47. Mickey, M.R. (1959). Some finite population unbiased ratio and regression estimators. *Jour. Amer. Statist. Assoc.,* 54, 594-612.

48. Mohanty, S. (1967). Combination of Regression and Ratio Estimate. *J. Ind. Statist, Assoc.,* 5, 16-29.

49. Mukherjee, R., Rao. T.J. and Vijayan, K. (1987). Regression type estimators using multi-auxiliary information. *Austral. J. Statist.,* 29, 244-254.

50. Murthy, M.N. (1964). Product method of estimation. *Sankhya,* Ser. A. 26, 294-307.

51. Naik, V.D. and Gupta, P.C. (1991). A general class of estimators for estimating population mean, using auxiliary information. *Metrika,* 38, 11-17.

52. Naik, V.D. and Gupta, P.C. (1996). A note on estimation of mean with known population of an auxiliary character. *J. Ind. Soc. Agri. Statist.,* 48(2), 151-158.

53. Neyman, J. (1934). On the two different aspects of representative method: The method of stratified sampling and the method of purposive selection. *J. Roy. Statist. Soc.,* 97, 558-606.

54. Neyman, J. (1938). Contributions to the theory of sampling human populations. *J. Amer. Statist. Assoc.,* 33, 101-116.

55. Nieto De Pascual, J. (1961).Unbiased ratio estimators in stratified sampling. *Jour. Amer. Statist. Assoc., 56, 70-87.*

56. Olkin, I. (1958). Multivariate ratio estimation for finite populations. *Biometrika,* 45, 154-165.

57. Pandey, B.N. (1979). On shrinkage estimation of normal population variance. *Commu. Stat., Theory-Method* A 8(4), 359-365.

58. Pradhan, B.k. (2005). A chain regression estimator in two-phase sampling using multi-auxiliary information. *Bull. Malays. Math. Sci. Soc.,* (2), 28(1), 81-86.

59. Prasad, B. (1989). Some improved ratio-type estimators of population mean and ratio in finite population sample surveys. *Comm. in Statist.Theory and Methods.,* 18, 379-392.

60. Prasad, B. Singh, R.S., and Singh, H.P. (1996). Some chain ratio-type estimators for ratio of two population means using two auxiliary characters in two-phase sampling. *Metron,* 54, 95-113.

61. Quenouille, M.H. (1956). Note on bias in estimation. *Biometrika,* 43, 353-360.

62. Radhey, B.P., Singh, S. and Singh, H.P. (2002). Modified chain ratio estimators for finite population mean using two auxiliary variables in double sampling. *Statistics in Transition,* Vol.5, No.6, 1051-1066.

63. Rao, J.N.K. (1973). On double sampling for stratification and analytic surveys. *Biometrika,* 6,125-133.

64. Rao, J.N.K. and Pereira, N.P. (1968). On double ratio estimators. *Sankhya,* Ser A.-bf. 30, 83-90.

65. Rao, P.S.R.S. and Mudholkar, G.S. (1967). Generalized multivariate estimators for mean of finite population. *Jour. Amer. Statist. Assoc.,* 62, 1009-1012.

66. Rao, P.S.R.S. (1969). Comparision of four ratio type estimators under a model. *Jour. Amer. Statist. Assoc.,* 64, 574-580.

67. Robson, D.S. (1957). Applications of multivariate polykays to the theory of unbiased ratio-type estimators. *J. Amer. Statist. Assoc.,* 52, 511-522.

68. Roy, D.C. (2003). A regression type estimator in two-phase sampling using two auxiliary variables. *Pak. J. Statist.,* 19(3), 281-290.

69. Sahai, A. (1979). An efficient variant of the product and ratio estimator. *Statist. Neerlandica,* 33, 27-35.

70. Sahai, A. and Ray, S.K. (1980). An efficient estimator using auxiliary information. *Metrika,* 27, 27-275.

71. Sahoo, J. and Sahoo, L.N. (1993). A class of estimators in two-phase sampling using two auxiliary variables. *J. Ind. Soc. Agri. Statist.,* 31, 107-114.

72. Sahoo, J. and Sahoo, L.N. (1994). On the efficiency of four chain-type estimators in two-phase sampling under a model. *Statistics,* 25, 361-366.

73. Sahoo, J., Sahoo, L.N. and Mohanty, S. (1993). A regression approach to estimation in two-phase sampling using two auxiliary variables. *Current Sciences,* 65, 1, 73-75.

74. Samiuddin, M. and Hanif, M. (2006). Estimation in Two-Phase Sampling with complete and incomplete information. *Proc. 8th Islamic Countries Conference on Statistical Sciences,* Vol. 13, 479-495.

75. Samiuddin, M. and Hanif, M. (2007). Estimation of population mean in single and two phase sampling with or without additional information. *Pak. J. Stat.,* 23(2): 99-118.

76. Sarkar, S.K. (1994). Shrinkage domination of some usual estimators of the common mean of several multivariate normal populations. *J. Statist. Plann. Inference,* 39, 43-55.

77. Sastry, K.V.R. (1965). Unbiased ratio estimators. *J. Indian Soc. Agri. Stat.,* 17, 19-29.

78. Searls, D.T. (1964). The utilization of a known coefficient of variation in estimation procedure. *Jour. Amer. Statist. Assoc*., 59, 1225-1226.

79. Shabbir, J. and Gupta, S. (2007). On estimating the finite population mean with known population proportion of an auxiliary variable. *Pak. J. Statist.,* 23(1), 1-9.

80. Shahbaz, M. Q. and Hanif M. (2009). A general shrinkage estimator in survey sampling. *World Applied Science Journal, 7(5), 593-596.*

81. Singh, A.K. and Singh, H.P. (2001). Dual to chain ratio-type estimator in double sampling using two auxiliary variables. *J. Ravishankar Uni.,* Vol.14, No B, 99-106.

82. Singh, A.K., Singh, H.P. and Upadhyaya, L.N. (2001).A generalized chain estimator for finite population mean in two-phase sampling. *J. Indian Soc. Agri. Stat.,* 34(3), 370-375.

83. Singh, G.N. (2001). On the use of transformed auxiliary variable in the estimation of population mean in two-phase sampling. *Statistics in Transition,* 5(3). 405-416.

84. Singh, G.N. and Upadhyaya, L.N. (1995).A class of modified chain type estimators using two auxiliary variables in two-phase sampling. *Metron,* 1, III, 117-125

85. Singh, H.P. and Biradar, R.S. (1994).A class of unbiased ratio and product estimators in two-phase sampling. *Statistica*, anno LIV, n.3, 349-359.

86. Singh, H.P. and Espejo, M. R. (2007). Double sampling ratio-product estimator of a finite population mean in sample survey. *J. Applied Stat.,* Vol. 34, No. 1, 71-85.

87. Singh, H.P. (1987). On the estimation of population mean when the correlation coefficient is known in two-phase sampling. *Assam statist. Review,* Vol.1, No.1, 17-21.

88. Singh, H.P. (1993).A chain ratio-cum difference estimator using two auxiliary variables in double sampling. *J. Ravishankar Uni.,* Vol.6, No B (Science), 79-83.

89. Singh, H.P., Katyar, N.P. and Gangwar, D.K. (1996). A class of almost unbiased regression-type estimators in two-phase sampling applying Quenouille's method. *J. Indian Soc. Agri. Stat.,* 48(1)), 98-104.

90. Singh, H.P., Singh, S. and Kim, J. (2006). General families of chain ratio type estimators of population mean with known coefficient of variation of the second auxiliary variable in two-phase sampling. *J. Kor. Statist. Soc.,* 35(4), 377-395.

91. Singh, H.P., Tripathi, T.P. and Upadhyaya, L.N. (1989).Improved estimators for population mean based on double sampling. *J. Indian Statist. Assoc.,* 27, 89-99.

92. Singh, H.P., Upadhyaya, L.N. and Chandra, P. (2004). General families of estimators for estimating population mean using two auxiliary variables in two-phase sampling. *Statistics in Transition.* Vol. 6, 1055-1077.

93. Singh, M.P. (1967). Ratio cum product method of estimation. *Metrika,* 12, 34-43.

94. Sisodia, B.V.S. and Dwivedi, V.K. (1981). A class of ratio cum product type estimators. *Biometrical J.,*23, 133-139.

95. Srivastava, S., Rani, S., Khare, B.B. and Srivastava, S.R. (1990). A generalized chain ratio estimator for mean of finite population. *J. Ind. Soc. Agri. Statist.,* 42, 108-117.

96. Srivastava, S.K. (1967). An estimator using auxiliary information in sample surveys. *Cal. Statist. Assoc. Bull.,* 16, 121-132.

97. Srivastava, S.K. (1970). A two-phase sampling estimator in sample surveys. *Austral. J. Statist.,* 12(1) 23-27.

98. Srivastava, S.K. (1971). A generalized estimator for the mean of a finite population using multi-auxiliary information. *J. Amer. Statist. Assoc.,* 66, 404-407.

99. Srivastava, S.K. and Jhajj, H.S. (1981). A class of estimators of population mean in survey sampling using auxiliary information. *Biometrika,* 68, 341-343.

100. Srivastava, S.K. and Jhajj, H.S. (1983). A class of estimators of the population means using multi-auxiliary information.*Cal. Statist. Assoc. Bull.,* 32, 47-56.

101. Srivastava, S.K. (1980). A class of estimators using auxiliary information in sample surveys. *Canadian J. Stat.,* 8,253-254.

102. Srivenkataraman, T. and Srinath, K.P. (1976). Ratio and product methods of estimation in sample surveys when the two variables are moderately correlated. *Vignana Bharthi.,* 22, 54-58.

103. Stein, C. (1956), "Inadmissibility of the usual estimator for the mean of a multivariate distribution. *Proc. Third Berkeley Symp. Math. Statist. Prob.,* Vol. 1, 197–206, University of California press, *Berkeley.*

104. Stien, C. (1962). Confidence sets for the mean of a multivariate normal distribution (with discussion). *J. Roy. Statist. Soc., Ser. B,* 24, 265-296.

105. Stigler, S.M. (1990). A Galtonian perspective on shrinkage estimators. *Statist. Science*, 5, 147-155.

106. Sukhatme, B.V. (1962). Some ratio-type estimators in two-phase sampling. *Jour. Amer. Statist. Assoc.,* 57, 628-632.

107. Tahir, A. (2008). *Empirical comparison of some estimators for two-phase sampling.*M.Phil. Thesis, National College of Business Administration & Economics, 40E-I, Gulberg III, Lahore, Pakistan.

108. Theil, H. (1963). On the use of incomplete prior information in regression analysis. *Jour. Amer. Statist. Assoc.,* 58, 401-414.

109. Thompson, J. R. (1968).Some Shrinkage Techniques for Estimating the Mean**.** *Jour. Amer. Statist. Assoc.,* Vol. 63, No. 321 113-123.

110. Tin, M. (1965). Comparison of some ratio type estimators. *Jour. Amer. Statist. Assoc.,* 60, 294-307.

111. Tiripathi, T.P. (1970). *Contributions to the sampling theory using multivariate information*. Ph.D. thesis submitted to Punjabi University, Patiala, India.

112. Tracy, D.S. and Singh, H.P. (1999a). Efficient use of two auxiliary variables in two-phase sampling as well as in successive sampling. *Pak. J. Stat.,* 15(1), 27-39.

113. Tracy, D.S. and Singh, H.P. (1999b).A general class of chain regression estimators in two-phase sampling. *J. Applied Statist. Sci.,* 8, 205-216.

114. Tripathi, T.P. (1980). A general class of estimators of population ratio. *Sankhya*, Ser. C. 42, 63-75.

115. Tripathi, T.P. (1987).A class of estimators for population mean using multivariate auxiliary information under general sampling designs. *Aligarh, J. Stat.,* 7, 49-62.

116. Upadhyaya, L.N. and Singh, G.N. (2001). Chain type estimators using transformed auxiliary variable in two-phase sampling. *Advances in Modeling and Analysis,* 38. (1-2), 1-10.

117. Upadhyaya, L.N., Dubey, S.P. and Singh, H.P. (1992). A class of ratio-in-regression estimators using two auxiliary variables in double sampling. *J. Scientific Research, 42, 128-134.*

118. Upadhyaya, L.N., Singh, H.P. and Tailor, R. (2006).Estimation of mean with known coefficient of variation of an auxiliary variable in two-phase sampling. *Statistics in Transition, Vol.7 (6), 1327-1344.*

119. Vos, J.W.E. (1980). Mixing of direct ratio and product method estimators. *Statist. Neerlandica,* 34, 209-218.

APPENDIX

#	DISTRICTS	ALL FRUITS					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
1	Attock	434	477	518	3014	3464	3885
2	Rawalpindi	513	516	523	3735	3841	3953
3	Islamabad	73	72	73	406	422	432
4	Jhelum	105	106	107	661	653	674
5	Chakwal	216	214	215	1123	1160	1184
6	Sargodha	45264	45341	45319	474469	497043	499434
7	Khushab	4620	4603	4734	42599	43444	45070
8	Mianwali	4802	4759	4746	40566	43162	43365
9	Bhakkar	6424	6639	6659	54486	57772	57891
10	Faisalabad	22770	22786	22873	212080	214511	216685
11	T.T. Singh	19552	19566	19717	214371	214345	215548
12	Jhang	10417	10416	10455	94408	95999	95566
13	Gujrat	2468	2456	2451	17703	18401	18544
14	M.B. Din	7651	7780	7803	74068	76009	76618
15	Sialkot	3520	3546	3546	28722	29090	29287
16	Narowal	2497	2557	2566	19935	20406	20510
17	Gujranwala	3650	3778	3799	34040	36685	36885
18	Hafizabad	1406	1439	1441	13217	13829	13967
19	Sheikhupura	11986	12057	12152	109565	110498	112088
20	Lahore	5321	5370	5436	39820	41199	42224
21	Kasur	11126	11268	11271	94599	96045	96790
22	Okara	12052	12196	12242	105745	108284	109228
23	Sahiwal	18347	18305	19476	182779	183710	196640
24	Pakpattan	2845	2864	2905	25946	26441	27136
25	Multan	14067	14069	14812	147694	148118	162530
26	Lodhran	2282	2270	2445	23538	23617	27707
27	Khanewal	17893	17838	17912	192355	199862	200334
28	Vehari	14152	14220	14183	141677	146701	146441
29	Muzaffargarh	8345	8396	8473	79423	80052	80941
30	Layyah	6780	6775	6810	61849	61218	64412

#	DISTRICTS	ALL FRUITS					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
31	D.G. Khan	1884	1901	1906	30833	31866	32014
32	Rajan Pur	1370	1377	1374	13170	14116	13665
33	Bahawalpur	22206	22206	22205	256312	256523	255080
34	Bahawalnagar	20253	20268	20317	153019	153893	154850
35	R.Y. Khan	38070	38065	38345	471471	474957	477475
36	Kairpur	28381	28660	29343	79254	79853	85816
37	Ghotki	1219	1275	1295	5678	5882	6070
38	Sukkur	2794	2887	2938	6790	6986	7250
39	N-Feroze	5963	6075	6312	46669	46829	49107
40	Nawab Shah	4913	4993	5163	35874	36056	37925
41	Jacobabad	4	4	4	18	20	20
42	Shikarpur	560	613	619	2272	2369	2452
43	Larkana	2243	2408	2488	13923	14719	15294
44	Sanghar	8433	8662	9018	56859	57753	61137
45	Tharparkar	222	0	0	1679	0	0
46	Umerkot	5502	5641	5888	35532	35992	38324
47	Mirpur Khas	8109	8366	8754	54373	55397	58793
48	Dadu	822	887	897	3958	4213	4340
49	Hayderabad	19622	20034	20725	103959	105616	113000
50	Badin	3017	3086	3180	15329	15647	16683
51	Thatta	3488	3555	3620	9384	9684	10749
52	Karachi	853	865	865	3066	3107	3186
53	Peshawar	1607	1613	1611	17833	17893	17867
54	Charsada	1554	1563	1562	16726	16817	16810
55	Nowshera	1659	1667	1659	19229	19313	19225
56	Mardan	2563	2593	2635	25752	26052	26423
57	Swabi	1073	1093	1112	10360	10540	10738
58	Kohat	1952	1974	2039	20061	20235	20852
59	Karak	6	5	8	34	28	45
60	Mansehra	1160	1169	1176	11036	11141	11212

#	DISTRICTS	ALL FRUITS					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
61	Battagram	162	165	167	1529	1569	1588
62	Abbotabad	664	666	668	5460	5477	5498
63	Haripur	1161	1168	1173	10199	10266	10320
64	Kohistan	95	97	98	473	485	501
65	Malakand	972	958	955	8737	8658	8627
66	Swat	6166	6408	6633	51312	52975	54647
67	Bunir	195	234	276	1634	1966	2314
68	Shanglapar	185	193	219	1691	1759	2001
69	Dir	1649	1627	1634	15565	15302	15338
70	Chitral	415	437	443	3769	3958	4053
71	D.I.Khan	9533	9559	9619	135059	135246	136314
72	Tank	156	137	380	1999	1731	4539
73	Bannu	1817	1840	1852	23492	23654	23755
74	Lakki Marwat	1887	1986	1919	30870	31041	31376
75	Mohmand	193	231	246	1613	1922	2047
76	Khyber	104	117	134	1019	1117	1278
77	Khurram	488	497	526	5635	6110	6502
78	Orakazi	102	106	113	1222	1269	1349
79	Bajour	378	455	526	3988	4017	4550
80	N-W	486	413	534	5278	5494	5707
81	S-W	4452	4449	4554	63011	62955	64446
82	FR. Peshawar	5	10	10	36	72	72
83	FR. Kohat	5	5	6	30	30	38
84	FR. Bannu	0	50	92	0	389	798
85	FR. D.I. Khan	83	95	99	539	653	690
86	Quetta	6230	6267	6252	88931	89415	89062
87	Pishin	20726	21268	21845	267915	291139	293669
88	Chagai	4739	4774	4831	63095	63246	63633
89	Loralai	14758	15216	15303	178732	173466	174413
90	Musakhail	226	241	265	2067	2046	2416

#	DISTRICTS	ALL FRUITS					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
91	Barkhan	1622	1673	1692	17593	17492	17622
92	Zhob	9845	9890	10003	134256	132446	132921
93	K.S.Ullah	6221	6996	7233	77103	77677	78004
94	Sibi	2191	2197	2204	33916	34176	34201
95	Ziarat	3394	3453	3494	48584	48287	48369
96	Kohlu	1127	1040	1043	11344	10324	10299
97	Dera Bughti	117	119	129	1295	1375	1535
98	Nasirabad	838	825	851	7035	7351	7604
99	Jafarabad	315	341	356	3680	3811	4121
100	Bolan	1381	1476	1611	22654	24434	26559
101	Jhal Magsi	120	138	183	910	880	917
102	Kalat	4858	5453	6155	72886	72634	72927
103	Mastung	4426	4492	4594	51601	51869	52120
104	Khuzdar	4567	4624	4665	67494	68958	69273
105	Kharan	3543	3250	3265	42697	38711	38876
106	Lasbela	2970	3423	3723	43741	44150	44401
107	Turbat	27289	27353	27655	268013	268845	270034
108	Panjgoor	13987	14059	14127	134522	134897	135514
109	Gwadar	2593	2611	2909	23011	23032	25392

#	DISTRICTS	VEGETABLE					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
1	Attock	2521	2690	2757	28202	31569	33566
2	Rawalpindi	1033	1063	1090	13808	14225	14934
3	Islamabad	309	307	302	3936	3807	3769
4	Jhelum	420	437	442	5801	5969	6016
5	Chakwal	358	365	371	4320	4297	4396
6	Sargodha	3119	3128	3196	51229	50098	51272
7	Khushab	1074	1075	1085	15667	15134	15311
8	Mianwali	747	1028	1072	11200	15147	15871
9	Bhakkar	1116	1139	1147	14931	15111	15362
10	Faisalabad	10340	10504	10544	167647	170859	173830
11	T.T. Singh	3123	3286	3327	52647	53918	55333
12	Jhang	4538	4584	4627	76862	75960	77801
13	Gujrat	3217	3237	3268	52672	51396	52486
14	M.B. Din	4029	4392	4421	67780	72286	72731
15	Sialkot	2786	2977	3037	43159	45878	46563
16	Narowal	1761	1996	2049	25762	30004	31113
17	Gujranwala	9977	10878	10990	174801	183189	187264
18	Hafizabad	1456	1593	1631	23921	25785	26758
19	Sheikhupura	14016	14096	14150	242741	242381	245108
20	Lahore	6580	6711	6754	105054	107695	108230
21	Kasur	6512	6556	6689	94646	95073	97276
22	Okara	5845	5925	6047	100700	100243	104245
23	Sahiwal	3661	3689	3743	56417	55728	56837
24	Pakpattan	1735	1828	2052	27927	29193	33594
25	Multan	3726	3807	3836	55006	55912	57386
26	Lodhran	992	1095	1201	16496	17951	19592
27	Khanewal	5127	5256	5504	81198	82587	86742
28	Vehari	2216	2434	2507	33668	37391	39588
29	Muzaffargarh	2024	2054	2064	27457	27921	28189
30	Layyah	1511	1541	1554	17170	17363	17593

#	DISTRICTS	VEGETABLE					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
31	D.G. Khan	2404	2419	2430	36784	37211	37857
32	Rajan Pur	957	965	985	11757	11612	11984
33	Bahawalpur	4188	4218	4228	56913	57404	58121
34	Bahawalnagar	3814	3838	3874	50033	50343	51706
35	R.Y. Khan	5561	5588	5637	70270	71021	72656
36	Kairpur	1097	1122	1137	7539	7700	7830
37	Ghotki	1127	1155	1161	8515	8695	8705
38	Sukkur	1142	1158	1173	9484	9573	9671
39	N-Feroze	1558	1579	1593	14163	14407	14622
40	Nawab Shah	1295	1311	1326	11598	11791	11963
41	Jacobabad	162	167	168	1124	1148	1158
42	Shikarpur	583	588	599	3977	4006	4086
43	Larkana	635	639	660	4263	4295	4411
44	Sanghar	3157	3221	3256	19622	20105	20339
45	Tharparkar	1141	0	0	3279	0	0
46	Umerkot	1768	1781	1789	10860	10942	11002
47	Mirpur Khas	2110	2125	2135	13633	13757	13854
48	Dadu	1272	1293	1322	6881	7001	7137
49	Hayderabad	7791	7922	7985	52656	53785	54372
50	Badin	2898	2924	2940	14534	14737	14936
51	Thatta	3761	3782	3882	13774	13904	14308
52	Karachi	3454	3453	3450	14503	14543	14654
53	Peshawar	1818	1829	1734	25162	25323	24420
54	Charsada	1440	1448	1371	19634	19725	18925
55	Nowshera	1338	1346	1251	18030	18123	17195
56	Mardan	2043	2065	2084	25457	25745	26053
57	Swabi	717	731	735	8134	8297	8331
58	Kohat	1461	1303	1149	16432	14850	12834
59	Karak	17	21	21	91	108	116
60	Mansehra	1145	1172	1186	17761	18147	18278

#	DISTRICTS	VEGETABLE					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
61	Battagram	22	25	27	215	244	264
62	Abbotabad	201	205	205	2726	2797	2815
63	Haripur	984	996	1022	9736	9940	10125
64	Kohistan	64	70	76	657	690	749
65	Malakand	1537	1535	1531	17724	17692	17641
66	Swat	5828	6218	6324	66184	70422	72562
67	Bunir	163	215	239	2139	2787	3074
68	Shanglapar	44	182	206	459	2011	2264
69	Dir	906	900	903	11084	11000	11010
70	Chitral	916	984	1233	6343	6812	8472
71	D.I.Khan	872	926	982	10461	10908	11380
72	Tank	901	918	941	11764	11893	12093
73	Bannu	767	727	716	8051	7478	7112
74	Lakki Marwat	163	231	241	1437	2372	2456
75	Mohmand	231	251	273	1582	1722	1879
76	Khyber	414	430	446	2924	3045	3164
77	Khurram	641	630	654	4490	4510	4688
78	Orakazi	0	0	0	0	0	0
79	Bajour	1456	1499	1482	10473	10796	10654
80	N-W	796	799	832	5140	5164	5380
81	S-W	1707	1751	1792	10537	10784	11034
82	FR. Peshawar	12	18	18	100	152	152
83	FR. Kohat	5	5	5	38	38	38
84	FR. Bannu	46	47	50	331	337	355
85	FR. D.I. Khan	137	147	158	963	1034	1112
86	Quetta	1766	1793	1765	25350	25955	25905
87	Pishin	2488	2477	2611	37370	37360	39870
88	Chagai	762	1335	3360	10440	20010	48165
89	Loralai	480	480	505	7070	7070	7450
90	Musakhail	305	305	360	4750	4750	5700

#	DISTRICTS	VEGETABLE					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
91	Barkhan	277	275	550	4460	4440	8900
92	Zhob	241	213	238	3880	3330	3760
93	K.S.Ullah	1400	1520	1560	20330	22150	22700
94	Sibi	1906	1945	1997	30730	31325	32230
95	Ziarat	0	0	0	0	0	0
96	Kohlu	466	434	451	7450	7080	7360
97	Dera Bughti	121	134	136	1680	1890	1930
98	Nasirabad	234	234	195	4070	4090	3515
99	Jafarabad	830	831	468	15070	15130	8510
100	Bolan	1057	1096	1180	16980	17670	19125
101	Jhal Magsi	110	122	126	1970	2200	2280
102	Kalat	577	630	668	9400	10345	11085
103	Mastung	1158	1169	1311	18930	19210	19975
104	Khuzdar	1990	2468	2490	29620	35155	35775
105	Kharan	733	700	660	10710	10235	9705
106	Lasbela	1853	1498	1171	28040	20820	16480
107	Turbat	3885	4703	5001	43310	54420	58180
108	Panjgoor	404	427	456	5240	5590	6225
109	Gwadar	104	74	137	1170	840	1245

#	DISTRICTS	WHEAT					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
1	Attock	151.4	155.8	156.2	220.1	159.7	267.7
2	Rawalpindi	123	121.8	122.2	192.2	138.2	238.9
3	Islamabad	16.2	15.8	15	25	17.5	28.8
4	Jhelum	63.1	63.1	65.1	84.1	74.6	119.3
5	Chakwal	134	135.2	137.2	160.1	137.4	235.1
6	Sargodha	197.1	195.9	178.5	450.2	401.6	390.7
7	Khushab	76.5	76.1	83	109.6	107.7	145.9
8	Mianwali	142.4	140.4	154.6	221.3	216.5	281.1
9	Bhakkar	140.4	134	142.4	244.4	250.1	309.2
10	Faisalabad	260.6	255	246.5	559.8	623.7	574.1
11	T.T. Singh	143.7	139.6	140.4	318.3	327.6	375.2
12	Jhang	332.6	331	330.6	860.1	714.8	755.4
13	Gujrat	114.9	116.5	111.3	196.3	189.3	210.7
14	Sialkot	180.9	179.3	178	373.5	365.1	386.3
15	Gujranwala	202.3	197.1	195	446.9	453.9	458.2
16	Narowal	115.3	114.1	116.2	211.9	213.6	236.8
17	M.B. Din	112.1	108.9	106.9	225.6	220.9	254.1
18	Hafizabad	117.8	116.9	116.5	261.2	270.6	247.6
19	Sheikhupura	264.7	266.3	263	565	610.4	604.3
20	Lahore	53	50.6	51	124	125.4	112.8
21	Kasur	171.6	157	159.4	403.5	390.1	396.6
22	Okara	215.7	208	206.4	536.5	556.2	614.3
23	Sahiwal	154.2	148.9	148.9	390.3	427	434.6
24	Multan	178.5	177.7	184.6	366.7	398.6	479.5
25	Khanewal	205.6	204.8	208.8	484.6	503.1	565.9
26	Vehari	236.7	227	240	570.5	602.6	668.7
27	Lodhran	164.7	149.7	163.9	342.3	353.4	408.1
28	Pakpattan	154.6	148.5	154.6	393.4	425.9	475.5
29	Muzaffargarh	272.7	268.7	275.2	540.8	551.5	655
30	Layyah	178.5	174.5	176.4	331.2	291.8	372.3

#	DISTRICTS	WHEAT					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
31	D.G. Khan	147.7	149.3	152.6	283	292.5	345.8
32	Rajan Pur	119	115.3	128.7	240.6	214	295.5
33	Bahawalpur	243.6	234.3	252.5	478.7	556.7	553.2
34	Bahawalnagar	305.6	290.5	299.5	629.9	654.1	720.3
35	R.Y. Khan	282.9	272.3	273.5	588.4	534.9	589.5
36	Kairpur	114.8	113.9	115.8	274.6	284.7	305.7
37	Sukkur	134.3	43.2	44.8	333.3	110.9	117.1
38	Nawab Shah	93.4	94	94.6	250	256.5	282.4
39	N-Feroze	127.6	126.4	127.5	352	349.9	375.2
40	Ghotki		90.7	92.3	0	237.7	249.2
41	Jacobabad	53.6	52.9	49.7	77.4	79.4	79.1
42	Shikarpur	29.8	30.4	31.4	43.6	48.1	51.5
43	Larkana	70.8	70.9	71.9	104.9	113.5	120.1
44	Sanghar	139.6	138.9	143.6	273.1	274.4	349.2
45	Tharparkar	0	0	0	0	0	0
46	Mirpur Khas	108.2	64.7	67.6	228.8	146.5	147.6
47	Umer Kot	0	44.8	45.9	0	95.3	102.6
48	Dadu	73.4	74.9	77.4	126.4	139.2	150.7
49	Hayderabad	117	115.3	114.1	217.5	233.7	263.9
50	Badin	33.4	33.8	32.5	48.5	55.1	48.7
51	Thatta	10.3	11.9	11	14.5	18.8	16.3
52	Karachi	0.2	0.1	0.1	0.2	0.2	0.1
53	Peshawar	30.6	31.9	37.4	65	58.4	86.1
54	Charsada	27.3	35	29	73.6	69	81.8
55	Nowshera	29.4	30	30.3	53	41.6	57.2
56	Mardan	47.7	47.2	49.5	84.1	76.1	107.1
57	Swabi	51.8	53.7	57.1	65.1	60.4	97.6
58	Kohat	47.7	46.7	55.7	42.7	27.5	48.6
59	Karak	31.1	29.9	33.9	24.3	18.5	33.2
60	Mansehra	35.8	36	37.7	50.8	51.9	56.8

#	DISTRICTS	WHEAT					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
61	Abbotabad	15.6	15.6	22.6	18.9	19.6	32
62	Kohistan	2.1	2.1	2.1	2.4	2.4	2.5
63	Haripur	37	36	37.2	46.4	46.8	54.8
64	Battagram	6.9	6.9	8.1	9.8	9.9	11.6
65	Malakand	26.1	26.2	26.7	35.6	32.7	38.7
66	Swat	57.4	47.2	57	76	55.2	77.3
67	Dir	39.8	39.8	41.5	47.1	39.4	50.5
68	Chitral	8.6	8.7	8.7	13.1	13.3	13.7
69	Bunir	45.3	45.3	45.1	72.5	61.6	66
70	Shanglapar	17.3	17.2	18.5	19.8	15	19.6
71	D.I.Khan	69.1	54.8	67.6	90.1	74.5	100.8
72	Tank	16.1	14.3	16.3	21.1	19	24.9
73	Bannu	37.4	31.7	39	56.6	47.8	57.6
74	Lakki Marwat	78.5	78.9	84.8	93	86.3	84.1
75	Mohmand	9.9	9.9	10.3	11.6	10.8	13
76	Khyber	9.9	10	11.1	14.4	14.4	16.5
77	Khurram	12.4	12.4	13.1	18.4	17.7	20.2
78	Orakazi	6	5.9	6	6.6	5.4	7
79	Bajour	41.8	41.8	43.1	54.8	54.9	58.1
80	N-W	8.9	9	9.5	12.4	12.2	13.5
81	S-W	7.4	7.4	7.6	8.6	8.6	9.4
82	FR. Peshawar	1.8	1.9	2	3.3	3.1	3.6
83	FR. Kohat	1.5	1.5	1.4	2.2	1.9	2.3
84	FR. Bannu	5.3	5.3	5.5	6.2	5.6	6.6
85	FR. D.I. Khan	2.6	2.6	2.7	3	2.9	3.3
86	Quetta	3.5	3.5	3.4	7.6	7.5	7.4
87	Pishin	22.5	22.4	24.6	47.6	47.7	49.7
88	Chagai	11.3	11.5	16.2	20.5	20	29.9
89	Loralai	17.9	12	12.1	33.8	22.9	22.8
90	Zhob	2.5	1.1	4	5	2.4	7.1

#	DISTRICTS	WHEAT					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
91	K.S.Ullah	10	14	13	18.1	28.6	22.3
92	Musakail	1.7	1.8	1.9	2.8	3.2	3.3
93	Barkhan	10.7	4.1	8.2	18.7	9	16.3
94	Sibi	16.1	14.2	16.1	35.3	32.9	34.1
95	Kohlu	13	1.7	10.2	18.2	3.8	13.5
96	Dera Bughti	6.1	0.5	0.6	9.6	1	1.3
97	Ziarat	0	0	0	0	0	0
98	Nasirabad	59.5	60.2	69.9	158.9	161.1	188.2
99	Jafarabad	79	83.9	77.4	211.3	224.8	208.3
100	Bolan	11.1	11.2	11.7	28.5	28	29.5
101	Jhal Magsi	12	11	13.3	27.1	26	32.3
102	Kalat	12.8	7.5	12	24.1	14.8	22.7
103	Khuzdar	68	40.3	42.9	146.2	101.7	106.9
104	Kharan	21.5	5.4	5.4	26.9	7.6	7.6
105	Lasbela	4	1.6	1.2	10.5	3.9	3
106	Mastung	40.5	9	31.2	69.4	20	55.8
107	Panjgoor	4.8	1.8	4	6.2	2.3	5.7
108	Gwadar	0.1	0.1	0.2	0.1	0.1	0.2
109	Turbat	1.9	0.8	2.2	3.7	1.9	3.7

#	DISTRICTS	ONION					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
1	Attock	227	229	227	2195	2218	2278
2	Rawalpindi	85	86	97	806	829	974
3	Islamabad	0	0	0	0	0	0
4	Jhelum	28	29	24	287	292	255
5	Chakwal	10	10	11	107	107	123
6	Sargodha	405	413	418	4852	4987	5210
7	Khushab	38	39	40	443	453	468
8	Mianwali	324	330	335	3732	3868	4013
9	Bhakkar	101	103	105	1120	1142	1189
10	Faisalabad	648	660	664	6569	6701	6856
11	T.T. Singh	709	724	728	6870	7007	7256
12	Jhang	1124	1146	1170	10883	10891	11326
13	Gujrat	24	25	30	246	255	308
14	Sialkot	85	87	97	862	927	1030
15	Gujranwala	850	875	893	8230	8477	8645
16	Narowal	34	36	45	317	328	415
17	M.B. Din	32	33	34	328	331	374
18	Hafizabad	231	238	242	2234	2235	2277
19	Sheikhupura	1254	1292	1335	11571	11918	12563
20	Lahore	243	250	258	2239	2422	2500
21	Kasur	2125	2189	2266	19595	22204	22922
22	Okara	1214	1250	1336	11757	12110	12933
23	Sahiwal	731	760	769	8426	8411	8652
24	Multan	1052	1094	1093	12130	12818	13000
25	Khanewal	1295	1347	1376	14333	14285	14848
26	Vehari	1214	1251	1270	13997	14647	15236
27	Lodhran	546	568	567	6299	6341	6427
28	Pakpattan	251	261	284	2893	2769	3070
29	Muzaffargarh	648	660	647	7465	7614	7286
30	Layyah	283	289	304	3005	3118	3443

#	DISTRICTS	ONION					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
31	D.G. Khan	1234	1259	1260	14230	14282	14650
32	Rajan Pur	283	289	324	3135	3198	3494
33	Bahawalpur	617	623	647	7400	7472	7644
34	Bahawalnagar	1154	1166	1214	13297	13110	13661
35	R.Y. Khan	890	899	911	10264	10450	10330
36	Kairpur	228	230	231	2345	2387	2398
37	Sukkur	2139	521	527	28259	6077	6148
38	Nawab Shah	633	640	645	7291	7971	8030
39	N-Feroze	1409	1424	1441	18765	19424	19659
40	Ghotki	0	1640	1658	0	21073	21308
41	Jacobabad	203	205	209	1967	1986	2032
42	Shikarpur	1295	1309	1321	15543	15526	15678
43	Larkana	702	709	717	7188	7358	7446
44	Sanghar	3846	3887	3928	52058	49947	50503
45	Tharparkar	0	0	0	0	0	0
46	Mirpur Khas	6619	3011	3044	89985	39881	40346
47	Umer Kot	0	3677	3713	0	49066	49572
48	Dadu	1484	1500	1517	15621	14827	15003
49	Hayderabad	10688	9800	9903	144820	130771	132193
50	Badin	2896	3926	3964	36340	53164	53691
51	Thatta	445	450	455	2678	4226	4296
52	Karachi	204	206	209	2341	1934	1974
53	Peshawar	51	38	56	510	383	561
54	Charsada	32	36	31	288	318	281
55	Nowshera	70	105	62	627	954	547
56	Mardan	80	64	62	908	725	713
57	Swabi	77	78	64	887	897	736
58	Kohat	135	111	79	1260	978	716
59	Karak	7	7	3	62	64	24
60	Mansehra	93	31	70	917	311	717

#	DISTRICTS	ONION					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
61	Abbotabad	0	0	0	0	0	0
62	Kohistan	0	0	0	0	0	0
63	Haripur	34	19	22	380	217	262
64	Battagram	2	4	0	21	42	0
65	Malakand	232	240	245	4016	3936	3910
66	Swat	3537	3841	3121	54364	46913	52588
67	Dir	680	700	650	7091	6713	6715
68	Chitral	20	30	31	124	180	217
69	Bunir	80	89	30	1062	1036	261
70	Shanglapar	0	1	0	0	11	0
71	D.I.Khan	17	7	8	253	105	120
72	Tank	10	15	17	120	180	204
73	Bannu	24	23	3	355	335	44
74	Lakki Marwat	42	40	70	614	560	984
75	Mohmand	480	490	500	6125	6260	6395
76	Khyber	270	275	293	3133	3191	3400
77	Khurram	132	136	136	1785	1835	1846
78	Orakazi	7	5	5	101	72	80
79	Bajour	191	195	195	2282	2330	2419
80	N-W	115	115	123	1395	1395	1492
81	S-W	210	215	215	2512	2597	2622
82	FR. Peshawar	25	30	30	265	318	314
83	FR. Kohat	16	20	20	172	215	207
84	FR. Bannu	65	65	70	926	934	1054
85	FR. D.I. Khan	60	60	60	747	747	746
86	Quetta	547	728	647	9875	13840	9850
87	Pishin	392	275	287	8072	5670	4740
88	Chagai	2025	5630	5715	36035	112600	91440
89	Loralai	65	71	75	1000	1093	930
90	Zhob	210	200	250	3260	3106	3110

#	DISTRICTS	ONION					
		AREA			PRODUCTION		
		1995-96	1996-97	1997-98	1995-96	1996-97	1997-98
91	K.S.Ullah	800	950	953	12800	15200	12200
92	Musakail	10	71	70	160	1136	900
93	Barkhan	70	90	100	1088	1400	1250
94	Sibi	294	298	298	5000	5070	4060
95	Kohlu	94	95	95	1155	1168	940
96	Dera Bughti	31	32	32	465	480	385
97	Ziarat	24	0	200	340	0	2400
98	Nasirabad	705	780	780	11550	12778	10225
99	Jafarabad	600	500	652	9800	8167	8520
100	Bolan	160	170	182	2560	2720	2330
101	Jhal Magsi	70	70	100	1100	1100	1250
102	Kalat	4200	2230	2363	98620	52905	44850
103	Khuzdar	600	480	480	15510	12410	9930
104	Kharan	810	810	800	12960	12960	10240
105	Lasbela	915	300	272	14630	4800	3480
106	Mastung	4180	4474	4464	98235	106370	84905
107	Panjgoor	75	85	82	1600	1814	1400
108	Gwadar	0	0	41	0	0	460
109	Turbat	1408	1708	1708	21150	25656	20525

Appendix Table-1
Description of Populations and Main Variable

Pop #	Description	Main Variable (In tones)
1	District-wise area and production of Vegetables for year 1995-96	Production of Vegetables
2	District-wise area and production of Vegetables for year 1996-97	Production of Vegetable
3	District-wise area and production of Vegetables for year 1997-98	Production of Vegetables
4	District-wise area and production of all Fruits for year 1995-96	Production of all Fruits
5	District-wise area and production of all Fruits for year 1996-97	Production of all Fruits
6	District-wise area and production of all Fruits for year 1997-98	Production of all Fruits
7	District-wise area and production of Wheat for year 1995-96	Production of Wheat
8	District-wise area and production of Wheat for year 1996-97	Production of Wheat
9	District-wise area and production of Wheat for year 1997-98	Production of Wheat
10	District-wise area and production of Onion for year 1995-96	Production of Onions
11	District-wise area and production of Onion for year 1996-97	Production of Onions
12	District-wise area and production of Onion for year 1997-98	Production of Onions

Appendix Table-2
Description of Auxiliary Variables

Pop #	Attribute-I (τ_1)	Attribute-II (τ_2)
1	Districts of N.W.F.P	Area of Districts less than 500 hectares
2	Districts of Punjab	Area of Districts less than 401 hectares
3	Districts of Punjab	Area of Districts greater than 1000 hectares
4	Area of Districts greater than 1000 hectares	Districts of Punjab
5	Districts of Sindh	Area of Districts less than 1000 hectares
6	Districts of N.W.F.P	Area of Districts less than 500 hectares
7	Area of Districts greater than 30 hectares	Districts of Punjab
8	Districts of Punjab	Area of Districts greater than 35 hectares
9	Districts of N.W.F.P	Area of Districts greater than 25 hectares
10	Area of Districts greater than 40 hectares	Districts of N.W.F.P
11	Area of Districts greater than 50 hectares	Districts of N.W.F.P
12	Districts of Punjab	Area of Districts greater than 60 hectares

Appendix Table-3
Mean Square Errors of Suggested Estimators for Different Populations for Single-Phase Sampling (Full Information)

Pop #	$\hat{MSE}(\bar{y})$	$\hat{MSE}(\hat{T}_{4(1)})$	$\hat{MSE}(t_{6(1)})$	$\hat{MSE}(\hat{T}_{32(1)})$	$\hat{MSE}(t_{69(1)})$
1	73840080.17	67884434.67	61483113.40	62468707.80	57007399.00
2	76354303.56	53675166.545	49788208.04	51065758.22	47535111.13
3	79554829.16	56011691.24	52024803.30	55481537.90	51567080.80
4	384075608.97	314047103.1	285360657	308335898.8	280656923.70
5	401786346.73	392008472.0	349501413.31	318837555.5	290136921.0
6	408141286.64	366347618.9	330086762.8	356217572.05	321840021.01
7	2250.18	1534.81	1449.04	938.05	905.30
8	1979.82	878.02	847.30	738.5	716.60
9	2271.61	1811.61	1710.113	1214.9	1168.40
10	37192365.83	35120899.97	27070342.53	34715576.1	26828899.20
11	35211000.70	32885361.5	25873612.5	32675808.60	25708740.04
12	29929209.6	29399791.9	23164324.15	27235609.2	21799474.30

Appendix Table-4
Mean Square Errors of Suggested Estimators for Different Populations for Two-Phase Sampling (No Information)

Pop#	$\hat{MSE}(\bar{y}_2)$	$\hat{MSE}(\hat{T}_{5(2)})$	$\hat{MSE}(t_{7(2)})$	$\hat{MSE}(\hat{T}_{53(2)})$	$\hat{MSE}(t_{71(2)})$
1	73840080.170	70582144.50	64872103.40	67620233.80	61268580.70
2	76354303.560	63949771.70	57630103.60	62522191.01	57310384.60
3	79554829.160	66677478.60	56900722.60	66387184.70	60859331.65
4	384075608.97	345768874.2	320036600.6	342650465.50	308805394.30
5	401786346.73	397356375.4	357863331.5	356416810.3	320928533.10
6	408141286.64	385281395.0	334060543.6	379740530.70	340920432.40
7	2250.1800000	1954.104000	1817.150000	1603.50	1510.10
8	1979.8200000	1338.600000	1268.450000	1257.30	1195.20
9	2271.6100000	2003.600000	1880.450000	1656.60	1571.30
10	37192365.833	35970027.05	27572028.40	35730604.0	27431125.10
11	35211000.670	33842218.03	8890333.220	33718826.70	26350244.40
12	29929209.620	29617612.17	23299306.30	28343782.30	22503722.10

Appendix Table-5
Mean Square Errors of Suggested Estimators for Different Populations for Two-Phase Sampling (Partial Information)

Pop #	$\hat{MSE}\left(\hat{T}_{42(2)}\right)$	$\hat{MSE}\left(t_{70(2)}\right)$
1	65321317	59373408
2	53438726	49584522
3	55890287	51918134
4	3.12E+08	283286944
5	3.53E+08	317975862
6	3.62E+08	326907257
7	1473.861	1394.6
8	867.613	837.6
9	1487.609	1418.5
10	34830727	26897456
11	32818141	25796815
12	28135134	22371854

Appendix Table -6 (a)
Optimum Values of Unknown Constants

Pop #	α_1	α_2	d_0	$d_1 = \delta_1$	$d_2 = \delta_2$
1	3124.50	6958.50	0.9126	-10318.70	-26151.10
2	-17043.4	-3040.90	0.9307	53078.01	13258.20
3	-11968.9	-5492	0.9245	37274.61	9070.05
4	-40523.50	-8761.80	0.9103	58900.42	27286.71
5	6074.05	25673.4	0.9010	-38936.22	-79979.45
6	9714.01	9813.20	0.9035	-32080.61	-46508.10
7	235.10	-227.20	0.9651	-391.83	682.30
8	-222.30	215.80	0.9704	697.80	-378.60
9	31.71	-100.70	0.9616	-102.80	156.12
10	-7111.89	1896.69	0.7728	8867.70	-6386.20
11	-6690.51	1736.68	0.7868	8512.10	-5769.77
12	2376.9	-11095.02	0.8004	-7202.7	13765.80

Appendix Table -6 (b)
Optimum Values of Unknown Constants

Pop #	λ_1	λ_2	γ_0	γ_1	γ_2
1	5123.80	6046.90	0.908944	-16921.40	-22724.20
2	-15096.80	-1864.90	0.927880	47015.70	8129.50
3	-14157.80	-3080.50	0.928930	44091.40	5086.90
4	-46475.10	-7061.50	0.909359	67551	21991.50
5	5712.70	25644.20	0.901330	-36619.90	-79888.50
6	15023.50	6928.60	0.901960	-49615.30	-32837
7	-105.10	-38.80	0.946220	175.13	116.52
8	-107.12	-54.70	0.965410	327.60	95.96
9	44.40	-138.22	0.9535434	-143.97	214.30
10	-8935.12	1541.3	0.7722340	11141.05	-5189.60
11	-9325.80	952.50	0.7805350	11864.90	-3164.40
12	1616.90	11014.5	0.7951572	-4899.70	-13665.60

Appendix Table -7
Various Population Parameters

Pop #	C_{τ_1}	C_{τ_2}	C_y	ρ_{Pb_1}	ρ_{Pb_2}	Q_{12}
1	1.524	1.67	1.457	0.284	0.376	0.50
2	1.461	1.84	1.443	0.545	0.389	-0.76
3	1.4606	0.81	1.423	0.544	0.474	0.79
4	0.6759	1.46	1.518	0.427	0.35	0.59
5	2.3366	1.46	1.529	0.142	0.424	-0.05
6	1.5245	1.94	1.515	0.32	0.479	0.69
7	0.8203	1.42	1.154	0.631	0.742	0.96
8	1.4413	0.87	1.194	0.746	0.651	0.96
9	1.5045	0.745	1.139	0.45	0.645	-0.38
10	0.4994	1.55	2.071	0.236	0.225	-0.60
11	0.5243	1.53	6.298	0.257	0.249	-0.79
12	1.3988	0.551	1.930	0.133	0.249	0.13