Mathematical models for the three-dimensional flow problems in the non-Newtonian fluids

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IN

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CERTIFICATE

A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE DOCTOR OF PHILOSOPHY

We accept this dissertation as conforming to the required standard

1. Prof. Dr. Muhammad Ayub (Chairman)	2. Prof. Dr. Tasawar Hayat (Supervisor)
3. Prof. Dr. Nawazish Ali Shah (External Examiner)	4. Prof. Dr. Muhammad Ozair Ahmad (External Examiner)

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2013

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May Allah bless all those who pray for me (Aameen)

Muhammad Awais

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Dedicated

to

my loving family

and

my supervisor (Prof. Tasawar)

Whose prayers, love and support have helped me in completing this degree.

Preface

Dynamics of boundary layer flows (BLF) is a topic of major interest both in sciences and engineering. Such interest of the recent researchers is generated because of importance of BLF in polymer processing and electrochemistry. In particular, the flow caused by a continuous stretching surface occurs in glass fiber and paper production, wire drawing, crystal growth, drawing of plastic films, food processing, metal spinning process, cooling of metallic plate in a cooling bath etc. Motivated by such applications, a rapidly increasing number of research papers dealing with the flow over a stretching surface have been published to understand either the sole effects of rotation, heat and mass transfer, chemical reaction, MHD, suction/blowing or their various combinations. Much attention in the reported studies has been given to the two-dimensional stretched flows of viscous fluids. Such flow analysis even in the non-Newtonian fluid mechanics is scarcely investigated. However, there is void in the literature for the three-dimensional boundary layer flows of non-Newtonian fluids over a stretching surface. Main purpose of this thesis is to fill such void through different possible combinations of heat and mass transfer, MHD and suction/injection. For non-Newtonian fluids, the boundary theory is still incomplete. Major obstacle in such completion is diversity of the rheological properties of non-Newtonian fluids. Viscoelastic effects in these fluids give rise to additional nonlinearities which offer formidable mathematical task that cannot be performed even through numerical simulations. Having these challenges in mind, the present thesis is organized as follows:

Chapter one consists of the literature survey regarding the flows of nonlinear fluids. Boundary layer equations for various non-Newtonian fluids are proposed in the three-dimensional flow situations. Advantages of homotopy analysis method (HAM) are also pointed out.

Chapter two addresses the steady three-dimensional flow of an incompressible Maxwell fluid. Boundary layer approach is adopted in the mathematical modeling. Constructed nonlinear differential system is reduced into a system containing ordinary differential equations. Series solutions are developed. Convergence of the derived series solutions is discussed in detail. Error analysis is presented for the validity of the obtained solutions. Graphical results are displayed to analyze the effects of Deborah number on the axisymmetric, two- and three-dimensional cases.

Main findings of this chapter are published in **"International Journal for Numerical Methods in Fluids, 66 (2011) 875-884"**.

Chapter three extends the flow analysis of chapter two for unsteady case. Comparison with the limiting results of the steady case is shown. The results are accepted for publication in **"Meccanica"**.

In chapter four, we have examined the mixed convection boundary layer flow of upper-convected Maxwell (UCM) fluid. The flow is induced due to a bidirectional stretching plate. Mainly the magnetic field, diffusion-thermo (Dufour) and thermal-diffusion (Soret) effects are addressed. The appropriate transformations are utilized to reduce the partial differential system into the coupled system of nonlinear ordinary differential equations. The arising problems are solved by homotopy analysis method. Results are obtained and discussed for velocity, temperature, concentration, local Nusselt and local Sherwood numbers. The main points of this chapter are published in **"ASME: Journal of Heat Transfer, 134 (2012) 044503"**.

Chapter five explores the three-dimensional flow of an Oldroyd-B fluid over a stretching surface. Mathematical modeling is developed for the boundary layer equations in the three-dimensional flow. Resulting boundary layer equations along with the subjected boundary conditions are transformed into coupled system of ordinary differential equations. Computations for the series solutions are made. Effects of Deborah number in the axisymmetric, two- and three-dimensional flows are graphically presented and analyzed. Major observations are published in **"International Journal for Numerical Methods in Fluids, DOI: 10.1002/fld.2716"**.

In chapters' six to nine, the flows of Jeffery fluid are modeled. Here, the following four problems are formulated and solved.

a) Three-dimensional boundary layer flow over a linear stretching surface.
b) Three-dimensional channel flow when lower wall exhibits stretching property.
c) Three-dimensional magnetohydrodynamic shrinking flow in a rotating frame
d) Axisymmetric flow due to rotating disk.

Convergence intervals in the series solutions are determined. Impact of key parameters entering into flow analysis is discussed in each problem. The results of chapters six, eight and nine are already published in the journals **"Communications in Nonlinear Science and Numerical Simulations, 17 (2012) 699-707; ASME: Journal of Fluid Engineering, 133 (2011) 061201; International Journal for Numerical Methods in Fluids, DOI: 10.1002/fld.2714"**, respectively whereas the contents of chapter seven are submitted for publication in "**The European Physical Journal Plus**".

Chapter ten presents the three-dimensional unsteady flow over a stretching surface. Constitutive relationships for the second grade fluid model have been utilized in the problem formulation. Nonlinear partial differential equations are reduced into a system of ordinary differential equations using the similarity transformations. The homotopy analysis method (HAM) has been implemented for the series solutions. Graphs are displayed for the effects of sundry parameters on the velocity field. **The findings of this chapter are published in "Zeitschrift Fur Naturforchung A, 66 (2011) 635-642".**

The problem of unsteady three-dimensional flow, which results due to stretching of a surface, is studied in chapter eleven. Flow analysis is advanced in view of mass transfer and chemical reaction effects. The corresponding boundary value problems are computed by HAM. Conclusions for velocity and concentration fields are drawn. Comparison of present investigation is found in an excellent agreement with the existing limited studies. The observations of this chapter are published in **"Nonlinear Analysis: Modeling and Control, 17 (2012) 47-59".**

Influence of Soret and Dufour effects in three-dimensional boundary layer flow of viscoelastic fluid bounded by a stretching surface is examined in chapter twelve. The resulting partial differential system is converted into the ordinary differential systems, which are then computed analytically using homotopic approach known as the homotopy analysis method. The flow quantities of interest are significantly influenced by the sundry parameters in the computations. The conclusions of this chapter are published in **"International Journal of Heat and Mass Transfer, 66 (2012) 2129-2136".**

Contents

Chapter 1

Introduction

This chapter contains the survey of literature relevant to the two- and three-dimensional flows in steady and unsteady cases. Brief idea of methodology adopted and boundary layer equations are also presented.

1.1 Literature survey

Rheological characteristics of non-Newtonian fluid differ a lot than the Newtonian fluids. No doubt, the rheological properties of all the non-Newtonian fluids cannot be predicted by one constitutive equation between shear rate and rate of strain. For non-Newtonian fluids, there is always a nonlinear relationship between the stress and the rate of strain. The constitutive equations in non-Newtonian fluids are much complicated, more nonlinear and higher order in comparison to the Newtonian fluids. This is because of the elastic features in addition to the viscosity. Despite all the challenges, several researchers are involved in the discussion of such flows. For examples, Rajagopal et al. [1] presented the flow of viscoelastic fluid over a stretching surface. They have considered an incompressible second order fluid and concluded that such flow analyses are important for the applications involving polymer procession. Separation and reattachment of non-Newtonian fluid flows in a sudden expansion pipe has been analyzed by Pak et al. [2]. Rheological properties of the non-Newtonian fluids including shear-rate dependent viscosity and the viscoelasticity have been discussed. Lockett et al. [3] investigated the stability of the inelastic non-Newtonian fluid in Couette flow for concentric cylinders. Authors