Higher Order Sliding Mode Based Parameter Estimation and Control of a Research Reactor

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A dissertation submitted in partial fulfillment of the degree of Doctor of Philosophy (PhD) in Computer Engineering

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DECLARATION

The substance of this thesis is the original work of the author and due references and acknowledgements has been made, where necessary, to the work of others. No part of this thesis has been already accepted for any degree, and it is not being currently submitted in candidature of any degree.

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The author acknowledges the enabling role of the Higher Education Commission Islamabad, Pakistan and appreciates its financial support through “Merit Scholarship for PhD studies in Science and Technology (200 Scholars)”, and also for providing travel grants for presenting my work in two international conferences.
This thesis pertains to model validation and novel higher order sliding mode observers and controller design for Pakistan Research Reactor-1 (PARR-1). The observers have been designed for estimating precursor concentration and reactivity and the controller is designed for regulating output power. Linear and non-linear models of PARR-1 have also been tuned and validated with experimental data. These models are subsequently used for higher order sliding mode observer/controller design and performance evaluation. As a first step of model development the neutronic model available in literature has been tuned to PARR-1 environment. Secondly a simulation model of control rod drive mechanism (CRDM) has been developed and implemented in SIMULINK®. This model has been validated with a lab-based CRDM model, which is similar to PARR-1 CRDM system. Finally neutronic model is appended with the CRDM model to get a complete model from control rod motion to output power change. The complete model is validated on the actual reactor. The mathematical model of the overall system, from control rod movement to output power change is validated with different sets of data at different operating points. Based on the validated model, a robust nonlinear observer and controller have been developed by using second order sliding mode technique. Linear observers have been used in the past to estimate reactor variables, but the bandwidth is limited and performance degraded as the operating point is changed.
The nonlinear observer can cater for this problem in a more efficient way. Similarly sliding mode controller has the added advantage of robustness against parameter variations in addition to increased bandwidth. Although other sliding mode techniques have been used for estimating reactor variables, but use of higher order sliding mode observer for this applications has not been reported in the literature, according to the author’s knowledge. The higher order sliding mode observer is more efficient and has the advantage of reduced chattering. The observer estimates reactor parameters with only the measurement of neutron flux. The estimated value is in close agreement with the theoretically calculated value. In addition to HOSM controller two other types of robust controllers are also designed for bench-marking purpose. These controllers include Linear Matrix Inequality (LMI) based robust controller and $H_{\infty}$ loop shaping controller.
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NOMENCLATURE

\( n_r \) \( n/n_0 \), neutron density relative to initial equilibrium density.

\( n \) Neutron density (n/cm³)

\( n_0 \) Initial equilibrium (steady-state) neutron density.

\( c_{ri} \) \( c_i/c_{i0} \), relative density of \( i \)th group precursor.

\( c_i \) Core averaged \( i \)th group precursor density (atom/cm³)

\( c_{i0} \) Initial equilibrium (steady-state) density of \( i \)th group precursor.

\( \rho \) \( (k-1)/k \), reactivity (\( \Delta k/k \)).

\( k \) \( k_{\text{eff}} \), effective neutron multiplication factor.

\( \Lambda \) Effective prompt neutron life time (s).

\( \lambda_i \) Radioactive decay constant of \( i \)th group neutron precursor (s⁻¹).

\( \beta \) Total delayed neutron fraction.

\( \beta_i \) ith group delayed neutron fraction.

\( T_f \) Average reactor fuel temperature (°C).

\( T_l \) Temperature of the water leaving the reactor (°C).

\( T_e \) Temperature of the water entering the reactor (°C).

\( T_c \) \( (T_l+T_e)/2 \), average reactor coolant temperature (°C).

\( f_f \) Fraction of reactor power deposited in the fuel.

\( P_0 \) Initial equilibrium power (MW).

\( \mu_f \) Total heat capacity of the fuel = weight of fuel times its specific heat (MW.s°C).

\( \mu_c \) Total heat capacity of the reactor coolant = weight of coolant times its specific heat (MW°C).

\( \Omega \) Heat transfer coefficient between fuel and coolant (MW°C)
$M$ Mass flow rate multiplied by heat capacity of the coolant (MW\(\cdot\)C).

$\delta \rho_r$ Reactivity due to control rod movement.

$Z_r$ Control input, control rod speed in units of fraction of core length per second.

$G_r$ Total reactivity worth of control rod.

$\alpha_f$ Fuel temperature reactivity coefficient (\(\Delta k/k^0C\)).

$\alpha_c$ Coolant temperature reactivity coefficient (\(\Delta k/k^0C\)).

$T_{f0}$ Initial equilibrium (steady-state) fuel temperature.

$T_{c0}$ Initial equilibrium (steady-state) coolant average temperature.
CHAPTER: 1
INTRODUCTION

For the past few decades nuclear energy has been widely used in diverse areas, such as power generation, medical, agriculture, and defense etc. Power and research reactors are used in many countries for generating electricity and for radio isotopes production intended for medical applications. Nuclear reactors are used to control fission process for producing thermal energy and consequently electrical energy or for providing propulsion energy to nuclear submarines. Now a days nearly 14% of world electricity is produced by nuclear reactors [1]. Nuclear reactors are very complex systems, comprising of many large subsystems. There are many parameters which need to be measured and controlled continuously for safe and proper operation of these reactors. Although most of the parameters are measurable, but there are some important parameters which cannot be measured. Precursor concentration and reactivity are two very important parameters which are immeasurable due to unavailability of adequate sensors. In this thesis novel higher order sliding mode observers are designed for estimating these parameters. Controlling reactor power is another extremely important problem. If due to any reason the fission process becomes uncontrollable then it leads to dangerous accidents such as Chernobyl or three mile Iceland accidents. Robust controllers are used extensively for controlling different industrial processes. The main advantage of robust controllers is the ability to withstand sudden disturbances and these controllers can withstand parameter variations. In this thesis Higher Order Sliding Mode (HOSM) based robust controllers are designed for output power regulation. Robust controllers can significantly increase system availability, which is extremely important for countries like Pakistan. In electricity generating reactors unscheduled
shutdown costs millions of rupees. By using robust controllers this factor can be minimized to a great extent.

PARR-1 was first made critical on 21st December 1965, and has been continuously used for research experiments and isotopes production. The instrumentation and control system was upgraded in 1986. The system has again become quite old and due to added advantages and robustness of new controllers, a need for improving existing system always remains in place. Keeping these requirements in mind, higher order sliding mode controller is designed for regulating output power. As discussed above precursor concentration and reactivity cannot be measured directly, so nonlinear observers for estimating these parameters have been designed for PARR-1. Before performing these tasks, mathematical model of PARR-1 was developed and validated with experimental data, as accurate mathematical model is essential for proper controller or observer design. The reason for using higher order sliding mode technique is outlined in the next paragraph.

Sliding mode technique has gained much importance in industrial applications and has been widely applied in a variety of applications. Due to its simple structure, inherent robustness and capability to control non linear systems, it has gained researcher’s attention and a variety of new algorithms have been proposed. Sliding mode technique has been in use since early sixties in the former USSR and in modern world since late seventies after a book by Itkis (1976) and a survey paper on variable structure systems by Utkin in 1977 [2]. This technique has two major advantages which attract control community, namely robustness against parameter variations and model reduction. Another advantage is easy implementation of the controller. The controller brings states from any arbitrary initial conditions to a sliding surface and then the states are bound to remain on sliding surface onwards. The first phase is called reachability phase and the second one is called sliding phase. It is this sliding phase where system becomes insensitive to parameter variations and the system order is reduced.
The sliding phase is maintained by switching control action very rapidly in the direction where switching surface is steered towards origin. The main drawback of this technique is high frequency variation, which causes chattering phenomenon in relay control system. This drawback has been catered for by using higher order sliding modes, which use derivatives of sliding surface instead of normal sliding mode. Sliding mode technique has been used for synthesizing robust controllers, observers, and for parameter estimations. Second order sliding mode control has been proposed by Levant [3-4] and is being applied in new applications. The main advantage of this algorithm is that higher order derivatives of the states are not required and a chattering free control law is derived. Another advantage of sliding mode control (SMC) is that it can be easily implemented due to its simple structure.

In this thesis higher order sliding mode observers are used to estimate precursor concentration and reactivity and higher order sliding mode controller is used for regulating output power. Sliding mode observer works by minimizing the error between plant model and the observer model by using a switching function. The observer gain is adjusted by the error such that the plant output matches with observer output or in other words, error surface moves towards minimum. Precursor concentration and reactivity are very important parameters for nuclear reactors, since they are used in many reactor physics calculations [5], also accurate measurement of reactor parameters help in condition monitoring of the reactor systems and thus help in avoiding accidents. Higher order sliding mode observers are used to determine these variables, with only the neutron flux measurement available. Due to unavailability of adequate sensors precursor concentration and reactivity cannot be measured directly and hence there is a need for observers for estimating these variables. The first approach for estimation could be use of linear observers, but it is not a viable solution as far as nuclear reactor is concerned. The dynamic behavior of nuclear reactor changes considerably at different power levels and is also dependent on many factors such as Xenon production, fuel
burnup etc. The linear observer behaves satisfactorily only around small neighborhood of reactor operating point, and for large variables change its performance is much degraded. So to cover a large operating range a need for nonlinear observer is obvious. In addition to nonlinearities, the reactor state estimation becomes more difficult due to measurement noise, parameter variations and presence of model uncertainties. Nonlinear observer such as sliding mode observer can cater these problems satisfactorily and due to inherent robustness it is insensitive to parameter variations and can also withstand model uncertainties. The second order sliding mode observer/controller is used for estimating reactor parameters and for regulating output power of Pakistan Research Reactor-1 (PARR-1).

1.1 Thesis Contribution

Higher order sliding mode technique has been used for the first time for parameter estimation and controller design in a nuclear reactor application. HOSM technique is selected due to many advantages not found in other techniques. The HOSM control law keeps robustness properties of first order sliding mode and has additional advantage of chattering removal. Sliding mode technique is ideally suited for systems having relays as actuators, which is the case for PARR-1 so this technique is selected for parameter estimation and controller design for PARR-1. First order sliding mode controllers and observers have been reported in literature, but use of HOSM has never been reported according to best knowledge of author. Linear and nonlinear models of PARR-1 have been validated on real reactor data.

Detailed thesis contribution is as follows, first the reactor model found in literature was tuned and validated for PARR-1, and then higher order sliding mode observers
were designed for parameter estimation and finally higher order sliding mode controller was
designed for power regulation. The details of all these steps are given next.

(a) Reactor model development and validation

General reactor model found in literature was tuned to PARR-1 environment. Some
model parameters were again calculated by using basic physics laws. A complete
SIMULINK/MATLAB model was developed for simulating mathematical model[6]. The
complete simulation model was developed in two steps, first a Control rod drive mechanism
(CRDM) model was developed, which was later appended with neutronic model. The CRDM
model was first validated with lab based CRDM system, which is similar to PARR-1 CRDM
system. For acquiring validation data from actual reactor, a data acquisition system, using NI
data acquisition card and labVIEW control software was used. The control rods were moved
in a chirp sequence and the output power variations were recorded with the help of labVIEW.
The SIMULINK model was given the same input signal and the output of model and
measured data was compared for model validation. Different sets of data were taken at
different power levels, which were latter used for validating nonlinear and linear models.

(b) Design of HOSM observer for parameter estimation

Higher order sliding mode observer was designed for estimating reactor parameters
such as precursor concentration and reactivity[7]. Neutron measurement was used as input.
Super twisting algorithm was employed for designing precursor and reactivity observers. As
derivative of neutron flux was needed for observer design, double derivative of neutron was
estimated by using HOSM observer.

(c) Design of Robust controllers for reactor power control

The HOSM controller was designed for regulating output power of PARR-1[6]. The
HOSM controller was successfully implemented for controlling speed and position of lab-
based CRDM system. Linear Matrix Inequality (LMI) based robust controller[8] and $H_\infty$ loop shaping controllers[9-10] were designed and simulated for benchmarking purpose. Performance comparison of all designed controllers carried out and HOSM chattering free controller found most suitable for PARR-1 environment.

1.2 Thesis Structure

Thesis structure has been formulated; keeping in view three main tasks performed namely model development and validation, higher order parameter estimation and higher order controller design. Logically the thesis can be divided in two sections. Section 1 comprises of chapters 1 to 3 and section 2 includes chapters 4 to 7. First section covers introduction, reactor system and reactor model, while second section discusses the techniques used for parameter estimation and controller design, the brief details of each chapter is discussed next. Chapter 2 is about reactor systems. Here a detailed description of PARR-1, the system on which experiments were performed, is given along with general reactor parameter estimation and control problem. Chapter 3 discusses in detail about nonlinear and linear models development and validation for the reactor under study. A brief discussion about linear system identification is also given in this chapter. Sliding mode technique is discussed in chapter 4. Here basic concept is illustrated with examples and computer simulations. Basic concept of higher order sliding mode is also discussed in this chapter. Details about HOSM parameter estimation, with experimental results is given in chapter 5. Chapter 6 deals with reactor controller design and computer simulations. Here $H_\infty$ loop shaping controller, LMI based controller and HOSM based controllers are discussed and performance of each controller is compared. Finally conclusion and future work is given in chapter 7.
CHAPTER: 2
NUCLEAR REACTOR SYSTEMS

This chapter is an introduction to basic nuclear reactor system. The chapter starts with a brief history of nuclear reactors and then basic components, classification and types of reactors are discussed. After discussing basic reactor system a detailed description of research reactors is given. Pakistan Research Reactor-1 (PARR-1) is discussed in detail afterwards.

2.1 A BRIEF HISTORY OF NUCLEAR REACTORS

Initially reactors were built for producing $^{239}\text{Pu}$, but later on they have been used in many diverse applications, of which electricity generation may be considered as the most important application. Other uses are ship propulsion, radio isotope production such as Iodine-131 for medical applications, and to supply heat in some cases. Other reactors called research reactors are built for teaching and research. The latest use of research reactors involves study of materials under neutron irradiation and for finding out basic properties of neutrons and other such particles.

On December 20, 1951, at the Experimental Breeder Reactor EBR-I in Arco, Idaho, USA, the nuclear energy was used first time ever in history for illuminating four light bulbs. EBR-I was not intended for producing electricity but for proving the viability of breeder reactor concept.

On June 26, 1954, the nuclear power plant APS-1 with a net electrical output of 5 MW was connected to the power grid in Russia. It was the world's first nuclear power plant that has been used ever in the history for commercial use of nuclear energy. On August 27, 1956 Calder Hall 1, the UK first commercial nuclear power plant was connected to the grid. The capacity of this reactor was 50 MW, which was later increased to 200 MW. By considering data since 30 June 2009, 436 nuclear power plant units with an installed capacity of about 370 GW are in operation in 31 countries and 48 plants with an installed capacity of 42 GW are under construction in 15 countries [1].
2.2 CLASSIFICATIONS OF REACTORS

2.2.1 Nuclear fission

Most reactors, and all commercial ones, are based on nuclear fission [11-13]. Uranium and its product plutonium are generally used as nuclear fuel; however a thorium fuel cycle may also be used. Fission reactors can be classified into two groups; thermal and fast reactors depending on the neutron energy used for maintaining the fission chain reaction. Fusion power is a new technology, in which hydrogen is generally used as fuel. Currently fusion technology is not used for power production, however it is used in neutron radiation applications.

2.2.2 Thermal and Fast Reactors

Reactors designed to operate with thermal neutrons, the neutrons whose energy have been slowed down to thermal energy ranges are called thermal reactors. Almost all of the world powers generating reactor are of this type. Reactors can also be operated with fast neutrons. No moderator is needed in such types of reactors and very few neutrons reach thermal energies by elastic collision. These types of reactors are called fast reactors. The liquid metal reactor is the only example of an operating fast neutron power reactor, though other types of fast reactors have also been constructed for experimental use.
2.2.3 Homogeneous and Heterogeneous Reactors

All power reactors in use today are of heterogeneous type i.e. fuel, coolant and moderator, if present, are separate physical units. However, in the early days of nuclear power, there was considerable search for another configuration, the homogeneous reactor characterized as a reactor whose small scale composition is uniform. Homogeneity can be achieved by using the fuel in liquid form, where heat is transferred by circulating this liquid fuel to a steam generator. Presently homogenous reactors for electricity generation is used nowhere in the world, therefore all the references given in the future will be of heterogeneous type.

2.3 COMPONENTS OF CONVENTIONAL REACTOR

All nuclear reactors consist of a range of structural components and systems comprising of mechanical and electrical controls [14-15]. In typical reactors, these are the fuel, the moderator, the coolant, and neutron absorbing materials such as cadmium used to control the reactor power level. The key difference between different reactor types is due to the choice of fuel, coolant and moderator.

2.3.1 Fuels

Naturally available elements which could be used directly as reactor fuel are Uranium and Thorium, additionally $^{233}$U and $^{239}$Pu can be made from $^{232}$Th and $^{238}$U. Considering uranium as a fuel, a number of options are available for the fabrication of fuel type. A significant range of enrichment of $^{235}$U can be used in reactors but usually enrichment in the vicinity of 4% is common in Light Water Reactors (LWRs), which are used for producing most of world’s current nuclear energy. Now the trend is shifting towards higher enrichment and more burn-up of the fuel. Solid fuel is used in today’s heterogeneous reactors, which is mostly in an oxide form as UO$_2$, but metallic fuel can also be used.

2.3.2 Moderators

For operating reactor at thermal neutron energies, a moderator is essential. Thus most of today’s operating reactors use moderator with an exception of fast breeder reactors. Usually
light water, heavy water, graphite and beryllium are used as moderators. Any of these can be used with uranium enriched in $^{235}\text{U}$. When natural uranium is used as fuel, chain reaction cannot be achieved with light water as a moderator, but heavy water or graphite can be used due to their high moderating ratios.

2.3.3 Coolants

In power generation plants, coolant is used for transferring heat energy from fuel to electrical turbine. Cooling is an essential feature of energy transfer, but it has a special additional importance in a nuclear reactor, because heat production is continued even after the reactor is shut down and electricity generation has stopped due to the radiation decay phenomenon. Hence there is still a need for maintaining cooling to evade core melting.

Either liquid or gas can be used as a coolant. Commonly used coolants in thermal reactors are the light water, heavy water, helium and carbon dioxide. Reactors are generally designated by the type of coolant used hence the names light water reactors (LWRs), heavy water reactors (HWRs) and gas-cooled reactors (GCR). The coolant can also be used as a moderator, as in the case of LWRs and HWRs. In gas cooled reactors, due to the low density of the coolant it can not be used as the principal moderator, and hence graphite is used.

2.3.4 Control Materials

Reactor operation is regulated with the help of control materials which also provide a means for reactor scram or quick shutdown. Due to high cross section for the absorption of thermal neutrons, Boron and Cadmium are the mostly used control materials. These control materials are generally used in the form of rods, commonly known as control rods. Control rods of Pressurized Water Reactors (PWRs) are in the form of boron carbide ($\text{B}_4\text{C}$) or cadmium in a silver indium alloy with 5% cadmium. In Boiling Water Reactors (BWRs) boron carbide is usually used as a control material. Boron may also be mixed into the circulating cooling water for regulating reactor operation.
2.4 Types of Reactors

Different types of reactors are in operation in the world today, though the diversity was more in earlier days of the reactor design. The main reactor type in use today is the light water reactor (LWR) in which ordinary water is used as both the coolant and moderator and fuel comprises of enriched uranium in UO₂ pellets. Light water reactors are of two types: the pressurized water reactor (PWR) and the boiling water reactor (BWR). Currently there are about 79% of LWR (57% PWR and 22% BWR), 8% gas cooled reactors (GCR) and 13% other types of reactors [1]. Some common reactor types are described below.

2.4.1 Pressurized Water Reactors (PWRs)

Nuclear fuel, control rods, moderator, and coolant are kept in a pressure vessel in these types of reactors. High pressurized liquid water is used as a coolant and moderator in these reactors. The water leaving the pressure vessel is circulated through a steam generator for producing steam by a secondary loop that can run turbines. As this hot primary loop water is radioactive so heat is transferred to a secondary loop by conduction which contains non-radioactive water used for steam production. Majority of the current reactors are of this type, and are generally considered the safest and most reliable technology at present in large scale deployments. The block diagram of PWR is shown in Fig. 2.2 [1]

![Block diagram of Pressurized Water Reactor (PWR)](image)
2.4.2 Boiling Water Reactors (BWRs)

A BWR shown in Fig. 2.3 is much like a PWR with the exception of a steam generator [1]. Like PWR it is cooled and moderated by light water, but the pressure is kept low. Due to this low pressure, water boils inside the pressure vessel which in turn produces steam used for running the turbine. As opposed to PWR, no primary or secondary loops are present. The design of these reactors can be simpler, potentially more stable and safe with a better thermal efficiency. Advanced Boiling Water Reactor and the Economic Simplified Boiling Water Reactor are the latest versions of this thermal neutron reactor design.

![Block diagram of Boiling Water Reactor (BWR)](image)

Fig.2.3. Block diagram of Boiling Water Reactor (BWR)

2.4.3 The CANDU Pressurized Heavy Water Reactor (PHWR)

CANDU (Canadian Deuterium Uranium) is basically a Canadian design. These reactors are a special type of Pressurized-Water Reactors, cooled and moderated by heavy water. The fuel is contained in hundreds of pressure tubes as opposed to a single large pressure vessel in PWR. Natural uranium is used as a fuel in these types of reactors and hence these reactors are categorized as thermal neutron reactors. As PHWRs can be fueled at full power, the efficiency in use of uranium is very high. This feature also helps in precise flux control in the core. The block diagram of PHWR is shown in Fig. 2.4.[1]
2.4.4 Gas Cooled Reactor (GCR) and Advanced Gas Cooled Reactor (AGR)

Figure 2.5 shows the block diagram of gas cooled reactors [1]. GCRs are generally moderated by graphite and CO₂ is used as a coolant. Owing to high operating temperature, the thermal efficiency is more than PWRs. As this design was originated in United Kingdom, the maximum numbers of operating reactors are in UK. Magnox station, an older design of this type is either shut down or going to be shut down in near future; however the AGCRs, a bit modern design have an estimated life of a further 10 to 20 years. GCR is also classified as a thermal reactor.
2.4.5 Liquid Metal Fast Breeder Reactor (LMFBR)

LMFBR is cooled by liquid metal and does not require moderator. It is characterized by producing more fuel than it consumes. The block diagram is shown in Fig. 2.6 [1]. The term breeder reactor is used due to the fact that fissionable fuel is produced during operation because of neutron capture phenomenon. In terms of efficiency, these reactors function much like a PWR, but as there is no requirement for high pressure for liquid metal, so high pressure containment is not required. Superphénix a French design and Fermi-I in the United States were these types of reactors. Another reactor of this type called Monju (a Japanese design) was shut down after sodium leak accident in 1995. It was approved for restart in 2008. Liquid sodium is used in all these reactors. This type of reactors is categorized as fast neutron design. LMFBR are of two types:

(a) Lead cooled

Excellent radiation shielding and very high temperature operation are achieved by using lead as a liquid metal. Due to transparency of lead to neutrons, the other advantages are very little amount of neutrons loss and also coolant does not become radioactive. In contrast to sodium, lead is typically inert, so a chance of explosion is rare, but to manage these large quantities of

Fig.2.5. Block diagram of Gas Cooled Reactor (GCR)
lead is another important task, which needs special care due to toxic behavior of this material. Normally lead-bismuth eutectic mixture is used in these types of reactors.

(b) Sodium cooled

Most LMFBRs are cooled by sodium because sodium is easily available and also it helps in preventing corrosion on the various immersed parts of the reactor. But, extra care is required in handling sodium, as it explodes violently when exposed to water. However, this explosion is not as much severe as, for example, a leak of superheated fluid from a PWR.

![Block diagram of Liquid Metal Fast Breeder Reactor (LMFBR)](image)

Fig.2.6. Block diagram of Liquid Metal Fast Breeder Reactor (LMFBR)

**2.5 Research Reactors**

In contrast to power reactors, these reactors are called non-power reactors. Power reactors are mainly used for electricity production, heat generation, or for providing propulsion energy to nuclear submarines, where as research reactors are used primarily for neutron production, intended to be used in non-destructive testing, material analysis and for radioisotope production. The reactors used for isotope production are also called isotope
reactors. The isotopes produced by these reactors are used in medical and industrial applications. As opposed to power reactors, the design of research reactors is much simpler with low operating temperature, also very less fuel is required and the amount of fission products as a result of fuel burn-up is also low. As far as enrichment is concerned, normally high enriched fuel is used in research reactors, most of the reactors use enrichment close to 20%, but some use as high as 93% U-235. Power density in the core is also very high, requiring special design features. As in power reactors, cooling is required for maintaining temperature of the core under desired limits. Normally natural or forced convection with water and a moderator is used to slow-down the neutrons for enhancing fission.

2.5.1 Research Reactor Classes

(a) Aqueous homogeneous reactor

Aqueous homogeneous reactors (AHR) use fuel in liquid form, produced by solving uranium sulfate or uranium nitrate in light or heavy water. Since the fuel is mixed with coolant and moderator, the term “homogeneous” is used in its name. Criticality can be achieved in heavy water aqueous homogeneous reactor by dissolving natural uranium as uranium sulfate, so enriched uranium is not required for this reactor. AHR with heavy water have the lowest specific fuel requirements and the neutron economy is the highest. Similar is the case with light water version where less than 454 grams of plutonium-239 or uranium-233 is required for operation. The main features, which make AHR unique among others, are its self-controlling feature and the capability to manage very large increase in reactivity which also makes it safest. AHR were also termed as water boiler, because water appeared to be boiling, though in actual this bubble formation is due to the production of hydrogen and oxygen as water is dissociated in its constituent gases due to radiation and fission particles.
Owing to its self controlling feature, high neutron flux and easy handling, AHR were widely used as research reactors.

(b) Argonaut class reactor

The Argonaut (Argonne Nuclear Assembly for University Training) was built at Argonne National Laboratory USA. This reactor went critical for the first time on February 9, 1957 and was shut down in 1972. A variety of this type reactors have been built all over the world at different power levels. It is basically a small research reactor design, intended for teaching nuclear physics at universities or for performing engineering laboratory experiments.

(c) DIDO class

DIDO reactor was built by Atomic Energy Research Establishment at Harwell, Oxfordshire in England. In this reactor enriched uranium metal was used as fuel, and heavy water as both primary coolant and neutron moderator. A graphite neutron reflector was used around the core. This reactor was also called AE334, depicting its engineering design code. The main design feature of this reactor was high neutron flux intended for minimizing the time required for material testing before their use in power reactors. Due to high flux another advantage was the formation of powerful neutron beams which could be used in neutron diffraction applications. Over all, six DIDO class reactors were built based on this design:

- DIDO.
- PLUTO (Harwell).
- HIFAR (High Flux Australian Reactor).
- FRJ-II at Jülich Research Centre, Germany.
- DR-3 at Riso National Laboratory, Denmark.

The last reactor of this type, HIFAR was shut down in 2007.

(d) TRIGA Reactors

TRIGA (Training, Research, Isotopes, General Atomics) reactor, built by General Atomics is the most widely research reactor in the world. It is a pool type reactor which does not require a containment building. The main applications of TRIGA are isotope production, non-destructive testing, nuclear education and private commercial research etc. The fuel used in TRIGA is Uranium-zirconium-hydride (UZrH), having large prompt negative thermal coefficient of reactivity which means that reactivity is inversely proportional to temperature, i.e. the reactivity decreases sharply by increase in core temperature. This feature greatly reduces the chance of core meltdown.

About 66 reactors of this type are in use or under construction all over the world at universities, medical centers and research institutes. TRIGA can function over a wide range of thermal powers, ranging from less than 0.1 MW to 16 MW. In pulse mode, it can be given a pulse of about 22,000 MW. This high power pulse is possible due to the unique features of UZrH fuel.

(e) Material Testing Reactors (MTR)

Material Testing Reactor (MTR) was built at the National Reactor Testing Station at Idaho National Engineering Laboratory, USA. This reactor started operation in March 1952 and was designed to provide a high neutron flux for studying radiation effects on reactor materials. Ordinary water was used as moderator and coolant and also to some extent as reflector. The fuel elements were slightly curved “sandwich” plates consisting of an alloy of aluminum and enriched uranium clad on both sides with aluminum. A number of parallel
plates were brazed into aluminum side members to form a long box-like assembly, roughly 76 mm by 76 mm across. A large heat removing area provided by the plate-type fuel elements make operations at fairly high power possible.

The use of these MTR-type elements facilitated the construction of the Bulk Shielding Reactor (BSR) at Oak Ridge National Laboratory by the end of 1950. The core comprised of 16 to 25 box-like assemblies which were arranged in the form of a parallelepiped with spaces in between for the control rods. The whole assembly was suspended to a depth of some 6.1 m in water which served as moderator, coolant, and reflector, and also for providing radiation shield. The BSR was nicknamed the “swimming pool” reactor because of the large tank of water in which it was enclosed. The simplicity and convenience of the pool design has led to its adoption in a number of reactors around the world for use in university and other research institutes.

(f) SLOWPOKE Reactors

The SLOWPOKE (Safe Low-Power Kritical Experiment) is a pool-type research reactor, designed for low-energy applications. It was designed in late sixties by Atomic Energy of Canada Limited (AECL). In this reactor very low critical mass is used, with beryllium as reflector, but the neutron flux produced is greater than obtainable from radioactive sources or small particle accelerator. Uranium enriched up to 93% in the form of 28% uranium-aluminum alloy with aluminum cladding is used in SLOWPOKE-2 reactor. There are about 300 fuel pins in the core with diameter of only 22cm and height equal to 23 cm. The core is surrounded by fixed beryllium annuls and a beryllium slab at the bottom. Beryllium plates are added in a tray at the top of the core to maintain criticality. The pool of light water with 2.5 m diameter and 6 m depth is used to place the core, which is cooled by natural convection. Besides cooling, this reactor has some other safety features, as fission
process slowing down, if water heats up or forms bubbles. Due to these safety features the SLOWPOKE-2 reactor can be operated unattended overnight. The nominal power of most of the Slowpoke reactors is around 20 kW, but operations at higher power for short durations can also be performed.

The main uses of these reactors are neutron activation analysis (NAA), irradiation studies, education, training of reactor personnel, fabrication of radioactive tracers and neutron radiography etc. Reliability, ease of use and reproducibility of the neutron flux are the key advantages of this reactor design. As fuel is unmodified for at least 20 years, the neutron spectrums in the irradiation sites remain unchanged and the flux can be reproduced within 1% accuracy.

**(g) Miniature neutron source reactor (MNSR)**

Miniature Neutron Source Reactor (MNSR) is fundamentally a Chinese version of the Canadian Slowpoke reactor design. It is a small and compact design with a nominal power of 27 kW and the maximum nominal power is approximately 30 kW. Other characteristics and performances are also similar. The MNSR is tank-in-pool type, where the core is immersed in a tank and the tank itself is immersed in a large pool. The fuel used is high enriched U235 with enrichment approximately equal to 90%. Natural convection is used for cooling the core. There are about 347 fuel rods, 4 tie rods and 3 dummy elements in the core, which are distributed in 10 circles containing fuel rods in a range varying from 6 to 62. The core is surrounded radially by a thick beryllium reflector.

**(h) Nuclear marine reactors**

Nuclear energy is also used for providing propulsion energy for submarines or cargo ships. The majority naval reactors are of pressurized water reactors (PWR) with some exceptions of liquid metal cooled reactors. These reactors are very compact and rugged in
design, so they can withstand violent forces exerted by vessel movement or harsh weather. Most of the naval reactors have highly enriched uranium as fuel. Steam turbine propulsion is used in the Russian, U.S. and British designs, while the French and Chinese designs use the turbine to generate electricity which, in turn is used for providing propulsion energy.

After giving a general introduction of nuclear reactors, especially power reactors, we will now give a thorough introduction of Pakistan Research Reactor-1 (PARR-1), the system used for thesis related experiments.

2.6 **Pakistan Research Reactor-1 (PARR-1)**

Pakistan Research Reactor-1 (PARR-1) is a 10MW pool type research reactor with a parallelepiped core which comprises of low enriched uranium (LEU) fuel. It was first made critical on 21st December 1965. The instrumentation and control system was upgraded in 1986 and the reactor power from 5MW with high enriched uranium (HEU) to 10 MW (LEU) was upgraded in 1992. Its core has been redesigned with material testing reactor (MTR) type silicide Low Enriched Uranium (LEU) fuel for operation[5, 16].
Fig. 2.7. PARR-1 Block Diagram

Fig. 2.8. PARR-1 Internal view (Operating at 10 MW)
The MTR fuel element is basically a stack of straight fuel plates supported by side plates at both ends throughout its height. PARR-1 core is assembled on a grid plate, having 54 holes arranged in 9x6 array, with a lattice pitch of 81.0x77.1 mm. It is a combination of fuel elements, control rods, graphite reflector elements, water boxes for irradiation of samples and fission chambers with guide tubes. One side of the parallelepiped core is reflected by graphite, i.e. thermal column, while opposite side is reflected by a blend of graphite reflector elements and light water. The bottom side is reflected by a combination of aluminum and water. Rest of the three sides, i.e. top and two lateral sides, are reflected by light water only. At PARR-1, five (Ag-In-Cd alloy) control rods are employed for: (a) startup (b) shutdown, and (c) reactor operating power level control. In addition to this PARR-1 core is designed to have negative coefficients of reactivity, a reflection of the fail-safe principle. These control rods can be relocated in any future core configuration as per design requirements [5, 17-18].

PARR-1 has two cooling modes: (a) cooling by natural convection when reactor power < 100 kW, and (b) cooling by gravity-driven downward flowing coolant when reactor power ≥ 100kW. The heat produced in PARR-1 core is carried away by the primary coolant system (PCS). The PCS consists of core, plenum, outlet piping & valves, hold-up tank, centrifugal pumps, heat exchangers and inlet or return piping & valves. This PCS of PARR-1 transfers its excessive heat to the secondary cooling system, which dissipates this energy to atmosphere via cooling tower. Both primary and secondary cooling systems have auxiliary systems for the makeup of the coolant, both quantitatively and qualitatively. For quantitative losses water make-up systems are incorporated while water quality is maintained through water purification system. There are five control rods. The reactor becomes critical when these rods are withdrawn about 56% from their fully inserted position. After reactor becomes critical, a single rod is used to regulate the reactor power [5]. The block diagram of PARR-1 is shown in Fig 2. The inside view of pool and bridge is shown in Fig 3. The design parameters
of the reactor are shown in Table 1. For the safe operation of PARR-1 several engineered safety features are incorporated. The PARR-1 core has been appropriately and adequately instrumented to seek information about its operational states and also for suitable control of these states. To adequately present the operational states PARR-1 control console consists of the reactor control system, the radiation monitoring system readouts, a monitoring panel of auxiliary system and power supply system of the console.

Table 1: PARR-1 Design parameters

<table>
<thead>
<tr>
<th>Type</th>
<th>Swimming Pool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal core power (MW)</td>
<td>10</td>
</tr>
<tr>
<td>Lattice pitch (mm)</td>
<td>81.0 x 77.11</td>
</tr>
<tr>
<td>Fuel material and enrichment</td>
<td>U₃Si₂-Al (19.99 % by wt)</td>
</tr>
<tr>
<td>Cladding material</td>
<td>Aluminum</td>
</tr>
<tr>
<td>Coolant/Moderator</td>
<td>Light water (H₂O)</td>
</tr>
<tr>
<td>Coolant flow rate (m³/h)</td>
<td>950</td>
</tr>
<tr>
<td>Reflector</td>
<td>Light water and Graphite</td>
</tr>
<tr>
<td>Fuel element description</td>
<td>Straight plate MTR type fuel element</td>
</tr>
<tr>
<td>U₂³⁵ contents per fuel plate (g)</td>
<td>12.61</td>
</tr>
<tr>
<td>Control rods</td>
<td>Oval shaped 5 rods</td>
</tr>
<tr>
<td>Compositions of control rods</td>
<td>80% Ag, 15% In, 5% Cd</td>
</tr>
<tr>
<td>Operational Modes</td>
<td>Manual and Automatic</td>
</tr>
<tr>
<td>Feedback coefficients:</td>
<td></td>
</tr>
<tr>
<td>Doppler coefficient</td>
<td>-1.91x10⁻³</td>
</tr>
<tr>
<td>Moderator coefficient</td>
<td>-5.97x10⁻³</td>
</tr>
<tr>
<td>Void coefficient</td>
<td>-0.34x10⁻³</td>
</tr>
<tr>
<td>Irradiation sites:</td>
<td>Neutron Flux:</td>
</tr>
<tr>
<td>Beam ports</td>
<td>~1.0x10¹³</td>
</tr>
<tr>
<td>Thermal column</td>
<td>depends on the depth in thermal column</td>
</tr>
<tr>
<td>Pneumatic rabbit</td>
<td>~3.0x10¹³</td>
</tr>
</tbody>
</table>
2.7 INSTRUMENTATION AND SENSORS DESCRIPTION

The instrumentation and control of PARR-1 can be divided in three main categories[5]

Nuclear Instrumentation
Process Instrumentation
Radiological Monitoring Instrumentation

2.7.1 Nuclear Instrumentation

Nuclear Instrumentation is basically used for measuring nuclear power. As the control rods are withdrawn fission process starts and neutron flux starts increasing. This flux is measured by using Fission chambers (In startup range) and uncompensated ionization chamber (UIC) and compensated ionization chamber (CIC) in power range. Five types of nuclear channels are used for measuring neutron flux (reactor power) at startup and power ranges of the reactor:

- Startup channels
- Linear Channels
- Logarithmic dc channels
- Linear Safety channels
- Nitrogen –16 channels

Different channels used are meant for keeping diversity, and also multiple channel of each type is installed for keeping redundancy. The Fission chambers are used to detect neutrons at the startup range. Fission Chamber (FC) is made by filling noble gas in a uranium coated cylinder polarized with high voltage. As the control rods are withdrawn, fission process starts and number of neutrons passing through the chamber increases. As a neutron passes through the cylinder, it ionizes the gas, and a pulse is generated from anode to cathode.
These pulses are directly proportional to the number of neutron passing through the cylinder. These pulses amplified by a preamplifier, are counted by Logarithmic Count Rate Meter (LCRM). As the flux increases then, instead of pulses a current start flowing between anode and cathode, but FC is not capable of bearing this current (there is a chance of burning of the uranium coating), so FCs are withdrawn from the core with the help of rod drive mechanism. In power ranges Uncompensated Ion chamber (UIC) and Compensated Ion chamber (CIC) are used.

The Ionization chambers are also constructed by filling noble gas in a cylinder. The cylinders are polarized with high voltage, so one end becomes cathode and other becomes anode. The noble gas is normally unionized, but when radiation passed through it, makes the gas ionized so a current starts flowing between anode and cathode. This current, although very much small, is directly proportional to the amount of neutron flux. As neutron flux can not be measured alone due to presence of gamma radiation, so in compensated ionization chamber, instead of one, two cylinders are used, one is used for measuring gamma radiations and the other for combined gamma and neutron. The gamma radiation output current is then subtracted from the current due to combined flux to get the current only due to neutron flux. The UIC output current corresponds to cumulative effect of neutron flux and gamma radiations.

2.7.2 Process Instrumentation

In PARR-1 different process parameters are measured. The most common are listed as follows

Thermocouples and RTDs for measuring temperature of different locations (such as core inlet and outlet temperatures)

Level sensors for measuring water level of main reactor pool, holdup tank etc
Flow sensors for measuring primary and secondary loop flow
Pressure sensors for measuring Reactor building inside pressure
Water quality measurement
Earthquake measurement

2.7.3 Radiological Monitoring

Radiation monitoring channels are installed for measuring radiation level at different parts of the plant. Usually Geiger Muller (GM) tubes and ionization chambers are used for measuring radiations at different places.

2.8 PARR-1 Fault Detection / Protection Scheme

The reactor protection is implemented in such a manner that it automatically initiates the required protective action, which will go to completion without hindrance from the operator or any other means. Currently three types of actions, to avoid any catastrophic condition are implemented at PARR-1. These are classified as follows

Reactors Scram (Trip)
Control rods fall under action of gravity to the fully seated position thus ceasing fission reaction.

Reverse Action
Instead of moving upward, control rod starts moving downward, thus giving an indication to the operator to take corrective action

Inhibit Action
Control rod stops moving and remain where they are
Annunciation

Only alarm is generated, no action is performed on control rods.

For serious faults scram action is implemented, similarly for less serious faults reverse, inhibit or only annunciations are implemented. In all cases an alarm window starts blinking, thus indicating probable cause of the fault. The parameters values estimated by the proposed observer could be much helpful in future PARR-1 fault diagnostic scheme.

2.9 Summary

This chapter has given a brief introduction of nuclear reactors. After giving a brief history, the classification of reactors is discussed, followed by components of the reactor system. After it different types of reactors are discussed and finally a detailed description of PARR-1 is given. After having a general idea of reactor systems, next chapter gives a detailed analysis of the mathematical model of PARR-1. This model will be used subsequently for observer and controller design.
3.1 INTRODUCTION

This chapter is about reactor modeling, covering both linear and nonlinear models. The model obtained by using system identification techniques has also been discussed. After discussing the general model, the step by step development of simulation model is discussed. First of all control rod drive mechanism (CRDM) model has been developed using MATLAB/SIMULINK and then complete model is formed by appending neutronic model with the CRDM model. The developed model is then verified in steps. First CRDM model is validated with lab based CRDM prototype system, and then the whole model is verified by experimental data taken from the actual reactor. The details will be discussed in coming paragraphs, after discussing briefly about the related work.

3.2 PREVAILING MODELING TECHNIQUES

Cs. Fazekas et al. have derived a low order nonlinear dynamic model using first engineering principles, for primary circuit of a VVER-type nuclear power plant [19]. For ensuring accuracy of the developed model, it was validated by using measured data from three VVER-440 units of Paks nuclear power plant. Some unknown parameters were also estimated using Nelder-Mead Simplex method. The developed model was used for the redesign of primary loop controllers. Vanttola et al. validated neutron kinetic and thermal hydraulic codes by using experimental data from different VVER type nuclear power plants [20]. Two transients of control rod drop data at nominal power were used for modal validation. Eight institutes contributed to the validation with 10 calculations using 5 different combinations of coupled codes. Z. Dong et al. developed a dynamic model for a low temperature power reactor (LTR) [21]. The lumped parameter model was developed for primary-loop and U-tube steam generator. The model was developed by using fundamental laws of fluid mass, energy and
momentum. The point kinetics with reactivity feedback and thermal hydraulics of the reactor were coupled to formulate the dynamic model. Simulations of the model were performed on MATLAB/SIMULINK. The numerical simulation results showed that steady-state performance results were in acceptable limits and the transient response trend was also correct. D. Lathouwers et al. developed a theoretical model describing the coupling of neutronics, thermo-hydraulics and fluidization in a fluidized bed nuclear reactor [22]. The stability of the system was also explored. Simulations of transient conditions were also carried out by a theoretical start-up with transient and a quasi-static transient correlated to noise resulting from stochastic movement of fuel particles. It was proved by simulations that although reactor power reached higher values, the fuel temperature remained under safety limits. T. Adali et al. developed a recurrent multilayer perception model (RMLP) model for the simulation of core neutronic phenomena in a nuclear power plant, consisting of large number of state variables [23]. The training and testing data was produced by REMARK, a first principal neutronic core model. For speeding up training, modified back-propagation learning algorithm with an adaptive steepness factor was employed. M.E. Pomerantz et al. devised a scheme for improving theoretical model by using experimental data [24]. In-core neutron detectors were used for taking experimental data, which was used later for validating and improving theoretical model. Flux mapping was tested on CANDU-600 reactor for several reactivity device configurations. The comparison of flux mapping using in-core detectors showed improved performance as compared to flux mapping calculated by DIFFUSION computer code. A. Tanaka et al. developed a 3-D virtual analysis system to analyze the motion of control rod drive mechanism (CRDM) [25]. The 3-D simulator was verified by mock-up tests using the same equipment as the actual product. The parameter study was also performed to establish the relationship between the parameters and the CRDM motion. Y.P. Chyou et al. developed a prototype of control rod drive mechanism (CRDM) [26]. A simulation model using MATLAB/SIMULINK was also developed to verify the prototype design parameters. Test results were used to benchmark the model and then parametric sensitivity studies were carried out through model calculations. Computational and experimental studies showed the improvement of impact force on the modified design as
compared with the counterpart on the original design. T. Dudley et al. used REMARK code to develop neutronic core model of Pebble Bed Modular Reactor (PBMR) [27]. It will be South Africa’s first power plant used for production and generation of electricity. The resulting neutronic model for PBMR is called PBMR-REMARK Reactor Core Neutronic Model. Robustness and stability of the model was verified by simulations performed for model operation under extreme and harsh conditions. H. Khalafi, and M.S. Terman developed a research reactor simulator using neural networks model [28]. The simulator can be used for predicting reactor power, and coolant, fuel and clad temperatures under normal and accident conditions. The main advantages of this neural model were rapid prediction of severe operational transients and accurate estimation of core safety margins. The model was validated by taking into consideration the fast and slow reactivity insertion scenarios for a 10-MW material testing (MTR) research reactor. The results of neural simulator were compared with the results obtained from PARET code and were found to be in close agreement. Kerlin, et al. used frequency response testing for evaluating the dynamic responses of the H.B. Robinson nuclear plant [29] and the Oconee nuclear plant [30]. Olsson applied three different identification methods (multiple input-single output model, vector difference model and vector state equation) on the Halden Boiling Water Reactor (HBWR) [31]. Fukunishi used multivariate autoregressive process for developing an identification method for the dynamic structure of a nuclear power plant. The method was also used for developing diagnostic tools of the boiling water reactor by means of time series data measured at a stationary operating plant [32]. P. Groudev and A. Stefanova validated computer model built for VVER 440 nuclear power plant, a 440 MWe PWR, using RELAP5/MOD3.2 computer code for reactor scram transient [33]. The validation was performed by comparing model results with experimental data obtained from Kozloduy NPP unit # 2. The comparison of model results and test data were in good agreement. E. Usparas et al. modeled RBMK-1500 reactor specific transients using RELAP5-3D code [34]. The experimental data being measured at Ignalina NPP was used to validate the model. RELAP5-3D model of the Ignalina NPP RBMK-1500 reactor was validated by the benchmark problem analyses including the simulation of the measurements of void and fast power reactivity coefficients and the change of graphite
cooling conditions transient recorded at the Ignalina NPP. The performed validation of RELAP-3D model of Ignalina NPP RBMK-1500 reactor helped in improving the model.

As discussed, many models can be found in literature, which varies in design, structure and complexity. The point kinetic model was selected from the work of Edwards et. al [35] because of similarity of the structure. Power reactors have very large models due to many more systems in addition to the neutronic models. There are also many types of research reactors, which vary according to construction and design. The mathematical models of these reactors also vary due to design, thermal hydraulic model variations, complexity etc. The selected point kinetic model is adequate enough for research reactors like PARR-1. The model was consequently adapted for PARR-1 as discussed in next section.

3.3 NONLINEAR MODEL

Before discussing point kinetic equations we need to understand fission process. Usually a couple of fission fragments, an average of 2.5 neutrinos and a collection of beta particles, gamma rays and neutrinos are produced by the fission of U-235. Two types of neutrons are produced as a result of fission process, prompt and delayed neutrons. Prompt neutrons are produced directly from the fission and appear almost instantly and majority of the neutrons produced in a reactor are of this type. The other types of neutrons called delayed neutrons are produced by certain fission fragments known as precursors. These precursors go through beta decay to a daughter nuclide emitting a new neutron. The time taken by the precursor for undergoing beta decay is the delay experienced by the delayed neutrons. An extremely small fraction of a reactor’s total neutron population comprises of delayed neutrons, approximately equal to 0.7 % of total neutron population, but they are much crucial because all of the reactor control depends upon them. The reason behind this phenomenon is that fast neutrons cannot sustain fission reaction and due to more half-life of delayed or thermal neutrons, the overall generation time is increased which makes sustainable fission process possible[36-37].

For tuning nonlinear model to PARR-1, the parameter values needs to be calculated. The parameter values are gathered from different sources and procedures. Many reactor
parameters are recalculated with PARR-1 new core and the current operating conditions. The parameters are determined considering the composition of the fuel, its enrichment and geometry of the reactor core. Some parameters calculated experimentally earlier [5, 38] are used as it is due to their proven surety. Total heat capacity ($\mu_f$) of the fuel is calculated by multiplying weight of fuel with its specific heat. Similarly total heat capacity ($\mu_c$) of coolant is calculated by multiplying weight of coolant with coolants specific heat. Other parameters such as fuel temperature reactivity coefficient ($\alpha_f$), coolant temperature reactivity coefficient ($\alpha_c$) and Heat transfer coefficient between fuel and coolant ($\Omega$) etc are calculated again by using basic physics laws and PARR-1 fuel dimensions and other measurable quantities.

For understanding time-dependent behavior of a reactor, the equations representing prompt and delayed neutron behavior to change in reactivity or control input are required. Mathematically it is a complex problem as the neutron population is a function of both space (position in the core) and time in a reactor. For many practical situations, it can be assumed that the spatial and temporal behaviors are separable. This allows us to write equations of reactor kinetics as a function of time alone. This approach is acceptable for the purposes of personnel training and for routine reactor operation including transients. The space-independent equations of reactor kinetics, also known as "point kinetics" equations[39-40], are:

$$\frac{dn_r}{dt} = \frac{\rho(t) - \beta}{\Lambda} n_r(t) + \frac{1}{\Lambda} \sum_{i=1}^{6} \beta_i c_i(t)$$

$$\frac{dc_i}{dt} = \lambda_i n_r(t) - \lambda_i c_i(t), \quad i, ..., 6$$

(3.1)

Where $n_r$ is the neutron density relative to initial equilibrium density and $c_i$ represents the relative density of $ith$ group precursor. Reactivity is represented by $\rho$ and $\Lambda$ is the effective prompt neutron life time, $\lambda$ represents the radioactive decay constant of $ith$ group neutron precursor. $\beta$ is fraction of delayed fission neutrons. The first equation portrays the behavior of neutrons i.e. the rate of change of the total neutron population is equal to the sum of the rate of change of the prompt and delayed neutrons. The term $(\rho(t) - \beta)$ represents partial change in
the prompt neutron population per generation, and the term \( \frac{\rho(t)-\beta}{\Lambda} n_r(t) \) describes change in prompt neutron population per unit time and the second term as already discussed represents change in the population of delayed neutrons per unit time for six delayed neutron groups. The second kinetics equation is about the behavior of precursors and says that the rate of change of precursors is equal to the difference between its production and loss. In this equation \( \lambda_c c_d(t) \) represents rate of decay of delayed neutron precursors. This equals the rate of appearance of the delayed neutrons and first term represents rate of production of delayed neutron precursors. The other equations of nonlinear model represents rate of change of fuel temperature, coolant temperature and change in reactivity. Now we will discuss these equations one by one.

\[
\frac{dT_f}{dt} = \frac{1}{\mu_f} [f_f P_{a0} n_r(t) - \Omega(T_f - T_c)]
\] (3.2)

In equation (3.2) the term \( f_f P_{a0} n_r(t) \) describes fraction of reactor power deposited in the fuel, and the time dependent heat transfer from fuel to coolant can be represented as \( \Omega(T_f - T_c) \), thus the equation above represents net change in fuel temperature by subtracting heat removed from the fuel due to coolant. The whole term is divided by \( \mu_f \), which is total heat capacity of the fuel and structural material.

\[
\frac{dT_c}{dt} = \frac{1}{\mu_c} [(1-f_f) P_{a0} n_r(t) + \Omega(T_f - T_c) - M(T_i - T_c)]
\] (3.3)

The temperature of the coolant leaving the reactor \( (T_l) \) is calculated in (3.3) by adding \( (1-f_f) P_{a0} n_r(t) \), the fraction of heat removed from the fuel, which will be transferred to coolant, and \( \Omega(T_f - T_c) \), the heat transferred from fuel to coolant and subtracting net heat removal from the coolant which is represented by \( M(T_i - T_c) \), thus by combining these three terms and dividing the whole term by \( \mu_c \), the total heat capacity of coolant, the rate of change of temperature leaving the reactor is calculated by above equation.
Reactivity changes due to change in control rod position and temperature feedbacks due to change in coolant and fuel temperatures. The change in reactivity due to control rod motion can be represented by above differential equation (3.4). The total change in reactivity by adding temperature feedback terms is shown below in (3.5)

\[ \rho = \rho_r + \alpha_f (T_f - T_{f_0}) + \alpha_c (T_c - T_{c_0}) \]  

(3.5)

The control input for changing output power is given by control rods. The system controlling the movement of the control rods is called Control Rod Drive Mechanism or CRDM. The complete simulation model was developed by appending CRDM model with neutronic model. The CRDM model and complete model development is described below.

### 3.3.1 CRDM Model Development

![SIMULINK Model of Control Rod Drive Mechanism (CRDM).](image)

The CRDM system consists of a 110V synchro motor, rack and pinion mechanism for translating rotational motion to translational motion, and two relays. When one relay is energized, control rod moves upwards with a fixed speed of 1.66 mm/s. Similarly rod moves downwards with the same speed when other relay is energized.

The CRDM model is developed by using SIMULINK. For modeling this system an integrator block is used which gives position output. The input to the integrator is assumed to
be the voltage given to CRDM. The input voltage which can be chirp (or any sequence) is first
given to the differentiator block for determining the sign of the slope. Based on the input slope
the direction of the motion of the control rod is determined. The outcome is 1.66 mm/s
velocity with appropriate sign. This velocity is fed to the integrator, which gives the position
of the rod. The model is shown in Fig. 3.1. The model is validated by using lab based
experimental setup for the CRDM system, which is similar to the PARR-1 CRDM system.
Different test sequences given to the model are also given to the experimental setup, for
CRDM model validation. Mathematically it can be represented as
\[
\dot{x} = Z_r = \text{sign}(x)0.00166
\]
\[or\]
\[
x = 0.00166\int \text{sign}(x) dt
\]
(3.6)
Where \(x\) is the position and \(\dot{x}\) represents the rod velocity and \text{sign}(x) determines the direction
of motion.

3.3.2 Complete Model Development

The CRDM model was then appended with neutronic model to formulate the complete model.
The complete simulation model is shown in Fig.3.2. In this model neutronic portion can be
given input either by CRDM model or by experimental data. The manual switch can be used
for desired input. The control rod motion was changed to reactivity by multiplying control rod
worth with the active rod length. This reactivity was then given as input to the neutronic
model. Here again the model output was compared with the experimentally measured power
output of PARR-1, for complete model (from control rod motion to output power variations)
verification. As seen in the simulation section, both outputs are in good agreement, thus
ensuring model validity. The complete mathematical model with rod position model is shown
below.
Fig. 3.2 SIMULINK model for HOSM controller/observer simulations

$$\frac{dn_i}{dt} = \frac{\rho(t) - \beta}{\Lambda} n_r(t) + \frac{1}{\Lambda} \sum_{i=1}^{6} \beta_i c_n(t)$$

$$\frac{dc_n}{dt} = \lambda_i n_r(t) - \lambda_i c_n(t), \quad i, \ldots, 6$$

$$\frac{dT_f}{dt} = \frac{1}{\mu_f} \left[ f_j P_{a0} n_r(t) - \Omega (T_f - T_c) \right]$$

$$\frac{dT_c}{dt} = \frac{1}{\mu_c} \left[ (1-f_j) P_{a0} n_r(t) + \Omega (T_f - T_c) - M (T_f - T_c) \right]$$

$$\delta \rho = \delta \rho_r + \alpha_r (T_f - T_{f0}) + \alpha_c (T_c - T_{c0})$$

and

$$\frac{d \delta \rho_r}{dt} = G_r Z_r$$
3.4 Linearized Model

The nonlinear model was linearized by using perturbation technique. This model will be used for LMI based controller design in chapter-6. For developing state space model, the single group delayed neutron model has been considered. The nonlinear model given in (3.7) can be written for single group as

\[
\dot{n}_r = \frac{1}{\Lambda} (\rho - \beta) n_r + \frac{1}{\Lambda} \beta c_r,
\]
\[
\dot{c}_r = \lambda n_r - \lambda c_r,
\]
\[
\dot{T}_f = \frac{1}{\mu_f} f_f P_{ao} n_r - \frac{\Omega}{\mu_f} (T_f - T_c),
\]
\[
\dot{T}_i = \frac{1}{\mu_c} (1 - f_f) P_{ao} n_r + \frac{\Omega}{\mu_c} (T_f - T_c) - \frac{M}{\mu_c} (T_i - T_c),
\]
\[
\delta\dot{\rho} = G(Z_r)
\]

and

\[
\delta\dot{\rho} = \delta\dot{n}_r + \alpha_f (T_f - T_{f0}) + \alpha_c (T_c - T_{c0})
\]
\[
T_c = (T_i + T_c)/2
\]

In above model, for simplicity n_r(t) is written as n_r, similar treatment is made for other variables. For developing state space model, the linearized version of (3.8) can be written as

\[
\delta\dot{n}_r = -\frac{\beta}{\Lambda} \delta n_r + \frac{\beta}{\Lambda} \delta c_r + \frac{1}{\Lambda} \delta\dot{\rho}
\]
\[
\delta\dot{c}_r = \lambda \delta n_r - \lambda \delta c_r,
\]
\[
\delta\dot{T}_f = \frac{f_f P_{ao}}{\mu_f} \delta n_r - \frac{\Omega}{\mu_f} \delta T_f + \frac{\Omega}{2\mu_f} \delta T_i + \frac{\Omega}{2\mu_f} \delta T_c,
\]
\[
\delta\dot{T}_i = \frac{P_{ao}}{\mu_c} (1 - f_f) \delta n_r + \frac{\Omega}{\mu_c} \delta T_f - \left( \frac{\Omega + 2M}{2\mu_c} \right) \delta T_i - \frac{1}{\mu_c} \left( \frac{\Omega}{2} - M \right) \delta T_c
\]
\[
\delta\dot{\rho} = G(Z_r),
\]

Finally after making small substitutions and considering T\_c=T\_i due to low power and taking n_r = n_{r0} + \delta n_r, the state space form shown in (3.11) is achieved. For linearization, the small terms multiplied with other small terms are ignored. The five states of the model consist of neutron flux (n\_r), Precursor density (c\_r), Average fuel temperature (T\_f), Core Outlet temperature (T\_i) and reactivity (\rho\_r). The state, control and output vectors are shown below.
\[
A = \begin{bmatrix}
-\beta / \Lambda & \beta / \Lambda & nr \cdot \alpha_f / \Lambda & nr \cdot \alpha_c / \Lambda & nr \cdot \alpha_f / \Lambda \\
\lambda & -\lambda & 0 & 0 & 0 \\
0 & -\Omega / \mu & \Omega / \mu & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[B = [0 \ 0 \ 0 \ 0 \ G f \ G r] \]
\[C = [1 \ 0 \ 0 \ 0 \ 0] \]
\[D = [0] \]
\[X = [\delta n, \ c, \ \delta T, \ \delta T, \ \delta \rho] \quad \text{State Vector} \]
\[Y = [\delta n_r] \quad \text{Output} \]

Table 2: Parameter Values of PARR-1 [5]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Symbol</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.007275</td>
<td>(\mu_f)</td>
<td>0.333 (MW .s/0°C)</td>
</tr>
<tr>
<td>(\Lambda)</td>
<td>0.000041 (s)</td>
<td>(\mu_c)</td>
<td>0.1451 (MW .s/0°C)</td>
</tr>
<tr>
<td>(f_r)</td>
<td>0.95</td>
<td>(\lambda)</td>
<td>0.079147 (s/0°C)</td>
</tr>
<tr>
<td>(\alpha_f)</td>
<td>1.11e-6 (Δ k/k/0°C)</td>
<td>(T_f^0)</td>
<td>30 (°C)</td>
</tr>
<tr>
<td>(\alpha_c)</td>
<td>-5.97e-5 (Δ k/k/0°C)</td>
<td>(T_c^0)</td>
<td>30 (°C)</td>
</tr>
<tr>
<td>(P_0)</td>
<td>0.001 (MW)</td>
<td>(M)</td>
<td>5.7351e-4 (MW/0°C)</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>0.81577 (MW/0°C)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Parameter values of PARR-1 are shown in Table 1. As mentioned earlier, these values were gathered from different sources and procedures. Many reactor parameters such as total fuel heat capacity (\(\mu_f\)) and total coolant heat capacity (\(\mu_c\)) are recalculated with PARR-1 new core. Some parameters calculated experimentally earlier were used as it is due to their proved surety. Fuel temperature reactivity coefficient (\(\alpha_f\)), coolant temperature reactivity coefficient (\(\alpha_c\)) and Heat transfer coefficient between fuel and coolant (\(\Omega\)) etc are also evaluated by using PARR-1 thermo-hydraulic calculations.
3.5 Linear Identified Model

The PARR-1 model was also identified by using standard system identification techniques. This identified model is used for developing $H_\infty$ loop shaping robust controller as discussed in chapter 6. The experimental data was used for reactor model identification. We will first review about basic system identification techniques and then the techniques used for PARR-1 system identification.

3.5.1 System Identification

System identification is a process of acquiring mathematical model of a dynamic system by persistently exciting a system with a known input sequence, and analyzing the resultant output of the system. By using this input output data a linear model of the system can be formulated, which could be used for controller synthesis or system analysis [41]. System Identification can be performed by persistently exciting the system with a chirp, Pseudo Random Binary Sequence (PRBS) or White Gaussian noise signal. The main idea is to excite all modes of a dynamic system so that a model which contains all the dynamics of the system can be acquired.

3.5.2 Operating Point Selection

The linear model of the system is valid along a certain operating point. To identify a model, this operating point should lie well within the linear range of operation. This linear range is determined by conducting staircase experiment. For identifying the reactor model, the reactor was made critical and the power was maintained at 2 kW. The control rod mean position at this power was taken as the operating point. The resulting output voltage varies from 0 to 1 volt corresponding to reactor power covering a full decade. The chirp signal magnitude was taken as $\pm 1$ volts and $\pm 2$ volts for different experiments, which vary the control rod according to the given input voltage.
3.5.3 Preliminary Identification Experiments

Preliminary experiments were performed to determine the following parameters.

1. Static Gains
2. Time Constants
3. Delays
4. Static Nonlinearity

A brief outline of preliminary experiments is given below

(a) Free-Run Experiment

In this experiment process is allowed to run in open loop without activating the input. This time should be long enough so that output becomes stable. This experiment is performed to determine statistical parameters of the output disturbances [42]

(b) Staircase Experiment

This experiment is used to determine plant behavior at different operating points. Each

Fig.3.3 Stair case movement of control rod
step is applied long enough so that output becomes stable. The static gain at each step is computed as a ratio of settled output power with input voltage. The staircase experiment performed at PARR-1 is shown in Fig.3.3

(c) Step Experiment
This experiment is basically a part of the staircase experiment. It is performed to calculate parameters such as delays and system time constant. These time parameters help in determining the frequency for estimation experiment. Figure 3.4 shows step change in reactor power caused by step change of control rod position at PARR-1

![Graph showing step change in reactor power](image)

Fig. 3.4 Step change in reactor power due to step change in control rod position

(d) White Noise Experiment
In this experiment inputs are activated by white noise signals. This experiment is performed to determine system bandwidth and delays, which has already been determined in previous experiment.
3.5.4 System Identification by chirp signal

A chirp signal contains all the frequencies from a minimum frequency $\omega_1$ to a maximum frequency $\omega_2$ such that ($\omega_1 < \omega < \omega_2$). The minimum and maximum frequencies are determined by the system time constant by analyzing step response. The chirp signal for frequencies varying from 0.01 Hz to 0.5 Hz were generated by a MATLAB® program. The output of the program was written to a file. The actual system controlling movement of the control rod is built with a data acquisition card (as discussed in experimental setup), control computer, control rod drive mechanism and labVIEW® software for acquisition and controlling the control rod position. The input data is read by labVIEW® program for moving the control rod in a chirp sequence. The output power of the linear channel is also saved in another file which is further used in system identification. Fig. 3.5 shows input and output data used for identification. Fig. 3.6 show actual and simulated model outputs. Poles and zeros of identified models are shown in Fig. 3.7. The chirp signal can be generated by using the following relationship.

$$u(t) = A \cos \left( \omega_1 t + \left( \omega_2 - \omega_1 \right) \frac{t^2}{2M} \right) \quad (3.12)$$

Here $\omega_1$ represents the starting frequency and $\omega_2$ represents the ending frequency of the chirp signal, and $t$ depicts the time bounded between 0 and M. The input/output data is used for estimating system model by using system identification techniques. The steps involved in identification are as follows

(a) Remove means from the data
(b) De-trend the data
(c) Select data sets for estimation and validation
(d) Find spectral and correlation models for acquiring transient and frequency responses
(e) Select the appropriate model for estimating the actual model.
(f) Validate the model.
3.5.5. System Identification by Pseudo Random Binary Sequence (PRBS)

Pseudo Random Binary sequence (PRBS) is a periodic and deterministic signal with white-noise like properties. It can be generated by implementing following difference equation [41]

\[ u(t) = \text{rem} \left( a_1 u(t-1) + \cdots + a_n u(t-n), 2 \right) \]  

(3.13)

Where \( \text{rem}(x, 2) \) is the remainder as \( x \) is divided by 2, thus \( u(t) \) assumes the values 0 or 1 in a random sequence. The sequence \( u \) is periodic, with a maximum period of \( 2^n - 1 \). The PRBS has many advantages over random Guassian noise and is also easy for implementation. The reactor model was also estimated by using PRBS signal. Here also the PRBS is generated in software and the output sequence was written to an ASCII file. This sequence was read by labVIEW® program to move the control rod in random sequence. The output power signal from the linear channel was stored in another file, later these input and output data was used for system identification as discussed earlier. Here an 8 bit sequence has been used which could attain 256 distinct values.

Fig.3.5 Input ‘\( u_1 \)’ and Output ‘\( y_1 \)’ data for system identification
Fig. 3.6 Actual and linear simulated output of reactor power

Fig. 3.7 Poles and zeros of the identified model
3.5.6. System Identification by Duty Cycle Modulation

System identification was also performed by a new technique of duty cycle modulation. The duty cycle which varies from 0 to 100% was generated with the help of software. The randomly generated duty cycle signal was given to a labVIEW® program, which generated the desired square wave signal of the given duty cycle to move the rod up and down in desired sequence. Here logic 1 is used to move the rod upward and logic 0 is used for other direction. The time period of the square wave was set to 4 seconds, which was calculated with the help of time constant of the system. Here again the input output data was used to identify the model with the help of system identification tool box of MATLAB®. Figure 3.8 shows the actual and simulated output of the system determined with this technique. Figure 3.9 shows the error residuals of the model.

Fig. 3.8 Actual and simulated power output by duty cycle modulation
3.6 Model Validation

The complete model was validated in two steps, first the CRDM model was validated with lab based experimental setup, which is the same system installed at PARR-1. After CRDM model validation complete model was validated by experimental data gathered from PARR1 [9-10, 43], Fig. 3.10 shows overall experimental setup. The details of the two procedures are outlined below.
3.6.1. Lab Setup

The lab setup shown in figure 3.11 consists of a CRDM system, control computer with NI6024E data acquisition card, interface circuit between computer and CRDM system, and labVIEW control software. The CRDM system consists of 110V synchro motor, a rack and pinion mechanism and a potentiometer for position feedback. With this arrangement, control rod can move up or down depending upon the motor direction. The two relays controlled by the computer, are used for controlling the motor direction. The input profile was given by the control software and the position output of the control rod was saved in a file. The same input sequence was given to the SIMULINK model and output of the model was saved in another file. This model output and CRDM system outputs were in close agreement as seen in the simulation section. After CRDM model was verified this experiment was performed on PARR1.

Fig. 3.11 Lab based CRDM system
3.6.2. CRDM Model Validation

The data gathered from CRDM lab based setup was compared with the SIMULINK model output. The results of two different test sequences and model outputs are shown in Figs 3.12 and 3.13. As shown in Figures the model output is in close agreement with the prototype system output. Fig 3.12 depicts output variations with big step changes and Fig 3.12 demonstrates small step sizes. Maximum percentage error calculated from Figs. 3.12 and 3.13 depicts an error of 4% (for larger step sizes) and about 5% for smaller step variations. The close match between model and prototype system guarantees model authenticity.

3.7 PARR1 EXPERIMENTAL SETUP

PARR1 experimental setup is almost similar to the lab setup. As discussed earlier the overall structure of experimental setup is shown in Fig. 3.10. For acquiring data, first the reactor was made critical in manual mode and power was maintained at 3KW. After power was stabilized the auto mode was used for model validation experiment with the help of software based PARR-1 control system. Here also a control computer with NI6024 data acquisition card and LabVIEW control software is used to manipulate control rod position.

Fig.3.12. CRDM simulation model and measured rod position for large step changes
A chirp sequence was given to the control rod and resulting power variation was recorded in a file. Different sets of data were taken at different power levels. These sets of data were later used to validate the complete model of the system. The model was validated at low power. For validating simulation model same chirp sequence was given to the model and the output power variations were later compared with actual plant data.

3.8 COMPLETE MODEL VALIDATION

The performance of the observer or controller depends heavily on the accuracy of the system mathematical model. Model validation is performed to ensure the accuracy of the model, whether developed by basic physics laws or tuned to a particular system by using a general model. In our case as the model was tuned to PARR-1 from a general model found in literature, the need for model validation is obvious. Model validation is usually performed by giving a particular input to the actual system and the same input to the simulation model and comparing the outputs of both model and system. Ideally the error between plant output and
model output should be zero and practically it should be within a prescribed threshold level. For achieving this target, experiments were performed on PARR1 at low power. The reactor was made critical and a chirp sequence was given to the control rod. The resulting power variations measured with the help of neutron detector was saved in a file. For validating simulation model same chirp sequence, which was given to the actual system, was given to the model and the output power variations were recorded in another file. The actual power output and model output were matched closely thus ensuring the model validity. Fig 3.14 shows the input chirp sequence along with resulting control rod and neutron flux (output power) variations. Fig 3.15 demonstrates the measured output with linear model output. As shown the match is quiet satisfactory with only a small deviation on the last interval. Nonlinear model output and measured output is shown in Fig. 3.16. Here the match is better than linear model, because nonlinear model covers a wider operating range as compared to linear models. The maximum percentage error for linear model is less than 6% and about 4% for non-linear model. From these results model validation can be ensured.

![Fig:3.14](image)

Fig:3.14 (a) Chirp signal given to the control rod (b) The measured rod movement (c) The measured neutron flux
Fig. 3.15. Linear model and measured output.

Fig. 3.16. Nonlinear model and measured output
3.9 SUMMARY

The chapter has discussed reactor model in detail. First nonlinear six group point kinetic model is discussed. Point kinetic model is adequate enough for research reactors such as PARR-1. For large reactors nodal model is more suitable, which divide reactor core in different zones. Parameter calculations for tuning generalized model to PARR-1 are also discussed in this section. After nonlinear point kinetic model, linearized model is discussed. Linear model has also been identified for PARR-1 by using standard system identification techniques. Linear identified model is discussed after giving a brief introduction of system identification techniques. Next Simulation model development is considered by first describing CRDM model and then complete model development, by combining CRDM model with neutronic model. In last model validation is presented. First CRDM model is validated with lab-based CRDM prototype, which is similar to PARR-1 CRDM system and then linear and nonlinear models are validated with experimental data taken from actual reactor.

Section-1, comprising of introduction, reactor systems and reactor model concludes here. Next section of the thesis comprises of chapters 4 to 7. Chapter 4 discusses sliding mode technique used in parameter estimation and controller design. HOSM based parameter estimation is discussed in chapter 5 and controller design in chapter 6. Conclusion and future work is given in chapter 7
CHAPTER. 4.

SLIDING MODE TECHNIQUE

4.1 INTRODUCTION

In this chapter an introduction of sliding mode technique will be discussed, followed by the use of higher order sliding mode (HOSM) in control and parameter estimation problems. Sliding mode technique appears in control literature since late fifties and in industry since late seventies. It is a robust control technique with an added advantage of model reduction. As it is nonlinear technique, it works equally well in both linear and nonlinear systems. The main idea behind sliding mode control is to design an observable function of system states, also called switching surface and then to design a controller to regulate this surface to zero. Due to its inherent robustness in dealing with uncertainties and easy implementation it has been successfully applied in designing control systems for non-linear systems.

Variable structure control [2-4] is based on the concept of changing controller structure as the state variables are changed for keeping the desired or optimal control process in a sliding mode. High speed switching control law is used for realizing this concept, which forcefully moves the system trajectories into a specially designed manifold also called sliding manifold and then to keep the trajectories in that manifold subsequently [44-45]. One of the major features of sliding mode is that system’s response becomes insensitive to a number of parameter variations and unknown disturbances, which is the main reason behind controller robustness.

The main idea of sliding mode lies in first driving the system to stable manifold also called reaching phase and then sliding to origin (sliding phase). For example consider the double integrator system and a control law given by

\[
\dot{y} = u(t) \\
u(t) = -K y(t)
\] (4.1)

If a control law of the form given above is used then regulation can never be achieved as can be verified by drawing phase portrait (a plot of velocity against position). We will get ellipses
of different major and minor axes, and states will never move towards origin. Now consider
the following control law.

\[
u(t) = \begin{cases} 
-K_1 y(t) & y \dot{y} < 0 \\
-K_2 y(t) & y \dot{y} > 0 
\end{cases}
\]

\[0 < K_1 < 1 < K_2\] (4.2)

Now by applying different controllers in different quadrants, by considering that the distance
to origin is always minimized a spiral motion results as shown in Fig. 1 below.

Fig. 4.1. Phase portrait under variable structure control Law

It can also be verified by considering the following Lyapunov function

\[V(y, \dot{y}) = y^2 + \dot{y}^2\] (4.3)
If the time derivative of the above function becomes negative definite then it will be proved that the system will remain stable and the states will move asymptotically towards origin. By taking time derivative of (4.3) we get

\[
\dot{V} = 2\ddot{y} \dot{y} + 2\dot{y} \dddot{y} \\
= 2\dot{y} (\dot{y} + \dddot{y}) \\
= 2\dot{y} (\dot{y} + u) \\
= \begin{cases} 
2\dot{y} \left(1 - K_1\right) & \text{if } \dddot{y} < 0 \\
2\dot{y} \left(1 - K_2\right) & \text{if } \dddot{y} > 0 
\end{cases}
\] (4.4)

The last two terms will remain negative definite in any case. Since as seen in (4.2) \(K_1\) is less than 1 then if \(\dddot{y}\) is negative then overall equation will remain negative. Similarly when \(\dddot{y}\) is positive ,\(K_2\),which is greater than 1 will make the term negative and hence \(V\) will remain negative definite and the states will spiral towards origin as seen in Fig.4.1. The control law in (4.2) can better be written as

\[
u(t) = \begin{cases} 
-1 & \text{if } s(y, \dot{y}) > 0 \\
1 & \text{if } s(y, \dot{y}) < 0 
\end{cases}
\] (4.5)

Where \(s(y, \dot{y})\), called switching function is defined as

\[
s(y, \dot{y}) = cy + \dot{y}
\] (4.6)

Where \(c\) is a positive design scalar [46]. The control law in (4.5) can be written more precisely as

\[
u(t) = -\text{sgn} (s(t))
\] (4.7)
Where \( \text{sgn}(.) \) is called the signum function because it gives the sign of the switching function. It exhibits the property that when it is multiplied by \( s \) then it gives the absolute value of switching function. The line passing through the origin for which \( s(y, \dot{y}) = 0 \) is the sliding surface, here

\[
\lim_{s \to 0} \dot{s} < 0 \quad \text{and} \quad \lim_{s \to 0} \dot{s} > 0 \quad (4.8)
\]

The trajectories on both sides will point towards this line and hence a motion which is called sliding motion will occur [10]. As the trajectories repeatedly cross the line, a high frequency motion called chattering will occur. This high frequency switching is highly undesirable for control applications as it will wear out mechanical actuators. The techniques for reducing this phenomenon will be discussed later, when the states are sliding on this line then system behaves as a reduced order system and shows robustness against parameter variations. The condition in (4.8) can also be written as

\[
s \dot{s} < 0 \quad (4.9)
\]

The condition in (4.9) is called reachability condition [47]. In Variable Structure Control System (VSCS) design control law is designed to make sure that above condition is satisfied and the design variable \( c \) in (4.6) determines the performance response. The reachability is satisfied in the domain of the phase plane [46]

\[
\Omega = \{(y, \dot{y}) : c|\dot{y}| < 1\} \quad (4.10)
\]

### 4.2 Literature Review

In this section a brief literature survey, covering applications of sliding mode technique will be discussed. D. Qian et al. have used sliding mode technique for designing robust controller for a class of under-actuated systems with mismatched uncertainties [48]. This type of system consists of a nominal system, consisting of several subsystems, and the mismatched uncertainties. A hierarchical sliding mode controller has been designed for the
nominal system based on this structural characteristic. B. Bandyopadhyay et al. have presented a new approach for designing a sliding mode controller for mismatched uncertain systems [49] A method of designing switching surface in the presence of mismatched uncertainties is also proposed. For implementing sliding mode controller system states are reconstructed with the help of high gain sliding mode observer. The controller is implemented for slosh free motion of a container. Simulation and experimental results are used for validating the proposed controller. Q. Hu et al. have devised a time-optimal based discrete-time sliding mode control (DSMC) structure [50]. An approximate switching curve which is time-optimal is assumed for the time varying sliding mode design in this scheme. The controller consisting of an equivalent and a discontinuous part is designed so that trajectories from any initial condition are driven to the sliding region and are then bound to remain inside it. Simulations and experimental results on hard disk drives are conducted for proving the effectiveness of the designed controller. M.K. Chang has developed an adaptive self-organizing fuzzy sliding mode controller for controlling 2-DOF robot with Pneumatic Muscle Actuator (PMA) [51]. The robot is used in rehabilitation engineering applications. The stability of the designed controller is proved by employing Lyapunov theory. H. Alwi and C. Edwards have presented an on-line sliding mode control allocation scheme for fault tolerant control [52]. Control allocation scheme uses effectiveness level of the actuator for redistributing control signals to remaining redundant actuators. A detailed analysis of the proposed sliding mode control allocation scheme along with determination of nonlinear gains required for maintaining sliding motion has been discussed in the paper. The proposed scheme has been implemented on ADMIRE (The Aerodata Model in Research Environment, a non-linear, six degree of freedom simulation model of a rigid small fighter aircraft with a delta-canard configuration) model for showing that fault and total actuator failure can be handled directly, without controller reconfiguration. W.G. Gabin et al. have combined the sliding mode technique with model-based predictive control to get sliding mode predictive control (SMPC) [53]. The combined algorithm showed more robustness as compared to classical MPC and also showed better tracking and disturbance rejection capability. Experimental results are performed on an air-conditioning solar plant, which is a non-minimum phase
system with variable time delay. X.G. Yan et al. have presented a sliding mode control strategy for stabilizing a class of nonlinear interconnected systems with mismatched uncertainty, using static output feedback [54]. A composite sliding surface based on the method proposed by Edwards and Spurgeon is formed, and then a reachability condition and a decentralized control scheme are developed for the interconnected systems. The results are proved with a numerical example. G. Reddy et al. have presented a new approach for designing sliding mode control by using a variant of Multirate Output Feedback (MROF) [55]. This technique is applicable on systems in which some states are available as output. Computation of remaining states uses matrices of lower dimension, which makes controller computationally efficient. The proposed controller has been applied for spatial control of 14 zonal power levels of a large Pressurized Heavy Water Reactor (PHWR). The nonlinear model consists of 70 states variables including xenon and iodine dynamics and 14 inputs and outputs each. Nonlinear equations on full power operating point are linearized to obtain the linear nodal model of the reactor. Nonlinear simulations show improved performance as compared to other techniques. C. Edwards et al. have used sliding mode observer for fault detection and isolation problem. Fault signal is reconstructed by using equivalent output injection technique. Nonlinear simulation results are used for demonstrating the results.

The work carried out using HOSM is discussed now. A method for second order sliding mode control for multi-input multi-output (MIMO) nonlinear uncertain systems has been proposed by E. Punta [56]. Same properties of robustness and precision as compared to standard sliding mode control are found by application of second order sliding mode control on single-input single-output (SISO) systems and additionally it shows extra benefits of higher order accuracy and reduced chattering effect. Y.B. Shtessel et al. have developed a novel HOSM observation algorithm for estimating states and reconstructing unknown inputs simultaneously in non-minimum phase nonlinear systems [57]. A new approach for numerically solving the unstable differential equations is also developed and used for asymptotic estimation of the full state in NMP causal nonlinear systems. Finally authors have achieved asymptotic reconstruction of the unknown inputs by employing HOSM differentiator and the estimate of the internal state. L.M. Capisani et al. have presented a control strategy for
robot manipulators, by combining inverse dynamic method with second order sliding mode approach [58]. This method provides robustness against disturbances and bounded uncertainties. The main advantage of above approach is the mitigation of the chattering problem, so that this scheme could be applied to an industrial robot. A novel second order sliding mode control algorithm for a class of MIMO nonlinear systems have been presented by M.K. Khan and S.K. Spurgeon [59]. A dynamic control is produced by the proposed algorithm which does not require derivatives of sliding variables and thus eliminates the need for observers or peak detectors. The controller has been implemented on a laboratory rig for liquid level control of interconnected twin tank systems. Robustness of the controller against parameter variations such as the admittance coefficients of different pipes and tank area was proved by implementation results. A. Cavallo and C. Natale have proposed a control strategy based on second-order sliding MIMO approach for controlling a class of mechanical systems via output feedback [60]. A time-varying sliding surface is designed for dealing with peaking phenomenon caused by unmatched initial conditions of the reference trajectories. Robustness and tracking accuracy has been confirmed by implementing the controller on a 6 DOF industrial robot manipulator. J. Davila et al. have used second order sliding mode technique for finding robust exact differentiation by employing super twisting algorithm [61].

### 4.3 Sliding Motion Properties

The properties of sliding motion and control action needed to maintain this ideal sliding will be discussed in detail in this section. Consider the second order system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x,t) + g(x,t)u
\end{align*}
\]

Where \(f(x,t)\) and \(g(x,t)\) are nonlinear functions
\(g(x,t) > 0\) or positive definite
\(f(x,t)\) and \(g(x,t)\) can be continuous or discontinuous functions
4.3.1 Reaching Phase Dynamics

For considering reaching phase dynamics, it can be observed that $x_1$ is stable if

$$\dot{x}_1 = -ax_1, \quad a > 0 \quad (4.12)$$

Now let’s take sliding surface as

$$s = x_2 + ax_1 \quad (4.13)$$

so that

$$\dot{x}_1 = x_2 = -ax_1 + s \quad (4.14)$$

Above equation is stable only when $s = 0$

4.3.2. Sliding Phase Dynamics

Now for elaborating sliding phase dynamics, take time derivative of $s$

$$\dot{s} = \dot{x}_2 + ax_1$$

$$= f(x,t) + g(x,t)u + ax_2 \quad (4.15)$$

For evaluating stability, take the Lyapunov candidate function as

$$V = \frac{1}{2}s^2 \quad (4.16)$$

4.3.3 Reaching Phase Dynamics Cont’d

Now for considering reaching phase dynamics again, we take time derivative of

Lyapunov function as

$$\dot{V} = \dot{s}s$$

$$\Rightarrow \dot{V} = s[f(x,t) + g(x,t)u + ax_2] \quad (4.17)$$

$\dot{V}$ is negative definite if

$$f(x,t) + g(x,t)u + ax_2 \begin{cases} < 0 & \text{for } s > 0 \\ = 0 & \text{for } s = 0 \\ > 0 & \text{for } s < 0 \end{cases} \quad (4.19)$$

and stability is assured if
\[ u \begin{cases} < \gamma(x) & \text{for } s > 0 \\ = \gamma(x) & \text{for } s = 0 \\ > \gamma(x) & \text{for } s < 0 \end{cases} \] where \[ \gamma(x) = -\frac{f(x,t) + ax}{g(x,t)} \] (4.20)

The above condition can be assured by taking control law as
\[ u = \gamma(x) - K \text{sign}(s) \text{, where } K > 0 \] (4.21)

4.3.4. Example

For example consider a simple pendulum described by following equations
\[ \begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a \sin(x_1) + u_c
\end{align*} \] (4.22)

Switching surface is selected as
\[ s = cx_1 + x_2 \] (4.23)

By selecting control law as
\[ u_c = -k_d \text{sign}(s) - cx_2 \] (4.24)

And taking \( k_d = c = 1 \) and \( a = 0.2 \), with initial conditions \( x_0 = (0.8, 0) \) we get phase portrait of sliding motion, shown in Fig. 4.2. The control action is shown in Fig. 4.3

![Fig. 4.2. Sliding motion phase portrait](image-url)
Fig. 4.3. Controller effort

Fig. 4.4. Sliding motion phase portraits with \( a=0 \) and \( a=0.25 \)
The key result is that phase portrait intercepts sliding surface in finite time and then it is forced to remain there. Once sliding motion is established the system behaves as a reduced order system, where nonlinearity is completely rejected [46]. After sliding the system behaves as a double integrator system with no effect of sine term. This fact is shown in Fig. 4.4 where both plots with $a=0.25$ and $a=0$ are plotted in a single figure. Here it is clear that the only difference is in reaching phase and when the states intercepts sliding surface then both trajectories behaves similarly, thus proving the robustness of the system and the fact that sine term has no further contribution. It is, therefore called robust controller, because it is insensitive to plant and control law mismatches.

### 4.3.5. Equivalent Control Law

The purpose of the control law is to drive states to the sliding surface and then to force trajectories to remain on this surface onwards. By considering relation between control action and switching function, this fact can be better understood. When sliding surface is reached and ideal sliding takes place then $s(t) = 0$ for all future times, which also implies that $\dot{s}(t) = 0$ for all future values of $t$. Thus considering equation (4.1) and (4.6) it can be written

$$\dot{s}(t) = cy(t) + u(t)$$  \hspace{1cm} (4.25)

As argued earlier $\dot{s}(t) = 0$ for all $t \geq t_s$, the above equation can be written as

$$u_{eq}(t) = -cy(t)(t \geq t_s)$$  \hspace{1cm} (4.26)

This control law is called equivalent control law. This control signal is applied on average and comprises the low frequency component. This is not the actual applied signal but it gives the average of applied control signal. Equivalent control signal can be better visualized by passing control signal through a low pass filter to obtain low frequency
component. If the following low pass filter is used then the resultant response will be as shown in Fig. 4.5

\[ G_f(s) = \frac{1}{s + 25} \] (4.27)

It can be seen from figure that filtered control signal behaves same as equivalent control signal defined in (4.26). After sliding takes place the filtered signal behaves as first order decay as of (4.26).

Fig.4.5 . Equivalent Control Law
As argued before, the system behaves as a first order system, which guarantees that no overshoot will occur, when the system (pendulum) is moved to origin from any initial condition. This fact is shown in Fig. 4.6 which shows the angular output of the system, which is similar to exponential decay of first order system.

**Fig.4.6. Angular displacement**

### 4.4 EFFICIENT CONTROLLER DESIGNS

If the controller (4.5) is used with large initial conditions, then the resulting switching surface is as shown in Fig. 4.7. Here the initial conditions taken are \( x_1 = 3 \) and \( x_2 = 0 \).
The switching surface after intercepting the switching line moves to the other side and again intercepts it at about 3 seconds after which the sliding takes place. As it is clear that reaching phase should be as minimum as possible, because all robustness properties are exhibited in sliding phase, thus above phenomenon is undesirables. To overcome above problem lets consider another modified controller given as follows

$$u(t) = c_1 x_1 + c_2 x_2 - \rho \text{sgn}(s(t))$$  \hspace{1cm} (4.28)

Where $c_1$, $c_2$ and $\rho$ are design scalars. These variables should be chosen so that the inequality (4.9) is always satisfied. Let’s consider (4.23)
\[ s = cx_1 + x_2 \]
\[ ss' = s (cx_2 + x'_2) = s (cx_2 - a \sin(x_1) + u) \]  \hspace{1cm} (4.29)

By substituting value of (4.28) in (4.29) we get

\[ ss' = s (cx_2 - a \sin(x_1) + c_1 x_1 + c_2 x_2 - \rho \text{sgn}(s(t))) \]  \hspace{1cm} (4.30)

Now let \(c_1 = 0\) and \(c_2 = -c\) and using the fact \(s \ast \text{sgn}(s) = |s|\), above equation becomes

\[ ss' = s (cx_2 - a \sin(x_1) - cx_2 - \rho \text{sgn}(s(t))) \]
\[ ss' = -sa \sin(x_1) - \rho |s| < |s|(a - \rho) \]  \hspace{1cm} (4.31)

Finally by choosing \(\rho > a + \eta\) where \(\eta\) is a positive design scalar, following inequality is achieved

\[ ss' < -\eta|s| \]  \hspace{1cm} (4.32)

The above condition is known as \(\eta\)-reachability condition. Now it is guaranteed that by using control law (4.28) whenever sliding surface is reached ideal sliding will take place. The condition observed in Fig 4.7. i.e. trajectories piercing the switching line is no more possible. Here no overshoot will take place but the response will become slow. To overcome this problem a little modification is made in (4.28) by letting \(c_1 = -\varphi c\) and \(c_2 = -(c + \varphi)\) the following modified control law is achieved

\[ u(t) = -(c + \varphi)x_2 - \varphi cx_1 - \rho \text{sgn}(s(t)) \]  \hspace{1cm} (4.33)
Here again $\eta$-reachability condition is satisfied, which could be confirmed by plugging modified control law in (4.29). The switching surface is shown in Fig. 4.8 and is clear from the Fig. that no piercing of the sliding surface, as opposed to Fig. 4.7, is observed and ideal sliding motion is started as soon as the trajectories intercepts sliding surface. The main advantage of controller (4.33) over (4.28) is that the value of $\rho$ only needs to be greater than 0.25 (the coefficient of nonlinear term) to satisfy $\eta$-reachability condition and hence making the control law much less aggressive as shown in Fig.4.10. By comparing Fig.4.3 with Fig.4.10 it is clear that control effort has reduced from $\pm 1$ to $\pm 0.3$, which is much better for avoiding wear and tear of actuators. The output response (angular displacement) is shown in Fig.4.9; this is still similar to Fig.4.6 and hence control law (4.33) produces similar results.

![Switching function for modified control Law](image-url)
Fig. 4.9 Angular displacement with modified control Law

Fig. 4.10 Control Action with modified control law
4.5 SLIDING MODE CONTROLLER DESIGN EXAMPLE

Consider a simple pendulum consisting of a (weightless) shaft and a mass driven by a motor at point of suspension [62]. The objective of the controller is to regulate pendulum at its vertical downward equilibrium point, when the pendulum is left from an initial point near equilibrium. Settling time of the designed controller should be less than 3 seconds with no or very small overshoot as per desired performance requirements. Consider the simple pendulum system shown in Fig. 4.11 [62]

\[ T_c - mg/l \sin \theta = I \dot{\theta} \]  

(4.34)

Where \( \theta \) represents the angular displacement from the vertical axis, \( T_c \) is the applied torque, \( m \) is the mass, \( g \) is gravitational acceleration, and \( l \) is the length of the shaft. For this example parameter values are taken as \( l=1 \text{m}, \ m=0.2 \text{kg}, \ g=9.82 \text{m/s}^2 \). After linearizing the system at equilibrium point and substituting parameter values following state space model is achieved.

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-9.82 & 0
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
5
\end{bmatrix} u(t)
\]  

(4.35)
Where \( x_1(t) = \theta \) and \( x_2(t) = \dot{\theta} \) are angular displacement and angular velocity respectively.

### 4.5.1 Sliding Surface Design

The first step is to design sliding surface matrix \( S \). For the period of sliding motion, \( s(t) = Sx(t) = 0 \) and considering

\[
x_2(t) = -S^{-1}_2S_1x_1(t) = -Mx_1(t)
\]

(4.36)

Where \( x_1(t), x_2(t) \) and \( M \) are scalars. First equation of (4.35) can be written as

\[
x_1' = x_2
\]

(4.37)

By substituting (4.36) in (4.37) we get

\[
x_1' = -Mx_1(t)
\]

(4.38)

The solution of above equation is \( x_1(t) = x_1(t_s)e^{-Mt} \) where \( t_s \) is the time when sliding starts. By taking \( M=2 \), the required settling time of less than 3 seconds is achieved.

Finally for designing sliding surface take \( S_2=1 \) to get

\[
S = [2 \ 1]
\]

(4.39)

### 4.5.2 Control Law Design

Consider the following system.

\[
u(t) = -(SB)^{-1}(SAx(t) + \eta \text{sgn}(s)) = -(SB)^{-1}SAX(t) - (SB)^{-1} \eta \text{sgn}(s)
\]

(4.40)
Where \( \eta \) is a positive scalar and \( \text{sgn}() \) represents the signum function. \( S \) represents the sliding surface and \( A \) and \( B \) matrices as given in (4.35). The term \( u_{eq} \) is the average signal applied for maintaining sliding motion, while the second term \( \text{sgn}() \) ensures robustness, i.e. once sliding motion is induced, it should be maintained onwards. By taking time derivative of sliding surface, following relation is achieved

\[
\dot{s}(t) = S\dot{x}(t) = S\left(Ax(t) + Bu(t)\right) \tag{4.41}
\]

By substituting (4.40) in above equation yields

\[
\begin{align*}
\dot{s}(t) &= S\dot{x}(t) = S\left(Ax(t) + B \left(-\left(SB\right)^{-1}SAx(t) - \left(SB\right)^{-1}\eta \text{sgn}(s)\right)\right) \\
&= S\dot{x}(t) - SB\left(SB\right)^{-1}SAx(t) - SB\left(SB\right)^{-1}\eta \text{sgn}(s) \\
&= S\dot{x}(t) - S\dot{x}(t) - \eta \text{sgn}(s) \\
&= -\eta \text{sgn}(s) \tag{4.42}
\end{align*}
\]

Multiplying both sides of (4.43) with \( s \), we get

\[
ss = s\left(-\eta \text{sgn}(s)\right) \leq -\eta |s| \tag{4.44}
\]

Hence by using above control law \( \eta \)-reachability condition is satisfied. By letting \( \eta = 1 \) and substituting value of \( S \), following control law is obtained from

\[
u(t) = \left[1.964 - 0.4\right]x - 0.2 \text{sgn}(s) \tag{4.45}
\]

4.5.3 Simulation Results

Simulation results are shown in Fig. 4.12. The states, deflection angle and angular velocity are shown in Fig. 4.12 (a). Here it could be seen that deflection angle settles
within 3 seconds with no overshoot. Phase portrait, which is a plot of position versus velocity is shown in (b) part, the first phase demonstrates the reaching phase and other shows sliding phase. Switching function is shown in (c) part, which shows that sliding starts after 2 seconds, when the trajectories hit sliding surface. Here $t_s$ denote the time after which ideal sliding starts. Control law is shown in part (d) of Fig. 4.12 here it is also clear that when sliding surface is reached after 2 seconds very high control activity is observed. It is due to the fact of system trajectories repeatedly crossing switching surface. This phenomenon is called chattering and is highly undesirable. The methods of removing these high frequency oscillations will be discussed in coming chapters. As discussed high control activity is undesirable, a more practical control law, which smoothes out control action is discussed in next section.
Fig. 4.12 (a) Pendulum states (b) Phase portrait (c) Switching function and (d) Control law
4.5.4 Design with a practical control law

As discussed in previous section high frequency oscillation is undesirable, due to the wear and tear of mechanical actuators, a modification in the control law is made as follows.

\[
u(t) = -(SB)^{-1}(SA - \varphi S)x(t) - \rho(SB)^{-1}\frac{S}{|s| + \delta}
\]  

(4.46)

Here \text{sgn}(.) function is replaced by sigmoid function. Taking same values of \(S\) and choosing \(\varphi = -6\) and taking design parameters \(\rho = 1\) and \(\delta = 0.001\), following control law is achieved.

\[
u(t) = [-0.436\quad -1.6]x - 0.2\frac{s}{(|s| + \delta)}
\]

(4.47)

The results of simulation with this control law are shown in Fig. 4.13. Phase portrait looks similar to the previous example but now control signal has been changed considerably, also shape of switching function has also been changed. There is a trade off between robustness and chattering. As the value of \(\delta\) is increased the chattering is minimized but at the cost of reduced robustness. On the other hand by making \(\delta\) larger, robustness is increased but with more chattering. Another change in control law is the introduction of new design variable \(\phi\), which controls the speed of convergence i.e. how fast sliding surface is reached. Here it can be seen in part (c) of Fig. 4.13 that sliding surface is reached in 0.46 seconds. This time should be as small as possible because all robustness properties are exhibited in sliding phase. Although chattering problem has almost been controlled, but by compromising on robustness. After having a basic understanding of sliding mode technique, higher order sliding modes, which removes chattering, while at the same time maintain robustness are discussed in second part of this chapter.
Fig. 4.13(a) Pendulum states (b) Phase portrait (c) Switching function and (d) Practical control law
4.6 Higher Order Sliding Mode (HOSM)

Higher order sliding mode controllers have started to find useful applications in different control problems. The main difference between the conventional and higher order sliding-mode is that the higher order derivatives of the sliding variable (s) are used in place of first order derivatives [63-65].

The simple way to withstand uncertainty is to maintain some constraints through brute-force, i.e. to react at once to any divergence of the system moving it back to the constraint by a sufficiently vigorous effort. This approach leads directly to first-order sliding modes. Having advantages of higher accuracy and robustness, they have a very serious drawback: the chattering effect which means high frequency vibration of the controlled system. To cater for this problem many approaches were proposed [66]. The key idea of all these approaches were to modify the dynamics in small neighborhood of the discontinuity and to preserve the main properties of the whole system simultaneously [66]. However, the ultimate precision and robustness of the sliding mode were lost to some extent. The recently introduced higher order sliding mode (HOSM) generalizes the basic sliding mode idea, operating on the higher order time derivatives of the system deviations from the constraint instead of manipulating the first deviation derivatives as in the case of standard "(first-order) sliding modes[66].

The sliding order describes the dynamic smoothness degree in the neighborhood of the mode. In addition to the advantages of the first order sliding modes such as robustness, easy design and implementation etc, HOSM has additional advantage of chattering removal. The nth order sliding mode is determined by the equalities $s = \dot{s} = \ddot{s} = \cdots = s^{(n-1)} = 0$, forming the n-dimensional condition on dynamic system state.

To better understand this technique consider few basic definitions adopted from [66]

**Definition 4.6.1**

"The sliding order $r$ is the number of continuous total derivatives (including the zero one) of the function $s = s(x,t)$ whose vanishing defines the equations of the sliding manifold."

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**Definition 4.6.2**

“The sliding set of order ‘r’ associated to the manifold \( s(x,t) = 0 \) is defined by the equalities

\[
s(.) = \dot{s}(.\ldots) = s^{(r-1)} = 0
\]

(4.48)

Note that (4.48) represents an \( r \)-dimensional condition on the state of the corresponding dynamic system.”

**Definition 4.6.3**

“Let the \( r \)-th order sliding set (4.48) be non-empty and assume that it is locally an integral set in Filippov’s sense (i.e. it consists of Filippov’s trajectories of the discontinuous dynamic system) [56]. The resultant motion satisfying (4.48) is called an \( r \)-th order sliding mode with respect to the manifold \( s = 0 \).”

After the successful implementation of HOSM in different control application, and the main advantage of reduced chattering the attention of control community diverted towards this approach and a number of different algorithms have been proposed. The number of sliding variables and its derivatives determines the sliding order. If only \( s = \dot{s} = 0 \) is imposed then it becomes the case of first order sliding mode, similarly if \( s = \dot{s} = \ddot{s} = 0 \) is imposed then it is called second order sliding mode. Generalizing this sequence \( s = \dot{s} = \ddots = s^{(r-1)} = 0 \) sliding variables are required for rth order sliding mode. Although 3\(^{rd}\) order and higher order sliding mode techniques have been used in industrial processes, but mostly 2\(^{nd}\) order HOSM is used. Different algorithms of 2\(^{nd}\) order sliding mode [56] are now considered in next section.

**4.7 SECOND ORDER SLIDING MODE ALGORITHMS**

**4.7.1 Twisting Algorithm**

Taking local coordinates \( y_1 = s \) and \( y_2 = \dot{s} \), the second order sliding mode control problem is equivalent to the finite time stabilization problem for uncertain second-order system with relative degree 1 [66-67]
\[ \dot{y}_1 = y_2 \]
\[ \dot{y}_2 = \phi(t, x) + \gamma(t, x)u \]  

(4.49)

Here \( y_2(t) \) is immeasurable but the sign is possibly known. \( \phi(t, x) \) and \( \gamma(t, x) \) are uncertain functions such that

\[
\Phi > 0, \left| \phi \right| \leq \Phi, 0 < \Gamma_m \leq \gamma \leq \Gamma_M
\]  

(4.50)

This algorithm features twisting around the origin as shown in Fig. 4.14. The trajectories carry out an infinite number of rotations while converging to the origin in finite time. The vibration magnitude and rotation time decrease in geometric progression. The control law is defined as follows [66]

\[
\hat{u}(t) = \begin{cases} 
-u & \text{if } \left| u \right| > 1 \\
-V_m\text{sign}(y_1) & \text{if } y_1y_2 \leq 0; \left| u \right| \leq 1 \\
-V_M\text{sign}(y_1) & \text{if } y_1y_2 > 0; \left| u \right| \leq 1
\end{cases}
\]  

(4.51)

Fig. 4.14. Phase trajectory of Twisting algorithm
The corresponding sufficient conditions for the finite time convergence to the sliding manifold can be given as

\[ V_M > V_m \]
\[ V_m > \frac{4 \Gamma M}{s_0} \]
\[ V_m > \frac{\Phi}{\Gamma_M} \]
\[ \Gamma_M V_M - \Phi > \Gamma_M V_m + \Phi \]

(4.52)

Where \( V_m \) and \( V_M \) are proper positive constants such that \( V_m < V_M\) and \( V_M, V_m \) is sufficiently large. The above controller can be used when relative degree is 1, for relative degree 2 similar controller, given below can be used

\[
\begin{align*}
V_m &> 0 \\
V_M &< \frac{4 \Gamma M}{s_0} \\
\Gamma_M V_M - \Phi &> \Gamma_M V_m + \Phi
\end{align*}
\]

(4.53)

4.7.2 Sub-Optimal Algorithm

This algorithm was built as a sub-optimal feedback realization of a conventional time-optimal control for a double integrator system [66]. By letting relative degree 2, the auxiliary system can be defined as

\[
\begin{align*}
y_1' = y_2 \\
y_2' = \phi(t,x) + \gamma(t,x) u
\end{align*}
\]

(4.54)

The trajectories of this algorithm are restricted within limit parabolic arcs which includes the origin. Here both twisting and leaping behaviors, as shown in Fig. 4.15 are possible. After initialization phase the control law is defined as [58, 66]

\[
v(t) = -\alpha(t) V_m \text{sign}(y_1(t) - \frac{1}{2} y_{1M})
\]

\[
\alpha(t) = \begin{cases} 
\alpha^* & \text{if } \left[y_1(t) - \frac{1}{2} y_{1M}\right]\left[y_{1M} - y_1(t)\right] > 0 \\
1 & \text{if } \left[y_1(t) - \frac{1}{2} y_{1M}\right]\left[y_{1M} - y_1(t)\right] \leq 0
\end{cases}
\]

(4.55)
Where $y_{f_{1M}}$ is the final singular value of the function $y_1(t)$.

![Phase trajectories of Sub-optimal algorithm](image)

**Fig. 4.15. Phase trajectories of Sub-optimal algorithm**

The corresponding sufficient conditions for the finite-time convergence to the sliding manifold are as follows [58, 66]

$$\alpha^* \in (0,1] \cap \left\{ 0, \frac{3\Gamma}{m\Gamma M} \right\}$$

$$V_M > \max \left( \frac{\Phi}{\alpha \Gamma M}, \frac{4\Phi}{3\Gamma m - \alpha \Gamma M} \right)$$

(4.56)

### 4.7.3 Super Twisting Algorithm

Super twisting algorithm has been developed for systems with relative degree one for avoiding chattering phenomena in variable structure systems. Control law, $u(t)$ is formed by
combining two terms, the first one is a discontinuous time derivative whereas second term is the continuous function of the sliding variable. As shown in Fig. 4.16, the trajectories of the super twisting algorithm are characterized by twisting around the origin on the sliding variable phase portrait.

\[ \dot{S} = \ddot{S} \]

**Fig. 4.16. Phase trajectory of Super-twisting algorithm**

Consider the system of the form

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= \theta(t,x) + \gamma(t,x)u
\end{align*}
\]  

(4.57)

where \( y_1 = s, \ y_2 = \dot{s} \) and \( \theta(t,x), \gamma(t,x) \) are smooth uncertain function with \( |\theta| \leq \Theta > 0, \ 0 < \gamma \leq \Gamma_M \). The super twisting algorithm converges to the 2-sliding set \( s = \dot{s} = 0 \) in finite time and can be given as

\[ u(t) = u_1(t) + u_2(t) \]  

(4.58)
\[
\dot{s} = \theta(x) + \gamma(x) u
\]

where, \( \theta(x) = \tilde{\theta}(x) - \tilde{x}_{id} \).

The sufficient conditions for the finite time convergence to the sliding manifold are [63]

\[
W > \frac{\Theta}{\Gamma_m} > 0 \quad (4.62)
\]

\[
\rho_{sm} (\lambda_{sm} \Gamma_m)^{-\frac{1}{2}} > \left( \frac{1}{\Gamma_m W + \Theta (2\Gamma_m)^{\frac{1}{2}}} \right)^{\frac{1}{2}} \quad (4.63)
\]

There is no need for any information about time derivatives of the sliding variable, \( s \), and also about system parameters. This reduces the need for observer and hence computational complexity is reduced.

### 4.7.4 Drift Algorithm

In this algorithm the trajectories are steered to the 2-sliding mode \( s=0 \) while keeping \( s \) relatively small. In other words trajectories are made to drift towards the origin along \( y_1 \)-axis. As shown in figure 4.17, the phase trajectories of drift algorithm on 2-sliding plane are characterized by loops having constant sign of sliding variable \( y_1 \).
Fig. 4.17. Phase trajectories of drift algorithm

The control algorithm for the case of relative degree 1 is defined by the following control law

\[ u = \begin{cases} -u & \text{if } |u| > 1 \\ -V_m \text{sign}(\Delta y_{11}) & \text{if } y_{11} \Delta y_{11} \leq 0; |u| \leq 1 \\ -V_M \text{sign}(\Delta y_{11}) & \text{if } y_{11} \Delta y_{11} > 0; |u| \leq 1 \end{cases} \] (4.64)

Where \( V_m \) and \( V_M \) are proper positive constants such that \( V_m < V_M \) and \( V_M / V_m \) is sufficiently large, also \( \Delta y_{11} = y_{1}(t_i) - y_{1}(t_i - \tau), t \in [t_i, t_{i+1}) \). For relative degree 2, the control law is modified a bit as

\[ u = \begin{cases} -V_m \text{sign}(\Delta y_{11}) & \text{if } y_{11} \Delta y_{11} \leq 0 \\ -V_M \text{sign}(\Delta y_{11}) & \text{if } y_{11} \Delta y_{11} > 0 \end{cases} \] (4.65)
4.7.5 Algorithm with a prescribed convergence law

These types of controllers have the probability of selecting a transient process trajectory and the switching of \( \dot{u} \) depends on a suitable function of \( s \) [66]. The trajectories of this algorithm are shown in Fig 4.18 and the general form of the controller is given below.

\[
\dot{u} = \begin{cases} 
-u & \text{if } |u| > 1 \\
-V_M \text{sign}(y_2 - g(y_1)) & \text{if } |u| \leq 1 
\end{cases}
\] (4.66)

Where \( V_M \) is a positive constant and the continuous function \( g(y_1) \) is smooth everywhere but at \( y_1 = 0 \). For relative degree 2, the controller is formed as

\[
\dot{u} = -V_M \text{sign}[y_2 - g(y_1)]
\] (4.67)

Here function \( g \) must be chosen in such a way that all the solutions of the equation \( y_1' = g(y_1) \) vanish in finite time and the function \( g'g \) be bounded. The sufficient condition for the finite time convergence to the sliding manifold is defined as follows.

\[
V_M > \frac{\Phi + \sup \left[ g'(y_1)g(y_1) \right]}{\Gamma_m} \] (4.68)

In above inequality convergence time depends on the function \( g \) [66]

![Fig.4.18. Phase trajectories for prescribed control law algorithm](image-url)
4.8 HOSM SIMULATION EXAMPLES (APPLICATIONS)

In this section few application examples of the HOSM technique will be considered. First example is about stabilizing nonlinear uncertain system using 2nd order sliding mode controller and the second example is for using HOSM technique for tracking application of a nonlinear system.

4.8.1 Robust Stabilization of Nonlinear Uncertain System

Consider the system

\[\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
\dot{x}_3(t) &= f(x(t)) + g(x(t))u(t)
\end{align*}\]  

(4.69)

Where

\[
\begin{align*}
f(x,t) &= 3e^{x_1} + 5x_2\sin(20t + x_1)\cos(x_2^2 + x_3^2) \\
g(x,t) &= 3 + \sin(3t + x_1 + x_2)
\end{align*}\]  

(4.70)

The state vector is assumed to be completely available for measurement. The initial conditions are taken as \(x(0)=[2,2,2]\). The control objective is to asymptotically regulate the states to zero. The sliding manifold is taken as

\[s(x,t) = x_3(t) + 8x_2(t) + 16x_1(t)\]  

(4.71)

First order sliding mode strategy is applied with the control law

\[u(t) = -U(x(t))\text{sign}(s)\]  

(4.72)

where

\[U(x,t) = 1.5e^{x_1} + 4.5|x_2(t)| + 2|x_3(t)| + 5\]  

(4.73)
Similarly second order sliding mode controller using super twisting algorithm mentioned earlier in (4.59) and (4.60) is also applied on the above uncertain system. The results obtained by applying these controllers along with the results are discussed next. The phase portraits of first order and second order sliding mode controllers are shown in Figs 4.19 and 4.20. The controller efforts of first and second order controllers are shown in Figs 4.21 and 4.22. Here it is clear that the control effort has reduced considerably for SOSMC which implies reduced chattering. The states of First order sliding mode controller (FOSMC) and Second order sliding mode controller (SOSMC) are depicted in Figs 4.23 and 4.24, the responses of first and second states are nearly same with a transient effect observed in third state. Finally sliding surfaces of FOSMC and SOSMC are shown in Fig. 4.25, here also it is evident that chattering effect is reduced in higher order sliding mode technique.

![Phase portrait of First order sliding mode controller (FOSMC)](image)

Fig.4.19 Phase portrait of First order sliding mode controller (FOSMC)
Fig. 4.20 Phase portrait of Second order sliding mode controller (SOSMC)

Fig. 4.21 Control effort for FOSMC
Fig. 4.22 Control effort for SOSMC

Fig. 4.23 State trajectories for FOSMC.
Fig. 4.24 State trajectories for SOSMC.

Fig. 4.25 Sliding surfaces of FOSMC and SOSMC
4.8.2 Tracking Example of a Nonlinear System

As another example for investigating tracking performance of a second order controller consider the system [66]

\[ \dot{x} = u, \quad x, u \in \mathbb{R}, \quad s = x - f(t), \quad f : \mathbb{R} \to \mathbb{R} \quad (4.74) \]

As opposed to regulation problem, tracking problem is considered as a more difficult task in control literature. In tracking problem the reference is changing continuously and the controller should be efficient enough to follow this changing reference as quickly and as accurately as possible. The task in this example is to track \( f(t) \) in real time, the only values available are \( x, f \) and \( u \) and values of \( f \) and its first and second derivative are less than 0.5. Let the function be selected as

\[ f(t) = 0.08 \sin(t) + 0.12 \cos(0.3t), \quad x(0) = 0, v(0) = 0 \quad (4.75) \]

The following super twisting algorithm has been used for tracking application

\[
u = -2 \| s(t) \|^2 \text{sign}(s) + u_1, \]
\[
u_1 = \begin{cases} -u(t), & |u(t)| > 1 \\ -\text{sign}(s), & |u(t)| \leq 1 \end{cases} \quad (4.76)
\]

The simulation results are shown in Figs 4.26 and 4.27. As can be seen in Fig 4.26, the \( x(t) \) tracks \( f(t) \) within 0.5 seconds with no overshoot. Once it hits the surface then tracking is remarkable. The control effort signal is shown in Fig. 4.27. The maximum value of \( u=0.7 \) and after 0.5 seconds, when tracking has achieved, the value of \( u \) remains within ±0.12. By further increasing the value of \( \lambda \) from 2 in (4.76), the tracking is achieved earlier but control signal is increased.
Fig. 4.26 Tracking of $f(t)$ by $x(t)$ using second order sliding mode controller

Fig. 4.27 Second order super twisting controller effort
4.9 Summary

This chapter has discussed basics of sliding mode control and higher order sliding mode. An example of simple pendulum has been discussed in detail for designing sliding surface and control law. The chattering phenomenon caused due to high frequency oscillation was then dealt with the help of sigmoid function. The concept of equivalent control has also been discussed. Detailed simulations were performed for demonstrating sliding mode design technique and performance evaluation.

In second part of this chapter HOSM technique has been discussed thoroughly. After giving a brief introduction and main advantages of HOSM over classical sliding mode technique, few algorithms of second order sliding mode were discussed. Among other algorithms, super twisting technique which has been used in this thesis for parameter estimation and control implementation has been elaborated in detail. After this few simulation examples of HOSM were presented. In coming chapter this method will be used for nuclear reactor parameter estimation and control applications.
5.1 INTRODUCTION

As discussed in previous chapters sliding mode technique and especially higher order sliding mode have many advantages not found in other techniques. Among others are robustness, order reduction and easy implementation etc. The main problem faced in sliding mode technique i.e. chattering has been solved by higher order sliding mode. HOSM observers have been used in a variety of applications [69-72]. HOSM has been selected for reactor parameter estimation and controller design due to the advantages mentioned earlier. As sliding mode is nonlinear in nature, it is best suited for the control of systems like PARR-1 with inherent nonlinearities. As stated earlier the problem of chattering, the main bottleneck in classical sliding mode has been dealt with by using HOSM. Sliding mode observer works by minimizing the error between plant model and the observer model by using a switching function. The observer gain is adjusted by the error such that the plant output matches with the observer output as the error surface moves towards zero. Higher order sliding mode observers are used to determine precursor concentration and reactivity, with only neutron flux measurement available. Due to the unavailability of adequate sensors, precursor concentration and reactivity cannot be measured directly and hence there is a need for estimating these variables. The first approach for estimation could be use of linear observers, but it is not a viable solution as far as nuclear reactors are concerned. The nonlinear behavior of nuclear reactor changes considerably at different power levels and is also dependent on many factors such as Xenon production, fuel burnup etc. The linear observer behaves satisfactorily only around a small neighborhood of the reactor operating point, and for large changes its performance is much degraded. To cover a large operating range the need for a nonlinear
observer is obvious. In addition to nonlinearities, the reactor state estimation becomes more
difficult due to measurement noise, parameter variations and the presence of model
uncertainties. Nonlinear observer such as sliding mode observer can cater for these problems
satisfactorily and due to inherent robustness it is insensitive to parameter variations and can
also withstand a certain class of model uncertainties. This chapter is organized as follows; first
a brief literature survey about parameter estimation is discussed followed by existing
techniques of parameter estimation. After comparing sliding mode parameter estimation
technique, with other techniques, sliding mode observers for precursor concentration and
reactivity are proposed with their parameter estimation formulation. In proposed observers
second order super twisting algorithm is used for estimating reactor variables. The main
advantage of this algorithm is that higher order derivatives of sliding variable are not required
and hence computational complexity is reduced. The chapter is concluded by giving
experimental results, obtained by using data acquired from PARR-1.

5.2 Literature Review for Parameter Estimation

Reactivity and Xenon concentration were estimated by using SMO for point reactor kinetics
model and Cherinks’s model in [73]. Reactivity was estimated as a function of time with only
the neutron density (rated reactor power) being measured. Similarly Xenon and Iodine
concentrations were estimated with only information on flux \( \phi(t) \). Sliding mode neural
observer was used for the estimation of neutron precursor power and internal reactivity in
nuclear research reactor [74]. Humberto, et al used input and neutron density measurements
for estimation. For training differential neural network, they used simplified third-order non-
linear model of the nuclear reactor. Parameters of a nonlinear system with constant
coefficients were identified by traditional and higher order sliding modes in [75]. Sliding
mode parameter observer was based on least squares continuous-time technique. The effects
of measurement noise were also addressed. I. Castellanos et al. proposed a method for identification of any even number of parameters of the process transfer function [76]. Authors used modified twisting algorithm for parameter identification. Higher order SMO was proposed and used to estimate states in multi input multi output (MIMO) nonlinear systems with unknown inputs [77]. It was shown by numerical examples that the authors were able to estimate exactly the observable states and asymptotically the unobservable states and unknown inputs. Higher order SMO was used to estimate actuator faults in satellite leader/follower system [69]. The authors used broken super twisting observer and third order sliding mode observers for fault estimation. Second order SMO was used for robust fault detection in uncertain nonlinear systems in [78]. Stability of the observer for reduced order system was also proved. A feedback linearization-based controller with a high-order SMO was applied to quad rotor unmanned aerial vehicle in [79]. Here high order SMO was used as an observer for estimating the effects of the external disturbances such as wind and noise. D. Vashaee et al. developed an optimal estimation method for CANDU core reactivity estimation [80]. Authors used joint extended Kalman filters for generating robust estimates of unknown reactivity components such as that of coolant void feedback in loss of coolant accident (LOCA). Numerical simulations were used to demonstrate the capabilities of the proposed method.

5.3 Parameter Estimation

Parameter estimation deals with calculating approximately the values of parameters based on measured or empirical data. The parameters depict a fundamental physical setting in such a way that the values of the parameters affect the distribution of the measured data. In other words the subject of parameter estimation provides tools for the efficient use of data for helping in mathematical modeling of the phenomena at hand, and estimating the constants.
appearing in these models. The unknown parameters are approximated by using these measurements with the help of an estimator.

5.3.1 Existing Techniques of Parameter Estimation

(a) Least Squares Method
In least squares method, the curve having minimal sum of the deviations squared (also called least square error) from a given set of data is termed as the best-fit curve of a given type. Assume that the data points are \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) where the independent variable is \(x\) and \(y\) is the dependent variable. The fitting curve \(f(x)\) has the deviation (error) \(e\) from each data point, i.e., \(e_1 = y_1 - f(x_1), e_2 = y_2 - f(x_2), \ldots, e_n = y_n - f(x_n)\). According to the method of least squares, the best fitting curve has the property that:

\[
\Pi = e_1^2 + e_2^2 + \ldots + e_n^2 = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - f(x_i))^2
\]

is a minimum.

(b) Maximum likelihood Parameter Estimation

Maximum Likelihood Estimation (MLE) is a statistical method for estimating model’s parameters. The statistical method is used for fitting data to a selected model. MLE selects the value of the model parameters that formulate the data in such a way that it becomes “more likely” than any other chosen values of parameters would make them for a fixed data set and fundamental probability model.

(c) Recursive Least Square (RLS) Based Parameter Estimation

Recursive least squares (RLS) algorithm is used for finding filter/system coefficients by recursively producing minimum of sum of the absolute squared, also called least squares of the error signal. In contrast to means square error (MSE), which are dependent on signal
statistics such as auto and cross correlation of input and output signals, RLS are dependent on the signals itself.

(d) Least Mean Square (LMS)

Least mean squares (LMS) algorithms is basically an adaptive filter for producing replica of a desired filter/system by finding out the filter coefficients that produce the least mean square of the error signal. Since the filter is only adapted on current time error, it is essentially a stochastic gradient descent method. The filter tap weights of the LMS filter are adapted so that error is minimized in the mean square form. As conventional LMS algorithm is a stochastic realization of the steepest descent algorithm, it simply replaces the cost function $\xi = E\left[ e^2(n) \right]$ by its instantaneous coarse estimate $\hat{\xi} = e^2(n)$. The advantages of LMS algorithm include simplicity of implementation, stable and robust performance against different signal conditions, whereas the disadvantage is slow convergence time.

(e) Mean Square Error (MSE)

The mean square error or MSE of an estimator is a technique by which divergence between an estimator and the true value of the quantity being estimated is quantified. Principally it is a risk function which corresponds to the estimated value of the squared error loss or quadratic loss. The average of the square of the error is measured in MSE. Here error refers to the amount by which the estimator varies from the estimated value. The MSE incorporates both the variance of the estimator and its bias since it is the second moment of error. The MSE is the variance for an unbiased estimator and has the same unit of measurement as the square of the quantity being estimated.

(f) Kalman Filter Based Parameter Estimation

The Kalman filter was developed by Kalman in 1960 [81]. It is a recursive solution to the discrete data linear filtering problem. Kalman described the dynamic system with the
concept of state - some quantitative information, which represents the minimum amount of data required to be known about the past behavior for predicting its future behavior. The dynamics can then be explained in terms of state transition, i.e. how one state is transformed into another state by the passage of time. A key property of the Kalman filter is that it is the minimum mean-square (variance) predictor of the linear dynamical system states.

(g) Extended Kalman Filter Based Parameter Estimation

For nonlinear state models, Kalman filtering may be extended through a linearization process. The resulting filter is called the Extended Kalman Filter (EKF). The state space model is linearized at each time instant around the most recent time estimate in EKF. The standard Kalman filter equations are applied after linearization process. The EKF gives accuracy to the first order (First order Taylor series approximation) of any nonlinearity. For nonlinear systems the Extended Kalman Filter is the most widely used estimation algorithm. However EKF is difficult to implement, difficult to tune and only reliable for systems that are nearly linear on the updates time scale.

(h) Unscented Kalman Filter

The Unscented Kalman Filter has been proposed as an alternative to Extended Kalman filter. The UKF is provably superior to the EKF as the Jacobians or Hessians need not to be calculated explicitly. The UKF not only have more accuracy than EKF (Second order approximation versus first order approximation) but is also computationally efficient. A small number of cautiously chosen sample points when propagated in each estimation step offer a compact parametrization of the original distribution.
(i) Sliding mode based parameter estimation

In sliding mode parameter estimation techniques, the parameters are identified by direct application of equivalent control for estimating right hand side of differential equations [82]. Both traditional (FOSMC) and second order (SOSMC) techniques [76] can be used. The added advantage of second order sliding mode technique is that no low pass filter is required for estimating equivalent control, as continuous control is generated directly. Another advantage of second order sliding mode parameter estimation technique is that more parameters can be estimated by using fewer equations [83]. One more advantage is higher accuracy of identified parameters in presence of measurement noise.

5.3.2 Comparison of Existing Techniques with SM Technique

The main disadvantage of least square method is that of outliers, which are bad observations. These outliers having more weight can distort the result significantly. This impact is because of the fact that square of a number grows faster than the number. Recursive least square suffers from higher computational requirements, which makes its use problematic for online estimation where measurements are made at faster rate. The main weakness of Least Mean Square (LMS) algorithm is that it is sensitive to the scaling of the input x(n). This makes the selection of the learning rate $\mu$ that guarantees stability of the algorithm very hard (however this problem is solved in normalized LMS by normalizing with powers of the input). Mean squared error (MSE) has also the drawback of heavily weighting outliers. This results from squaring of each term, which in effect weights large errors more profoundly than small ones. Optimal structure and parameter adaptive estimators for continuous Gaussian process do not give surety about convergence in the presence of measurement noise. In the discrete time case Matrix forgetting factor is introduced in the least square method to ensure an acceptable level of non stationary parametric identification. Kalman filters are computationally expensive, the main problem with extended Kalman filter, is that it usually leads to higher
order equations, which have to be linearized in advance. Extended Kalman filter only gives good results when nonlinearity is small, for highly nonlinear system unscented Kalman filters are used. The problem with non model based parameter estimation techniques such as artificial neural network (ANN) is the need for extensive data and much more computational complexity.

Keeping in view the drawbacks of the above-mentioned techniques and the additional advantages gained by sliding mode technique, the parameter estimation problem of PARR-1 has been tackled by using sliding mode technique. Both first order and second order sliding mode techniques are robust against measurement noise, provides for global convergence of estimation error to zero with no immeasurable noises in the system dynamics. The main problem faced in classical first order sliding mode, the chattering effect has been solved by the use of higher order sliding mode. The major reasons for the use of the HOSM technique are: a better accuracy of resulting motions; the chance of using continuous second order sliding mode control laws for example super twisting or twisting algorithms, the finite time convergence for the systems having relative degree of arbitrary order.

5.4 HIGHER ORDER SMO FOR PARAMETER ESTIMATION

5.4.1 Sliding Mode Observer Design

Sliding mode observer works by minimizing the error between plant model and the observer model by using a switching function. The observer gain is switched by the error such that the plant output matches with observer output or in other words, error surface moves towards minimum. Higher order sliding mode technique has started to find useful applications in different control problems [7, 63, 83] and [84]. The main difference between conventional and higher order sliding-mode is that higher order derivatives of the sliding variable (s) are used in place of first order derivatives. In addition to the advantages of first order sliding
modes, HOSM has additional advantage of chattering minimization [63]. The $n^{th}$ order sliding mode is determined by the equalities $s = \dot{s} = \ddot{s} = \cdots = s^{(n-1)} = 0$, which forms the n-dimensional condition on dynamic system state. The injection term, $u(t)$ is defined as a combination of two terms in super twisting algorithm. The first one is defined as a discontinuous time derivative whereas second term is the continuous function of the sliding variable. The trajectories of the super twisting algorithm are characterized by twisting around the origin on the sliding variable phase portrait. Consider the system of the form [63].

$$\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= \theta(t, x) + \gamma(t, x) u
\end{align*}$$

(5.1)

Where $y_1 = s$, $y_2 = \dot{s}$, $s = n_r - \hat{n}$, and $\theta(t, x)$, $\gamma(t, x)$ are smooth uncertain functions [4, 63]. The super twisting algorithm converges to the 2-sliding set $(s = \dot{s} = 0)$ in finite time and can be given as

$$u(t) = u_1(t) + u_2(t)$$

(5.2)

$$\begin{align*}
\dot{u}_1 &= \begin{cases} 
-u, & |u| > 1 \\
-W \text{sign}(s), & |u| \leq 1
\end{cases} \\
\dot{u}_2 &= \begin{cases} 
-\lambda \left| s \right|^p \text{sign}(s), & |s| > s_0 \\
-\lambda \left| s \right|^p \text{sign}(s), & |s| \leq s_0
\end{cases}
\end{align*}$$

(5.3)

(5.4)

Where, $W > 0$, $0 < p \leq 1$ and $0 < s_0$ $|s(t,x)|$ and bound on $u$ is normalized. The sliding surface $s = n_r - \hat{n}$, is the error between measured and estimated neutron flux or reactor power. It satisfies the second order differential equation of the form

$$\ddot{s} = \theta(x) + \gamma(x) u$$

(5.5)

Where $\theta(x) = \tilde{\theta}(x) - \tilde{x}_{id}$. 

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For the range of interest

$$|\theta| \leq \Theta > 0;$$

$$0 < \Gamma_m \leq \gamma \leq \Gamma_M$$

The injection term can be defined as

$$u(t) = -\lambda_m |s| \text{sign}(s) + u_1, \quad \dot{u}_i = -W_s \text{sign}(s)$$

(5.6)

There is no need for any information about time derivatives of the sliding variable, s, and also about system parameters. This reduces the observer complexity and hence computational complexity is reduced.

5.4.2. Higher Order SMO for Parameter Estimation

Differential equations for neutron flux, precursor concentration and double derivative of neutron flux are shown in (5.7)-(5.9). For the estimation of reactor variables, derivative of neutron flux is needed which is estimated by using second order sliding mode observer. The sliding surface has been taken as

$$\hat{s} = n - \hat{n}, \text{ the error between measured and estimated neutron flux.}$$

$$\dot{n} = \frac{\rho}{\Lambda} n - \frac{\beta}{\Lambda} n + \lambda C, \quad (5.7)$$

$$\dot{C} = \frac{\beta}{\Lambda} n - \lambda C, \quad (5.8)$$

$$\ddot{n} = \left( \alpha^2 + \frac{\dot{\rho}}{\Lambda} + \lambda \frac{\beta}{\Lambda} \right) n + \left( \alpha \lambda - \lambda^2 \right) C, \quad (5.9)$$

Where

$$\alpha = \left( \frac{\rho}{\Lambda} - \frac{\beta}{\Lambda} \right)$$

Equations (5.7) and (5.9) are the basic single group delayed point kinetic equations, which are used for observer construction. The sliding mode observer structure is shown below.
\[ \dot{x}_1 = \dot{n}_r \]  \hspace{1cm} (5.10)

\[ \dot{x}_1 = \ddot{n}_r = \dot{x}_2 \]  \hspace{1cm} (5.11)

\[ \dot{x}_2 = \dddot{n}_r = -\chi \]  \hspace{1cm} (5.12)

Where \( \chi \) can be calculated, using super twisting algorithm as follows:

\[ \chi(t) = u_1(t) + u_2(t) \]  \hspace{1cm} (5.13)

\[ u_1 = \begin{cases} -\chi & |\chi| > 1 \\ -W_{so\text{sign}}(s), & |\chi| \leq 1 \end{cases} \]  \hspace{1cm} (5.14)

\[ u_2 = \begin{cases} -\lambda |s_0|^{\circ \text{sign}}(s) & |s| > s_0 \\ -\lambda |s|^{\circ \text{sign}}(s), & |s| \leq s_0 \end{cases} \]  \hspace{1cm} (5.15)

Here the estimate of neutron flux is taken as first state and derivative of neutron flux estimate is taken as second state. As seen in (5.16) and (5.17) \( \dot{n}_r \), the derivative of neutron flux estimate is required for both observer equations, so second derivative of neutron flux was directly estimated by using super twisting algorithm. For reactor variables estimation \( \dot{n}_r \) is deduced from \( \ddot{n}_r \). Equations (5.11) and (5.12) show the state estimates for \( \dot{n}_r \) and \( \ddot{n}_r \). Super twisting algorithm being applied to calculate \( \chi \), which is equal to \( \ddot{n}_r \), is shown in (5.13)-(5.15). As seen in (5.13), \( \chi \) consists of sum of two terms. The value of first term i.e. derivative of \( u_1 \) is calculated by applying a switching function depending upon the value of \( \chi \), similarly second term is calculated by applying switching function on the basis of sliding variable ‘s’. Note that we have estimated second derivative of neutron flux by directly evaluating \( \chi \). Now we have an estimate of the derivative of the neutron flux, so precursor concentration can be estimated by using the following relation.
\[
\hat{C}_r = \frac{1}{\lambda} \left[ \dot{n}_r - \frac{\rho}{\Lambda} n_r + \frac{\beta}{\Lambda} n_r \right]
\]

(5.16)

In (5.16) neutron flux \((n_r)\) is available from measurement data, reactivity \((\rho)\) is the input to the system and all other parameters except \(\dot{n}_r\) are known, so we have estimated \(\dot{n}_r\) and hence \(\hat{\dot{n}}_r\) using super twisting algorithm. As discussed earlier, precursor concentration is a very important reactor variable since it determines the amount of delayed neutrons, which makes reactor control possible. Now all the required values are available and hence estimate of the precursor concentration can be evaluated using above equation.

\[
\hat{\rho} = \frac{\Lambda}{n_r} \left[ \dot{n}_r + \frac{\beta}{\Lambda} n_r - \lambda C_r \right]
\]

(5.17)

For estimating reactivity, neutron flux \((n_r)\) is again available from measurement, \(\dot{n}_r\) is estimated as discussed and value of \(C_r\) is taken from theoretically calculated value, all other parameters are known and hence reactivity estimate is evaluated by using (5.17). The complete derivation is shown in A1. For finite time convergence proof of super twisting algorithm, one is referred to [85]. As the observers are nonlinear and also robust, a wide operating range is covered and also these observers withstand uncertainties or disturbances present in the reactor system.

**5.5 Test Results of HOSM Observer**

The derivation for the second order sliding mode observer for estimating precursor concentration and reactivity have been shown in appendix. The second order derivative of neutron flux has been derived for observer construction. As seen in the derived formulae for the estimation of precursor concentration and reactivity, the only unknown is the derivative of the neutron flux, which is estimated with higher order sliding mode observer. The sliding surface is taken as the neutron flux difference between estimated and measured neutron flux level. The input taken is the change in reactivity \(\rho\) and output is the neutron flux level \(n_r\). For simulation the controller coefficients are chosen as \(W=10\), \(\lambda_{sm}=2.5\) and \(\rho_{sm}=0.5\). The value of
\( \lambda_{sm} \) determines the overshoot and steady state error. Overshoot decreases by increasing the value of \( \lambda_{sm} \), but at the same time steady state error increases. Conversely by making the value of \( \lambda_{sm} \) small, perfect tracking is achieved at the cost of higher overshoot. So for making a compromise, the value of \( \lambda_{sm} \) has been selected such that, steady state error remains within 5% of desired value and overshoot also remains minimum. The factor W controls the speed of convergence, for small values of W the output converges slowly to its final value, whereas for bigger value it converges promptly. We have taken the value of W equal to 10, which gives satisfactory convergence time. Fig.5.1 shows the estimated and calculated values of precursor density and reactivity estimate along with calculated reactivity shown in Fig. 5.2. The measured neutron flux has been given to the precursor observers as input. As seen in Fig. 5.2 the observer tracks the true value in about 45 seconds. Neutron flux is also estimated just for catering measurement noise present in the system Fig. 5.3 illustrates estimated and measured values of neutron flux. Here after about 8 second observer for neutron flux tracks actual value. Fig. 5.4 depicts the error or sliding surface, as seen once the sliding surface is reached, then system remains on this surface onwards.

![Graph of Normalized Precursor Concentration](image)

Fig.5.1 Estimated and calculated values of precursor concentration (Cr)
Fig. 5.2 Estimated and calculated values of reactivity

Fig. 5.3 Estimated and measured values of neutron flux
5.6 SUMMARY

Higher order sliding mode observers are used to estimate precursor concentration and reactivity in a nuclear research reactor. The measured value of neutron flux is used as an input to estimate these values. The neutron flux is also estimated for catering measurement noise. The tuned mathematical models are validated with experimental data before observer construction. For observer construction double derivative of neutron flux is estimated by using super twisting algorithm. The estimated values are in good agreement with the theoretically calculated values from the model.
CHAPTER: 6

REACTOR CONTROLLER DESIGN

6.1 INTRODUCTION

Nuclear reactor is a very complex system, consisting of many subsystems. These systems need to be controlled properly for safe and continuous operation of the reactor, making reactor control a very important problem. Of many sub-systems, reactor power control is the main problem, which needs special consideration due to its safety critical nature. Also plant parameters vary with time, for example, the nuclear plant characteristics vary with operating power levels, ageing effects and changes in nuclear core reactivity with fuel burnup. These parameter variations make reactor power control a difficult task. There are many types of disturbances and uncertainties present in nuclear reactors. To cater for all these challenges robustness has become a key requirement in modern reactor controllers. Robust controllers have the capability to withstand disturbances and can also cater for parameter variation i.e. these types of controllers can ensure the stability of the system for a wide operating range in presence of disturbances like sudden addition or removal of a neutron absorbing material near reactor core or any change in parameters due to ageing effects etc. Higher order sliding mode controller selected for PARR-1 has the robustness properties of conventional sliding mode controller and in addition the control law is chattering free. HOSM can also be implemented easily at PARR-1. For bench marking purpose two other controllers, LMI based robust controller and H∞ loop shaping controller have also been designed and simulated.

6.2 REACTOR CONTROL PROBLEM

The basic operation of a reactor relies on fission process. The Fission is controlled by inserting control rods, which absorb neutrons, in parallel with fuel rods. As the control rods are moved up, the fission process starts automatically when the rods attain a particular height. After criticality, only single rod is moved up or down to control the reactor power. Light water is used as a moderator most of the time, which also helps in controlling the fission rate
and also acts as a coolant to remove heat from the core.

Neutron flux can be changed in a reactor by four general methods; by temporary adding or removing fuel, by changing moderator level, by changing position of the reflector, or by adding or removing neutron absorbing material. All of the schemes or a combination of them has been used for reactor control, but the mostly used method is the insertion or withdrawal of neutron absorbing material such as cadmium or boron which have large cross section area for neutrons absorption [11]. The materials used for neutrons absorption and fissile materials may be considered as competitors for neutrons; the larger the fraction absorbed by the control material, the smaller the portion available for fission, and vice versa. In reactors these control elements are used in the form of long cylindrical rods, called control rods. Reactor control is performed by manipulating position or speed of these rods.

6.3 LITERATURE REVIEW

Different types of controllers have been implemented in nuclear reactors for controlling power and other variables. $H_\infty$ optimal controller for regulating output power, primary heat transfer pressure, boiler pressure and boiler level were used in [86]. The authors first identified the models by using system identification techniques and then $H_\infty$ controllers were designed by using normalized co-prime factorization based plant description. Better performance as compared to classical controllers was reported, but the controller order was large. The main advantage of the designed controller was robustness against parameter variations. A fuzzy model predictive controller for regulating thermal power in a pressurized water reactor was dealt in [87]. Fuzzy model identified from subtractive clustering method was used for predicting future reactor power. Authors used a three dimensional nuclear reactor analysis code for verifying the proposed controller. The controller tracked step and ramp input sequences satisfactorily. A robust controller using loop shaping technique for regulating primary and secondary circuits pressures of an integrated nuclear power reactor was designed in [88]. The designed controller showed disturbance rejection capability and met required performance specifications. An observer based optimal state feedback controller for improved nuclear reactor temperature control was discussed in [89]. Instead of model based controller
diagonal recurrent neural networks were used. The designed controller showed better performance to account for uncertainties, unmodelled dynamics and parameter variations. Fuzzy controllers were used for automating the control of power axial offset by adding soluble boron and manipulating the volume flows through the water pumps in [90]. The performance of fuzzy controller seemed equal to an expert operator for controlling a pressurized water nuclear reactor. Model predictive controller for controlling integrated power level and axial power distributions was dealt in [91]. Due to time varying nature of nuclear reactor, model predictive controllers showed improved performance since the controller design model was identified and applied recursively. This controller showed fast tracking capability. The idea of state feedback assisted classical control (SFAC) was used to achieve better temperature response in [92]. A linear Quadratic Guassian with loop transfer recovery (LQR/LTR) controller was developed in SFAC control structure to achieve improved temperature performance for wide operating range condition. A robust feedforward- feedback controller for wide range nuclear reactor operation was developed in [93]. Feedforward element was used for providing optimized performance and feedback element was used for guaranteed robust stability and performance. The feedforward control law was synthesized by nonlinear programming and feedback control law was synthesized by structured singular value employing μ synthesis approach, which guaranteed robustness in the presence of disturbances and modeling uncertainties. Simulation results proved that designed controller was superior in robustness and stability to previous observer based LQR controller. Factors involved in design and implementation of digital control laws for research reactors are discussed in [94]. Authors have used model based control laws and compared its performance with classical PID controllers. The results are supported by experiments performed at 5MW MIT Research Reactor and Annular Core Research Reactor operated by Sandia National laboratories. The idea of state feedback assisted classical control (SFAC) was given and simulated in [35]. In SFAC configuration, classical control loop was modified by another state feedback loop. The demand signal was modified by this external loop to achieve optimal performance.

In addition to above mentioned controllers sliding mode controllers have also been
applied in nuclear reactor control applications. Fuzzy Adaptive Recursive Sliding-Mode Controller (FARSMC) for controlling reactor pressure, reactor water level and turbine power was developed in [95]. The chattering problem was minimized by the use of the recursive sliding mode algorithm. Sliding mode controller was applied in space nuclear reactor control, and showed good simulation results in [96]. Robustness and good tracking of reference thermal power profile were reported. Recursive sliding mode controller to control advanced BWR turbine throttle pressure by controlling turbine control valve opening was applied in [97]. It was shown by simulation results that RSMC algorithm results in milder control action and considerable less power surge than the conventional PI controller. The response time was also improved by application of RSMC.

Linear Matrix Inequalities have been used in nuclear reactor control applications for example Sharma et al. [98] used LMI formulation for controlling xenon induced spatial oscillations in pressurized heavy water (PHWR) reactors. M. Gyuan and I.J. Hwang [99] designed a model predictive controller for automatically controlling thermal power of PWR reactors. First they identified the core dynamics using recursive least square method. The future average coolant temperature was predicted by using identified reactor model which consist of control rod position and the core average coolant temperature. The objective function was solved by using LMI. The controller was verified by application on MASTER code (A three dimensional nuclear reactor analysis code). O. M. Kwon, et al. [100] proposed a design method of an observer-based controller for uncertain time-delay systems by delayed feedback. Based on the Lyapunov method, an LMI criterion was derived to design an observer-based controller which makes the system stable. P. Bendotti and B. Bodenheimer designed controller for regulating output power for a pressurized Water Reactor (PWR) [101]. The model was identified using experimental data. The \( H_\infty \) controller was synthesized after balanced model reduction. The results were validated on a linear simulator. Although the results were quiet good at 50% of nominal power but a remarkable difference exist at nominal power, for which the authors suggested the use of nonlinear or gain scheduled controller. They also suggested controller validation on a nonlinear simulator. G. Becker et al. [102] used LMI for designing controller for a linear parameter varying system (LPV). They used the
initial model identification work of O. M. Kwon et al. [100]. As the dynamics of PWR varies significantly at power range so a single $H_\infty$ controller cannot achieve desired closed loop performance so, they suggested the use of LPV system using LMIs to overcome this problem. G.L Sharma and B. Bandyopadhay [103] used LMIs for designing robust feedback for the uncertain systems. The fast output sampling technique already used for designing controllers for PWR was modified by using the LMI approach to overcome poor error dynamics and noise sensitivity of the original controller. In coming sections higher order sliding mode based robust controller will be designed and simulated for PARR-1. Due to its robustness and better performance specifications, its performance is much better than installed PID controller. For bench marking two other robust controllers are designed and performance is compared with HOSM based controller.

6.4 PARR-1 CONTROL PROBLEM

The control problem is to maintain output power at a desired level by changing the reactivity, which acts as a control input. Here for simulation purpose we have considered reactivity directly as a control input, which makes the relative degree of the system equal to one and hence super twisting algorithm can be applied directly. The problem can also be posed the other way round by considering control rod speed as input, which changes reactivity and hence output power. By considering control rod speed as input, the relative degree of the system becomes two and hence any other algorithm such as drift algorithm may be applied.

6.4.1 Control Specifications

The main control specifications are as follows:

- The desired power should be achieved in a minimum amount of time with no oscillations.
- The rate of change in reactivity should also be less than $\beta/3$ or 0.0024 as a sharp raise in reactivity causes the reactor to shut down for maintaining safety.
• The reactor period (the time in which reactor power increases ‘e’ times) should be more than 10 seconds for ensuring that power is not increasing exponentially. For reactor period less than 3.5 seconds the reactor will scram automatically as a safety measure. For normal operations period greater than 30 seconds is used.

• The settling time should be under 100 seconds with no oscillations [104].

• The controller should also be robust enough to withstand sudden disturbances such as addition of a neutron absorbing material such as cadmium, which are inserted near reactor core for irradiation purpose with the help of a pneumatic experimental facility.

• The output power should remain within ±5 % of the demanded power.

In addition to above mentioned specifications, the robustness of the designed controllers are verified by varying two reactor parameters β and Λ to 20% of their nominal values.

Higher Order Sliding Mode Controller Design for PARR-1

6.5 HOSM Controller Design

Sliding mode control has gained much importance in control applications and has been widely applied in a variety of applications. Due to its simple structure, inherent robustness and capability to control non linear systems ([3, 105-106]) it has gained researchers attention and a variety of new algorithms have been proposed. Second order sliding mode control has been proposed by Levant [3] and is being used in various applications. The main advantage of this algorithm is that higher order derivatives of the states are not required and a chattering free control law is derived. The use of higher order sliding mode controllers for controlling reactor power is not found in the literature. This work aims at presenting a novel second order sliding mode controller design and simulation for a properly validated nonlinear model of PARR-1.
Due to its inherent robustness against parameter variations and chattering free control law, this controller will be ideally suited for PARR-1.

Higher order sliding mode controllers have started to find useful applications in different control problems. [95-97] have applied sliding mode techniques in nuclear reactors, but no application of higher order sliding modes in nuclear reactors have been found to date. The main difference between conventional and higher order sliding-mode is that higher order derivatives of the sliding variable (s) are used in addition to first order derivatives. In addition to the advantages of first order sliding modes, HOSM has additional advantage of chattering removal ([3, 105]). The n<sup>th</sup> order sliding mode is determined by the equalities \( s = \dot{s} = \ddot{s} = \cdots = s^{(n-1)} = 0 \), which forms the n-dimensional condition on dynamic system state. Control law, \( u(t) \) is defined as a combination of two terms in the super twisting algorithm. The first one is defined as a discontinuous time derivative whereas the second term is a continuous function of the sliding variable. The trajectories of the super twisting algorithm are characterized by twisting around the origin on the sliding variable phase portrait. The advantages of this algorithm are that it removes chattering for relative degree one systems, another advantage is that derivate of the sliding surface is not required. The main advantage of this system is that it is not sensitive to sampling time interval \( \tau \). The super twisting algorithm converges to the 2-sliding set (\( s = \dot{s} = 0 \)) in finite time and can be given as

\[
 u(t) = u_1(t) + u_2(t) 
\]

\[
 u_1 = \begin{cases} 
 -u & |u| > 1 \\
 -W \text{sign}(s), & |u| \leq 1
\end{cases}
\]

\[
 u_2 = \begin{cases} 
 -\lambda |s|^\rho \text{sign}(s) & |s| > s_0 \\
 -\lambda |s|^{\rho} \text{sign}(s), & |s| \leq s_0
\end{cases}
\]

Where, \( W > 0, 0 < \rho_{sm} \leq 0.5 \) and \( 0 < s_0 \leq |s(t,x)| \) and bound on u is normalized. The sliding surface has been taken as \( s = n_r - n_d \), the error between desired and existing neutron flux or reactor power. As the relative degree of the system is ‘1’, super twisting algorithm can be applied
The control law can be defined as
\[
 u(t) = -\lambda_{sm} |s|^\rho_{sm} \text{sign}(s) + u_i, \\
 \dot{u_i} = -W \text{sign}(s)
\]  

(6.4)

The sufficient conditions for the finite time convergence to the sliding manifold are given in [19]. There is no need for any information about time derivatives of the sliding variable, \( s \), and also about system parameters. This eliminates the need for observer and hence computational complexity is reduced.

### 6.6 Controller Design and Performance

For achieving above mentioned performance specifications second order sliding mode controller using super twisting algorithm has been designed as discussed next. The nonlinear model (3.1-3.6) is used for controller development. The sliding surface is taken as the difference between desired and actual neutron flux level. The input taken is the change in reactivity \( \rho \) and output is the neutron flux level \( \mathbf{n} \). The controller coefficients are chosen as \( W=10, \lambda_{sm}=2.5 \) and \( \rho_{sm}=0.5 \). The controller is tested on the experimentally validated nonlinear simulation. The value of \( \lambda_{sm} \) determines the overshoot and steady state error. By increasing the value of \( \lambda_{sm} \), overshoot decreases, but at the same time steady state error increases. Conversely by making the value of \( \lambda_{sm} \) small, perfect tracking is achieved at the cost of higher overshoot. So for making a compromise, we have selected the value of \( \lambda_{sm} \) such that, steady state error remains within 5% of desired value and the overshoot also remains small. The factor \( W \) controls the speed of convergence, for small values of \( W \) the output converges slowly to its final value, whereas for bigger values it converges rapidly. We have taken the value of \( W \) equal to 10, which give a satisfactory convergence time. Fig.6.1. demonstrates the output tracking of the neutron flux to a normalized flux demand. As seen in Fig.6.1, the output reached final value in about 0.25 second without overshoot and chattering, from an arbitrary initial value. The controller effort signal is also shown in Fig.6.1. As seen, very less control input is required for reaching the required power level. Simulations were performed with different flux demands and initial conditions. In all cases performance
specifications were satisfactorily achieved. The disturbance rejection capability of the controller to a 30% pulse disturbance signal is shown in Fig. 6.2. This disturbance signal demonstrates the sudden insertion and removal of a neutron absorbing material, which changes the reactivity, inside the reactor core. The disturbance is rejected immediately with a minimal change in control input. The figure also shows the disturbance signal and control effort signal. The output remains stable thus proving the robustness of the controller. Sensitivity analysis, by varying parameters values was also verified. The controller still behaved well in dealing with parametric uncertainties. Fig 6.3 shows the graphs of output power when two reactor parameters beta and lambda were changed 20% from their nominal values. All the four graphs representing 20% variations (beta_min, lambda_min; beta_min, lambda_max; beta_max, lambda_min and beta_max, lambda_max) and the graph with nominal values seem to be overlapping in Fig. 6.3. Only by looking at a zoomed portion of the output a slight change in output is observed, proving the robustness of the controller against parameter variations.

Fig. 6.1 Output power tracking to a normalized demand power along with Controller Effort
Fig. 6.2. Disturbance Rejection to a pulse type output disturbance (addition and removal of neutron absorbing material)

Fig. 6.3. Output power when two parameters (beta and lambda) were changed to 20% of nominal value along with a zoomed portion of the output
LMI-based Robust Control System Design for PARR-1

Linear Matrix Inequalities (LMIs) have appeared as powerful design tools in diverse areas such as structural design, system identification and especially control engineering. Some important features of LMIs are as follows. A variety of design specifications and constraints can be expressed as LMIs. A problem can be solved exactly by efficient convex optimization algorithms when formulated in terms of LMIs. Another important feature of LMIs is that most problems with multiple constraints or objectives cannot be solved analytically in terms of matrix equations, but they are solvable in the LMI framework. Due to these features LMI-based design has become a valuable alternative to classical design methods. LMIs can be used for designing robust controllers. Robust controller design for nuclear research reactors has gained much popularity, due to the advantages gained by wide range of operation, more availability of the system and rejection of disturbances and model uncertainty.

Robust controllers for output power regulation for Pakistan Research Reactor (PARR1) have been synthesized using $H_\infty$, pole placement, $H_2$ and mixed $H_2/H_\infty$ optimization [8]. For controller synthesis the model was formulated as a linear matrix inequality (LMI) problem with 20% uncertainties added in two reactor parameters. The model was first parameterized as an affine model to be used in LMI framework. The uncertainties were added in fraction of delayed fission neutrons ($\beta$) and effective prompt neutron life time ($\Lambda$). Simulation results show that best performance is achieved by $H_\infty$ or mixed $H_2/H_\infty$ controllers. The nonlinear reactor model was also simulated with the controller synthesized from LMI formulation. The results show good performance.
6.7 AFFINE PARAMETERIZATION

The system can be written in parameter dependent affine dynamical model form as follows [107]

\[
E(p)\dot{x} = A(p)x + B(p)u \\
E(p)y = C(p)x + D(p)u
\]

(6.5)

where \( A(\cdot), \ldots, E(\cdot) \) are known functions of some parameter vector \( p = (p_1, \ldots, p_n) \).

\[
A(t) = A_0 + p_1(t)A_1 + p_2(t)A_2 \\
E(t) = E_0 + p_1(t)E_1 + p_2(t)E_2
\]

(6.6)

This form is called affine parameter-dependent model. Consider (3.11) given here again for reference.

\[
\begin{bmatrix}
-\beta / \Lambda & \beta / \Lambda & nr \cdot \alpha / \Lambda & nr \cdot \alpha / \Lambda & nr \cdot \alpha / \Lambda \\
\lambda & -\lambda & 0 & 0 & 0 \\
P \cdot f / \mu & 0 & -\Omega / \mu & \Omega / \mu & 0 \\
P \cdot (1-f) / \mu & 0 & \Omega / \mu & -(M + \Omega) / \mu & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

A

The parameter vector is taken as \( p = [\beta / \Lambda \quad 1 / \Lambda] \)

The \( A \) matrix can be affine parametrized by using (6.6) as given below. Here it should be considered that \( B \) and \( C \) matrices are independent of \( p \) vector and \( D \) matrix is equal to zero.

\[
A_0 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 \\
(P \cdot f) / \mu_c & 0 & -\Omega_m / \mu_c & \Omega_m / \mu_c & 0 \\
P \cdot (1-f) / \mu_c & 0 & \Omega_m / \mu_c & -(M + \Omega_m) / \mu_c & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(6.7)
Consider the control structure shown in Fig 6.4 below. The plant P(s) is a given LTI system and full measurement of its state vector $x$ is assumed.
Here by denoting $T_{\infty}(s)$ and $T_2(s)$ as the closed loop transfer functions from $w$ to $z_\infty$ and $z_2$ respectively, the objective is to design state-feedback law $u = kx$ that

(i) Maintain the RMS gain (H_{\infty} norm) of $T_{\infty}$ under some set value $\gamma_0 > 0$

(ii) Maintain the H_2 norm of $T_2$ (LQG cost) under some set value $\nu_0 > 0$

(iii) Minimizes an H_2/ H_{\infty} trade-off criterion of the form given below

$$\alpha \|T_{\infty}\|^2 + \beta \|T_2\|^2$$

(iv) Places the closed loop poles in the open left-half plane at the given region D.

Many practical situations are covered by this abstract formulation. For example, take into consideration a regulation problem with disturbance $d$ and white measurement noise $n$, and let $e$ represent the regulation error. Setting

$$w = \begin{bmatrix} d \\ n \end{bmatrix}, \quad z_\infty = e, \quad z_2 = \begin{bmatrix} x \\ u \end{bmatrix}$$
the mixed $H_2/H_\infty$ criterion covers both the disturbance rejection features (RMS gain from $d$ to $e$) and the LQG features ($H_2$ norm from $n$ to $z_2$). In addition, to obtain well-damped transient responses, the closed-loop poles can be forced into some sector of the stable half-plane.

6.8 LMI FORMULATION AND CONTROLLER SYNTHESIS

Assume that the system has state space representation:

$$\begin{align*}
\dot{x} &= Ax + Bw + Bu \\
z_\infty &= C_1x + D_{11}w + D_{12}u \\
z_2 &= C_2x + D_{22}u
\end{align*} \tag{6.17}$$

The closed loop system in state-space form can be represented as

$$\begin{align*}
\dot{x} &= (A + B_2K)x + Bu \\
z_\infty &= (C_1 + D_{12}K)x + D_{11}w \\
z_2 &= (C_2 + D_{22}K)x
\end{align*} \tag{6.18}$$

6.8.1 $H_\infty$ Performance:

The $H_\infty$ norm of the system $G_\infty(s)$ would not exceed a positive scalar $\gamma$ if and only if [107]

$$\begin{pmatrix}
(A + B_2K)X_\infty + X_\infty(A + B_2K)^\top & B_1 & X_\infty(C_1 + D_{12}K)^\top \\
B_1^\top & -I & D_{11}^\top \\
(C_1 + D_{12}K)X_\infty & D_{11} & -\gamma^2I
\end{pmatrix} < 0 \tag{6.19}$$

and $X_\infty$ is a symmetric matrix.

6.8.2 $H_2$ Performance:

Similarly $H_2$ norm of the system $G_2(s)$ would not exceed a positive $\psi$ if and only if [107]

124
\[
\begin{align*}
&\left( (A + B_2K)X_1 + X_2(A + B_2K)^T - I \right) < 0 \\
&\left( X_2(C_2 + D_{22}K)X_2^T \right) > 0 \\
&\text{Trace}(Q) < \nu^2
\end{align*}
\]

(6.20)

6.8.3 Pole Placement

The closed loop poles lie in the LMI region

\[
D = \left\{ z \in \mathbb{C} : L + Mz + M^Tz < 0 \right\}
\]

where \( L = L^T = \{ A_{ij} \} \) and \( M = \begin{bmatrix} \mu_{ij} \end{bmatrix} \) if and only if \( X_{pol} \), a symmetric matrix satisfies [107]

\[
\begin{bmatrix}
\lambda_{ij}X_{pol} + \mu_{ij}(A + B_2K)X_{pol} + \mu_{ji}X_{pol} + \mu_{ji}(A + B_2K)^T \lambda_{ij} \leq \mu_{ij} \leq \mu_{ij} \leq \mu_{ij} \leq \mu_{ij} < 0 \\
X_{pol} > 0
\end{bmatrix}
\]

(6.21)

For LMI tractability a single Lyapunov matrix \( X \) is sought such that

\[
X := X_\infty = X_2 = X_{pol}
\]

Which ensures above mentioned objectives. This leads to the subsequent suboptimal LMI formulation of the desired multi-objective state-feedback synthesis with change of variables as \( Y = KX \) [107]

Minimize \( \alpha \gamma^2 + \beta \text{Trace}(Q) \) over \( Y, X, Q, \) and \( \gamma^2 \) satisfying the following inequalities

\[
\begin{bmatrix}
AX + XA^T + B_2Y + Y^TB_2^T & B_1 & XC_1 + Y^TD_{12}^T \\
& & & B_1^T & -I & D_{11}^T \\
& & & & & -\gamma^2I \\
C_1X + D_{12}Y & D_{11} & \end{bmatrix} < 0
\]

\[
\begin{bmatrix}
Q & C_2X + D_{22}Y \\
& & X
\end{bmatrix} > 0
\]
\[
\begin{bmatrix}
\lambda_{i,j} + \mu_{i,j} (AX + B_2 Y) X_{pol} + \mu_{j,i} (X A^T + Y^T B_2^T)
\end{bmatrix}_{1 \leq i, j \leq m} < 0
\]

\[
\text{Trace}(Q) < \nu_0^2
\]

\[
\gamma^2 < \gamma_0^2
\]

Let the optimal solution be denoted by \((X_{opt}, Y_{opt}, Q_{opt}, \gamma_{opt})\), the corresponding state feedback gain is given by

\[
K_{opt} = Y_{opt} (X_{opt})^{-1}
\]

The above gain ensures the worst-case performances:

\[
\left\| P_x \right\|_\infty \leq \gamma_{opt}, \quad \left\| P_z \right\|_2 \leq \sqrt{\text{Trace}(Q_{opt})}
\]

### 6.8.4 Mixed H_2/H_\infty Performance

For multi-modal H_2/H_\infty synthesis, the pole placement objectives are expressed in LMI regions as

\[
D = \{Z \in C : L + Mz + M\tilde{z} < 0\}
\]

For the closed-loop transfer function from \(w\) to \(z\) and \(z_2\) to \(T\) and \(T^2\) the suboptimal solution is computed for the following mixed problem.

Minimize

\[
\alpha \left\| T_x \right\|_0^2 + \beta \left\| T_z \right\|_2^2
\]

subject to

\[
\left\| T_x \right\|_\infty < \gamma_0
\]

\[
\left\| T_z \right\|_2 < \nu_0
\]

Several mixed or unmixed designs can be performed with the help of MATLAB LMI toolbox commands [107]. The objective vector for various controllers is summarized in following table.
Table 3: Objective vector for various controllers

<table>
<thead>
<tr>
<th>Object Vector</th>
<th>Corresponding Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0 0 0 0]</td>
<td>Pole placement only</td>
</tr>
<tr>
<td>[0 0 1 0]</td>
<td>$H_\infty$ optimal design</td>
</tr>
<tr>
<td>[0 0 0 1]</td>
<td>$H_2$ optimal design</td>
</tr>
<tr>
<td>[g 0 0 1]</td>
<td>Minimize $|P_2|<em>2$ subject to $|P</em>\infty|_\infty &lt; g$</td>
</tr>
<tr>
<td>[0 h 1 0]</td>
<td>Minimize $|P_\infty|_\infty$ subject to $|P_2|_2 &lt; h$</td>
</tr>
<tr>
<td>[0 0 a b]</td>
<td>Minimize $a|P_\infty|_\infty^2$ subject to $b|P_2|_2^2$</td>
</tr>
</tbody>
</table>

6.9 Simulation Results

The model of the system (3.1) to (3.6) was simulated with the controller synthesized with LMI formulation. The model was simulated with the help of MATLAB S-function. The S-function was called in a SIMULINK model with the controller gain matrix ‘K’ for feedback. The nonlinear model responses were in accordance with the desired results. The results of simulation are shown in Fig 6.5. Here again 20% variation in two reactor parameters beta and lambda were introduced to see the effect of robustness against parameter variations. The graphs representing nominal values of beta and lambda along with graphs with varying parameters (beta_min, lambda_min; beta_min, lambda_max; beta_max, lambda_min and beta_max, lambda_max) are shown in Fig 6.6. The first graph showing output values seems to be overlapping, where the second graph of Fig. 6.6 shows a zoomed portion of the output. As seen in Fig 6.6 the output remains with in a narrow band and remains almost insensitive to parameter variations thus proving robustness against parameter uncertainty.
Fig: 6.5 Output power of Non linear simulation model

Fig: 6.6 Output power when two parameters (beta and lambda) were changed to 20% of nominal value along with a zoomed portion of the output
Robust Control Design by Using $H_\infty$ Loop Shaping Technique for PARR-1

6.10 INTRODUCTION

A robust controller using $H_\infty$ loop shaping technique has been designed for regulating thermal power of Pakistan Research Reactor 1 (PARR-1) [9]. The system model was acquired experimentally by using different system identification techniques [10]. The chirp sequence, Pseudo Random Binary sequence (PRBS) and a newer duty cycle modulation technique were used for exciting the system for acquiring input/output data for model identification. The linear model of the system was identified by ARX method. The identified model was used for robust controller synthesis. The reason behind identifying reactor model experimentally was to keep diversity in reactor models, which increases the system reliability. The nonlinear model discussed before may also be linearized, but it introduces inaccuracies due to approximations, where as identified model is inherently linear so modeling errors are reduced considerably. The robustness of the $H_\infty$ loop shaping controller was verified by observing stability of the controller by parameter variations and disturbance rejection capabilities. Controller behaved well against these disturbances, thus confirming the robustness of the system.

6.11 CONTROLLER DESIGN

6.11.1 Controller Design Procedure

The loop shaping design procedure is based on $H_\infty$ robust stabilization combined with classical loop shaping. This procedure proposed by McFarlane and Glover [108-109] is basically a design procedure with two stages. In first stage, a desired shape to the singular values of open-loop frequency response is given to the open loop plant by augmenting pre and post-compensators. In second stage, robust stabilization with respect to co-prime factor
uncertainty using $\mathcal{H}_\infty$ optimization is performed for the resulting shaped plant. The main advantage of this technique is that $\gamma$ iterations are not required and explicit formulae for corresponding controllers are available [110]. For implementing this technique, first of all plant $G$ is left co-prime factorized as

$$G = M^{-1}N \quad (6.23)$$

A perturbed plant model $G_p$ can be written as

$$G_p = (M + \Delta_M)^{-1}(N + \Delta_N) \quad (6.24)$$

Where $\Delta_M$, $\Delta_N$ are unknown stable transfer functions, used for representing the uncertainty in the nominal plant model $G$. The main objective of robust stabilization is to stabilize not only the plant model $G$, but a family of perturbed plants defined by

$$G_p = \{(M + \Delta_M)^{-1}(N + \Delta_N) : \|[\Delta_N \quad \Delta_M]\|_\infty < \varepsilon\} \quad (6.25)$$

where $\varepsilon > 0$ is the stability margin. Equation (6.26) basically describes the co-prime uncertainty as shown in Fig. 6.7 below. This uncertainty description is amazingly general as it allows both poles and zeros to cross the right-half plane. This uncertainty description has proved to be very useful in practical applications. The robust stabilization problem of normalized co-prime factored plant deals with maximizing this stability margin as suggested and solved by Glover and McFarlane [108]. For the perturbed feedback system shown in Fig 6.7, the stability property is robust if and only if the nominal feedback system is stable and

$$\gamma = \left\| \begin{bmatrix} K & I \end{bmatrix} (I - G K)^{-1} M^{-1} \right\|_\infty \leq \frac{1}{\varepsilon}$$

Where $\gamma$ is the $\mathcal{H}_\infty$ norm from $\Phi$ to $\begin{bmatrix} u \\ y \end{bmatrix}$ and $(I - G K)^{-1}$ is the sensitivity function for this positive feedback arrangement.
The lowest achievable value of $\gamma$ and the corresponding maximum stability margin, are given as [109]:

$$\gamma_{\text{min}} = \varepsilon_{\text{max}}^{-1} = \left\{1 - \|N \quad M\|_H^2 \right\}^{-1/2} = \left(1 + \rho(XZ)^{1/2}\right)$$  \hspace{1cm} (6.27)

where $\| \cdot \|_H$ denotes Hankel norm, $\rho$ denotes spectral radius (maximum eigen value), and for a minimal state-space realization $(A,B,C,D)$ of $G,Z$ is the unique positive definite solution to the algebraic Riccati equation

$$(A-BS^{-1}D^TC)Z + Z(A-BS^{-1}D^TC)^T - ZC^TR^{-1}CZ + BS^{-1}B^T = 0 \hspace{1cm} (6.28)$$

where

$$R = I + DD^T, \quad S = I + D^TD$$  \hspace{1cm} (6.29)

and $X$ is the unique positive definite solution of the following algebraic Riccati equation

$$(A-BS^{-1}D^TC)^TX + X(A-BS^{-1}D^TC) - XBS^{-1}B^TX + C^TR^{-1}C = 0 \hspace{1cm} (6.30)$$
A controller which guarantees that

$$\left\| \begin{bmatrix} K & L \end{bmatrix}^T (I - GK)^{-1} M^{-1} \right\|_\infty \leq \gamma$$

(6.31)

for a specified $\gamma > \gamma_{\text{min}}$, is given by

$$K = \begin{bmatrix} A + BF + \gamma^2 (l^T)^{-1}z c^T (c + DF) & \gamma^2 (l^T)^{-1} z c^T \\ B^T x & -D^T \end{bmatrix}$$

(6.32)

Fig. 6.8 Full structure of Controller ‘K’ and plant ‘G’, after weight insertion inside controller

**6.11.2 Weight Transfer Function Selection**

The weights for the shaped plant were calculated after several iterations, for achieving maximum gain and bandwidth. The transfer function for weights came out to be

$$W = \frac{(100s + 15)}{(s + 0.0001)}$$

(6.33)

For selecting weights, the consideration should be to have high gains at low frequencies and low gain at high frequencies. High gain at low frequencies improves tracking ability and low gain at high frequencies improves noise rejection capability. This transfer function was cascaded in series with the original plant to calculate controller gains of the system. When the desired loop shape is achieved then the weight transfer function is combined with the controller transfer function, to get the complete controller. Fig. 6.8 shows overall structure of plant and controller.
6.12 SIMULATION RESULTS AND DISCUSSIONS

The $H_\infty$ controller was synthesized by using robust stabilization technique. For controller synthesis the mathematical model of the system was found experimentally by persistently exciting the system with a chirp signal. ARX model was selected for generating model of the system with system identification toolbox. After identification was validated a robust controller was synthesized by using $H_\infty$ loop shaping technique. Figure 6.9 shows step response of the overall system with controller, as seen in figure, the output is stabilized within 20 seconds with about 17% overshoot. The response time is adequate enough for output power dynamics. The value of the gamma achieved of the robustified plant comes out to be ‘2.384’ which is suitable for this controller. The disturbance rejection capability of the controller was verified by inserting pulse type disturbance signal between ‘50’ to ‘80’ seconds. As seen in Fig. 6.10 the output was regulated within ‘10’ seconds by the controller. The normalized output graphs, when two parameters (beta and lambda) are varied to 20% are shown in Fig. 6.11. As seen in second part of Fig. 6.11, which shows a zoomed portion of part a, the effect of parameter variations are minimal ensuring robustness of controller against parameter variations. Fig 6.12 illustrates gain and phase margin of robustified plant. Here also the gain margin and phase margin are enough for ensuring stability of the overall system. The Sensitivity and Complementary Sensitivity plots of the closed loop system are shown in Fig. 6.13. As can be seen the initial value of Complementary Sensitivity is 0 db or the magnitude is unity in lower frequency range which ensures good reference tracking and Sensitivity is nearly equal to zero at higher frequencies which helps in noise rejection.
Fig. 6.9. Step response of the output power

Fig. 6.10 Step response of the output power with pulse type disturbance
Fig: 6.11 Output power when two parameters (beta and lambda) were changed to 20% of nominal value along with a zoomed portion of the output

Fig. 6.12. Gain and phase margin of the robustified Plant
Fig. 6.13. Sensitivity and Complementary sensitivity of the closed loop system

6.13 SUMMARY

Robust controller for output power regulation of PARR1 has been developed and simulated using HOSM technique. For HOSM controller nonlinear model of PARR-1 was used. The chattering free robust controller was developed and simulated for different power levels. The output gives good tracking without the controller chattering, common in first order sliding modes. For benchmarking, LMI and $H_\infty$ loop shaping controllers are also designed and tested for PARR-1. For LMI formulation linearized model was used with affine parameterization. Here the problem was posed in LMI framework to achieve maximum performance. The controllers were synthesized with $H_\infty$, pole placement, $H_2$ and mixed $H_2/H_\infty$ optimization. Of these controllers the $H_\infty$ controller has shown the best performance in disturbance rejection and reference tracking. Nonlinear reactor model simulations were also performed with the LMI controller. The results are quite satisfactory. Finally $H_\infty$ loop shaping
controller was designed and simulated for ARX and state space models of PARR-1, obtained experimentally by using system identification techniques. After identification a robust controller for regulating output power was synthesized using $H_\infty$ loop shaping technique. The results show that best performance is given by $H_\infty$ controller, both in output response and disturbance rejection. The robustness against parameter variation was also tested with 20% uncertainty introduced in two reactor parameters. For all designed controllers the outputs showed robustness against parameter variations, with best performance by HOSM controller and then by LMI controller and in last by loop shaping controller, but results of all controllers were well inside acceptable limits.

By comparing simulation results of all controllers, it comes out that best performance is given by HOSM controller, both in reference tracking and disturbance rejection. The final value is achieved within 0.3 seconds with minimal controller effort which is decades better than other controllers. Similarly disturbance is rejected within 0.2 seconds as opposed to more than 5 seconds by $H_\infty$ loop shaping controller or by LMI based controller. This controller has been tested successfully on lab based CRDM system, which is exact copy of PARR-1 CRDM system. As stated earlier the CRDM is controlled by relays, for which sliding mode techniques gives best results. The main problem of chattering is catered by the use of second order sliding mode controller. So this controller is planned to be implemented as given in future work chapter.
CHAPTER: 7

CONCLUSIONS AND FUTURE WORK

The thesis is about modeling, design and implementation of higher order sliding mode observer and controller in a research reactor. The research reactor model was developed, simulated and validated on Pakistan Research Reactor-1 (PARR-1), before higher order sliding mode observer and controller design and implementation. The observer has been designed for estimating precursor concentration and reactivity and controller for regulating output power. Linear and non-linear models of PARR-1 have also been tuned and validated with experimental data. These models are subsequently used for higher order sliding mode observer/controller design and performance evaluation. As a first step of model development a simulation model of control rod drive mechanism (CRDM) has been developed using SIMULINK®. This model was validated with a lab based CRDM model, which is similar to PARR1 CRDM system. This model was later appended with neutronic model to get a complete model from control rod motion to output power change. The complete model was validated on the actual reactor. The mathematical model of the overall system, from control rod movement to output power change is validated with different sets of data at different operating points. Based on the validated model robust nonlinear observer and controller have been developed by using second order sliding mode technique. Linear observers have been used in the past to estimate reactor variables, but the bandwidth is limited and performance degraded as the operating point is changed. The nonlinear observer can cater for this problem in a much efficient way. Similarly sliding mode controller has added advantage of robustness against parameter variations in addition to increased bandwidth. Although other sliding mode techniques have been used for estimating reactor variables, but use of higher order sliding mode observer for this applications were never reported in literature, according to the authors.
knowledge. The higher order sliding mode observer is much efficient and has the main advantage of reduced chattering. The observer estimate reactor parameters with only the neutron flux measurement available. For bench marking purpose LMI based robust controller and $H_\infty$ loop shaping controller are also designed and simulated for output power regulation. The LMI controller is developed by affine parameterization of PARR-1 linearized model and $H_\infty$ loop shaping controller is developed by using identified model of PARR-1. The robustness of the designed controllers is verified by observing disturbance rejection capability to different disturbances. Finally the performances of all designed controllers are compared. By comparing all specifications of the designed controllers, HOSM controller is found to be best suitable for PARR-1.

The tuned and validated mathematical model of PARR-1 will be useful in future control related applications. The successful application of HOSM technique for parameter estimation of reactor system proves that it is a viable and useful option for nuclear reactor systems and due to its robustness and other advantages can be used with better performance in harsh and rugged environment common in nuclear reactors. The controller designed by this technique has shown remarkable improvement over other such controllers and hence can be successfully used in reactor control related applications.

There are many new directions, which can be explored in future, such as integral sliding mode controller which has zero reaching phase and hence more robustness. It can also be applied for parameter estimation and control applications. A major future application could be the use of HOSM observer for reactor condition monitoring. Usually redundant sensors are used to measure a reactor variable and then by using decision logic such as 2 of 3 logic, the output of sensor is considered accurate if at least two measurements are same. Similarly any malfunction is also detected by observing the difference of the two sensors, measuring same variable. Ideally this difference should be zero and practically it should be inside a tolerance
bound. By estimating these variables, the hardware cost could be reduced substantially and also better performance could be achieved as software sensors are not prone to measurement noise. Due to robustness properties of HOSM observer, the parameter variation due to ageing effect or any other disturbance could be catered for and more accurate measurements can be achieved. Finally HOSM controller is planned to be implemented on actual reactor for power regulation. The controller has already been implemented on lab based CRDM system for chattering free control rod position control and in future it is planned to be implemented on actual reactor for output power control.
Appendix A1

The derivation for finding the second derivative of neutron flux. And then finding observers for neutron flux and ratio $\beta / \Lambda$

\[
\dot{n}_r = \frac{\rho}{\Lambda} n_r - \frac{\beta}{\Lambda} n_r + \lambda C_r,
\]

\[
\dot{C}_r = \frac{\beta}{\Lambda} n_r - \lambda C_r,
\]

\[
\ddot{n}_r = \frac{1}{\Lambda} \left[ (\rho \dot{n}_r + n_r \dot{\rho}) - \frac{\beta}{\Lambda} \dot{n}_r + \lambda \dot{C}_r \right],
\]

\[
\ddot{C}_r = \frac{1}{\Lambda} \rho \dot{n}_r + \frac{1}{\Lambda} n_r \dot{\rho} - \frac{\beta}{\Lambda} \dot{n}_r + \lambda \dot{C}_r,
\]

\[
\ddot{n}_r = \left( \frac{\rho}{\Lambda} - \frac{\beta}{\Lambda} \right) \dot{n}_r + \frac{1}{\Lambda} n_r \dot{\rho} + \lambda \dot{C}_r,
\]

\[
\ddot{C}_r = \left( \frac{\rho}{\Lambda} - \frac{\beta}{\Lambda} \right) \left[ \rho \dot{n}_r - \frac{\beta}{\Lambda} n_r + \lambda C_r \right] + \frac{1}{\Lambda} n_r \dot{\rho} + \lambda \left[ \frac{\beta}{\Lambda} n_r - \lambda C_r \right],
\]

\[
\ddot{n}_r = \left( \frac{\rho}{\Lambda} - \frac{\beta}{\Lambda} \right) \left[ \frac{\rho}{\Lambda} n_r - \frac{\beta}{\Lambda} n_r + \lambda C_r \right] + \left( \frac{\dot{\rho}}{\Lambda} + \lambda \frac{\beta}{\Lambda} \right) n_r - \lambda^2 C_r,
\]

Let

\[
\alpha = \left( \frac{\rho}{\Lambda} - \frac{\beta}{\Lambda} \right)
\]

\[
\ddot{n}_r = \alpha^2 n_r + \alpha \lambda C_r + \left( \frac{\dot{\rho}}{\Lambda} + \lambda \frac{\beta}{\Lambda} \right) n_r - \lambda^2 C_r,
\]

\[
\ddot{n}_r = \left( \alpha^2 + \frac{\dot{\rho}}{\Lambda} + \lambda \frac{\beta}{\Lambda} \right) n_r + \left( \alpha \lambda - \lambda^2 \right) C_r.
\]
Let \( \eta_1 = \alpha^2 + \frac{\dot{\rho}}{\Lambda} + \lambda \frac{\beta}{\Lambda} \)

and \( \eta_2 = \alpha \lambda - \lambda^2 \)

\( \ddot{n}_r = \eta_1 n_r + \eta_2 C_r \)

\( \dot{n}_r = -\kappa \)

\( \ddot{e} = \eta_1 n_r + \eta_2 C_r + \kappa \)

\( 0 = \eta_1 n_r + \eta_2 C_r + \kappa \)

\( -\kappa = \eta_1 n_r + \eta_2 C_r \)

\( \hat{n}_1 = \frac{-\kappa - \eta_2 C_r}{n_r} \)

\( \dot{C}_r = \frac{1}{\lambda} \left[ \frac{\ddot{n}_r - \frac{\dot{\rho}}{\Lambda} n_r + \frac{\beta}{\Lambda} n_r}{n_r} \right] \)

\( \hat{\rho} = \frac{\Lambda}{n_r} \left[ \ddot{n}_r + \frac{\beta}{\Lambda} n_r - \lambda C_r \right] \)
REFERENCES


