RESOURCE ALLOCATION IN COGNITIVE RADIO AD HOC NETWORKS: A POTENTIAL GAME PERSPECTIVE

By
Qurratul-Ain Minhas

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy
QUAID-I-AZAM UNIVERSITY
ISLAMABAD, PAKISTAN
SEPTEMBER 2014

© Copyright by Qurratul-Ain Minhas, 2014
Dedication

My Mother

and

The Memory of My Father
Abstract

In wireless cognitive radio networks, natural antagonism arises among unlicensed users when nodes opportunistically compete for unused frequency bands and the operations are seriously hampered by acute scarcity of resources. The transmitted power, which is inherently pertinent to the signal-to-interference-plus-noise ratio, cognition methodology, and lack of central management, must be preserved for longer network lifetime. In the midst of this struggle to acquire desired frequency band, where the performance of the entire network is dependent upon the behavior and etiquette exhibited by individual nodes, it is pivotal to introduce an effective cooperation mechanism in order to improve the vital network parameters. In this work, we employ the concepts of game theory to develop an efficient and sustainable cooperation mechanism for efficient cognition and improved spectrum utilization. Instead of focusing merely on the interference a user observes, cooperation is ensured by taking into consideration the amount of interference a user creates for other network users.

With the introduction of unlicensed users in licensed bands, the operations and interests of licensed users need to be protected, hence the spectrum owners are given an advantage and control over the multiple access policy. We address the problems in spectrum access and channel selection equilibrium in a leader-follower setup. In contrast to the game formulations that lack efficient power and pricing schemes, we present a cooperative Stackelberg potential game for cognitive players. A dynamic cost function is articulated to induce awareness in players to mitigate the effects of selfish choices in spectrum access while at the same time steer the distributive network towards achieving Nash equilibrium. The proposed scheme is mutually beneficial for
all players and focuses on improving the network performance and power efficiency. We design the network potential function such that the nodes have performance based incentives to cooperate and achieve a Nash equilibrium solution for efficient channel acquisition and capacity. Simulation results show fast convergence in channel selection strategies and increase in capacity for the entire network.

In order to avoid anarchy in this uncontrolled and sometimes hostile environment, it is important to inhibit the nodes in making potentially risky decisions that may eventually jeopardize the stability and performance of the entire network. We present a game theoretic approach to combat the effects of uncontrolled and selfish behavior exhibited by cognitive network nodes. A sustainable solution is proposed that employs nonlinear learning in conjunction with potential function to alleviate the implications of disruptive behavior that is usually demonstrated in the access of scarce spectrum resources. The regret information in decision making is exploited along with history statistics to minimize information exchange and achieve swift convergence of strategies. Moreover, incorporating learning allows the cognitive players to select the channels in a simultaneous fashion instead of waiting for their turns to change their channel choices. This considerably reduces the delay in achieving network stability.
Table of Contents

Abstract i

Table of Contents iii

List of Symbols and Abbreviations viii

Acknowledgements x

1 Introduction 1

2 Cognitive Radio Games 8
  2.1 Potential Games . . . . . . . . . . . . . . . . . . . . . . . . . . . 9
     2.1.1 Types of Potential Games . . . . . . . . . . . . . . . . . . . . 9
  2.2 Potential Games Formulation . . . . . . . . . . . . . . . . . . . . 12
     2.2.1 Optimality of Potential Function . . . . . . . . . . . . . . . . 12
     2.2.2 Spectrum Overlay and Underlay . . . . . . . . . . . . . . . . 17
  2.3 System Model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 18

3 Cooperative Potential Games for Cognitive Radios 29
  3.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 29
  3.2 System Model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 31
     3.2.1 Underlay Spectrum System . . . . . . . . . . . . . . . . . . . . 32
     3.2.2 Overlay Spectrum System . . . . . . . . . . . . . . . . . . . . 33
  3.3 Non-pricing Utility Function Deliberating Interference . . . . . . . . 34
  3.4 CR Potential Game . . . . . . . . . . . . . . . . . . . . . . . . . . . 36
     3.4.1 Overlay Case . . . . . . . . . . . . . . . . . . . . . . . . . . . . 36
     3.4.2 Underlay Case . . . . . . . . . . . . . . . . . . . . . . . . . . . 39
  3.5 Cost Based Potential Game . . . . . . . . . . . . . . . . . . . . . . 41
     3.5.1 Other Potential Game Formulations . . . . . . . . . . . . . . . 46
List of Figures

2.1 The cognitive radio network ........................................ 19

3.1 Plot of the average transmitted power for overlay and underlay cases 51
3.2 Convergence of strategies for minimum cost game .................. 52
3.3 Convergence of strategies for power control game ................. 53
3.4 Average transmission power for minimum cost game .............. 53
3.5 Transmit power for power control game ............................. 54
3.6 Average throughput comparison .................................... 55

4.1 Convergence of strategies for spectrum overlay system (Note the absence of SU on channel 3 due to the presence of PU) ............ 66
4.2 Convergence of strategies for spectrum underlay system .......... 67
4.3 Convergence of strategies in terms of channel switching for spectrum overlay system ............................................. 67
4.4 Convergence of strategies in terms of channel switching for spectrum underlay system ..................................................... 68
4.5 Transmission power for spectrum overlay system .................. 69
4.6 Transmission power for spectrum underlay system ................. 70
4.7 Comparison of average sum capacity of proposed work with [1] ... 70

5.1 Convergence of strategies for spectrum allocation in sequential learning 80
5.2 Iterative channel switching before convergence ..................... 81
5.3 Convergence of strategies for spectrum allocation .................. 89
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4</td>
<td>Iterative channel switching</td>
<td>90</td>
</tr>
<tr>
<td>5.5</td>
<td>Comparison of average capacity for different schemes</td>
<td>92</td>
</tr>
<tr>
<td>5.6</td>
<td>Comparison of convergence for sequential and simultaneous moves games</td>
<td>92</td>
</tr>
<tr>
<td>5.7</td>
<td>Average Transmission power of users on the channels</td>
<td>93</td>
</tr>
<tr>
<td>5.8</td>
<td>Average convergence of strategies for spectrum allocation for 20 different network topologies</td>
<td>93</td>
</tr>
</tbody>
</table>
List of Tables

2.1 Non-cooperative game model between CRs in underlay case  . . . . . . . . . . 23
2.2 Non-cooperative game model between a PU and SU in overlay case  . . 24
2.3 Non-cooperative game model between CRs in overlay case when channel 2 is occupied  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 24
2.4 Non-cooperative game model when both channels are vacant  . . . . . . 25
2.5 Overlay game model among cooperative SUs  . . . . . . . . . . . . . . 25
2.6 Interference aware game model for cooperative SUs  . . . . . . . . . . 27
List of Symbols and Abbreviations

SYMBOLS

- $a_i$: Action set for player $i$
- $a_{-i}$: Action set for player $i$ various
- $A$: Complete Action set for all players
- $s_i$: Strategy set for player $i$
- $u_i$: Utility function for player $i$
- $V$: Potential function for the network
- $p_i$: Transmission power of user $i$
- $N$: Total number of CR transmitter-receiver pairs
- $K$: Total number of available channels or PUs
- $h_{ij}$: Link gain between user $i$ and user $j$
- $h_{ii}$: Link gain between the transmitter and receiver of $i$th CR pair
- $p_0$: Transmission power of PU
- $\gamma_i$: Signal-to-Interference-plus-Noise Ratio (SINR) for user $i$
- $N_o$: Noise power
- $f_{ji}$: Indicator function for actions of players $j$ and $i$
- $I_{iv}$: Interference observed by SU $i$ in overlay
- $I_{iv}$: Interference observed by SU $i$ in underlay
- $I'_{iv}$: Interference created by SU $i$ in overlay
- $I'_{iv}$: Interference created by SU $i$ in underlay
- $I_p$: Interference observed by PU from all SUs on the channel
- $I'_p$: Interference created by PU from all SUs on the channel
- $\Psi_{kj}$: Bit-error-rate (BER) when CR $j$ is transmitting over channel $k$
- $\Psi_{p_0}$: Threshold BER for primary user
- $\Psi_{po}$: Original BER of a channel in the absence of CRs
$c_{ki}$ Cost function for the SU $i$ for accessing the PU channel $k$

$\gamma_{ko}$ Target SINR for channel $k$

$\alpha$ Probability of error in detecting the PU

$V_o$ Potential function for overlay CR network

$V_u$ Potential function for underlay CR network

$\mu_i$ Spectrum efficiency

$\rho$ Weight assigned to the interference a user creates over a channel

$I_{th}$ Interference level tolerated by a PU on a channel

$\zeta$ Threshold SINR level required by the SU for a channel

$U_i(t)$ Utility function for user $i$ at time $t$

$C_{ik}(t)$ Cost paid by SU $i$ to gain access to channel $k$ at time $t$

$W_i(t)$ Weight function based on learning evaluated by user $i$ at time $t$

$\theta(t)$ Probability of channel access in iteration $t$

$\alpha_k(t)$ Probability of channel usage in the iteration $t$ for channel $k$

$I_{ik}(t)$ Interference a user $i$ suffers on channel $k$ at time $t$

$I'_{ik}(t)$ Interference a user $i$ creates on channel $k$ at time $t$

$U_{ik}(t)$ Utility function for the CR $i$ with strategy $s_i$ over a channel $k$

**ABBREVIATIONS**

- **SINR**  Signal to Interference plus Noise Ratio
- **BER**  Bit Error Rate
- **CR**  Cognitive Radio
Acknowledgements

Praise be to Allah Who is the most gracious and exalted. First, I would like to express my deepest appreciation to my advisor, Dr. Hasan Mahmood. Being his first PhD student I enjoyed his affection and the freedom to express my views. Without his supervision and constant guidance this dissertation would not have been possible.

Special thanks to my most favorite teacher Dr Azhar A. Rizvi, who has always been a source of inspiration and a tremendous mentor for me. His teachings are extremely valuable both academically and practically. His generous support, coaching and companionship are the things I’ll treasure throughout my life.

I am indebted to Dr Qaiser A. Naqvi for his support and encouragement through the early years of chaos and confusion. He is the person who motivated me in any crisis with his profound kindness. I must also thank Dr Muhammad Zia who shared with me his knowledge and provided many useful references and friendly encouragement. His valuable suggestions greatly assisted in improving the manuscript.

Words cannot express how grateful I am to my mother and my brother Abdul Qudoos for all of the sacrifices that they have made on my behalf. Their prayers for me was what sustained me thus far. I can never thank my brother enough for supporting me through thick and thin. His wisdom and sense of humor made the difficult times more bearable and changed my life from worse to bad.

I wish to thank the following: Dr Aqeel A. Syed, Mr Musarrat Abbas, Dr Aqueel Ashraf, Dr Farhan Saif, Dr Zeeshan Akbar, Dr Arshad Fiaz, Farhat Majeed and Samina Gulistan for the support and guidance they provided me and for all the good and bad times we had together. They are wonderful people and their support made this research possible and above all their companionship added pleasure to the nerve-racking ordeal of my PhD.

I would like to thank all of my friends and my research group who supported and incentivized me to strive towards my goal. I would also like to extend my thanks and appreciation to the office staff for their assistance. Last, but not the least I’m also
grateful to my dear students who helped me evolve and who are my partners in the process of teaching and learning.
Chapter 1

Introduction

The concept of Cognitive Radio (CR) formulated by Joseph Mitola [2] and later refined by Haykin [3] present the CR as a dynamic and inexpensive way to improve spectrum efficiency by intelligently exploiting the vacant spectrum opportunities. When employed in ad hoc networks, the available opportunities cause conflicts among the unlicensed CRs striving for a suitable spectrum band or sub-channel. This struggle and lack of management leads to the depletion of CR’s energy resources and deterioration in the network performance. Moreover, the qualm of CRs by their respective opportunities leads towards network instability. To top it off, the licensed user or Primary User (PU) of a sub-channel can simply abolish the available spectrum opportunity, forcing the CR or Secondary User (SU) to disrupt its transmissions and search for new sub-channels.

The above scenario reveals the worth and woes of CR communication. On one end, CRs promise to reduce the underutilization of bandwidth and achieve spectrum efficiency [4]. On the other, the CR networks suffer from the imminent threat of conflicts, instability and poor performance. The main concern is to deal with these challenges so as to find a stable solution.
However, the stability of a solution does not guarantee its optimality or efficiency. For a stable network, the CRs must be contented with the opportunities achieved among the several available options. For a practical and efficient CR system, the CR user’s satisfaction is linked with reduced interference, improved network performance and low power consumption. These challenges are dominated by any constraints set by the PU. Most licensed transmissions simply terminate the SU’s communication (spectrum overlay). However, more tolerant PUs allow the cognitive transmissions to co-exist as long as they do not interfere with the licensed communication (spectrum underlay). In any case, the SUs must abide by the mode of communication established by the licensed owner.

In this thesis, we consider a CR ad hoc network, where several SUs are contesting for a limited number of available sub-channels. This competition is modeled as a game where certain participants play to achieve a particular goal or win a payoff. The mathematical formulation of a game comprising of players, their strategies and the payoffs is effectively described using game theory; a branch of economics. In a CR network, the SUs and PUs are the players, opting from the strategy set of available channels and earning a reward in the form of successful cognitive transmissions. The ultimate aim of every SU is to gain access to a channel that provides reduced interference for successful communication. This simple objective is entwined with the struggle to save power and achieve high performance. Since each CR follows the same objectives, they play as rivals to attain the most suitable channel. This rivalry causes SUs to create interference for each other in pursuing their ambitions selfishly. This non-cooperative setup causes SUs to fluctuate their choices resulting in conflicts and unstable network. This degraded network performance discourages the selfishness
and a long-term solution is sought by promoting cooperation among the CRs, which converges to provide efficient power consumption and improved network performance.

In the past, the competition among CRs is discussed using different models. The potential game implementation [5], which simultaneously ensure the individual and collective objectives of cognitive users, can drastically improve the performance of these type of systems. Below is a brief description of some of the noticeable works and their contributions. While these works contribute significantly to the game theoretic approach for CR networks, the application to resource allocation is is not effectively explored and lack the fair pricing structure, improved network performance and energy efficiency.

Several researchers have explored the cognition phenomenon from game theoretic perspective, including non-cooperative and cooperative games [3-15]. A non-cooperative game with linear pricing is discussed in [6], but the proposed game does not guarantee fairness. An interference temperature based pricing model for power control algorithm is discussed in [7], however, due to limited information access to users, the game does not provide an optimal solution. Some of the non-competitive two and three player games, thoroughly discussed in literature are given in [12-15].

Contrary to the non-cooperative game, the cooperative game theory delivers a more stable strategy set and faster convergence. Some form of cooperation involves the use of SUs as relays by the PU in return for channel use [8]. Other cooperative forms involve payments by CRs to their licensed owners, which set a price for channel access. Spectrum access issues through trading are discussed in [9] and [10], where channel price is set based on the supply and demand functions.

One of the most valuable work regarding CRs is [5]. This work is a comprehensive
study of different models that can be employed to solve various problems encountered in CRs. Potential games for CRs or adaptive radios alongwith system stability are discussed. However, this research is focused on the fact that these non-cooperative games reach Nash equilibrium, with less attention given to the properties of the equilibrium with respect to the players and the network objectives. [11] is a valuable work that defines and describes the applications of cognitive radios, and investigates the convergence behavior of these systems. It explores the cognition problem as a game theoretic system and provides different potential game formulations and their impact on the convergence behavior of these systems. [12] exploit the concepts of potential games to encourage cooperation among CRs. Their work, however, excludes the role of PU and lacks efficient allocation of power to individual users based on network performance.

In [13], the authors present the convergence of a cooperative game for temporal spectrum sharing, but users may enter or leave the network only at the beginning of game, and are controlled to wait for the existing players to establish their strategies if they try to enter at a later time. Their work also lacks the involvement of PU behavior.

Li and Xie [14] discuss a repeated game for improving transmission rate of CRs using clustering. The convergence is observed for two SUs, which, along with the transmit power and pricing affects the transmission rate. The cooperation in cognitive game for an incomplete channel information is discussed in [15]. Their work presents a Baesian game where SUs are allowed channel access at the expense of forwarding packets for the licensed users. This creates an additional delay and power consumption by the SUs for relaying as channel cost. These works do not incorporate
a competitive environment of a large number of CRs striving for a single channel and lack a dynamic pricing model.

Different variants of transmit power control algorithms exist, which include different approaches of iterative water-filling algorithms [16]. Several researches have employ the concept of iterative water filling to allocate power to cognitive radios [3], [17], [18]. Haykin [3], [19] presented mathematical formulation for distributed water-filling in CRs. A two-user, two-channel game for power control is discussed by Liao et al. [17] under cooperative and competitive scenarios. The famous prisoners dilemma and sub-optimality in water-filling algorithms is described by Laufer and Amir [18], presenting a new efficient algorithm. However, these works lack the effective cooperation required for stable and efficient network performance.

In [1], the authors discuss a Stackelberg potential game with PU as the leading player deciding the first move and addresses the channel and power allocation issue, where players opt from a joint set of channels and power levels. Their work, however, considers only the overlay system with four discrete power levels and channels, for a fixed channel cost determined by the leader. The pricing mechanism is not dynamic and fair, which restricts the system and does not effectively optimize transmit power and network performance.

Learning techniques are explored for improving performance by empowering players more information through history [20]. Learning algorithms are usually based on determining regret, which is a measure of loss encountered based on historical data [21]. External regret schemes predict the strategies via history, whereas internal regret mechanism adaptively assigns probabilities to action sets based on the regret for not playing other strategies (regret matching) [22]. [23] presents an adaptive regret
tracking which provides information about the users and their actions. [24] allocates transmit power according to the probabilities learned from distributed information. These games, however, do not provide fair and efficient utilization of resources.

No regret learning is based on assigning higher probabilities to better payoff strategies to reduce or eliminate regrets [3]. A no-regret game for channel allocation is discussed in [25], which applies a no-regret learning algorithm to the work presented by [1]. The results apparently provide a slightly improved payoff but do not provide an optimum solution. Hart and Mas-Colell showed in [26] that correlated equilibrium can be attained by a procedure of play called regret-matching. This work has inspired the application of no-regret to the problem of cognitive radio power control in some research papers, for example, [27], [24], [28]. The procedure of regret-matching leading to correlated equilibrium is applied for distributed access point selection in a wireless network [27] and distributed opportunistic spectrum access for cognitive radio networks [24]. However, in [28] a modified version of the regret-matching learning algorithm is considered, where each player only needs to know his own payoffs and actions. Most of these works in literature is focused on achieving an equilibrium for channel access among the cognitive players. The transmission power is usually fixed or allocated using water-filling, which may not provide improved performance.

In this thesis, we model the CR competition as a cooperative game which converges to a stable solution or Nash Equilibrium (NE), such that the individual motives of licensed and unlicensed users are satisfied along with the mutual objective of improved network performance. Moreover, the transmission power is also efficiently conserved to reduce interference and prolong battery life. The influence of licensed user and its implications are discussed in detail. We also present game theoretic schemes to
reduce delay in achieving an efficient and stable network by incorporating learning.

This thesis is divided into six chapters. Chapter 2 gives a brief account of potential games and the relevant game theoretic concepts to be employed in this thesis to solve several cognition issues. This chapter also acts as a primer for readers unfamiliar with game theory. It elaborates the reasons why we chose potential games for finding solutions to various problems.

Chapter 3 applies the concepts of potential games presented in Chapter 2 for solving resource allocation problems arising in spectrum access among CRs. The different formulations of potential games and their performance is compared to elaborate the significance of proposed scheme.

Chapter 4 explains a Stackelberg potential game where a cooperative pricing scheme is proposed. The cost function proposed encourages cooperation and results in an improved converging solution.

Chapter 5 explains different learning schemes for sequential and simultaneous games. Learning allows players to decide simultaneously, eliminating the need for sequential setup. While learning adopted in sequential games improve the convergence rate, it provides the Nash equilibrium for simultaneous games to reduce the delay and achieve improved performance in the proposed formulation.

Finally, in Chapter 6, we conclude our discussion and provide the core contributions of our work. The stability and efficiency of proposed scheme is described and the significance of using game theoretic concepts is established.
Chapter 2
Cognitive Radio Games

In the previous chapter, we describe the game as interactions between a group of decision makers, which impeccably models the spectrum access competition among CRs. A typical game for cognitive radios is defined by the SUs as a set of players, each employing a strategy from a strategy set, \( s_i \in S \). The action of CR \( i \) is given by \( a_i \), while the actions of opponent CR players are represented by \( a_{-i} \in A \). The CR \( i \) chooses an action \( a_i \in A \), based on the set of possible payoffs or obtained utilities \( u_i \). The decision is made in favor of the actions providing maximum payoff or minimum losses under the given set of conditions. Since the objectives of SUs can cause conflicts, in the absence of a game theoretic model or some other inhibiting mechanism, the CR players may achieve sub-optimal payoff and consequently never accomplish a stable solution (Nash equilibrium) [29].

A game can be modeled in several ways, depending on objectives of players that depend on the perspectives of players and their goals. In the proposed game theoretic model, these goals involve efficient access to channels and improvement in throughput. The objective of the game is to lead all the players to a Nash equilibrium point, where the CR users restrain from deviating from there channel choices in order to achieve
best throughput. In a CR setup, each SU desires to use the best available sub-channel, this emulation effect can be mitigated by exploiting potential game model. In the next section, we describe the conditions and types of the potential games employed in this dissertation.

2.1 Potential Games

In potential games, the incentives for strategy change of all players can be mathematically represented in a single comprehensive function. This is possible only when all players share a common objective. The potential games form a class of generic games that can be played cooperatively or otherwise, as long as the players pursue a common goal. These games include congestion and competitive games, where several players are competing for the limited resources, yet all players utilize common resources, the potential game formulation is suitable in CR implementations. However, if the players’ objectives are heterogeneous, with each player pursuing a different kind of ambition, a potential game cannot be established. Every potential game is known to have at least one Nash equilibrium [11]. Hence, the proposed potential game model is guaranteed to converge. The proposed repeated potential game allows the players to iteratively move for maximum individual utility as well as the potential function in order to achieve the desired Nash equilibrium solution.

2.1.1 Types of Potential Games

A potential game can be classified into several types: exact, ordinal, weighted and best potential games. For utilities $u_i$ and potential function $V$, $\forall a_i, a'_i, a''_i, a_{-i} \in A,$
these potential games must satisfy the conditions described below [30]:

**Exact Potential Game (EPG):**

For a potential function representing CR competition, defined by $V$, with discrete actions such that $\forall a_{-i} \in A, \forall a'_i, a''_i \in A$,

$$V(a'_i, a_{-i}) - V(a''_i, a_{-i}) = u_i(a'_i, a_{-i}) - u_i(a''_i, a_{-i}) \quad (2.1.1)$$

In case of an exact potential game, the effect on the utility of a CR player resulting from a unilateral change in strategy is the same as in the global utility. That is, when SU $i$ switches from action $a'_i$ to action $a''_i$, the change in the potential equals the change in the utility of that player [11]. If utilities and potential function are continuous and differentiable with respect to their actions $a_i$, the condition for the existence of EPG can be written as $\forall a \in A$:

$$\frac{\partial u_i(a)}{\partial a_i} = \frac{\partial V(a)}{\partial a_i} \quad (2.1.2)$$

**Ordinal Potential Game (OPG):**

An ordinal potential game is defined if there is a function $V$, such that $\forall a_{-i} \in A, \forall a'_i, a''_i \in A$,

$$u_i(a'_i, a_{-i}) - u_i(a''_i, a_{-i}) > 0 \iff V(a'_i, a_{-i}) - V(a''_i, a_{-i}) > 0 \quad (2.1.3)$$

A generalized ordinal potential game is expressed with an even more relaxed condition, such that $\forall a_{-i} \in A, \forall a'_i, a''_i \in A$ [30],

$$\text{sgn}[u_i(a'_i, a_{-i}) - u_i(a''_i, a_{-i})] = \text{sgn}[V(a'_i, a_{-i}) - V(a''_i, a_{-i})] \quad (2.1.4)$$
where, $\text{sgn}()$ is the sign function. In terms of continuous and differentiable utilities, we can write:

$$\text{sgn} \left[ \frac{\partial u_i(a)}{\partial a_i} \right] = \text{sgn} \left[ \frac{\partial V(a)}{\partial a_i} \right]$$

(2.1.5)

**Weighted Potential Game (WPG):**

The utility $u_i$ and potential function $V$ must satisfy the condition:

$$u_i(a_i', a_{-i}) - u_i(a_i'', a_{-i}) = w_i[V(a_i', a_{-i}) - V(a_i'', a_{-i})]$$

(2.1.6)

where, $w_i$ is the scaling factor, which may vary for each user $i$. For continuous payoffs, this leads to:

$$\frac{\partial u_i(a)}{\partial a_i} = w_i \frac{\partial V(a)}{\partial a_i}$$

(2.1.7)

The potential games are generally a combination of dummy games, unilateral games and multi-symmetric coordination games. The dummy game is defined by the part of potential game that does not depend on the action of CR $i$, but only on the actions of other CRs $-i$, making the utility of player independent of its own strategy, and the potential function takes the form of a constant.

The unilateral game involves the actions taken by only one SU, whereas, in a Bilateral Symmetric Interaction (BSI) game, the payoff of a strategy is independent of the SUs playing them. In such games, the objective function or utility is a sum of bilateral symmetric terms, which can be written as:

$$u_{ij}(s_i, s_j) = u_{ji}(s_j, s_i)$$

It can be seen that the change in identities of the CR players does not effect the payoffs.
For continuous and differentiable utility function, another condition for the existence of potential game is given as:

$$\frac{\partial^2 u_i}{\partial a_i \partial a_j} = \frac{\partial^2 u_j}{\partial a_i \partial a_j}$$  \hspace{1cm} (2.1.8)

### 2.2 Potential Games Formulation

The cognitive radio networks present an ideal environment for the implementation of potential games, as all CR players have identical objectives of availing suitable channel opportunity. The proposed formulation of potential function provides an improved network performance and a better utility for each player. These utilities are not focused on individual player’s point of view, but enable all network users to attain an optimum compromise, which is the essence of proposed cooperative communication. The combined influence of utilities and potential function assists in attaining a solution based on satisfied players and superior network performance. In order to encourage the cooperative mode, a pricing function is introduced to implement the self-governing cooperation.

#### 2.2.1 Optimality of Potential Function

The purpose of introducing the potential game for CR is to determine a channel providing successful communication as well as an optimum power level, so that the individual users are satisfied and the network performance is also improved. Thus the potential function is designed to accommodate these objectives. However, not all formulations of potential function incorporate the optimality of the solution. For this reason, we require a potential function that can provide an optimal solution in
terms of power consumption and channel access. Hence, the basic design of potential function must satisfy the conditions for the existence of potential game as well as the conditions for the existence of optimality [31] [32]. Maximizing this potential function should enable an optimum solution, which is possible only if a global maximum can be achieved.

The local maxima, providing maximum value of potential function in a certain interval only, are the Nash equilibria of the game. However, these Nash equilibrium points are not necessarily optimal and network performance might be compromised. In order to obtain optimum performance for the CR game, the Nash equilibrium must be attained at a global maximum. This is possible only when the formulated potential function is strictly concave.

For a general CR game, we can write the potential function for $N$ network users as the sum of individual utilities:

$$V = \sum_{i=1}^{N} u_i = \sum_{i=1}^{N} [g_i - c_i]$$

(2.2.1)

where, $u_i$ is the utility function of SU $i$, $g_i$ is the reward or gain function and $c_i$ is the cost function CR $i$ is required to pay for a channel of its choice. For the above equation to represent a potential game, $V$ must satisfy the condition:

$$[g_i(a_i', a_{-i}) - c_i(a_i', a_{-i})] - [g_i(a_i'', a_{-i}) - c_i(a_i'', a_{-i})] > 0$$

$$\Leftrightarrow \sum_{i=1}^{N} ([g_i(a_i', a_{-i}) - c_i(a_i', a_{-i})] - [g_i(a_i'', a_{-i}) - c_i(a_i'', a_{-i})]) > 0$$

(2.2.2)

For continuous and differentiable payoff functions, we can write:

$$\frac{\partial (g_i - c_i)}{\partial a_i} = \sum_{i=1}^{N} \frac{\partial (g_i - c_i)}{\partial a_i}$$

(2.2.3)
Nash proved in 1951 that at least one Nash equilibrium solution exists for finite player games having finite action set [29]. The game discussed here involves a finite number of players (SUs and PUs), and a finite number of action set (number of available channels and power), hence the cognitive radio game qualifies for at least one Nash equilibrium. For potential games, Nash equilibrium can be obtained by maximizing the potential function. Our motivation is to make the NE an optimal solution to the cognition problem.

**Theorem 2.1:**

For the function $V$ to be strictly concave, the reward function $g$ for a CR should also be concave, while the cost function $c$ should be convex. Then the CR potential games provides a global maximum which is an optimal Nash equilibrium providing stable and high performance network.

**Proof.** The existence of global maximum depends on the following condition:

$$V(\theta x + (1 - \theta)y) > \theta V(x) + (1 - \theta)V(y)$$

where, $0 < \theta < 1$. The above inequality implies a strict concavity. If a function is strictly concave, it is guaranteed to have a global maximum.

$$V(\theta x + (1 - \theta)y) = \sum_{i=1}^{N} u_i(\theta x + (1 - \theta)y)$$

which can be expanded as:

$$V(\theta x + (1 - \theta)y) = \sum_{i=1}^{N} [g_i(\theta x + (1 - \theta)y) - c_i(\theta x + (1 - \theta)y)]$$

For the function $V$ to be concave, we must have:
\[ \sum_{i=1}^{N} [g_i(\theta x + (1 - \theta)y) - c_i(\theta x + (1 - \theta)y)] > \sum_{i=1}^{N} \theta [g_i(x) - c_i(x)] + (1 - \theta) [g_i(y) - c_i(y)] \]

which implies:

\[ \sum_{i=1}^{N} g_i(\theta x + (1 - \theta)y) > \theta \sum_{i=1}^{N} g_i(x) + (1 - \theta) \sum_{i=1}^{N} g_i(y) \]

\[ \sum_{i=1}^{N} c_i(\theta x + (1 - \theta)y) < \theta \sum_{i=1}^{N} c_i(x) + (1 - \theta) \sum_{i=1}^{N} c_i(y) \]

Moreover, for a continuous and differentiable potential function, the condition for the concavity can be written as:

\[ \frac{\partial^2 V(a)}{\partial a_i^2} < 0 \quad (2.2.4) \]

The proposed CR potential game comprises of an action set containing channels and transmission power, i.e., \( a_i = [ch_i, p_i] \). Here, channel \( ch_i = 1, ..., K \) is a discrete quantity, while \( p_i \in P \) represents transmission power of user \( i \) and is a continuous quantity. Thus, the action set for CR game is continuous and differentiable with respect to power, but behaves as discrete quantity when decision is made to access a channel. Hence, the potential game conditions defined as differences in equations (2.1.1), (2.1.3), (2.1.4) and (2.1.6) are considered for channel, and the partial derivative conditions given in equations (2.1.2), (2.1.5), (2.1.7) and (2.1.8) are employed when opting power.
Theorem 2.2

For the power efficient cognitive potential game, the potential function is continuous and differentiable, and should satisfy the derivative condition for the existence of potential game given in equation (2.1.8).

Proof. The derivative condition for the existence of potential game is given by:

$$\frac{\partial^2 u_i}{\partial p_i \partial p_j} = \frac{\partial^2 u_j}{\partial p_i \partial p_j}$$

Since, the utility of CR player $i$ can be split into the gain and cost part, we can write:

$$\frac{\partial u_i}{\partial p_j} = \frac{\partial g_i}{\partial p_j} - \frac{\partial c_i}{\partial p_j}$$

The second partial derivative becomes:

$$\frac{\partial^2 u_i}{\partial p_i \partial p_j} = \frac{\partial^2 g_i}{\partial p_i \partial p_j} - \frac{\partial^2 c_i}{\partial p_i \partial p_j}$$

Similarly, for CR $j$, we can write:

$$\frac{\partial u_j}{\partial p_j} = \frac{\partial g_j}{\partial p_j} - \frac{\partial c_j}{\partial p_j}$$

$$\frac{\partial^2 u_j}{\partial p_i \partial p_j} = \frac{\partial^2 f_j}{\partial p_i \partial p_j} - \frac{\partial^2 g_j}{\partial p_i \partial p_j}$$

Hence, we have

$$\frac{\partial^2 g_i}{\partial p_i \partial p_j} - \frac{\partial^2 c_i}{\partial p_i \partial p_j} = \frac{\partial^2 g_j}{\partial p_i \partial p_j} - \frac{\partial^2 c_j}{\partial p_i \partial p_j}$$

$\square$
Lemma 1

Condition for the existence of optimum solution for a CR potential game implies that the potential function for the network must be concave.

Proof.

\[
\frac{\partial^2 u_i}{\partial p_i^2} = \sum_{i=1}^{N} \left[ \frac{\partial^2 (g_i - c_i)}{\partial p_i^2} \right] < 0
\]

\[
\frac{\partial^2 u_i}{\partial p_i \partial p_j} = \sum_{i=1}^{N} \left[ \frac{\partial^2 (g_i - c_i)}{\partial p_i \partial p_j} \right] < 0
\]

\[
\square
\]

2.2.2 Spectrum Overlay and Underlay

The implementation of cognitive radio can be performed in two different modes; spectrum overlay and spectrum underlay. In case of spectrum underlay, concurrent transmissions of SUs and PU are allowed, as long as PU’s transmissions remain unaffected by the SUs. The SUs typically use spread spectrum techniques and low transmission power [33]. By ensuring sufficiently low interference for PU, the CRs can successfully utilize the unlicensed spectrum.

The second mode of cognition is the spectrum overlay, where CRs only operate on completely vacant channels devoid of any licensed communication [31]. This scheme is feasible due to the fact that most licensed users are not active at all times and the incorporation of CRs reduce this underutilization. However, these spectrum opportunities are usually limited, generating a competition among the SUs. This competition dies along-with its transmissions whenever the PU reclaims the channel.
Hence, correct detection of PU activity is important to avoid undesirable situations in CR communications.

Underlay technique has the advantage of larger bandwidth flexibility with increased system complexity. Overlay scheme requires accurate detection of PU activity to identify vacant channels. In order to reap the benefits of both schemes, a hybrid approach can be employed, which emphasizes on accommodating multiple users in the limited bandwidth.

2.3 System Model

The network under consideration involves a finite number of SUs and fewer number of sub-channels each corresponding to a PU. We assume $N$ cognitive transmitter-receiver pairs in the proposed network. The number of available channels, $K$ is assumed to be less than the candidate CR pairs competing for the channels, i.e., $N > K$. This creates a strictly competitive environment where users are prone to make conflicting choices. This anarchy can be reduced by adopting certain self-evolving regulations for the game play. The network diagram is shown in Figure 2.1.

In the proposed work, we assume that the transmit power levels $p_i$, $i = 1,...,N$ are fixed for every user, though they may vary from user to user. $a_i \in A$, $i = 1,...,N$ is the action set for player $i$ according to its strategy $s_i \in S$, $i = 1,...,N$. These action sets comprise of the available channel choices and power levels, i.e., $a_i \in [k,p_i]$, $k = 1,...,K$. The path loss model for the networks considered in the proposed work is assumed as inverse squared distance and the link gain between node $i$ and $j$ is given by $h_{ij} \propto 1/d_{ij}^2$. The link gain $h_{ii}$ implies the path gain between a cognitive transmitter $i$ and the corresponding receiver. The transmission power of SU $i$ is given by $p_i$. 
while the transmission power of PU is given by $p_o$. The underlay system parameters are represented by the subscript $u$ and the overlay parameters are represented by subscript $v$.

The SUs are using Orthogonal Frequency Division Multiplexing (OFDM) during cognition, which allows simultaneous transmissions, as opposed to Time Division Multiplexing (TDM), where only one SU can transmit over a sub-channel at any particular time. The OFDM mode of transmission, considered in this dissertation, allows multiple SUs to transmit over a channel, but also results in interference due to these multiple transmissions. This higher interference level lowers the Signal-to-Interference-plus-Noise Ratio (SINR) and can disrupt the transmissions. The SINR $\gamma_i$ for SU $i$ is defined as:
\[ \gamma_i = \frac{p_i h_{ji}}{N_0 + \sum_{j=1,j\neq i}^{N} f_{ji} p_{ji} h_{ji}} \] (2.3.1)

where, \( N_o \) is the noise power, and the function \( f_{ji} \) is an indicator function, which demonstrates the fact that only the users utilizing the same channel cause interference for each other. This can be written as:

\[ f_{ji} = \begin{cases} 
1, & \text{if } s_j = s_i, j \neq i \\
0, & \text{if } s_j \neq s_i 
\end{cases} \]

The most important problem is to ensure the availability of channel, which should preferably provide a better SINR for transmissions. The game, intrinsically, is non-cooperative due to the rationality of selfishness in these types of games. We aim to convert this non-cooperation to a cooperative game. In order to motivate players to cooperate, we propose that players gain a reward in the form of successful channel access with better utility if they cooperate. As a first step to cooperation, all players consent to be considerate instead of being reckless towards each other. This leads to players transmitting in a way that regulates the amount of interference created over a channel. In order to further strengthen the concept of cooperation, we also incorporate a pricing function, which charges players according to their transmit power and interference created. This also discourages the malicious players by banishing and disrupting their transmissions. The pricing assists in self enforcing the cooperation and allows efficient selection of available channels.

The potential function we propose here is based on two simple yet important network parameters. One of them is the transmission power of every player, and the other is the interference level for each player. These two parameters are closely
related to each other, as increasing power raises the interference level for that channel and depletes the resources of the users.

In this thesis, we consider both types of cognitive modes. The first model involves co-existing PUs and SUs (spectrum underlay), while the second approach deals with competition among cognitive users for accessing vacant channels (spectrum overlay) where CRs can only gain access to a channel with no licensed transmissions. Moreover, the interference can be classified in two different types; observed interference and created interference. For these two models, we define four interference terms based on the fact that only the players which opt for the same strategy (or channel) are a source of interference for each other.

\[
I_{iv} = \sum_{j=1}^{N} p_j h_{ji} f_{ji}, \text{ observed by SU } i \text{ in overlay}
\]

\[
I_{iu} = \sum_{j=0}^{N} p_j h_{ji} f_{ji}, \text{ observed by SU } i \text{ in underlay}
\]

\[
I'_{iv} = \sum_{j=1}^{N} p_i h_{ij} f_{ij}, \text{ created by SU } i \text{ in overlay}
\]

\[
I'_{iu} = \sum_{j=0}^{N} p_i h_{ij} f_{ij}, \text{ created by SU } i \text{ in underlay}
\]

\[
I_p = \sum_{j=0}^{N} p_j h_{jo} f_{jo}, \text{ observed by PU from all SUs on the channel}
\]

\[
I'_p = \sum_{j=0}^{N} p_o h_{oj} f_{oj}, \text{ created by PU for all SUs on the channel}
\]

The primed terms serve to encourage cooperation, making the users considerate about their behavior towards other network users.

The game involves a set of primary and secondary players, which have different yet supportive and endorsing objectives. The PU plays to avail the spectrum availability
at any time as well as to increase the efficient utilization of its sub-channel via SUs. On the other hand, the SUs play to attain access to a sub-channel, which offers lower interference levels for successful communication.

The network model in this thesis creates a cooperative environment to play the resource allocation game. Since there is no centralized authority or other external factors to enforce cooperation, the distributed players must achieve a consensus to formulate the rules of the games and strategies in a way that encourages cooperation. This cooperation is enforced as a coalition, where the PU and SUs collaborate for a mutual objective of successful channel access and energy efficiency. The players choose a strategy based on mutual consensus. Hence, the decision making process becomes a cooperative game.

In overlay mode, the PU earns revenue by allowing the unlicensed users to access its channel when it is not transmitting. The SUs can also choose to transmit in the presence of PU as long as it creates low interference to the PU and still can attain a suitable SINR. This mode of cognition, also termed as spectrum underlay, can be transpired with or without introducing a cost function. The price acts as an incentive to PU from the SUs for a flexible interference limit.

The strategies involve the process of channel choices, while the actions comprise of the selected channels and power levels. The utilities involve the accessibility of channel opportunities with bearable interference at a reasonable price.

Table 2.1-2.5 present different cooperative and non-cooperative game scenarios for cognitive radios. The games incorporate PU and SUs. The tables provide a two-player game model, however, it can be extended to include any number of players. Table 2.1 depicts the non-cooperative game model for the underlay spectrum access
scenario. The cognitive players strive to access the channels in the presence of a licensed user, while keeping the created interference under the PU tolerance level. When two players choose the same channel, they create interference for each other and their utility in this case is lower than if they opt for different channels. Hence, $u_{ki} > u'_{ki}$, where $u_{ki}$ is the utility of SU $i$ when it uses channel $k$ and $u'_{ki}$ is the utility of user $i$ when the opponent $j$ chooses the same channel $k$, $i, j = 1, 2, \ldots N, k = 1, \ldots, K$.

The cost of choosing a channel $k$ is given by $c_k$, which reduces the utility of users due to the underlay scenario. In this case, the PU utility on the channel, which is not chosen by any SU is reduced due to lack of spectrum utilization, while the PU whose channel is utilized by both SUs has its utility reduced due to higher interference.

Table 2.1: Non-cooperative game model between CRs in underlay case

<table>
<thead>
<tr>
<th>SU $i$</th>
<th>SU $j$</th>
<th>Channel $k$</th>
<th>Channel $l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel $k$</td>
<td>$(u'<em>{ki} - c_k, u'</em>{kj} - c_k)$</td>
<td>$(u_{ki} - c_k, u_{lj} - c_l)$</td>
<td></td>
</tr>
<tr>
<td>Channel $l$</td>
<td>$(u_{li} - c_l, u_{lj} - c_k)$</td>
<td>$(u'<em>{li} - c_l, u'</em>{lj} - c_l)$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2 describes a cooperative underlay game between licensed and unlicensed users for a single PU-SU pair. If only two players are competing for two channels, the best strategy is to opt for different channels to maximize the payoff of all the players. This cooperation benefits the PU even when it is not using the channel and only cognitive users are accessing it. This is due to the efficient spectrum allocation and consequently avoids possible waste of bandwidth.
Table 2.2: Non-cooperative game model between a PU and SU in overlay case

<table>
<thead>
<tr>
<th>SU i</th>
<th>Channel in use</th>
<th>Channel not in use</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU j</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
</tbody>
</table>

The parameter $R$ in Table 2.2 shows that PU benefits when it is not transmitting and channel is solely under the use of SU, due to the utilization of its bandwidth resources and price paid by the SU. Nothing can be gained by keeping the channel un-utilized by PU and SUs.

For the spectrum overlay case, the two-channel non-cooperative game model when channel 2 is occupied and channel 1 is vacant, is depicted in Table 2.3. When both channels are vacant and game is non-cooperative, game model can be written as shown in Table 2.4.

Table 2.3: Non-cooperative game model between CRs in overlay case when channel 2 is occupied

<table>
<thead>
<tr>
<th>SU i</th>
<th>Channel k</th>
<th>Channel l</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
</tbody>
</table>

For the spectrum overlay case, the two-channel non-cooperative game model when channel 2 is occupied and channel 1 is vacant, is depicted in Table 2.3. When both channels are vacant and game is non-cooperative, game model can be written as shown in Table 2.4.

Table 2.3: Non-cooperative game model between CRs in overlay case when channel 2 is occupied

<table>
<thead>
<tr>
<th>SU i</th>
<th>Channel k</th>
<th>Channel l</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
</tbody>
</table>

The parameter $R$ in Table 2.2 shows that PU benefits when it is not transmitting and channel is solely under the use of SU, due to the utilization of its bandwidth resources and price paid by the SU. Nothing can be gained by keeping the channel un-utilized by PU and SUs.

For the spectrum overlay case, the two-channel non-cooperative game model when channel 2 is occupied and channel 1 is vacant, is depicted in Table 2.3. When both channels are vacant and game is non-cooperative, game model can be written as shown in Table 2.4.

Table 2.3: Non-cooperative game model between CRs in overlay case when channel 2 is occupied

<table>
<thead>
<tr>
<th>SU i</th>
<th>Channel k</th>
<th>Channel l</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
</tbody>
</table>

The parameter $R$ in Table 2.2 shows that PU benefits when it is not transmitting and channel is solely under the use of SU, due to the utilization of its bandwidth resources and price paid by the SU. Nothing can be gained by keeping the channel un-utilized by PU and SUs.

For the spectrum overlay case, the two-channel non-cooperative game model when channel 2 is occupied and channel 1 is vacant, is depicted in Table 2.3. When both channels are vacant and game is non-cooperative, game model can be written as shown in Table 2.4.

Table 2.3: Non-cooperative game model between CRs in overlay case when channel 2 is occupied

<table>
<thead>
<tr>
<th>SU i</th>
<th>Channel k</th>
<th>Channel l</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
</tbody>
</table>

The parameter $R$ in Table 2.2 shows that PU benefits when it is not transmitting and channel is solely under the use of SU, due to the utilization of its bandwidth resources and price paid by the SU. Nothing can be gained by keeping the channel un-utilized by PU and SUs.

For the spectrum overlay case, the two-channel non-cooperative game model when channel 2 is occupied and channel 1 is vacant, is depicted in Table 2.3. When both channels are vacant and game is non-cooperative, game model can be written as shown in Table 2.4.

Table 2.3: Non-cooperative game model between CRs in overlay case when channel 2 is occupied

<table>
<thead>
<tr>
<th>SU i</th>
<th>Channel k</th>
<th>Channel l</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
<tr>
<td>SU i</td>
<td>Channel k</td>
<td>Channel l</td>
</tr>
</tbody>
</table>

The parameter $R$ in Table 2.2 shows that PU benefits when it is not transmitting and channel is solely under the use of SU, due to the utilization of its bandwidth resources and price paid by the SU. Nothing can be gained by keeping the channel un-utilized by PU and SUs.

For the spectrum overlay case, the two-channel non-cooperative game model when channel 2 is occupied and channel 1 is vacant, is depicted in Table 2.3. When both channels are vacant and game is non-cooperative, game model can be written as shown in Table 2.4.
When both channels are vacant and game is cooperative, the SUs get maximum benefit when they coordinate and operate at separate channels, instead of both trying to access the same channel and reducing their payoffs. This is shown in Table 2.5.

Table 2.5: Overlay game model among cooperative SUs

<table>
<thead>
<tr>
<th>Channel k</th>
<th>Channel l</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel k</td>
<td>((u_{ki}, u_{lj}))</td>
</tr>
<tr>
<td>Channel l</td>
<td>((u_{li}, u_{kj}))</td>
</tr>
</tbody>
</table>

We can see from Table 2.5 that SUs gain some non-zero payoff under all possible moves and there is no case where a zero payoff is obtained by any player resulting in a complete waste of bandwidth. Hence, cooperation results in better payoffs for all involved stakeholders.

Table 2.6 presents the interference aware cooperative game model for the cognitive
radios. If channel $k$ already has $m$ users and channel $l$ has $N - m$ users, and a game is played among player $i$ and its opponents $-i$, competing for the same channel, then the interference can be expressed as shown in Table 2.6. If both players choose the same channel, they face the interference by already existing $m$ users plus the interference created by the new entering opponent. The term of opponent simply implies the other players in the game. On the other hand, if players opt for different strategies, they face the interference by the already existing users ($m$ users in case of channel $k$ and $N - m$ users in case of channel $l$). They, however, do not create interference for each other by cooperating and choosing different strategies. The players that choose the same channel, create interference for other users. The utility function for each player depends on the number of users sharing the same channel. If more users are accessing a channel, larger interference is created over that channel and all users suffer from this high interference level.

Table 2.6 provides the interference each channel offers its users. If a user choose to transmit over a channel, it must tolerate this interference from the existing users. For two competing players, the best option is to opt for low interference channels, otherwise they become an additional source of interference for each other. If users cooperate, they can avoid causing additional interference to each other. This, however, depends on the number of users already existing on a channel, as more users on a channel lead to higher interference levels.

In any cognitive game, the primary user is the sole proprietor of the channel. The SUs must identify vacant channels and transmit data over it. If several SUs strive to access the same channel, they must compete among each other. If they transmit simultaneously, they create interference for each other. If players are selfish,
they do not cooperate. This non-cooperative behavior can be modeled as a game, where players are concerned only with their own benefits regardless of the payoffs for their opponent. This kind of behavior may prove to be beneficial temporarily, but eventually deteriorates the network performance. If all nodes behave selfishly, they create more interference, which causes others to increase their transmit power resulting in higher interference for itself. Although the equilibrium point can be reached, the benefit from persistently selfish behavior is reduced.

On the other hand, if SUs compete for the channel in a way so as to cause minimum possible interference for their competitors, more users can benefit. This behavior formulates a cooperative game and takes into account the benefits of individual users and the performance degradation they can cause for the network. The cooperative game

Table 2.6: Interference aware game model for cooperative SUs

<table>
<thead>
<tr>
<th>SU $-i$</th>
<th>Channel $k$</th>
<th>Channel $l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU $i$</td>
<td>$\left( \sum_{j=1, j\neq i, s_j=s_i}^{m+1} p_j h_{ji}, \sum_{j=1, j\neq i, s_j=s_i}^{m+1} p_j h_{ji} \right)$</td>
<td>$\left( \sum_{j=1, j\neq i, s_j=s_i}^{N-m} p_j h_{ji}, \sum_{j=1, j\neq i, s_j=s_i}^{N-m} p_j h_{ji} \right)$</td>
</tr>
<tr>
<td>Channel $k$</td>
<td>$\left( \sum_{j=1, j\neq i, s_j=s_i}^{N-m} p_j h_{ji}, \sum_{j=1, j\neq i, s_j=s_i}^{N-m} p_j h_{ji} \right)$</td>
<td>$\left( \sum_{j=1, j\neq i, s_j=s_i}^{N-m+1} p_j h_{ji}, \sum_{j=1, j\neq i, s_j=s_i}^{N-m+1} p_j h_{ji} \right)$</td>
</tr>
<tr>
<td>Channel $l$</td>
<td>$\left( \sum_{j=1, j\neq i, s_j=s_i}^{m} p_j h_{ji}, \sum_{j=1, j\neq i, s_j=s_i}^{m} p_j h_{ji} \right)$</td>
<td>$\left( \sum_{j=1, j\neq i, s_j=s_i}^{N-m+1} p_j h_{ji}, \sum_{j=1, j\neq i, s_j=s_i}^{N-m+1} p_j h_{ji} \right)$</td>
</tr>
</tbody>
</table>
considers individual benefits and the overall network performance can be expressed in terms of a potential function. These cooperative potential games are discussed in detail in the subsequent chapters.
Chapter 3

Cooperative Potential Games for Cognitive Radios

The natural antagonism among unlicensed users, hampered by acute scarcity of energy resources and lack of central mechanism demands an effective cooperation mechanism in order to improve the vital network parameters. In this chapter, we employ the potential game to develop an efficient and sustainable cooperation mechanism for efficient cognition and improved spectrum utilization. The nodes exhibit autarchthonous pattern in opting for spectrum choices, which results in acceptable level of cooperation and consequently improvement in spectrum utilization. In order to achieve this global benefit, the users are motivated to carefully analyze the impact of their own choice in selecting a channel for transmission as well as for the peers. We discuss economical and power efficient solutions.

3.1 Introduction

The efficiency of spectrum utilization significantly depends upon the sharing mechanism, as the entities in wireless ad hoc networks are prone to selfish behavior. We
discuss a cognitive radio network, where the nodes opportunistically try to utilize un-
used spectrum, irrespective of the type of cognition mechanism (underlay or overlay) and the presence of cooperation at some level is critical. A cooperative game envi-
ronment can effectively resolve conflicts among different players and keep the system stable while leading to improved performance.

The cognitive radio transmissions increase interference and the Bit Error Rate (BER), degrading the channel performance. If the users behave in a non-cooperative manner, they choose their strategies selfishly and do not consider the impact of their actions on the opponents or the overall network. This behavior may initially prove beneficial, but deteriorates network performance in the long run. We employ coop-
erative potential game theory that encourages the users to carefully utilize resources for mutual benefits expressed as a potential function. The users cooperate with each other by taking into account the influence of their actions for all opponents. This promotes efficient sharing of resources and leads to a controlled use of available band-
width.

In this chapter, we discuss two different potential game models for cognitive radio systems. In the first part of this chapter, we formulate a potential function based solely on interference and BER and no pricing scheme is adopted. The second part of this chapter involves a cost based potential game, which takes into account the channel cost as well as interference. The impact of non-pricing model is compared with the price-based model. Moreover, the performance of economical or low cost channel choices are compared with power efficient choices. The convergence of the network is analyzed and the improvement in performance is also discussed.
3.2 System Model

We elaborate our problem for two paradigms; one excludes the PU (overlay) while the other includes it during simultaneous secondary transmissions (underlay). The later system can avail more spectrum opportunities as long as the parameters set by PU are satisfied. The overlay problem addresses the issue of channel access among SUs that are involved in playing the game to achieve better payoff for improved potential function. The underlay problem discusses the channel acquisition by SUs co-existing with the PU by taking into account PU’s tolerance limit that is incorporated in the potential game formulation. In the overlay approach, the channel is accessed only when it is considered as vacant, while the underlay scheme includes the presence of PU and its transmissions. The cognitive users must access the channel in such a way so as not to disrupt the PU’s transmissions. The cognitive users monitor these channels, keeping a record of vacant bands (overlay case), or observe the interference level for every channel in order to decide for the minimum interference channel (underlay case). The payoff of cognition is different for these cases [34], [35].

In the proposed formulation, we consider two different games for the above mentioned scenarios. The first game lacks any pricing mechanism and only interference is responsible for a suitable solution. The second part of the proposed formulation considers a pricing game, which includes a cost function to trigger cooperation. The utility function for each of these cases is discussed below. Nodes avoid choosing the same strategy as its competitors due to interference. Each channel has a single PU owner, and is not utilized all the time.
3.2.1 Underlay Spectrum System

The mode of cognition in this case is slightly complex due to the involvement of the licensed user. The SUs must not hinder the transmission of PU and keep their interference level lower than the tolerable amount regulated by the PU.

Every SU decides its strategy according to the utility achieved based on the transmission success. This can be determined using the ratio of number of bit errors to the total bits transmitted. Every channel operates for a target BER otherwise the communication is compromised due to unacceptable signal conditions. The higher the interference observed over a particular channel, higher number of errors are detected in transmissions and greater is the BER. Each PU, being a priority user, benefits from a BER suitable for successful transmissions. The SUs accessing this channel must consider this criterion to avoid disrupting PU’s transmission. A channel with a better BER attracts more cognitive users. However, if more users start transmitting over the same channel, the BER is increased causing more noise for the PUs. The PU’s main concern is its target BER, if satisfied, allows its transmission to remain unaffected by these CR users. This provides an efficient utilization of bandwidth.

We define the utility function of \(i^{th}\) SU when it uses channel \(k\) as \(u_{ki}\) for underlay system. When the users behave in a cooperative mode, they provide each other incentives to trigger such behavior. This causes the payoffs of these players to be written as:

\[
   u_{ki} = - \sum_{j=1,j\neq i}^{N} \Psi_{kj} f_{ji} - \Psi_{ph} - c_{ki} \tag{3.2.1}
\]

where, \(\Psi_{kj}\) is the bit error rate when \(j^{th}\) CR is transmitting over the \(k^{th}\) channel. Every cognitive user suffers from interference created by its cognitive opponents and
licensed user over that channel. In order to encourage cognition, PU prefers to transmit at the threshold BER, $\Psi_{th}$. The utility of licensed user over channel $k$ with $N_k$ cognitive users accessing its channel is given by:

$$u_{pk} = -\sum_{i=1}^{N_k} \Psi_{ki} - \Psi_{po} + r_k$$  \hspace{1cm} (3.2.2)

where, $\Psi_{po}$ is the BER of channel in the absence of cognition. Without the loss of generality, we assume that all channels provide the same BER without cognition. The factor $r_k$ is the reward function for licensed user and $c_{ki}$ is the cost paid by the SU $i$ for accessing the PU channel $k$. If we assume that licensed and unlicensed users are selfish and no player considers the impact of their actions on other players, then the cost and reward functions become zero. The licensed user’s payoff also increases with the reward paid by SUs, but higher price attracts lower number of cognitive users. The objective of SUs is to increase their payoff by obtaining higher chances of accessing the channel and paying a smaller amount to PU as a reward. Higher number of unlicensed users on a channel increases its BER and the corresponding licensed user procures lower payoffs.

### 3.2.2 Overlay Spectrum System

In this case, the cognitive users search for vacant spectrum bands. This scheme is only applicable when PU is not transmitting. The competing players involve only the unlicensed users. The main objective of these players is to choose channels that offer minimum interference. The interference level of the network is continuously monitored and it changes with the adopted strategies of players. This mode of implementing cognition is considered simpler due to the absence of the crucial licensed
player. As the number of CRs accessing a channel is increased, their utility is reduced due to increased BER and interference. These SUs search for vacant channels and compete to gain access. The users, when playing in a cooperative mode, encourage resource sharing by providing incentives to their competitors by lowering their created interference. Due to the absence of PU in this game, the utility for SUs can simply be written as:

$$u_{ki} = - \sum_{j=1, j \neq i}^{N} \Psi_{kj} f_{ij} - c_{ki}$$  \hspace{1cm} (3.2.3)

### 3.3 Non-pricing Utility Function Deliberating Interference

We develop a utility function for a cooperative scenario such that the nodes contemplate the payoffs of their opponents besides maximizing their own utilities [35]. No pricing scheme is adopted in this game. The users consider the amount of interference caused to others and keep it lower in order to earn more payoffs. This kind of behavior in strategy building improves the performance of other players and also increases the individual payoff. In this way, the user causing higher interference to its opponents is discouraged via lower payoff.

Given the target SINR $\gamma_{ko}$ for kth channel in dBs, the payoff function for overlay case can be written as:

$$u_{iv} = 1 - [-\gamma_{ko} + p_i - I_{vi} - I'_{vi} - N_o]^2$$  \hspace{1cm} (3.3.1)

where, $p_i$ is the power of $i$th cognitive transmitter, and $N_o$ is the noise power. The received interference for $i$th CR is represented by $I_{vi}$ and interference created by $i$th
cognitive transmitter for its counterparts in the network is $I'_{vi}$, (these terms are defined in Chapter 2). All selfish users are considered with the term $I_{vi}$. However, cooperation encourages the users to be considerate and take into account the interference $I'_{vi}$. Hence, $I'_{vi}$ is responsible for initiating cooperation among SUs.

For overlay, we must take into account the probability of error $\alpha$ in detecting PU, which affects the performance of the SUs by unknowingly adding to the interference in addition to other sources such as jamming devices. Moreover, the payoff depends on the amount of interference observed and created. Based on these observations, the utility function can be modified as:

$$u_{iv} = 1 - [-\gamma_{ko} + p_i - (1 - \alpha)I_{vi} - (1 - \alpha)I'_{vi} - \alpha p_i h_{io} - \alpha p_o h_{oi} - N_o]^2$$  \hspace{1cm} (3.3.2)

where, $p_i h_{io}$ is amount of interference PU observes and $p_o h_{oi}$ is the interference it creates for PU over its channel.

In case of spectrum underlay scheme, the presence of PU must also be incorporated with highest priority and cognitive users adjust accordingly. This scheme requires two payoffs to be optimized, one of licensed user and the other of CR. The PU’s utility for transmit power $p_o$ is written as:

$$u_{pk} = 1 - [-\gamma_{ko} + p_o - I_p - I'_{p} - N_o]^2$$  \hspace{1cm} (3.3.3)

The SU’s utility for the underlay case is expressed as:

$$u_{ui} = 1 - [-\gamma_{ko} + p_i - I_{ui} - I'_{ui} - I_{pi} - I'_{pi} - N_o]^2$$  \hspace{1cm} (3.3.4)

where, the primed terms represent the interference created by PU and unprimed terms are interference suffered.


3.4 CR Potential Game

The potential game represents a global formulation of a game, represented by a potential function, which combines the individual payoffs in a way that are associated with the network performance. Incorporating cooperation is more effective and comprehensive in a potential game. In order to improve network performance, this single function is optimized instead of individual payoffs. The corresponding strategies for optimum performance are determined based on the optimized potential function.

In the subsequent discussion, we develop the potential function suitable for the underlay and overlay access scenarios for cognitive radio networks.

3.4.1 Overlay Case

The potential function in this case is based on the accumulated payoffs of individual players over a channel. There is no PU involved to create interference or govern the rules of the game. The challenges a CR must face are created by the peer SUs. In order to ensure a suitable environment for all stake holders, the players attempt to keep their transmission power low, serving the dual purpose of reducing interference for others and conserving their resources. For the \(j\)th CR over channel \(i\), the utility function is given by:

\[
 u_{ki} = 1 - [-\gamma_{ko} + p_i - \sum_{j=1, j\neq i}^N p_{ji}h_{ji}f_{ji} - \sum_{j=1, j\neq i}^N p_{ij}h_{ij}f_{ij} - N_0]^2 \]  

(3.4.1)

**Theorem 3.1**

The potential function for the overlay case is given by \(V_o = \sum_{i=1}^N u_{ki}\), where \(u_{ki}\) is as given in equation (3.4.1).
Proof. As stated in Chapter 2, the potential game formulation is valid, if the utility functions associated with it can be split into dummy, BSI and unilateral parts [11]. In the BSI, the payoff of a strategy is independent of the player, and depends only on the other strategies. In such games, the objective function or utility is a sum of bilateral symmetric terms, which can be written as:

\[ w_{ij}(s_i, s_j) = w_{ji}(s_j, s_i) \]

It can be seen from the above expression that the change in identities of the players does not affect the payoffs. The unilateral game involves the actions taken by only one party or player. Thus the potential game is represented as a combination of a dummy game, unilateral game and BSI games. The expression of these component games are formulated as:

\[ U_{\text{dummy}} = 1 - \gamma_k \gamma_o - N_o^2 - (\sum_{j=1,j\neq i}^N p_j h_{ji}^2 f_{ji} - 2\gamma_k \sum_{j=1,j\neq i}^N p_j h_{ji} f_{ji} - 2\gamma_k N_o) \]

The unilateral game payoff can be separated as:

\[ U_{\text{uni}} = -p_i^2 + 2\gamma_k p_i + 2p_i N_o \]

Similarly, the bilateral symmetric part of the game can be written as:

\[ U_{\text{BSI}} = -(\sum_{j=1,j\neq i}^N p_i h_{ij}^2 f_{ij} - 2\gamma_k \sum_{j=1,j\neq i}^N p_i h_{ij} f_{ij} - 2N_o \sum_{j=1,j\neq i}^N p_j h_{ji} f_{ji} + 2\sum_{j=1,j\neq i}^N p_i p_j h_{ji} f_{ji} + 2\sum_{j=1,j\neq i}^N p_j h_{ji}^2 f_{ij} - 2\sum_{j=1,j\neq i}^N p_j h_{ji} f_{ij} \sum_{j=1,j\neq i}^N p_i h_{ij} f_{ij}) \]

The potential function for overlay game can be written as:

\[ V_o = \sum_{i=1}^N \sum_{j=1}^N (U_{\text{uni}} + U_{\text{BSI}}) \quad (3.4.2) \]

which implies that the proposed function is the sum of unilateral and BSI game utilities. \qed
The potential function for overlay case can now be written as:

\[
V_o = \sum_{k=1}^{K} \sum_{i=1}^{N} \left[ -p_i^2 + 2\gamma_{ko}p_i + 2p_iN_o - \left( p_i \sum_{j=1, j\neq i}^{N} h_{ij} \right)^2 f_{ij} - 2\gamma_{ko} \sum_{j=1, j\neq i}^{N} p_i h_{ij} f_{ij} \\
- 2N_o \sum_{j=1, j\neq i}^{N} p_i h_{ij} f_{ij} + 2 \sum_{j=1, j\neq i}^{N} p_i p_j h_{ji} f_{ji} + 2 \sum_{j=1, j\neq i}^{N} p_i^2 h_{ij} f_{ij} \\
- 2 \sum_{j=1, j\neq i}^{N} p_j h_{ji} f_{ji} \sum_{j=1, j\neq i}^{N} p_i h_{ij} f_{ij} \right]
\]  

(3.4.3)

In order to determine a maxima for this potential game, we determine whether the second derivative of our potential function is negative. This condition can be mathematically written as:

\[
\frac{\partial^2 V_o}{\partial p_i^2} < 0
\]

which implies that the power level maximizes the potential function under the condition:

\[
(1 - \sum_{j=1, j\neq i}^{N} h_{ij}) < 0
\]

We maximize this potential function by differentiating it with respect to \( p_i \) and determine the optimum power level. By equating the result to zero, we evaluate the transmit power expression for the optimum case as:

\[
p_i^* = \frac{1}{(1 - \sum_{j=1, j\neq i}^{N} h_{ij} f_{ij})^2} \left[ \gamma_{ko} + N_o - (\gamma_{ko} + N_o) \sum_{j=1, j\neq i}^{N} h_{ij} f_{ij} + \sum_{j=1, j\neq i}^{N} p_j h_{ji} f_{ji} - \sum_{j=1, j\neq i}^{N} p_j h_{ji} f_{ji} \sum_{j=1, j\neq i}^{N} h_{ij} f_{ij} \right]
\]

(3.4.4)
3.4.2 Underlay Case

The underlay access case must incorporate the primary user as a priority player in its utility and potential function formulations. The utility of CR is influenced by the presence of PU transmission and the interference created by the competitors as well as the interference a CR creates for its opponents and PU. The utility function for the PU over \( k \)th channel, in this case, can be written as:

\[
U_{pk} = 1 - \left[ -\gamma_k - \sum_{j=1}^{N} p_j h_{jo} f_{jo} - \sum_{j=1}^{N} p_o h_{oj} f_{oj} - N_o \right]^2
\]  

(3.4.5)

where, interference suffered by PU is represented as \( \sum_{j=1}^{N} p_j h_{jo} f_{jo} \).

The SU’s payoff depends on the interference created by the opponents and the interference due to the presence of PU. This utility must also incorporate the interference the SU creates for its opponents and corresponding PU. The utility function for SUs in this case is given by:

\[
U_{skj} = 1 - \left[ -\gamma_k + p_j - I_{sj} - I'_{sj} - p_o h_{oj} - p_j h_{jo} - N_o \right]^2
\]  

(3.4.6)

Theorem 3.2:

The underlay potential function can be written as:

\[
V_u = \sum_{k=1}^{K} \sum_{i=1}^{N} (U_{pk} + U_{ski})
\]

(3.4.7)

Proof. The unilateral part of the potential game utility in equation (3.4.5) is given by:

\[
U_{up-un} = -p_o^2 + 2\gamma_k p_o + 2p_o N_o
\]
Similarly, the BSI measure of the proposed utility can be written as:

\[
U_{up-BSI} = -\left( \sum_{j=1}^{N} p_{j}^{2} h_{oj} f_{oj} \right)^{2} - 2\gamma_{ko} p_{o} \sum_{j=1}^{N} h_{oj} f_{oj} + 2p_{o}^{2} \sum_{j=1}^{N} h_{oj} f_{oj} \\
+ 2 \sum_{j=1}^{N} p_{o}^{2} p_{j} h_{jo} f_{jo} - 2 \sum_{j=1}^{N} p_{o}^{2} p_{j} h_{jo} f_{jo} \sum_{j=1}^{N} h_{oj} f_{oj} - 2N_{o} \sum_{j=1}^{N} p_{o} h_{oj} f_{oj}
\]

The unilateral payoff for SUs in spectrum underlay case represented by equation (3.4.6) is given by:

\[
U_{us-uni} = -p_{i}^{2} + 2\gamma_{ko} p_{i} + 2p_{i} N_{o}
\]

Similarly, the bilateral payoff for underlay potential game can be written as:

\[
U_{us-BSI} = -\left( \sum_{j=1}^{N} p_{i} h_{ij} f_{ij} \right)^{2} - (p_{o} h_{oi} f_{oi})^{2} - (p_{i} h_{io} f_{io})^{2} - 2\gamma_{ko} \sum_{j=1,j\neq i}^{N} p_{i} h_{ij} f_{ij} \\
- 2\gamma_{ko} p_{o} h_{oi} f_{oi} - 2\gamma_{ko} p_{i} h_{io} f_{io} + 2p_{i} \sum_{j=1,j\neq i}^{N} p_{j} h_{ji} f_{ji} + 2p_{i} \sum_{j=1,j\neq i}^{N} p_{i} h_{ij} f_{ij} \\
+ 2p_{i} p_{o} h_{oi} f_{oi} + 2p_{i}^{2} h_{io} f_{io} - 2 \sum_{j=1,j\neq i}^{N} p_{j} h_{ji} f_{ji} \sum_{j=1,j\neq i}^{N} p_{i} h_{ij} f_{ij} - 2p_{i} h_{io} f_{io} N_{o} \\
- 2p_{i} h_{io} f_{io} \sum_{j=1,j\neq i}^{N} p_{j} h_{ji} f_{ji} - 2p_{o} h_{oi} f_{oi} \sum_{j=1,j\neq i}^{N} p_{i} h_{ij} f_{ij} - 2p_{i} h_{io} f_{io} \sum_{j=1,j\neq i}^{N} p_{i} h_{ij} f_{ij} \\
- 2N_{o} \sum_{j=1,j\neq i}^{N} p_{i} h_{ij} f_{ij} - 2p_{o} h_{oi} f_{oi} \sum_{j=1,j\neq i}^{N} p_{j} h_{ji} f_{ji} - 2p_{o} p_{i} h_{oi} h_{io} f_{io} - 2p_{o} h_{oi} f_{oi} N_{o}
\]

Hence the proposed potential function corresponds to a valid potential game.

\[
\square
\]

Based on the potential game proposed in equation (3.4.7), the power level for users is given by:
\[ p_i^* = \frac{1}{\left(1 - \sum_{j=0,j\neq i}^{N} h_{ij} f_{ij}\right)^2} \left(\gamma_{ko} + N_o - (\gamma_{ko} + N_o)(1 - h_{io})\right) - \frac{2h_{io} \sum_{j=1,j\neq i}^{N} h_{ij} f_{ij}}{\left(1 - \sum_{j=0,j\neq i}^{N} h_{ij} f_{ij}\right)^2} \]

\[ - (\gamma_{ko} + N_o + p_o h_{oi}) \sum_{j=1,j\neq i}^{N} h_{ij} f_{ij} + \sum_{j=1,j\neq i}^{N} p_j h_{ji} f_{ji}(1 - h_{ij} - h_{io}) \] (3.4.8)

This power solution is found for the case where no pricing or cost based scheme is implemented. No revenue is generated for PU and no coordinating mechanism is employed for the SUs.

### 3.5 Cost Based Potential Game

In order to achieve a stable solution for the competition faced by the introduction of cognition, players are encouraged to cooperate. The nodes must take into account the amount of interference they are causing for their opponents. A parameter providing the interference measurement is given by the function \( \eta_j \). This function considers the level of interference a node must bear as well as the level of interference it creates over a channel. Thus we can write this as:

\[ \eta_i = \frac{p_i h_{ii}}{(1 - \alpha) \left( \sum_{j=1,j\neq i}^{N} p_j h_{ji} f_{ji} + \sum_{j=1,j\neq i}^{N} p_i h_{ij} f_{ij} \right) + \alpha f_{io}(p_i h_{io} + p_o h_{oi})} \] (3.5.1)

Here, \( \alpha \) is the probability of incorrect PU detection (false alarm). This leads to the involvement of PU interference according to the transmit power level \( p_o \).
The previous formulation of potential function involves only the level of interference for the player and its opponents. Next we develop a potential game, where players are charged for their channel choices. The cost function is developed to discourage high interference creating users. This allows revenue generation for the license holder and promotes a more robust cooperative scenario. In this game, the utility function depends on the interference levels and the cost of selected channel. This accommodates the transmit power of CRs. We can write the potential function for the pricing game as:

\[ V = \sum_{i=1}^{N} (\eta_i - c_i) \]  

(3.5.2)

where, the cost function \( c_i \) is the cost paid by the \( i \)th CR to gain channel access.

This kind of game presents a player with two different approaches for selecting a strategy. One of the approach allows the channel selection based on lowest cost, which leads to an obvious increase in the utility. The other approach provides strategies according to the transmit power levels that improves the utilities.

The cost function must be carefully devised to accommodate the requirements of a CR network. The users that create higher level of interference must be discouraged by charging a higher price, while the users providing lower interference are enticed by offering a lower price. This results in an increase in competition among channels for low interference users. Hence, the channel cost reduces with the number of available channels and the interference level, and increases with the amount of interference an entering user creates over the channel. Mathematically, we define the proposed cost function as:
43

\[ c_i = \frac{\sum_{j=1,j \neq i}^{N} p_i h_{ij} f_{ij}}{K \sum_{j=1,j \neq i}^{N} p_j h_{ji} f_{ji}} \]  

(3.5.3)

**Theorem 3.3:**

In order to develop a valid potential game, we propose the following utility function:

\[ U_i = \ln \left( \frac{p_i h_{ii}}{N_o + \sum_{j=1,j \neq i}^{N} p_j h_{ji} f_{ji}} \right) - K \ln \left( \sum_{j=1,j \neq i}^{N} p_i p_j h_{ij} f_{ji} \right) \]  

(3.5.4)

The potential function for the overlay case can then be written using (3.5.2) as:

\[ V_o = \sum_{i=1}^{N} \left[ \frac{p_i h_{ii}}{\sum_{j=1,j \neq i}^{N} p_j h_{ji} f_{ji} + \sum_{j=1,j \neq i}^{N} p_i h_{ij} f_{ij}} - \frac{\sum_{j=1,j \neq i}^{N} p_i h_{ij} f_{ij}}{K \sum_{j=1,j \neq i}^{N} p_j h_{ji} f_{ji}} \right] \]  

(3.5.5)

**Proof.** For the existence of a valid potential game, the proposed utility function must satisfy the condition in equation (2.1.8). The partial derivative of proposed utility \( U_i \), with respect to transmission power of user \( j \) can be written as:

\[ \frac{\partial U_i}{\partial p_j} = \frac{-h_{ji}}{N_o + \sum_{k=1,k \neq i}^{N} p_k h_{ki} f_{ki}} - K \frac{h_{ji}}{\sum_{k=1,k \neq i}^{N} p_k h_{ki} f_{ki}} \]

The second partial derivative with respect to power level of user \( i \), yields:

\[ \frac{\partial^2 U_i}{\partial p_i \partial p_j} = 0 \]
Similarly, the partial derivative of the utility of user \( j \) with respect to its power \( p_j \) is given by:

\[
\frac{\partial U_j}{\partial p_j} = \frac{1}{p_j} - K \frac{\sum_{k=1,k\neq j}^{N} p_k h_{kj} f_{kj}}{\sum_{k=1,k\neq j}^{N} p_j p_k h_{kj} f_{kj}}
\]

The partial derivative of the above expression with respect to power \( p_i \) of user \( i \), leads to:

\[
\frac{\partial^2 U_j}{\partial p_i \partial p_j} = 0
\]

Moreover, the derivative of potential function \( V_o \) is written as:

\[
\frac{\partial V_o}{\partial p_i} = \frac{h_{ii}}{\sum_{k=1,k\neq i}^{N} p_k h_{ki} f_{ki} + \sum_{k=1,k\neq i}^{N} p_i h_{ik} f_{ik}}
\]

\[
- \frac{p_i h_{ii} h_{ij}}{(\sum_{k=1,k\neq i}^{N} p_k h_{ki} f_{ki} + \sum_{k=1,k\neq i}^{N} p_i h_{ik} f_{ik})^2} - K \frac{\sum_{k=1,k\neq i}^{N} h_{ik} f_{ik}}{\sum_{k=1,k\neq i}^{N} p_k h_{ki} f_{ki}}
\]

Similarly, the derivative of utility \( U_i \) with respect to power \( p_i \) is given by:

\[
\frac{\partial U_i}{\partial p_i} = \frac{1}{p_i} - K \frac{\sum_{k=1,k\neq i}^{N} p_k h_{ki} f_{ki}}{\sum_{k=1,k\neq i}^{N} p_i p_k h_{ki} f_{ki}}
\]

Now,
\[
\frac{\partial U_i}{\partial p_i} < 0
\]
and,
\[
\frac{\partial V}{\partial p_i} < 0
\]

Hence,
\[
\text{sgn} \left( \frac{\partial V}{\partial p_i} \right) = \text{sgn} \left( \frac{\partial U_i}{\partial p_i} \right) < 0
\]

Hence, the condition for the existence of potential game is satisfied and the proposed potential function signifies an ordinal potential game.

The potential function in equation 3.5.5 is responsible for evaluating the network performance. The objective in selecting strategies allows access to the most suitable channel with maximum utilities. For the cooperative benefit, the potential function is optimized by equating its first derivative to zero, which yields the optimized transmit power for \( i \)th player \( p^*_i \) as:

\[
p^*_i = \frac{\sum_{j=1, j \neq i}^N p_j h_{ij} f_{ij} \left[ \sqrt{K h_{ii} \sum_{j=1, j \neq i}^N h_{ji} f_{ji} - \sum_{j=1, j \neq i}^N h_{ji} f_{ji}} \right]}{\left( \sum_{j=1, j \neq i}^N h_{ji} f_{ji} \right)^2}
\]  

(3.5.6)

For underlay systems, the potential function can be defined as:

\[
V_u = \sum_{i=1}^N \left[ \frac{p_i h_{ii}}{\sum_{j=0, j \neq i}^N p_j h_{ji} f_{ji} + \sum_{j=0, j \neq i}^N p_i h_{ij} f_{ij}} - \frac{\sum_{j=0, j \neq i}^N p_i h_{ij} f_{ij}}{K \sum_{j=0, j \neq i}^N p_j h_{ji} f_{ji}} \right]
\]  

(3.5.7)
which provides the power levels as:

\[
p_i^* = \frac{\sum_{j=0,j\neq i}^N p_j h_{ij} f_{ij}}{\left( \sum_{j=0,j\neq i}^N h_{ji} f_{ji} \right)^2}
\]

(3.5.8)

where, the subscript \( o \) represents the contribution of PU.

### 3.5.1 Other Potential Game Formulations

The previous potential game model provides a valuable solution for cognitive network. For the sake of completeness, we formulate this problem for other types of potential games as well. Below is a description of each along with the proofs.

**Weighted Potential Game**

**Theorem 3.4:**

The weighted potential function for the cost-based cognitive game in spectrum overlay can be written as:

\[
V = \frac{1}{N} \sum_{i=1}^N \left[ \ln \left( \frac{p_i h_{ii}}{N_0 + \sum_{j=1,j\neq i}^N p_j h_{ji} f_{ji}} \right) - \frac{p_i}{K \sum_{j=1,j\neq i}^N p_j f_{ji}} \right]
\]

(3.5.9)

**Proof.** From equation (2.1.2) in Chapter 2, we have:

\[
\frac{\partial V}{\partial p_i} = \frac{1}{N} \frac{\partial U_i}{\partial p_i}
\]

Taking the partial derivative of equation (3.5.9), we obtain:
\[
\frac{\partial V}{\partial p_i} = \frac{1}{N} \left( \frac{1}{\sum_{j=1, j\neq i}^{N} p_j f_{ji}} \right)
\]

The partial fraction of the utility function for the proposed potential game in equation (3.5.9) is given by:

\[
\frac{\partial U_i}{\partial p_i} = \frac{1}{N} \sum_{j=1, j\neq i}^{N} p_j f_{ji}
\]

Hence, the condition for weighted potential function is satisfied.

The second step involves the condition for the existence of potential function, as given in equation (2.1.8). In order to prove the existence of potential game, we differentiate the utility function of player \(i\) with respect to power of opponent player \(j\):

\[
\frac{\partial U_i}{\partial p_j} = \frac{h_{ji}}{N_o + \sum_{k=1, k\neq i}^{N} p_k h_{ki} f_{ki}} + \frac{p_i}{K \left( \sum_{k=1, k\neq i}^{N} p_k f_{ki} \right)^2}
\]

The above expression for player \(i\) is differentiated with respect to the transmission power of player \(i\) to yield:

\[
\frac{\partial^2 U_i}{\partial p_i \partial p_j} = \frac{1}{K \left( \sum_{k=1, k\neq i}^{N} p_k f_{ki} \right)^2}
\]
Similarly, for player $j$, we obtain:

$$
\frac{\partial^2 U_j}{\partial p_i \partial p_j} = \frac{1}{K(\sum_{k=1, k \neq i}^{N} p_k f_{ki})^2}
$$

which satisfies the existence of potential game:

$$
\frac{\partial^2 U_i}{\partial p_i \partial p_j} = \frac{\partial^2 U_j}{\partial p_i \partial p_j}
$$

\[\square\]

**Exact Potential Game**

**Theorem 3.5:**

The exact potential function for the proposed spectrum overlay game can be written as:

$$
V = \sum_{i=1}^{N} \left[ \ln \left( \frac{p_i h_{ii}}{N_o + \sum_{j=1, j \neq i}^{N} p_j h_{ji} f_{ji}} \right) - K \ln \left( \sum_{j=1, j \neq i}^{N} p_i p_j h_{ji} f_{ji} \right) \right]
$$

(3.5.10)

**Proof.** For the potential function $V$:

$$
\frac{\partial V}{\partial p_i} = \frac{1}{p_i} - K \sum_{k=1, k \neq i}^{N} \frac{p_k h_{ki} f_{ki}}{\sum_{k=1, k \neq i}^{N} p_k p_k h_{ki} f_{ki}}
$$

Similarly, for the utility function:
\[
\frac{\partial U_i}{\partial p_i} = \frac{1}{p_i} - K \frac{\sum_{k=1, k \neq i}^{N} p_k h_{ki} f_{ki}}{\sum_{k=1, k \neq i}^{N} p_k p_i h_{ki} f_{ki}}
\]

\[
\frac{\partial V}{\partial p_i} = \frac{\partial U_i}{\partial p_i}
\]

This proves the condition for the exact game.

In order to prove the condition for the existence of potential function, we can write:

\[
\frac{\partial U_i}{\partial p_j} = - \frac{\sum_{j=1, j \neq i}^{N} h_{ji} f_{ji}}{N_o + \sum_{j=1, j \neq i}^{N} p_j h_{ji} f_{ji}} - K \frac{\sum_{j=1, j \neq i}^{N} p_j h_{ij} f_{ij}}{\sum_{j=1, j \neq i}^{N} p_j p_i h_{ij} f_{ij}}
\]

\[
\frac{\partial^2 U_i}{\partial p_i \partial p_j} = 0
\]

Similarly,

\[
\frac{\partial^2 U_j}{\partial p_i \partial p_j} = 0
\]

Satisfying the potential game formulation:

\[
\frac{\partial^2 U_i}{\partial p_i \partial p_j} = \frac{\partial^2 U_j}{\partial p_i \partial p_j}
\]

3.6 Numerical Simulations

The proposed simulation model comprises of two dimensional uniformly distributed network of dimension 200$\text{m}^2$. The number of licensed users with their dedicated
Randomly choose initial actions (channel and transmission power) of all users;
For i=1:N if two CRs choose the same channel then
    Compute interference and utilities based on initial actions;
end
for t=1:T do
    for i=1:N do
        Players decide their strategies one-by-one (or with probability 1/N);
        Player i makes a move and computes interference and utilities at ALL
        channels;
        Choose the channel with the maximum utility value;
        Compute the transmission power of user i for the selected channel;
        Repeat for all players;
        Compute potential function for every move;
    end
    Compute the maximum value of the potential function among all moves;
    Choose the action set corresponding to the maximum potential function for
    the next iteration;
    Repeat for all iterations or till players no longer change their strategies and
    convergence is achieved;
end

Algorithm 1: Iterative algorithm for the convergence of strategies in sequential
potential game
Figure 3.1: Plot of the average transmitted power for overlay and underlay cases

channel is $K = 4$ and the number of CR users is $N = 36$. We observe two different potential games. The first game excludes any pricing scheme while the other game is cost-based. The users opt for their strategies by monitoring the potential function at every iteration and consider power options that eventually lead to an efficient solution. The designed algorithm that is used to implement the proposed game is presented in Table 3.1.

The game is played for overlay and underlay spectrum access scenarios and the corresponding power levels for the two schemes are observed. It is seen that the underlay scheme, besides providing more spectrum opportunities, also requires lower transmission power. On the other hand, the transmit power levels for the overlay access is much higher as shown in Figure 3.1.
For the potential game based on pricing, the convergence of strategies is established as in Figures 3.2 and 3.3. Figure 3.2 provides convergence for minimum cost potential game, where potential function is improved by lowering the cost of the channel. This provides better utility with the benefit of low cost channels. However, this leads to sub-optimal channel choices in terms of power consumption. Due to low cost, the players may choose high interference channels, which require more transmit power. This can also lead to an increased packet loss. The power levels for this case are depicted in Figure 3.4.
Figure 3.3: Convergence of strategies for power control game

Figure 3.4: Average transmission power for minimum cost game
Figure 3.3 shows the convergence of strategies for a power optimum cost based potential game. In this game, the players choose strategies, which allow conservation of resources by employing power efficient choices. The chosen channels may not be economical in terms of cost, but the main advantage is offered in terms of optimum transmit power. Figure 3.5 provides the power level achieved at convergence. The average number of iterations after which the convergence is achieved is approximately equal to the number of users competing in the game.

![Figure 3.5: Transmit power for power control game](image)

Figure 3.6 is an interesting comparison of the average throughput for all three games; non-pricing, minimum cost and power control games. From the figure, we can see that power control game provides better performance compared to the minimum cost and non-pricing potential games. The games lack any cost mechanism and provide the lowest performance.
In this chapter, we discuss different cooperative potential games and their solutions for cognitive radio networks. The cooperation is encouraged by employing a cost function that provides better network performance compared to non-pricing games. The introduction of cost function allows users to opt for inexpensive or high performance channels according to their requirements and the ability to afford.
Chapter 4

Stackelberg Potential Games for Cognitive Radio Networks

As discussed in the previous chapters, the mode of cognition is always governed by the spectrum license holders, which control the conditions for cognitive users. This creates a natural environment for a leader-follower setup, also known as Stackelberg game. In this chapter, a cooperative strategy mechanism for cognitive players is proposed that is based on a dynamic cost function and efficient power allocation. The proposed scheme is mutually beneficial for all players and focuses on improving the network performance. Previous formulations explore the cooperative cognition for fixed powers and static cost, which limits the network performance. We design the network potential function such that the nodes have performance based incentives to cooperate and achieve a Nash equilibrium solution for efficient channel acquisition and power levels.

One of the main concerns that arise in the implementation of cognition occurs due to the restrictions imposed by PU. Moreover, as all SUs attempt to access channels, their actions create conflicts and deteriorate performance. This leads to a depletion of resources including bandwidth and power. We attempt to address the conservation
of these resources conforming to the PU’s requirements by employing the Stackelberg (or leader-follower) game model.

The previous works incorporating cooperation lack the improvement in network performance, and the pricing methodology is not fair and dynamic [1]. The presented approach differs from previous formulations by incorporating performance based and efficient pricing and power allocations. We present the competition among cognitive users for the overlay and underlay systems as a cooperative Stackelberg potential game. The potential function incorporates spectrum efficiency along with fair pricing to improve network performance and power consumption. We explore the convergence or strategies of proposed Stackelberg potential game formulated to incorporate cooperation among all players (PUs and SUs) so that a Nash equilibrium solution for improved network performance is achieved.

The proposed methodology, adopted for achieving improved performance, considers the PU as leader of Stackelberg game, making the first move and causing the successive SUs to obey the rules. The proposed potential function includes performance affecting parameters and maximizing this function ensures improved performance with proficient resource consumption.

The Stackelberg model comprises of a strategic game, where players compete as a leader and follower. The leader has the knowledge of followers that are observing the actions. This allows the leader to govern the system in the best possible interest. The followers, being rational, observe the leader’s action and act accordingly that leads to an equilibrium solution. It should be noted that although the follower can only act after the leader, its actions may damage the leader. Hence, leader must be careful about its chosen action so as to lead the follower along a Nash equilibrium path [36].
The followers have an advantage of having knowledge of leader’s actions. Since PU are the mandatory players, being committed to the spectrum monopoly, they are obliged to make the first move. The followers, in this case, are the new entrants striving for a possible abode in this game for transmission bandwidth. The Stackelberg game is applied to illustrate the dynamics of competition.

The game discussed here for the implementation of cognition is a sequential game, similar to the game played in Chapter 3, and players are required to take turns, which allows them to have knowledge about the strategies of the preceding players. This makes the players more informed and they can choose their strategies wisely, according to the payoff or utility achieved as a result. An intelligent design of a cost function can significantly alleviate the effects of inappropriate channel selection by the sensing nodes. The players are motivated to opt for low interference channels that are economical.

4.1 System Model

The $K$ available channels, are owned by their respective leader PU, which sets the performance criteria for their respective channels. However, not all PUs necessarily play the role of the active leader; some of the PUs may choose to remain idle. The total number of cognitive transmitter-receiver pairs (followers) competing for these channels is $N$, where $N > K$.

We consider two different types of system models. The first model involves co-existing PUs and SUs (spectrum underlay), while the second approach deals with accessing vacant channels (spectrum overlay) where CRs can only compete for a
channel with no licensed transmissions. For these two models, we define four interference terms based on the fact that only the players that opt for the same channel are a source of interference for each other. Based on these interference terms, we can write the spectral efficiency of a cognitive radio system from [4] as:

\[ \mu_i = \log_2(1 + \beta \gamma_i) \]  

(4.1.1)

where, \( \gamma_i \) is the SINR modified accordingly for underlay or overlay. \( \beta = \frac{15}{\ln(0.2/B_o)} \), and \( B_o \) is the target bit-error-rate required for successful transmission. This spectral efficiency is a measure of spectrum utilization. Higher spectral efficiency is a desirable feature for licensed and unlicensed users. Hence, the payoff of a player is measured by the spectral efficiency offered by a particular strategy. The spectrum efficiency or gain for overlay case is thus defined as:

\[ \mu_{iv} = \ln \left( \frac{\beta p_i h_{ii}}{I_{iv} + N_o} \right) \]  

(4.1.2)

Similarly, the gain or spectral efficiency function in underlay case is simplified to:

\[ \mu_{iu} = \ln \left( \frac{\beta p_i h_{ii}}{I_{iu} + N_o} \right) \]  

(4.1.3)

4.2 Problem Formulation

We model the overlay and underlay spectrum access problems as cooperative Stackelberg games. If the leader PU is transmitting over a channel or reclaims it in the middle of the game, the corresponding action set of CRs in overlay case is to wait for transmission to end or to opt for an alternate channel from other vacant options. In case of underlay, all CRs may not vacate the channel if they can satisfy the condition imposed by PU.
In order to introduce cooperation in the game, we propose a dynamic cost model. The cost function is responsible for providing spectrum opportunities to SUs by generating revenue for PU. Instead of charging all users with the same price, which may not be fair for some players, we model a pricing scheme where the cost of every channel varies for each user according to the performance. The users creating low interference levels are encouraged by offering a discounted price. The cost function also depends on the number of competing cognitive users, more users create more competition and higher interference, increasing the channel cost. The channel cost increases with the transmit power over that channel. Users that transmit with higher power must pay a higher price due to higher level of induced interference. The PU’s goal is to sell the channel at a cost that is profitable for it but at the same time must enable it to attract more SUs. The objective of cost function is not just to earn revenue for the PU, but to enable more SUs to be accommodated.

4.3 Utilities and Potential Function

In overlay systems, the cost increases with the threshold SINR level $\zeta$ required by the SU for a channel. If the number of available channels is large, the competition among users is greater and correspondingly the cost must be kept low. Based on this discussion, the cost function for overlay model of a channel required from a cognitive user trying to access it is given by:

$$C_{iv} = \frac{p_i N\zeta}{K} \left( \rho I'_{iv} - (1 - \rho) I_{iv} \right)$$  \hspace{1cm} (4.3.1)
where, $\rho$ is the weight assigned to the interference a user creates over a channel, which incorporates the case of imperfect channel estimation for the Bayesian game [15]. The primed term represents the interference created by a user, which acts as a cooperating factor. This cost function depicts that the cost for a CR increases with the interference it creates and decreases with the interference it observes.

In underlay scheme, the SUs are required to keep the interference level within a certain limit so as not to hinder PU’s transmissions. PU charges the SUs for a channel based on the interference level created by them. The higher interference level tolerated by a PU on a channel, $I_{th}$, reduces the cost by encouraging SUs to choose that particular channel. In this case, the CRs are not required to monitor the presence of PU and interference by PU is always accommodated. This encourages high performance CRs and the others that create higher interference for the PU are discouraged. However, setting the threshold $I_{th}$ too stringent increases the cost and discourages the users to opt for it. Thus, the cost function for underlay case becomes:

$$C_{iu} = \frac{p_i N_0}{K} \left( \rho I'_{iu} - (1 - \rho)I_{iu} \right) + I_{th} \quad (4.3.2)$$

The payoff or utility of the leader PU increases with increase in spectral efficiency and the revenue generated from follower SUs. The payoffs of SUs increase with higher spectral efficiency but decrease with the price paid for channel access. The utility functions for the overlay and underlay cases can be respectively summarized as:

$$U_{iv} = \mu_{iv} - C_{iv} \quad (4.3.3)$$
$$U_{iu} = \mu_{iu} - C_{iu} \quad (4.3.4)$$

The PU’s utility increases with the revenue generated by SU, i.e., price paid for channel access, and decrease with the interference created by SUs. The PU on channel
\[ U_{pj} = \sum_{k=1,s_k=s_j}^{N} (\mu_{kv} + C_{kv}) \]  \hspace{1cm} (4.3.5)

The leader PU \( j \)'s utility in underlay cases can be obtained as:

\[ U_{pj} = \sum_{k=1,s_k=s_j}^{N} (\mu_{ku} + C_{ku}) \]  \hspace{1cm} (4.3.6)

Potential games have an added advantage of providing the complete behavior of all players in a single comprehensive global function. This function is useful for catering individual player’s needs as well as the overall performance of the network. The proposed potential function incorporates the spectral efficiency measure and cost in its design. Higher spectral efficiency improves the potential function, whereas higher cost deteriorates performance by lowering the value of potential function.

In the proposed case, the overall network performance decreases with the interference level set by PU. In spectrum overlay approach, the potential function is simplified as there is no interference limit set by the PU due to its absence.

**Theorem 4.1**

The designed potential function for the overlay case is given as:

\[ V_v = \sum_{i=1}^{N} (\mu_{iv} - C_{iv}) \]  \hspace{1cm} (4.3.7)

**Proof.** In order to show that equation (4.3.7) represents a valid potential game, we prove the condition given in equation (2.1.8). The partial derivative of the utility of player \( i \) with respect to the power of player \( j \) can be written as:
\[
\frac{\partial U_{iv}}{\partial p_j} = \frac{\partial}{\partial p_j} \left[ \ln \left( \frac{\beta p_i h_{ii}}{I_{iv} + N_o} \right) - p_i \frac{N\zeta}{K} \left( \rho I'_{iv} - (1 - \rho)I_{iv} \right) \right]
\]

\[
\frac{\partial U_{iv}}{\partial p_j} = \frac{h_{ii}}{(I_{iv} + N_o)} + \frac{N\zeta}{K} (1 - \rho) p_i h_{ji}
\]

The second partial derivative of the above expression with respect to the power of player \(i\) can be written as:

\[
\frac{\partial^2 U_{iv}}{\partial p_i \partial p_j} = \frac{N\zeta}{K} (1 - \rho) h_{ji}
\]

Now, the utility function for player \(j\) can be written as:

\[
U_{jv} = \left[ \ln \left( \frac{\beta p_j h_{jj}}{I_{jv} + N_o} \right) - \frac{p_j N\zeta}{K} \left( \rho I'_{jv} - (1 - \rho)I_{jv} \right) \right]
\]

The partial derivative of player \(j\)'s utility with respect to its power is given by:

\[
\frac{\partial U_{jv}}{\partial p_j} = \frac{1}{p_j} - \frac{N\zeta}{K} \left( 2\rho \sum_{k=1,k\neq j}^N p_j h_{jk} - (1 - \rho) \sum_{k=1,k\neq j}^N p_k h_{kj} \right)
\]

The second partial derivative with respect to power of player \(i\) becomes:

\[
\frac{\partial^2 U_{jv}}{\partial p_i \partial p_j} = \frac{N\zeta}{K} (1 - \rho) h_{ij}
\]

Since the channel is assumed to be symmetric, the link gain \(h_{ij}\) is equal to the link gain \(h_{ji}\). Hence, equation (2.1.8) is satisfied

\[
\frac{\partial^2 U_{iv}}{\partial p_i \partial p_j} = \frac{\partial^2 U_{jv}}{\partial p_i \partial p_j}
\]

Moreover, the second order derivative of potential function yields:
\[
\frac{\partial^2 V_v}{\partial p_i^2} = \frac{-1}{p_i^2} - \frac{N \zeta}{K} (2 \rho \sum_{k=1,k\neq j}^{N} h_{ik}) < 0
\]

Thus, \(V_v\) represents a valid potential game.

Similarly, for underlay case, the SUs are required to keep their interference level below the level \(I_{th}\), yet achieve a sufficient SINR level to establish successful transmissions. The potential function formulated for spectrum underlay scheme is given by:

\[
V_u = \sum_{i=1}^{N} \left( \mu_{iu} - C_{iu} \right)
\] (4.3.8)

These potential functions relate the SUs’ gain in the form of spectrum efficiency with the price paid to achieve the gain to ensure fair allocation of resources.

### 4.4 Power Allocation

The proposed potential function is a convex function, which considers power and spectrum efficiency besides cost and interference. In order to implement power control at the transmitter nodes, we optimize the potential function for transmission power of nodes in an effort to increase overall network performance. This allows efficient power allocation to users, creating a balance between successful transmission and acceptable interference. The potential function given in (8) and (9) can be optimized with respect to power. In order to achieve this, we evaluate the first derivative of potential function with respect to power, which can be written as:

\[
\frac{\partial V}{\partial p_i} = \frac{\beta h_{ii}}{N_o + I_i + \beta p_i h_{ii}} - 2M \rho p_i h_i + M(1 - \rho)I_i
\] (4.4.1)
where, $h_i = \sum_{j=0}^{N} h_{ij}$, $M = N \zeta / (KI_{th})$ for underlay systems, and $h_i = \sum_{j=1}^{N} h_{ij}$, $M = N \zeta / K$ for overlay systems. Equating the above expression to zero yields the optimum solution as:

$$p^*_i = \frac{1}{4B_p h_i} \left[ \rho' B_i I_i - 2\rho (I_i + N_o)h_i + \sqrt{4\rho h_i \{ I_i + N_o \} \{ \rho (I_i + N_o)h_i + B_i \rho' I_i \} + 2B_i^2 / M} + B_i^2 \rho'^2 I_i^2 \right]$$

(4.4.2)

where, $B_i = \beta h_{ii}$, and $\rho' = 1 - \rho$. The above expression provides the transmit power level required by the cognitive users to ensure better network performance.

### 4.5 Numerical Results and Simulations

The simulation setup considers $K=3$ identical channels and $N=5$ cognitive users uniformly distributed in an area of $200m^2$. The noise variance $N_o$ is assumed to be $10^{-5}$mW and threshold SINR $\zeta$ is taken as 20dB. The probabilistic parameter $\rho$ is considered to be 0.5 for these simulations. Initially, the PU is assumed to be absent and all three channels are vacant. The game is played among SUs only that initially choose a channel randomly at the beginning of the game and then decide their actions according to the proposed potential game. After some time, when the game reaches 30 iterations, the PU appears on channel 3. If the game is played in the overlay mode, the SUs transmitting at channel 3 must vacate it and switch to other available channels. This creates additional interference to the users already transmitting on channels 1 and 2, and some of them may also switch their strategies. The Nash equilibrium for this case is depicted in Figure 4.1. This figure shows that whenever a PU appears (in this case, after 30 iterations over channel 3), the cognitive users
must re-adjust their strategies and the choice of reclaimed channel 3 becomes non-existent. The game is now played for the remaining 2 channels instead of 3. In case of underlay mode, however, all players do not completely switch to other channels, some may continue using that channel while others can make a switch so as to retain the tolerable interference level for PU. This scenario is shown in Figure 4.2, where the channel reclaimed by the PU is not ignored. Instead, this case modifies its strategy according to the level of interference suffered by SUs and tolerance level of the PU. The convergence in underlay cases may take longer time to establish as compared to overlay. However, the convergence time for both the cases is much better than the overlay case as discussed in [1]. The interference and power levels achieved at stability are also improved. The underlay scheme provides more spectral opportunities as the PU’s channel can still be used and is not completely excluded from the possible set.

Figure 4.1: Convergence of strategies for spectrum overlay system (Note the absence of SU on channel 3 due to the presence of PU)
Figure 4.2: Convergence of strategies for spectrum underlay system

Figure 4.3: Convergence of strategies in terms of channel switching for spectrum overlay system
Another interesting observation is that in [1], the users keep changing their power levels even after the convergence of channel selection, which means that power levels converge much later than the channel choices. No reason is provided for this kind of behavior. In the proposed model, power levels converge as soon as channel acquisition achieves Nash equilibrium. The results are valid for any number of users and channels. A comparison of average sum capacity of the proposed work with [1] is shown in Figure 4.3. This comparison reveals that the proposed scheme performs better. The performance is improved for a congested network in case of the presented method as compared to the previous methods.

Figure 4.4: Convergence of strategies in terms of channel switching for spectrum underlay system

Figure 4.5 and Figure 4.6 provide the transmission power for overlay and underlay
Figure 4.5: Transmission power for spectrum overlay system

cognition schemes, respectively. The comparison of the proposed scheme with the works in literature is shown in Figure 4.7. It is evident from the figure that the proposed scheme performs better than the previously discussed schemes.

This chapter explains the effects of PU on SU competition by employing the Stackelberg game. It is seen that PU is the leader administrating the CRs. We formulate the problem as a potential game based on power and cost. The CR nodes choose their strategies for the maximum value of proposed potential function. These strategies are opted for the improved potential function to provide better choices for the CR players, the PU, as well as the entire network. The convergence and average capacity for proposed scheme is improved as compared to existing methods [1]. The underlay and overlay scenarios are separately discussed with underlay providing more spectrum opportunities. The action set comprising of transmission power and
Figure 4.6: Transmission power for spectrum underlay system

Figure 4.7: Comparison of average sum capacity of proposed work with [1]
channel opted in these games provide better network performance with efficient power consumption.
Chapter 5
Learning in Cognitive Radio Games

In the previous chapters, we discuss games, where only one user decides its action at a certain time. These type of games are called sequential moves games. There is another, more realistic scenario of playing a cognitive game, where all the players choose their actions simultaneously based on the current information available. This is called a simultaneous moves game or a Cournot game and may lead to conflicting choices. The sequential game allows other users in the network to choose their actions keeping in view the strategies of previous players to avoid conflicts. Hence, users have better chance of choosing suitable strategies, avoiding low performance actions. In sequential moves game, the players have information about the preceding players’ strategies that is not possible in simultaneous moves game.

5.1 Introduction

In a typical application of a cognitive radio network, the CRs are unable to coordinate in deciding the individual actions. Most real systems involve SUs that concurrently
opt for a channel instead of making a queue and waiting for their turn. This scenario increases the risk of conflicting choices and needs to be resolved by providing appropriate insight to users regarding their decisions. One method to achieve this harmony is by applying learning methods suitable for CR systems.

Learning provides a more dynamic approach for users with different objectives and selfish attitude. Various techniques are used to implement learning in a game [20]. One form of learning involves introspection, where players ponder over their situation by analyzing the opponents' behavior. Another form of learning involves imitation, where players learn from others and mimic the strategies of other players involved in the game. These types of learning involve coordination among players or some kind of pre-play communication, which is not possible in case of ad hoc networks.

Another type of learning scheme, suitable for ad hoc networks, involves history and is employed in the repeated games. The history based learning schemes analyze the strategies the opponents have played in the past. This assists the player to devise its best strategy learning from past experiences and observations. Learning algorithms are functions based on history obtained from the past strategy sets. Most learning algorithms are based on probabilities assigned to the previous strategies. The potential function takes into account the collective benefits of users while learning also allows to cater the individual needs [37].

A preferred solution would be if the players can make decisions simultaneously without having to wait for their turn according to some probability distribution. The main concern in simultaneous decisions is to avoid conflicts due to being uninformed of other player’s strategy [38]. The simultaneous moves game structure eliminates the need to be informed about other players’ strategies and instead only requires a player
to consider its own strategy from previous plays. This process of attaining information is called learning that serves to minimize regret, where regret is the difference in payoffs from the previous iterations. If regret for a certain game increases, the utility or payoff for that game decreases.

Game theory allows the concept of learning as a way to reduce the regret in decision making using historical data. Learning is utilized in repeated games to provide a solution based on prior knowledge for improved system stability. Learning is adopted in several previous researches discussing CR games. An interesting work where the authors develop a modified learning technique for regret matching is discussed in [39]. Regret is a measure of performance against the best strategy. Another work applies the no-regret learning to [1], which yields a slight improvement in the utility but requires more time to converge [25]. [40] presents an algorithm for improved channel and resource allocation, through a channel priority table. The players opt for the best choices by learning from the history. All these systems take a much longer time to converge that increases with the increase in the number of cognitive players. Some of them require a central system or channel table for information exchange. In this chapter, we incorporate a learning mechanism employing past data in the proposed potential function design. This allows players to be informed about history without added overload of information exchange or a separate learning algorithm.

In a repeated game, the information gained via history is significant if the players are contented with their choices and do not suffer from a regret at some stage during the game. In order to overcome the implications of unwise moves, a regret function is defined that computes the amount of regret a player undergoes during the game. This is computed from the amount of payoffs for the previous moves. Minimizing
this regret is the key to employing learning efficiently. The strategies are assigned based on regrets computed from history and players decide in favor of the moves, which offer minimum regret. Several no-regret learning algorithms are employed for cognitive radios, the most popular being the Freund and Schapire (also called Hedge) algorithm [41]. This algorithm provides on-line learning for strategic games.

The no-regret algorithms can be implemented to informed and uniformed/naive users. However, it must be noted that learning algorithms do not necessarily lead to Nash equilibrium solution. Another useful work, which proposes a no-regret solution converges to Nash equilibrium using exponential learning probabilities [37].

Almost all cognitive games discussed in literature involve a sequential playing mode where players take turns in opting for channels. This adds a lot of delay for the network to stabilize [2-7]. The delay increases with the number of players. In [42], the authors discuss the CR game as simultaneous moves game but the problem is simplified by assuming that at a certain time, only one player makes a decision with a fixed and equal probability, and the sequential model is followed. Similarly, other works also assign equal probabilities for making a move and the game reduces to a sequential play [6-10].

[12] considers a learning algorithm to assign weights to suitable strategies for a simultaneous setup. The learning mechanism is used to alter these probabilities according to history in order to encourage players to opt for high probability strategies [12]. This eliminates the need for a sequential potential game. However, due to practical implications of information exchange, the players make moves based on a weighted probability to avoid collisions.

[41] presents a learning based simultaneous moves game, where players take actions
simultaneously by computing a weight function to minimize regret. It implements the combined channel and power allocation problem as the Freund and Schapire Informed (FSI) algorithm. This, however, takes a much longer time to converge, though with a reduced overhead. The FSI or hedge algorithm reduces the loss by playing a safe strategy, i.e., the strategy with the highest probability of providing a better gain.

The simultaneous moves inherently cannot be incorporated in potential game scenarios. To the best of our knowledge, learning and potential games are treated independently in literature [12]. Learning allows to discuss heterogeneous players, but the cognitive system mostly involves players with shared objectives. We aim to combine the two formulations to achieve the desired solution, which reaps the benefits of learning by allowing simultaneous moves and the advantage of potential game by providing improved network performance. The potential game scenario presented in previous chapters requires a sequential move setup. Learning eliminates this restriction and does allow simultaneous moves.

5.2 Sequential Learning Model

In a sequential set up, the $N$ cognitive players take turns to decide their respective strategies from the strategy set $s \in [1, K]$. In order to ensure that only one player moves at a certain time, it is assumed that each player decides its strategy with equal probability $(1/N)$ at any time. Thus, the players have knowledge of their opponents’ actions besides the information from history.

We model the cognitive competition as a learning based cooperative game, where SUs cooperate to avoid conflicts and accomplish the task of suitable channel acquisition. We develop a potential game, which exploits learning based incentives. Learning
allows users to choose more wisely based on the history. The potential game triggers a cooperative environment by focusing on collective optimality and represents the incentives for every strategy for all players. When learning is included in the potential function, the players employ history to improve network performance. The Nash equilibrium solution for the system can be found by maximizing the global potential function [34], [35]. The previous moves of the players assist in determining the best solutions, by avoiding the sub optimal choices.

The proposed cost function depends on the history, if the utility for a user in the current iteration is greater than the previous one, user may opt for that channel even for a higher price. The weight function this is responsible for price controls the channel choices by varying cost. This weight function is also responsible for assigning higher probabilities to channels that are chosen less frequently in the previous iterations and thus provide lower interference. The individual utility function of a cognitive user can be written as:

$$U_i(t) = \eta_i(t) - c_i(t) - W_i(t)$$  (5.2.1)

where, the cost function $c_i(t)$ is the cost paid by the $i$th CR to gain channel access and $W_i(t)$ is the weight factor at time $t$. These quantities employ history to determine channel cost and the weight associated with an action. We define the function $\eta_i$ that involves the interference created by the opponents and also the interference created by a user for its opponents. Representing the probability of false detection as $\alpha$, we can write the function $\eta_i$ as:

$$\eta_i(t) = \ln \left( \frac{p_i(t)h_{ii}}{N_\alpha + (1 - \alpha) \sum_{j=1, j \neq i}^N p_j(t)h_{ji}f_{ji}(t) + \alpha p_\alpha(t)h_{\alpha \alpha}f_{\alpha \alpha}(t)} \right)$$  (5.2.2)
where, $p_o(t)$ is the transmitted power of the primary user at time $t$ and $p_i(t)$ is the transmission power of SU $i$. We can notice from the above equation that power is the only factor that depends on time $t$, as the channel conditions $h_{ij}$ remains constant at least until the establishment of convergence.

We now mathematically develop the cognitive environment in the form of a potential game model. The potential function involves transmitted power of CRs and the price of transmission over a channel.

**Cost based allocation**

We devise our cost function based on the number of available channels and the level of interference presented. The higher the number of users on a channel, higher interference is offered by the channel and less users are attracted to it. Hence, cost of this channel becomes lower. Cost is also determined by the amount of interference an entering user adds to the channel. Higher interference users are discouraged by charging a high price. When the competition is reduced due to large number of available channels, the price is low and if the number of SUs is high, the channel price is high. Hence, we write the proposed cost function as:

$$c_i(t) = \frac{p_i(t)N\zeta}{K} \left( \rho \sum_{j=1,j\neq i}^{N} p_i(t)h_{ij}f_{ij}(t) - (1 - \rho) \sum_{j=1,j\neq i}^{N} p_j(t)h_{ji}f_{ji}(t) \right)$$  \hspace{1cm} (5.2.3)

An added factor affecting the utility is the weight function based on learning data. This added information assists in decisions by making the users more informed via previous history. The players, being informed, chose more wisely and acquire better payoffs. This weight function is given by:

$$W_i(t) = \theta(t) \frac{I_i(t)}{I_i(t-1)}$$  \hspace{1cm} (5.2.4)
where, $\theta(t)$ is the probability of channel access in iteration $t$ and $I_i(t)$ is the interference observed by $i$th node at iteration $t$. If players make smart choices, the weight factor is less than one, and the utility for the next iteration is improved. The payoff function can be written as:

$$U_i(t) = \eta_i(t) - c_i(t) - W_i(t) \quad (5.2.5)$$

### 5.3 Convergence of Sequential Game

In order to determine the solution for transmission power, the potential function takes into account the power consumption of each user and the interference it suffers from its colleagues. The potential function for a power optimized case yields:

$$V(t) = \sum_{i=1}^{N} [\eta_i(t) - c_i(t) - W_i(t)] \quad (5.3.1)$$

The players primary target is to opt for the channel providing maximum payoff with a suitable transmit power. Due to their cooperative nature, the players objective is to maximize the potential function instead of just the individual payoffs. In order to determine a Nash equilibrium solution for this problem, we evaluate the maximum of potential function value by taking the first partial derivative. Hence, we can write:

$$\frac{\partial V}{\partial p_i} = \frac{1}{p_i} - \frac{2N\zeta}{K} \sum_{j=1, j \neq i}^{N} p_i h_{ij} + \frac{N\zeta}{K} \sum_{j=1, j \neq i}^{N} p_j h_{ji} \quad (5.3.2)$$

Based on this derivative, the value of transmit power at Nash equilibrium is given by:

$$p^*_i(t) = \frac{1}{4} \left[ \sum_{j=1, j \neq i}^{N} p_j(t) h_{ji} +\sqrt{\left( \sum_{j=1, j \neq i}^{N} p_j(t) h_{ji} \right)^2 + \frac{8K}{N\zeta} \sum_{j=1, j \neq i}^{N} h_{ji}} \right] \quad (5.3.3)$$
The simulation environment involves $N=20$ players competing for $K=4$ channels for the network dimensions $D=200m^2$. The transmit power and the convergence of proposed strategies is observed for the users. The convergence is established for lower cost to maximize the user payoffs. The transmit power, however, is not optimized and some users transmit at higher power levels and create higher interference for other network users. The results are shown in Figure 5.1 and Figure 5.2.

Next, the game is played by the SUs according to optimized power levels required for transmissions. This ensures that the unlicensed users conserve their power resources required for a successful transmission, while being considerate about the interference level this power is creating for opponents. The power levels achieved at
Figure 5.2: Iterative channel switching before convergence

equilibrium as a result of the opted strategies, show that the transmit power consumed is higher when low cost channels are selected as compared to the games where strategies are chosen according to power control instead of cost. The power control game also minimizes the interference level created and suffered, hence improving the network performance.

The convergence of these strategies is established in both the games. When economizing for cost, the system converges after the number of iterations equal to the number of players. This is due to the sequential nature of the game. The convergence in case of minimum cost game makes some channels over crowded due to their inexpensive nature, while some channels remain underutilized because they are more expensive. When players opt for power efficient channels, the chosen strategies
converge to almost uniform channel choices. The convergence, however, is delayed due to learning overhead but the payoff is improved.

5.5 Learning in Simultaneous Moves Games

We introduce a repeated game to alleviate the spectrum allocation problem and facilitate the users to make spectrum selection decision simultaneously or asynchronously. In contrast to sequential games, the proposed simultaneous move multi-stage game model is appropriate for practical applications, where paucity of central spectrum management resources is common. In order to avoid the conflicts arising from coinciding concurrent decisions, we incorporate learning via history statistics to attain a stable and efficient equilibrium point. Every player computes the feasibility of playing a strategy from opponents’ actions in the previous iterations via proposed learning rule. This learning process assists in decision making for the next iteration and eventually the Nash equilibrium is obtained.

The lack of information created due to switching from sequential to simultaneous moves is compensated in a way so as to feed the information by other means. This source of information is generated through history. The information gained from previous moves is utilized as a learning tool for players during decision making. We present a new learning methodology that computes the actions of opponents by a forecast rule. This creates some additional overhead for the players but decreases the convergence time. This faster convergence reduces the processing time to establish stability and hence the processing overhead is eventually reduced.
5.6 Simultaneous Learning Model

The objective of learning is to devise one’s strategy by constantly updating its belief about the opponents’ strategies. This allows a more informed and calculated action which may provide a better performance over time. The PU is a silent player in this game and is assumed to be not transmitting for the duration of the game, leaving the game to be played among SUs only. If, however, a PU chooses to transmit or reclaims its channel, the game is simply played for the remaining available channels.

The cost function $C_{ik}(t)$ at time $t$ for $i$th player over channel $k$ depends on the amount of interference created; greater cost is charged from a player, which creates higher interference over a channel. Similarly, the interference suffered by a user on a channel makes that channel less attractive, thus lowering the cost of that channel. The cost also lowers depending on the number of available channels. We have incorporated the cost based on the fact that all players are attracted to an empty channel. This fact, however, is also known to the players in the game and every player knows that its opponent can opt for it and the seemingly vacant channel may become over occupied. This lead to conflicts, which can be avoided by keeping a cost function that every user must pay for channel access. The channels that are vacant provide the best utility but at a much higher cost than the channels which are pre-occupied by a few users. This eventually lowers the higher utility of a vacant channel. Hence the cost function employed is given by:

$$C_{ik}(t) = \alpha_k(t - 1) \frac{I'_{ik}(t - 1)}{KI_{ik}(t - 1)} + \frac{I_{ik}(t)}{I_{si}(t - 1)} - \frac{I_{ik}(t - 1)}{I_{si}(t - 2)}$$

(5.6.1)

where, $I_{ik}(t) = \sum_{j=1, j \neq i, s_i = s_j}^{N} p_{jk} h_{ji} f_{ji}(t)$ is the amount of interference a user $i$ suffers on
channel $k$, and $I_{ik}(t) = \sum_{j=1, j\neq i, s_i=s_j}^{N} p_{ik} h_{ij} f_{ij}(t)$ is the amount of interference a user $i$ creates over a channel $k$. This term behaves as a cooperative parameter, as it allows the players to behave considerately towards other network users. $I_{si}(t) = \sum_{\tau=1}^{t} I_{ik}(t-\tau)$ is the sum of interference in all previous iterations. $\alpha_k(t - 1) = \frac{\sum_{l=1,l\neq k}^{K} U_{il}(t-1)}{K}$ is the probability of channel usage in the previous iteration for channel $k$. This factor increases or decreases the channel cost according to its suitability. The game learns by evaluating the probability of channel access from previous iteration and utilizes this probability to scale the price for next iteration. Hence, the first term scales the cost of channel according to the probability of channel access and the ratio of interference created to the interference suffered. The second term adds to the channel cost according to the interference it provides in the current iteration as compared to the interference observed in the history. The third term reduces the channel cost by determining the wisdom of player in recent history. If a player is successful in achieving an improved solution in the previous iteration, it is rewarded by a lower price in the next iteration.

The utility function for $i$th node with strategy $s_i$ over a channel $k$, when $s_{-i}$ is the strategy of its opponents, is given by:

$$U_{ik}^{t}(s_i, s_{-i}) = \ln \left( 1 + \frac{p_{ik}}{N_0 + I_{ik}(t)} \right) - C_{ik}(t) \quad (5.6.2)$$
Lemma 1:

For the function $U_{ik}$ to be concave, the second derivative of utility should be negative, i.e.,

$$\frac{\partial^2 U_{ik}}{\partial p_{ik}(t)} < 0$$

Proof. The utility function is given by the expression:

$$U_{ik}(s_i, s_{-i}) = \ln \left(1 + \frac{p_{ik}}{N_o + I_{ik}(t)}\right) - \alpha_k(t-1) \frac{I_{ik}(t-1)}{K I_{ik}(t-1)} - \frac{I_{ik}(t)}{I_{si}(t-1)} + \frac{I_{ik}(t-1)}{I_{si}(t-2)}$$

The first partial derivative of utility with respect to power $p_{ik}(t)$, can be written as:

$$\frac{\partial U_{ik}}{\partial p_{ik}(t)} = \frac{1}{p_{ik} + N_o + \sum_{j=1,j \neq i,s_i=s_j}^N p_{jk} h_{ji} f_{ji}(t)}$$

The remaining three terms of the utility vanish after derivative as they are functions of previous time or of the power of other users $p_{jk}$. This simplifies the second derivative as:

$$\frac{\partial^2 U_{ik}}{\partial p_{ik}^2(t)} = \frac{-1}{\left(p_{ik} + N_o + \sum_{j=1,j \neq i,s_i=s_j}^N p_{jk} h_{ji} f_{ji}(t)\right)^2}$$

Hence,

$$\frac{\partial^2 U_{ik}}{\partial p_{ik}^2(t)} < 0$$
Thus the proposed utility function is concave with respect to the transmission power of CRs.

The proposed work is based on determining a stable strategy set for a simultaneous moves CR game. Usually when all players make their decisions simultaneously, they tend to go for the best choice at that moment. Since all players are doing the same, the best choice no longer remains the best due to conflicts leading to higher interference levels. This problem can be solved by making the players patient. Instead of jumping off for the apparently best choice, players are encouraged to analyze their situation and estimate the behavior and likely action of other players. Since the players are cooperative, they are ready to accommodate other users, instead of greedily searching for an even better payoff. Based on history, players are made to be contended with their choices if the previous actions provides a better interference than the currently available choices. Hence, players do not feel the need to change their choices unnecessarily.

Due to simultaneous decisions of players, the best response utility function formulations incorporates the previous performance to reduce regret. The best iterative utility function provides the facility of computing the value of utility function with each move of players, and decides in favor of the best strategy set among all moves by comparing the utility function at each move. In this work, we decide in favor of strategy set providing the maximum value of utility function among all previous iterations. Thus the decision function for this game can be written as:

$$W(t) = \max_{\tau} \left\{ \sum_{i=1}^{N} \sum_{k=1}^{K} U_{ik}^{t-\tau}(s_i, s_{-i}) \right\}$$  \hspace{1cm} (5.6.3)
where, \( \tau = 0, 1, ..., t - 1 \). The basic idea is to make all channels comparable so that players do not get biased towards one particular channel. Higher number of users leads to even better performance due to simultaneous nature of decision making process. Instead of waiting for all users to one-by-one choose their strategies, our algorithm empower users to make their decisions at the same time as their opponents by analyzing the historical data.

The transmission power for simultaneous moves game is given by:

\[
p_i^* = K \sum_{j=1, j \neq i}^{N} p_i(t-1)h_{ij} - N_o - \sum_{j=1, j \neq i}^{N} p_j(t)h_{ji} \tag{5.6.4}
\]

When assigning power according to the water-filling, the strategy set involves jointly opting for channel and transmission power. From [12], the \( k \)th channel users have the power vector given by:

\[
P_k = (I - H_k)^{-1} \gamma_k \tag{5.6.5}
\]

where, \( I \) is the identity matrix and \( \gamma_i = \frac{2p_i N_o}{h_{ii}} \). The channel vector \( H_k \) is given by:

\[
H_k = (h_{ij}), s_i = s_j = k
\]

These power levels are assigned to the cognitive users based on their respective channel choices.

The basic algorithm for the proposed scheme is given in Algorithm 1. The algorithm explains the iterative procedure of channel choice by the CRs. Each player observes the interference over a channel and computes its suitability according to the payoff obtained from the channel. The players make their moves only after determining the highest payoff channel. However, this payoff is determined based on the
Randomly choose initial actions (channel and transmission power);
Define T as number of iterations, K available channels and N number of CRs;
for $i=1:N$ do
    if two CRs choose the same channel then
        Compute interference and utilities based on initial actions;
    end
end
for $t=1:T$ do
    Players decide their strategies simultaneously;
    Players make moves and compute interference and utilities for all channels K;
    Choose the channel with the maximum utility value;
    Compute the best utility of all players till the current iteration;
    if previous iteration yields higher utility than the current utilities then
        move to the channel with higher utility;
    end
    Choose the action set corresponding to the weight function $W(t)$ for the next iteration;
    Repeat for all iterations or till players no longer change their strategies and convergence is achieved;
end

Algorithm 2: Iterative algorithm for the convergence of strategies
knowledge of previous iterations and may actually lead to a lower value due to the moves made by the opponents. Thus, an apparently suitable channel may not remain so after the completion of simultaneous game and convergence becomes hard to achieve. This algorithm allows players to utilize the knowledge of previously played games and use it as the initialization of next game. Since all players perform this operation, a weight function $W(t)$ can be formulated which is employed as the decision function, achieving network convergence.

5.7 Numerical Results for Simultaneous Learning

The proposed simulation setup consists of $N=20$ nodes, which are competing for $K=4$ available channels. The network topology is two-dimensional uniformly distributed
nodes in a square region of $200m^2$. The game is played repeatedly for some time to observe the convergence. Each iteration serves as a source of learning for the next iterations. Based on each iteration, the users evaluate the probability of each channel according to the number of players opting for it in the previous iteration and utilize this probability to increase or reduce the price of that channel in the next iteration. The final decision is made by the SU based on the best utility provided by a channel among all possibilities. The results are shown in Figure 5.3 that demonstrates the steady state solution for simultaneous game. It can be seen from the figure that the convergence time for the game is less than or equal to $N$ and the players are able to establish their strategies according to the network size.

Figure 5.4 is a demonstration of iterative channel switching observed over time.
We can see that initially almost all users change their strategies. As the time passes, the players get more informed through learning and fewer users need to change their strategies. This leads to convergence, observed as zero times switching in the figure.

Figure 5.5 provides a comparison of average network capacity obtained by employing learning schemes. The figure elaborates the learning schemes when a fixed transmission power is assigned to SUs (shown in red). It is seen that network performance is considerably improved when learning is employed in combination with the water-filling power allocation. The third graph shown in green represents the no-learning scenario and performs better than fixed power learning scheme. However, it should be noted that the no-learning game is a sequential moves game and provides better performance at the cost of additional delay. This is depicted in Figure 5.5, which demonstrates the increase in convergence time with increasing number of SUs.

Figure 5.7 shows the average transmission power of SUs on each channel.

Figure 5.8 depicts the average convergence rate for twenty different network topologies. It is to demonstrate the fact that network topology does not effect the convergence behavior of the proposed formulation. Hence the proposed game model is valid for all kinds of networks with any number of PUs and SUs.

This chapter presents a potential game for spectrum allocation in ad hoc cognitive radios. The game is played in a cooperative environment with all players deciding their strategies simultaneously. The cooperation is enforced among selfish players by devising a pricing scheme based on the level of interference for each channel. In order to achieve a stable and meaningful solution to the spectrum allocation problem, learning technique is employed combination with the pricing. This learning, based on the weighted knowledge of historical data assists in avoiding conflicting and imprudent
Figure 5.5: Comparison of average capacity for different schemes

Figure 5.6: Comparison of convergence for sequential and simultaneous moves games
Figure 5.7: Average Transmission power of users on the channels

Figure 5.8: Average convergence of strategies for spectrum allocation for 20 different network topologies
choices. To achieve a stable solution for an otherwise unsteady simultaneous moves system, we employ the concepts of learning to compensate the lack of information. This assists in achieving convergence by determining a stable solution. The time taken by the players to establish their strategies is directly proportional to the number of SUs involved in the game, which is a considerable improvement as players do not have to wait for making a decision. In sequential games, the convergence time for each user is proportional to the square of network size $O(N^2)$ due to the delays in decision making. This work deals with fixed transmit power levels and optimal power allocation through water-filling. The water-filling power allocation performs better in terms of network performance and reduced delay.
Chapter 6

Conclusion

This thesis presents game theoretic methods and algorithms to resolve bandwidth allocation and energy utilization issues in cognitive radio ad hoc networks. The spectral efficiency and power allocation are dependent upon interference levels in the network, therefore we emphasize on controlling these vital parameters and reducing conflicts. The proposed solution is obtained by employing cooperative potential game theory that assists in achieving cooperation as a two step algorithm. In the first step each player keeps a check on the interference level it creates for the peers. In the second step of cooperation, the players are required to pay a price according to the performance in the network. In this way, players that can harm the network are charged a higher price. This pricing scheme also assists in discouraging malicious players. The objective is to find a stable solution that allows players to conserve energy and efficiently avail spectrum opportunities.

We discuss various potential games for ad hoc cognitive radio networks and formulate the potential functions for these games. The underlay and overlay access schemes are considered for optimal network performance in a cooperative environment through the potential function as the decision making parameter. In this way
the players that are in a pursuit to opt for the most suitable strategy achieve optimum network performance, benefiting all players. The formulation of the potential function changes for the overlay and underlay schemes. The main advantage offered by underlay systems is the additional opportunities that can be availed even in the presence of PU, and the transmit power levels must be kept so as to minimize the interference for PU. We observe the performance of network for two different pricing games. In the first game, we use price as the parameter for channel choice. The unlicensed users opt for a cheaper channel in order to maximize their profit. This game converges for a higher transmission power and some channels become more congested than others. The second game involves users, which improve their payoff not by price but by optimizing their power levels. The opted strategy may cost slightly higher, but it is more resource optimized. In this game, the users converge to channels, which are not overcrowded, making more efficient utilization of all bandwidth. The optimum power allocation ensures most appropriate interference levels.

The effects and constraints of licensed users are considered in order to avoid disruptions in the licensed users transmission. The problem of licensed communication is approached as a Stackelberg game, where the PU is the leading authority to set transmission parameters for the follower cognitive players. The CR nodes choose their strategies for the maximum value of the proposed potential function. These strategies are opted for the improved potential function to provide better choices for the CR players, the PU, as well as the entire network. The convergence and average capacity for proposed scheme is improved as compared to existing methods [1]. The underlay and overlay scenarios are separately discussed with underlay providing more spectrum opportunities. The power and action set opted in these games provide better network
performance in terms of potential function.

We employ learning in cooperative potential game for cognitive radio ad hoc networks, which enables users to choose wisely based on the history. Moreover, the learning function is included as a potential function parameter. The players opt for a suitable strategy and achieve Nash equilibrium by utilizing history to improve network performance. Learning provides the benefit of incorporating simultaneous moves, which is otherwise not possible in a conventional potential game setup. Hence, the incorporation of learning algorithm in CR games reduces the delay in decision making process and provides all users the freedom to choose channels independently. This learning, based on the weighted knowledge of historical data, assists in avoiding conflicting and imprudent choices. In order to achieve a stable solution for an otherwise unstable simultaneous moves system, the concepts of learning compensate the lack of information about the strategies of the opponents. The time taken by the players to establish stable strategies is directly proportional to the number of SUs involved in the game. In sequential games, the convergence time for each user is proportional to the square of network size due to the delays in decision making, which is reduced considerably in simultaneous moves game.
Bibliography


