# Table of Contents

Abstract v

Acknowledgements vii

Appendix ix

Table of Contents xi

1 Introduction & Preliminaries 1

1.1 Introduction ............................................. 1

1.1.1 Brief history of differential equations .................. 1

1.1.2 Different approaches for the conservation laws ........... 5

1.2 Preliminaries ........................................... 6

1.2.1 Conserved quantities by using partial Lagrangian ........ 6

1.2.2 Geometry ............................................ 8

1.2.3 \((1 + n)\)-dimensional Laplacian on curved surfaces ...... 10

1.3 Outline of the thesis ..................................... 12

2 Conservation Laws for Heat Equation on Curved Surfaces 13

2.1 Introduction ............................................. 13

2.2 Partial Noether operators of \((1 + n)\)-dimensional heat equation .... 14

2.2.1 \((1 + 2)\)-dimensional heat equation .................... 14

2.2.2 \((1 + 3)\)-dimensional heat equation .................... 17

2.2.3 \((1 + n)\)-dimensional heat equation .................... 19

2.3 Conservation laws for heat equation on different curved surfaces .... 20

2.3.1 Conservation laws for heat equation on cone ................ 21

2.3.2 Conservation laws for heat equation on a sphere ........... 23

2.3.3 Conservation laws for heat equation on a torus ............ 25

2.3.4 Conservation laws for heat equation on 2-dimensional flat surface .... 28

2.3.5 Conservation laws for heat equation on 3-dimensional sphere ...... 31

2.3.6 Conservation laws for heat equation on 3-dimensional flat surface .... 33

2.4 Concluding Remarks ...................................... 35
3 Conserved Quantities for a Class of \((1+n)\)-Dimensional Heat Equation

3.1 Introduction .......................................................... 37
3.2 Conserved vectors for \((1+n)\)-dimensional linear evolution equation .... 41
  3.2.1 The \((1+1)\)-dimensional linear diffusion equation .................. 41
  3.2.2 For the \((1+2)\)-dimensional linear evolution equation ............. 43
  3.2.3 Extension to the \((1+n)\)-dimensional linear evolution equation .... 45
3.3 Application .......................................................... 45
3.4 Concluding Remarks ................................................ 54

4 Derivation of Conservation Laws for \((1+2)\)-dimensional Wave Equation on Curved Surfaces

4.1 Introduction .......................................................... 55
4.2 Determining equations for the \((1+2)\)-dimensional wave equation on curved surfaces ................................................... 56
  4.2.1 Partial Noether operators and conservation laws for Eq. (4.1.1) on different curved surfaces ................................. 58
  4.2.2 Conservation laws for wave equation on sphere ....................... 58
  4.2.3 Conservation laws for the wave equation on a cone ................... 64
  4.2.4 The wave equation on a flat surface ................................ 71
4.3 Concluding Remarks ................................................ 77

5 Effect of Background Geometry on Symmetries of the Nonlinear \((1+2)\)-dimensional Heat Equation and Reductions of the TDGL Model

5.1 Introduction .......................................................... 79
5.2 Group classification of Eq. (5.1.1) .................................... 82
  5.2.1 The nonlinear \((1+2)\)-dimensional heat equation on plane .......... 82
  5.2.2 The nonlinear \((1+2)\)-dimensional heat equation on sphere ........ 84
  5.2.3 The nonlinear \((1+2)\)-dimensional heat equation on torus .......... 86
5.3 “\(f(u)\)” vs “underlying geometry” ................................... 87
  5.3.1 For sphere ......................................................... 87
  5.3.2 For torus .......................................................... 87
5.4 The TDGL model on curved surfaces .................................. 88
  5.4.1 The TDGL model on sphere ........................................ 88
  5.4.2 The TDGL model on torus ........................................ 92
5.5 Conclusion .......................................................... 94

6 Group Classification of \((1+n)\)-dimensional Klein-Gordon Equation and the Nonlinear Wave Equation on Curved Surfaces

6.1 Introduction .......................................................... 98
6.2 Group classification of the \((1+n)\)-dimensional Klein-Gordon equation .... 101
  6.2.1 Case 1: \(n = 1\) .................................................. 101
  6.2.2 Case 2: \(n = 2\) .................................................. 102
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2.3</td>
<td>Case 3: $n = 3$</td>
<td>105</td>
</tr>
<tr>
<td>6.2.4</td>
<td>General Case</td>
<td>108</td>
</tr>
<tr>
<td>6.3</td>
<td>Lie point symmetry generators</td>
<td>109</td>
</tr>
<tr>
<td>6.3.1</td>
<td>$f(u) \neq ae^{bu} + c$</td>
<td>109</td>
</tr>
<tr>
<td>6.3.2</td>
<td>$f(u) = ae^{bu} + c$</td>
<td>111</td>
</tr>
<tr>
<td>6.3.3</td>
<td>$\tau_t = 0$</td>
<td>113</td>
</tr>
<tr>
<td>6.4</td>
<td>Group classification of the $(1 + 2)$-dimensional Klein-Gordon equation on curved surfaces</td>
<td>113</td>
</tr>
<tr>
<td>6.4.1</td>
<td>The $(1 + 2)$-dimensional Klein-Gordon equation on a sphere</td>
<td>114</td>
</tr>
<tr>
<td>6.4.2</td>
<td>The $(1 + 2)$-dimensional Klein-Gordon equation on torus</td>
<td>119</td>
</tr>
<tr>
<td>6.5</td>
<td>Remarks</td>
<td>123</td>
</tr>
<tr>
<td>6.5.1</td>
<td>Conclusion</td>
<td>123</td>
</tr>
<tr>
<td>7</td>
<td>Conclusions</td>
<td>125</td>
</tr>
<tr>
<td>7.1</td>
<td>Future Directions</td>
<td>126</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>128</td>
</tr>
</tbody>
</table>