THE CHOU-YANG MODEL FOR HADRON-NUCLEUS AND NU insert NUCLEUS-NUCLEUS ELASTIC REACTIONS AT HIGH ENERGIES

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Abstract

This thesis deals with the various characteristics of nucleus-nucleus and hadron-nucleus elastic scattering at high energies. After surveying the literature on experimental measurements of these reactions, the form factors of proton, $\alpha$-particle and deuteron which are used as input in the Chou-Yang model have been discussed in detail. It is shown that nucleus-nucleus and hadron-nucleus elastic scattering can be explained by using electromagnetic form factors as input in the pristine Chou-Yang model only up to the diffraction peak region and breaks down in the vicinity of the dip and beyond. The generalised Chou-Yang model which is based upon multiple diffraction theory is used to explain the existing experimental data for these reactions at high energies. It is observed that the data for $aa$ and $pa$ elastic scattering is explained by using generalized Chou-Yang model. However, the model is unable to reproduce the entire structure in $dd$ elastic scattering at high energies. The basic conjecture of Chou and Yang that the hadronic form factor is proportional to the electromagnetic form factor, is then critically examined. We have proposed energy dependent hadronic form factors of deuteron and alpha, different from their electromagnetic form factors. It is shown that $dd$, $pd$, $aa$ and $pa$ elastic scattering can be explained over the entire measured region by using energy dependent hadronic form factors of deuteron, proton and alpha in the pristine Chou-Yang model. The agreement between the predicted values and the experimental data for the elastic scattering at all momentum transfers suggests that the basic conjecture of Chou and Yang that the hadronic form factor is proportional to the electromagnetic form factor is only an approximation of the true picture.
To my parents,

who inspired me to achieve all that I can and
taught me to never lose my sense of humour
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Statement

This thesis which is being submitted for the degree of Ph.D. in the University of the Punjab does not contain any material which has been submitted for the award of any other degree or diploma in any University and, to the best of my knowledge and belief, neither does this thesis contain any material published or written previously by another person, except when due reference is made to the source in the text of the thesis.

Following is the list of my research publications based on the material presented in this thesis:

1. Dr. Fazal-e-Aleem, Shaukat Ali, Mohammad Saleem, Aziz-ul-Haq Qureshi and Mohammad Rafique

2. Fazal-e-Aleem and Shaukat Ali

3. Fazal-e-Aleem, Shaukat Ali and Mohammad Saleem

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5. Fazal-e-Aleem, Shaukat Ali and Mohammad Saleem
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CHAPTER 1

INTRODUCTION
INTRODUCTION

The study of high energy hadron-nucleus and nucleus-nucleus elastic scattering has produced very fascinating results which are very significant in understanding the dynamics of strong interactions. For $p\alpha$, $\alpha\alpha$, $pd$ and $dd$ elastic scattering, the measurements of total and elastic differential cross sections, $\sigma_t$ and $d\sigma/dt$, and the local slope parameter $B$ at various energies have made the problem very appealing.

The efforts of theoretical physicists are to develop a model which can explain various characteristics of hadron-nucleus and nucleus-nucleus elastic scattering at high energies. The Chou-Yang model [1] which is based on the eikonal picture, is one such model. This model is based on geometrical considerations in which hadrons are treated as extended objects with internal structure.

Geometrical (or eikonal) picture has firm theoretical foundations in different areas of physics. In 1968, Chou and Yang [1] extended the scope of the geometrical picture to High Energy Physics. Since then a large number of papers have been written to explain various aspects of hadron-hadron, hadron-nucleus and nucleus-nucleus elastic scattering. Although a piecemeal explanation of the experimental data exists for these reactions, a comprehensive explanation is elusive.

The Chou-Yang model employed an eikonal (or opacity or blackness) which was related to the distribution of matter inside the colliding particles. In its pristine form, the constancy of the total cross section was a built in feature. Later on the rising trend of the total cross-section was accommodated in this model in a natural way by Hayot and Sukhatme [2]. The most prominent feature of the pristine Chou-Yang model is the absence of free parameters in the model. The model has been applied successfully to explain hadron-hadron, hadron-nucleus and nucleus-nucleus elastic scattering in the diffraction peak region [3]. Beyond the dip region the
pristine Chou-Yang model gives multi-dip structure which is inconsistent with the experimental data.

In order to explain the experimental data beyond the diffraction peak region, Glauber and Velasco [4] have generalised the Chou-Yang model and fitted the data for pp elastic scattering at $\sqrt{s} = 546$ GeV. The generalised Chou-Yang model [4] is based on the multiple diffraction scattering theory and takes into account anisotropy of the scattering. The generalised Chou-Yang model has been successful in explaining the data for hadron-hadron elastic scattering at high energies and up to large momentum transfers. It also explains the dip structure observed in these reactions. In view of the success of the generalised Chou-Yang model, we have used the same to explain the entire structure for hadron-nucleus and nucleus-nucleus elastic scattering. It is observed that the data for pa and aa elastic scattering is explained by using generalised Chou-Yang model [5]. However, the generalised model when applied to dd elastic scattering does not give satisfactory results [6]. The model is unable to reproduce the entire structure in dd elastic scattering at $\sqrt{s} = 53$ (GeV) [6]. Thus in the framework of the Chou-Yang model with electromagnetic form factor as input, the entire structure in dd elastic scattering can not be explained.

As form factors of the interacting particles play an important role in calculating the scattering amplitude, careful analysis of these is very essential. The hadronic form factor is related to the distribution of matter inside a hadron/nucleus. Chou and Yang initially assumed [1] that the matter distribution inside a hadron is the same as its charge distribution. Failure of pristine and generalised Chou-Yang models poses a question of considerable importance whether or not the electric charge distribution is the same as the nuclear matter distribution. Several different parameterizations [7] of opacity which have been proposed in the literature to accommodate the experimental data, point towards this problem.

Failure of the geometrical picture to explain the entire structure in dd elastic scattering thus suggests that either the assumption regarding the
multiple scattering needs modifications or the concept of proportionality between the electromagnetic form factor and hadronic form factor is only an approximation. Saleem et al [8] while explaining the data for $\bar{p}p$, $aa$ and $pa$ elastic scattering followed the second line of arguments. In their recent work, Chou and Yang [9], also following the second line of arguments, have proposed an energy dependent hadronic form factor. They have thus extended the scope of the geometrical model and have proposed hadronic form factor of the proton which was used in the geometrical model to fit the elastic scattering data at high energies. They have given a good fit to the $\bar{p}p$ data at ISR and Collider energies. On similar lines, by assuming the hadronic form factor of deuteron to be energy dependent, we have fitted the data for $dd$ elastic scattering at all available energies and in the entire measured region. Our expression for the hadronic form factor of the deuteron is different from its electromagnetic form factor. We have then extended this concept to $aa$ elastic scattering and again find a good fit to the available experimental data. The results obtained are compared to those of the generalized Chou-Yang model which takes into account the anisotropy of the scattering.

In order to further verify the validity of the proposed hadronic form factors of deuteron and alpha we have also fitted the data for $pd$ and $pa$ elastic scattering at various energies. A good fit to these reactions thus strongly suggests that form factors are energy dependent i.e the hadronic matter is expanding with energy. The hadronic form factors of $a$, deuteron and proton have also been compared to their electromagnetic form factors.

Physics underlying the energy dependent form factors has also been taken up in our study of hadron-nucleus and nucleus-nucleus elastic scattering.

In order to give a detailed account of this problem we have subdivided this thesis into five chapters. Second chapter gives a review of the experimental data on these reactions while in third chapter the form factors of deuteron, alpha and proton has been discussed. The Chou-Yang
model, a review of theoretical explanations by various authors, predictions of generalised Chou-Yang model along with electromagnetic form factors and predictions of pristine Chou-Yang model along with proposed energy dependent hadronic form factors are taken up in chapter 4. Final chapter envisages the physical ideas which are underlying in elastic scattering of nucleons and nuclei.
Chapter 2

SURVEY OF LITERATURE ON EXPERIMENTAL MEASUREMENTS
Survey of Literature on Experimental Measurements

A series of experiments on alpha-alpha, proton-alpha, deuteron-deuteron and proton-deuteron elastic scattering at high energy have been performed at CERN, Fermilab, Brookhaven, Surpukho and PPA (Princeton-Pennsylvania Accelerator). These experiments have been performed for alpha-alpha elastic scattering at the centre of mass energy $\sqrt{s} = 126$ GeV in the momentum transfer squared range $0.05 < -t < 0.8$ (GeV/c)$^2$ and for proton-alpha elastic scattering at $\sqrt{s} = 89$ GeV for $-t$ ranging from 0.052 to 0.7125 (GeV/c)$^2$ and at the incident laboratory energy from 45 to 400 GeV in the momentum transfer squared region $0.003 < -t < 0.52$ (GeV/c)$^2$. For deuteron-deuteron elastic scattering, these experiments cover laboratory momenta from 0.68 to 7.9 GeV/c in the range $0.03 < -t < 2.0$ (GeV/c)$^2$ and at $\sqrt{s} = 53, 63$ GeV in the region from $-t = 0.06$ (GeV/c)$^2$ to 1.4 (GeV/c)$^2$ and from $-t = 0.08$ (GeV/c)$^2$ to 1.41 (GeV/c)$^2$, respectively, and for proton-deuteron elastic scattering, at incident proton beam momenta from 50 to 400 GeV/c in the range $0.013 < -t < 0.14$ (GeV/c)$^2$ and at $\sqrt{s} = 53, 63$ GeV in the range $0.06 < -t < 1.65$ (GeV/c)$^2$ and $0.08 < -t < 1.85$ (GeV/c)$^2$, respectively. In the present section we describe these measurements briefly.

2.1 Elastic differential cross section and other characteristics of Alpha-Alpha scattering

The storage of $\alpha$ particles in the CERN ISR enabled Ambrosio et al [10] to study $\alpha \alpha$ interaction at unprecedentedly high centre-of-mass energies. The first high energy results, at $\sqrt{s} = 126$ GeV, were published by them [10] in 1982. The measurements were made in the range $0.05 < -t < 0.8$ (GeV/c)$^2$ with a resolution of $\pm 0.02$ (GeV/c)$^2$. The differential cross
section data are shown in Fig.1. The errors are statistical only. The figure exhibits a first minimum at \( t = 0.10 \pm 0.01 \) (GeV/c)\(^2\) and a second one at \( t = 0.38 \pm 0.02 \) (GeV/c)\(^2\). The slope \( B \) in the range \( t = 0.05 \) to 0.07 (GeV/c)\(^2\) is found to be 100 \pm 10 (GeV/c)\(^2\). This is given in Table 1. This group [10] obtained the total and integrated cross-section values as \( \sigma_\tau = 250 \pm 50 \) mb and \( \sigma_{\mu} = 45 \pm 15 \) mb, which have also been given in Table 1. An independent estimate of this cross section, based on the measured luminosity-monitor cross section, is \( \sigma_\tau = 295 \pm 40 \) mb. This is also given in Table 1.

In 1985, Akesson et al [11] also published results, which come from the final ISR measurements of the \( \alpha \)-particle collisions, at \( \sqrt{s} = 126 \) GeV, but in the limited range 0.05 < \( t < 0.19 \) (GeV/c)\(^2\). These are shown in Fig.2. The \( t \) range covered, though small, includes the interesting region of the minimum in the differential cross-section, and the resolution in \( t \) of \( \pm 0.005 \) (GeV/c)\(^2\) represents a significant improvement over previous data. The dip occurs at \( t = 0.098 \pm 0.002 \) (GeV/c)\(^2\). The differential cross-section measurements of Ambrosio et al [10] and Akesson et al [11] are in excellent agreement above \( t = 0.14 \) (GeV/c)\(^2\), but are not mutually consistent for \( t < 0.14 \) (GeV/c)\(^2\). It may be noticed, however, that the measurements in Ref.11 are of superior resolution. Akesson et al [11] also obtained a slope \( B = 72 \pm 6 \) (GeV/c)\(^2\) and a total cross section \( \sigma_\tau = 280 \pm 70 \) mb, where the error includes both the statistical and scale uncertainties. These values of \( \sigma_\tau \) and \( B \) are also given in Table 1. Their result for \( \sigma_\tau \) agrees well with the value 250 \pm 50 mb obtained earlier by Ambrosio et al [10].

Owen et al [12] have measured the total interaction cross section for \( \alpha \alpha \) scattering at \( \sqrt{s} = 126 \) GeV. The result obtained for the total cross section, viz., \( \sigma_\tau = 315 \pm 18 \) mb, is an improvement on the precision of earlier measurements. The slope \( B \) for \( t < 0.07 \) (GeV/c)\(^2\) is 87 \pm 4 (GeV/c)\(^2\), while the integrated cross section is \( \sigma_{\mu} = 58 \pm 6 \) mb. The statistical and systematic errors have been combined in these results.
Having measured both the total and the elastic cross sections, they estimated the ratios \( \sigma_e/\sigma_T \) and \( B/\sigma_T \), and obtained the following results:
\[
\sigma_e/\sigma_T = 0.184 \pm 0.004,
\]
\[
B/\sigma_T = 0.276 \pm 0.020 \text{(GeV/c)}^{-2}/\text{mb},
\]
\[
<r^2>^{1/2} = 2.60 \pm 0.06 \text{ fm},
\]
where \( r \) is the radius of interaction. The values of \( \sigma_T, \sigma_e, \) and \( B \) are given in Table 1.

Bell et al [13] have also measured the differential cross section for \( \alpha \alpha \) elastic scattering at \( \sqrt{s} = 126 \text{ GeV} \) for \(-t\) from 0.056 to 0.7625 \text{(GeV/c)}^2. Since these results are not consistent with other measurements, we have not considered them.

### 2.2 Elastic differential cross section and other characteristics of Proton-Alpha scattering

A study of \( p\alpha \) elastic scattering at small angles was made in the WA9 experiment at CERN SPS by Burq et al [14] in 1981. The differential cross-section was measured for \(-t\) ranging from 0.008 to 0.05 \text{(GeV/c)}^2 at momenta from 100 to 300 GeV/c. The cross sections were normalized absolutely with a precision of 2%, which made it possible to determine, through the optical theorem, the total cross section of the \( p\alpha \) interaction. Representative experimental \( d\sigma/dt \) data for \( p\alpha \) are shown in Fig.3. The values of the total cross section \( \sigma_t \), and the slope parameter \( B \) for \( p_L = 100 \) to 300 GeV/c are given in Table 2.

In 1981, Bujak et al [15] determined the differential cross-section for \( p\alpha \) elastic scattering for incident laboratory energy from 45 to 400 GeV in the range \( 0.003 < -t < 0.52 \text{ (GeV/c)}^2 \) by means of the internal gas-jet target technique. Specimen differential cross section data at 301 and 393 GeV are shown in Figs.4(a) and 4(b). The differential cross section drops 4-5 orders of magnitude in the dip at \(-t = 0.22 \text{ (GeV/c)}^2 \) and has a subsequent rise to
a secondary maximum at $-t = 0.33 \text{ (GeV/c)}^2$. The values of the total cross section $\sigma_t$ and the ratio $\rho$ of the real and imaginary parts of the forward scattering amplitude are shown in Table 2. Average values of the slope parameter $B$ of the diffraction peak in different $t$ intervals are also given in Table 2. The shrinkage in the differential cross-section is found to be twice as fast as that in the proton-proton case.

The differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV was measured in 1982 by Ambrosio et al [10]. This has a slope $B = 41 \pm 2 \text{ (GeV/c)}^2$ for $0.05 < -t < 0.18 \text{ (GeV/c)}^2$ and a minimum at $-t = 0.20 \pm 0.02 \text{ (GeV/c)}^2$. The extrapolation of this slope gives $\sigma_t(p\alpha) = 130 \pm 20 \text{ mb}$ via the optical theorem (assuming that the real part of the forward amplitude is negligible) and $\sigma_e(p\alpha) = 20 \pm 4 \text{ mb}$. The numerical values of $p\alpha$ differential cross section are shown in Fig.5. The values of $\sigma_t$ and $B$ are given in Table 2.

Bell et al [13] have also measured the differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV for $-t$ ranging from 0.2375 to 0.7125 (GeV/c)$^2$. Their results are also shown in Fig.5.

### 2.3 Elastic differential cross section and other characteristics of Deuteron-Deuteron Scattering

The first experiment [16] on deuteron-deuteron elastic scattering was performed in 1969 at the Princeton-Pennsylvania Accelerator (PPA) from 0.68 to 2.12 GeV/c laboratory momenta. The second one [17] was performed at 7.9 GeV/c in the 80-in bubble chamber at the Brookhaven National Laboratory in the electrostatically separated beam. The measurements were made in the range $0.03 < -t < 0.7 \text{ (GeV/c)}^2$. The differential cross sections at 7.9 GeV/c are shown in Fig.6. For momentum transfers below 0.2 (GeV/c)$^2$, there is a sharp forward peak which falls off approximately as $e^{-4st}$. In the region immediately above 0.2 (GeV/c)$^2$ the low
statistic of the data indicates only the presence of a break in the slope. The cross section decreases exponentially as $e^{-2.5t}$ beyond the break out to the limit of $t \sim 0.8$ (GeV/c)$^2$. In 1979, Goggi et al [18] reported experimental results on deuteron-deuteron elastic scattering at the two highest ISR energies at $\sqrt{s} = 53$ GeV and $\sqrt{s} = 63$ GeV. The data cover the single- and multiple-scattering regions over a wide interval of four-momentum transfer $t$. They observed clear evidence for a substantial $t$-dependent contribution of inelastic intermediate states in the multiple-scattering region, as well as in single scattering. The $t$-dependence of the elastic cross section was measured in the range from $-t = 0.06$ GeV$^2$ to 1.4 GeV$^2$ at $\sqrt{s} = 53$ GeV and from $-t = 0.08$ GeV$^2$ to 1.41 GeV$^2$ at $\sqrt{s} = 63$ GeV. A narrow interference minimum is clearly observed at both energies. The exponential slopes below and above the interference region are about 56 GeV$^2$ and 6.5 GeV$^2$, respectively. The position of the minimum and the corresponding differential cross section are found to be

$$-t_{\text{min}} = (0.179 \pm 0.005) \text{ GeV}^2,$$

$$d\sigma/dt_{\text{min}} = (37.28 \pm 7.86) \mu\text{b}/\text{GeV}^2,$$ at $\sqrt{s} = 53$ GeV, and

$$-t_{\text{min}} = (0.183 \pm 0.005) \text{ GeV}^2,$$

$$d\sigma/dt_{\text{min}} = (33.78 \pm 8.57) \mu\text{b}/\text{GeV}^2,$$ at $\sqrt{s} = 63$ GeV

The differential cross section measurements at $\sqrt{s} = 53$ GeV and at $\sqrt{s} = 63$ GeV are shown in Fig.7 and Fig.8, respectively.

### 2.4 Elastic differential cross section and other characteristics of Proton-Deuteron Scattering

Several experiments [19] were performed in the late sixties to measure the differential cross section for pd elastic scattering in the 1-10 GeV region, the main idea being to test in a simple case the Glauber eikonal approach [20] to hadron-nucleus scattering and to extract information on the real part of the hadron-nucleus amplitude. In 1973, Bartenev et al [21]
observed shrinkage of the deuteron diffraction cone with increasing energy during the measurement of pd elastic scattering up to 70 GeV/c incident momentum at Serpukhov. At Fermilab, in 1975, Akimov et al [22] extended these measurements from 50 to 400 GeV/c for incident proton beam momenta in the four-momentum transfer squared region $0.013 < -t < 0.14$ (GeV/c)$^2$ using the same basic technique of a gas-jet target [23] and the detection of slow recoils by solid state detector [24,25]. The jet could be pulsed at any desired time in the acceleration cycle, thus allowing data to be taken over a wide range of incident proton momentum. They observed shrinkage of the diffraction cone with increasing energy equal to $(0.94 \pm 0.04) \ln (s/1 \text{ GeV}^2)$ (GeV/c)$^2$. This shrinkage was greater than that observed in pp elastic scattering. The ratio of the elastic to the total cross section was approximately 0.1 and independent of energy above $\sim 150$ GeV. In 1978, Armitage et al [26] published the results of $d\sigma/dt$ measurements of pd elastic scattering at $s = 2800$ GeV$^2$. The measurements were made at the CERN ISR with a single arm spectrometer. The final-state deuteron were detected at angles between 14 and 25 mrad with respect to the incident deuteron direction. This corresponds to a range of $0.42 < -t < 0.15$ GeV$^2$, where $t$ is the square of the four-momentum transfer between the initial and final-state deuterons. This is the region in which rescattering processes (i.e. those in which the emerging proton has interacted with both the nucleons in the deuteron) are expected to play a significant role. The semi-classical picture of these rescattering processes is given by the Glauber theory [20], but divergences from this picture are expected at high energies. It was therefore of interest to explore this $t$ region, something which had not previously been done at high energies. The data for pd elastic scattering showed a gradually flattening $t$ dependence between $0.15 > -t > 0.42$ GeV$^2$. Armitage et al [26] compared the data with the FNAL data of Akimove et al [22] and observed very little energy dependence in the $t$ distribution between $\sqrt{s} = 38$ and $\sqrt{s} = 53$ GeV$^2$. 
In 1979, Goggi et al [18] reported experimental results on proton-deuteron elastic scattering measured at the two highest ISR energies, $\sqrt{s} = 53$ GeV and $\sqrt{s} = 63$ GeV. The data were obtained using the split Field Magnet detector at the CERN intersecting storage rings. The data covered the single and multiple-scattering regions over a wide interval of four-momentum transfer i.e. from $-t = 0.06$ GeV$^2$ to $-t = 1.65$ GeV$^2$ at $\sqrt{s} = 53$ GeV and from $-t = 0.08$ GeV$^2$ to $-t = 1.85$ GeV$^2$ at $\sqrt{s} = 63$ GeV. Exponential fits to the data in limited t intervals yield approximate values of the slopes in the single- and double-scattering regions, about 30 GeV$^2$ and 5 GeV$^2$, respectively. They compared lower energy data points (at $\sqrt{s} = 53$ GeV) with Fermilab [22] and ISR [26] results on an expanded t scale. The data match up well with one another. They found clear evidence for a substantial t-dependent contribution of inelastic intermediate states in the multiple-scattering region, as well as in single scattering. The differential cross section over the entire t range at $\sqrt{s} = 53$ and $\sqrt{s} = 63$ GeV are shown in Fig.9 and Fig.10, respectively.
Chapter 3

FORM FACTORS
Form factors play a very important role in the calculation of the scattering amplitude within the framework of the Chou-Yang model, which in turn yields various characteristics of the measured parameters. We will therefore discuss them in detail in this chapter.

The electrons and muons are point objects, within the small-distance limits of $10^{-14}$ to $10^{-15}$ cm. Their interaction with the electromagnetic field is correspondingly simple. The hadrons, on the other hand, do not exhibit pointlike behaviour in high-energy interactions, but rather show structure on a scale of $10^{-13}$ cm. Historically the idea of drawing the pions and protons as a spatially extended objects is due to Fermi [27]. There is compelling evidence which suggests that there can be no explanation other than that the hadron is a composite object and has internal structure. The form factor was introduced historically to describe the scattering of point particles of extended objects, thus giving us useful information about the structure of the hadrons. These form factors, which can be determined experimentally, occur in the expressions for the corresponding scattering amplitudes and represent the effect of the finite size of the target.

In general, the form factor is related to the distribution of matter in space. The hadronic form factor consequently gives us information about the distribution of matter inside hadron. Thus any theory that claims to explain the structure of hadron should be able to give some idea of the hadronic form factors. Large amount of data exists on the electromagnetic form factors of hadrons (proton, pion, kaon, etc) and nuclei (deuteron, alpha, etc.). This consequently gives us some information about the charge distribution of a hadron. Unfortunately, there does not exist any expression of the hadronic matter distribution obtained from the first principles.
In the eikonal picture of Chou and Yang, it is assumed that the matter distribution inside a hadron is proportional to its charge distribution. We can then use the information about the charge distribution obtained from the experimental data for evaluating the scattering amplitude of the interacting particle. Comparison of the theoretical predictions and the experimental data for the differential cross section supports this assumption only in the diffraction peak region. Beyond the diffraction peak region (dip region) the disagreement of the theory and experiment suggests further modifications about this assumption.

It thus gives rise to a question of considerable importance whether or not the electric charge distribution is the same as the hadronic/nuclear matter distribution. As pointed out earlier, although at ISR energies the Chou-Yang Model with electric charge form factor as input has yielded impressive quantitative agreement with experiments, yet as the energy increases, deviation from the strict interpretation of the geometrical picture become quite noticeable. Several different parametrizations of opacity have been proposed in the literature to accommodate the experimental fact that the percentage of increase for the opacity is much faster at larger $b$ than that at smaller $b$. Lombard and Wilkin [28] suggested that it was possible that the matter distribution of the proton differed by a few percent from its charge distribution. They also suggested that there is probably some energy-dependent secondary mechanism, which plays an important role for large $t$. The comparison of results obtained by using the Chou-Yang model with the experimental data shows that the Chou-Yang conjecture which states that the hadronic form factor is proportional to the electromagnetic form factor is an approximation only and needs modification. Detailed discussion of these modifications will be taken up in the next chapter where we will be comparing the predictions of our calculations with the experimental data.

We will now describe the experimental data for the form factors of alpha, deuteron and proton. We will also take into account various parametrizations fitting the experimental data of these form factors.
3.1 Alpha Form Factors

For low momentum transfer, the electromagnetic form factor of alpha has been measured by a number of groups [29] while an extensive study up to $Q^2 = 20$ fm$^{-2}$ was made by Frosch et al [30] who observed a diffractive minimum and a secondary maximum around $Q^2 = 11.50$ and 18.0 fm$^{-2}$ respectively. The alpha form factor had already been found by Repellin et al [31] to deviate from a smooth Gaussian. McCarthy et al [32] studied the charge form factor of isotopes of helium and have verified the low-momentum-transfer measurements of Erich et al [29] and Frosch et al [30]. The experimental data on the alpha form factor have been parameterized by McCarthy et al [32] as

$$G_{1\alpha} = \{1 - (a^2 Q^2)^{0.5}\} \exp(-b^2 Q^2),$$

where $a = 0.316$ fm and $b = 0.675$ fm and by Lombard and Tellez-Arenas [33] as

$$G_{2\alpha} = 0.6345 \exp(-0.9 Q^2) + 0.1555(1 - 0.1157 Q^2) \exp(-0.1736 Q^2).$$

Both of these parameterisations agree with the experimental data quite reasonably, except that the parameterisation of McCarthy et al [32] gives a zero near $Q^2 = 10$ fm$^{-2}$ whereas the experimental data show only a dip in that region. A comparison of these two parameterisations with the data is shown in Fig.11. We have used the one proposed by McCarthy et al [32]. These expressions of McCarthy and Lombard and Tellez-Arenas when used in the eikonal picture give a satisfactory fit to the experimental data for the differential cross section only in the small momentum transfer region.

We have recently proposed that the hadronic form factor of alpha is only approximately equivalent to its charge form factor [34]. A simple
expression for the hadronic form factor of alpha particle, different from the electromagnetic form factor, which depends on energy is given below:

\[ G_\alpha = s^{-0.81}(1.03s^{14.23r} + 3.25t e^{14.73r} - 0.03s^{8.18t}) \]

In Fig.12 we have plotted the results obtained from this expression for three different energies i.e. √s = 47.68, 89 and 126 GeV. The data point have been taken from McCarthy et al [32] and Frosch et al [30]. The data obtained from the alpha hadronic form factor increases with the increase of energy. The proposed alpha hadronic form factor has a form similar to the alpha charge form factor measured experimentally except that the hadronic form factor has a dip at 7.5 fm⁻² where as the charge form factor has a dip at 11.50 fm⁻², as shown in Fig.12. This choice of the hadronic form factor gives a very good fit to the experimental data and suggests a change in the radius of the nucleus/hadron as energy increases.

### 3.2 Deuteron Form Factor

The deuteron is the simplest of the definitely known composite systems amongst nuclei and its electromagnetic form factor provides an ideal illustration of the continuity between nuclear and particle physics at the microscopic level [35]. The electromagnetic form factor of the deuteron at high momentum transfer has long been of interest for the information they contain on the short range nucleon-nucleon interaction and the transition from nucleon to quark degrees of freedom. Several groups have measured the structure functions A(t) and B(t) involving the deuteron form factors. The structure function A(t) which is a combination of the charge form factor \( G_c(t) \), the magnetic form factor \( G_M(t) \) and the quadrupole form factor \( G_Q(t) \) is given below:
\[ A(t) = G_c^2(t) + \frac{\{t^4G_0^2(t)\}/18M^4}{(t/6M^2)(1-t/4M^2)}G_m^2(t). \]

where \( M \) is the mass of the deuteron. The results of experimental measurements of \( A(t) \) [36] are plotted in Fig.13. It can be shown that, up to a good approximation, the above equation may be written as

\[ G_c(t) = \sqrt{A(t)} \]

for \(-t < < 4M^2 = 14 \text{ (GeV/c)}^2\). Extensive data on \( A(t) \) are available [36] in the spacelike region extending up to 6 \((\text{GeV/c})^2\). Therefore a formula for parametrizing \( A(t) \) can be developed. Parida [37], using a modified N/D method where \( N \) and \( D \) are functions representing anomalous and two point cut contributions, parameterised the deuteron charge form factor as follows:

\[ G_c(t) = (A(t))^{1/2} \]

where

\[ A(t) = e_0 \exp(-aZ_s(t))/(e_0 + e_1t + e_2t^2 + e_3t^3 + h(t) + m_{\pi}^2/m), \]

\[ Z_s(t) = \{\ln[\sqrt{-t/ta} + \sqrt{-t/ta + 1}]\}^2, \]

\[ a = 0.931, \quad t_a = 0.033 \text{ (GeV/c)}^2, \]

\[ e_0 = 1.0 \text{ (GeV/c)}^2, \quad e_1 = -6.507, \]

\[ e_2 = 74.289 \text{ (GeV/c)}^{-2}, \quad e_3 = 0.192 \text{ (GeV/c)}^{-4}, \]

\[ h(t) = 2q^3/(m^2) \{\ln[\sqrt(t/4m^2_{\pi}) + \sqrt(t/4m^2_{\pi} - 1)] -i\sqrt{3}/\sqrt{t} \]

with \( q = \sqrt(t/4 - m^2_{\pi}) \).
This parameterisation has quite good agreement with the experimental data which is available in the range $0 \leq -t \leq 6$ (GeV/c)$^2$. A simpler form of the deuteron electromagnetic form factor was also proposed by Fazal-e-Aleem and Shaukat Ali [6]. We propose the following expression:

$$ A(t) = \sum a_i \exp(b_i t), \quad i = 1, 2, 3 $$

where

$$ a_1 = 0.9691, \quad a_2 = 0.03, \quad a_3 = 0.0009 $$

$$ b_1 = 9.1, \quad b_2 = 1.35, \quad b_3 = 0.46 $$

The results of our parameterisation of the deuteron form factor are plotted in Fig. 13 together with the results of Parida et al [37]. It can be seen in the figure that the results of two parameterisations are in good agreement with the experimental data. It also has an asymptotic behaviour of the form

$$ \exp[-a(\ln t)^2]/t^3 $$

which is consistent with the power fall off $t^{-10}$ predicted by the dimensional counting rule [38] taking the deuteron as a six-quark system, and is supported by the asymptotic freedom and the quantum chromodynamics.

Before 1984, the experimental information regarding the magnetic form factor $B(t)$ of the deuteron had been limited to momentum transfers smaller than 14 fm$^{-2}$ [36, 39]. In 1985, Auffret et al [40] published the results of the deuteron magnetic form factor $B(t)$ for values of the momentum transfer squared between 7 and 28 fm$^{-2}$. The resulting values, along with the previous data, are shown in Fig. 14. The overall precision between 7 and 14 fm$^{-2}$, where previous data are available, is significantly improved.
The predictions of most of the theoretical models and the extrapolation of the data show that at higher $Q^2 = -t$, the magnetic form factor $B(Q^2)$ becomes more than two orders of magnitude smaller than $A(Q^2)$. For this reason, Arnold et al. [41] made measurements of $B(Q^2)$ close to $180^\circ$, where the contribution from $A(Q^2)$ is small. The deuteron magnetic form factor $B(Q^2)$ was measured at momentum transfers $Q^2 = 1.21$ to $2.77$ (GeV/c)$^2$ at the Stanford Linear Accelerator Centre by detecting electrons back scattered at $180^\circ$ in coincidence with the recoiling deuterons at $0^\circ$. These results for $B(Q^2)$ along with the data of Auffret et al. [40] and Cramer et al. [42] are also shown in Fig. 14. The new data for $B(Q^2)$ joins smoothly on to the previous data and show that the magnetic form factor of the deuteron continues to fall rapidly above $Q^2 = 1.2$ (GeV/c)$^2$. A shallow diffraction minimum beginning around $Q^2 = 1.8$ (GeV/c)$^2$ and secondary maximum around $Q^2 = 2.5$ (GeV/c)$^2$ can be seen. Several models have been proposed [43-52] to describe the magnetic deuteron form factor. However, quantitative agreement is not found for any prediction; those that agree below $Q^2 = 2$ (GeV/c)$^2$ are too low at higher $Q^2$, while those predictions that agree with the data above $Q^2 = 2$ (GeV/c)$^2$ are too low at lower $Q^2$.

As in the case of alpha form factor we have also proposed an expression for the deuteron matter distribution which is different from the electromagnetic form factor and depends on energy. We propose the following expression for the deuteron matter form factor:

$$G_d = s^{et+b\tau^2}(\Sigma d_i \exp(f_i t) \exp(\tau^2^2), \quad i = 1,2,3$$

where

$$a = 0.246 \quad b = 0.024 \quad c = -0.19$$

$$d_1 = 0.85 \quad d_2 = 0.14 \quad d_3 = 0.01$$

$$f_1 = 20.07 \quad f_2 = -0.47 \quad f_3 = -1.47$$
To compare the results obtained from newly proposed deuteron matter form factor with the experimental data for deuteron electromagnetic form factor, we have plotted the results for $\sqrt{s} = 6.12$ and 53 GeV in Fig. 15. The data points are from Ref. 36. As shown in Fig. 15, the results obtained from newly proposed matter form factor of the deuteron are higher than the corresponding values for its electromagnetic form factor.

3.3 Proton Form Factor

The electromagnetic form factors of the proton have been measured by a number of groups [53]. The specimen data from earlier measurements of electric form factor of proton are shown in Fig. 16. In 1986, Arnold et al. [54] have reported new measurements of elastic electron scattering from protons which significantly increase the precision of the data at large values of the square of the four-momentum transfer $(Q^2)$. The data, which are in agreement with previous measurements at low $Q^2$, extend to $Q^2 = 31.3 \text{ (GeV/c)}^2$. With some modest assumptions, these cross section measurements can be used to extract the proton magnetic form factor $G^M_p$, and consequently electric form factor $G^E_p$ by virtue of the scaling relation $G^E_p = G^M_p/\mu_p$, where $\mu_p$ is the magnetic moment of the proton with sufficient precision to allow a significant comparison with recent predictions of perturbative quantum chromodynamics.

At low momentum transfers [$Q^2 < 3 \text{ (GeV/c)}^2$], $G^E_p$ has been found to scale with $G^M_p$. If the form factor scaling continues, then at high $Q^2$ the contribution of $G^M_p$ to the cross section dominates over that of $G^E_p$. The contribution of $G^E_p$ to the cross section under this assumption is typically a few percent above $Q^2 = 5 \text{ (GeV/c)}^2$, and so moderate deviations from form factor scaling would have little effect on the extracted value of $G^E_p$ for most of the data of Ref. 54.
The results for $G_p^M/\mu_p$ are plotted in Fig. 16, scaled by $Q^2$. The data agree with previous measurements at low $Q^2$, reaching a broad peak near $Q^2 = 8 \text{ (GeV}/c)^2$, and then exhibit a significant decrease with increasing $Q^2$. A straight-line fit to the data between $Q^2 = 12.0 \text{ (GeV}/c)^2$ and $Q^2 = 31.3 \text{ (GeV}/c)^2$ shows a slope of $(-4.1 \pm 0.8) \times 10^{-3} \text{ (GeV}/c)^{-2}$ in that range.

This experiment was motivated in part by perturbative QCD predictions for the asymptotic behaviour of proton form factors. Lepage, Brodsky and their collaborators [55, 56] were able to calculate the evolution of $G_p^M$ with $Q^2$, but not its overall magnitude. Subsequently, Isgur and Llewellyn Smith [57] calculated the overall normalisation for the perturbative contribution to proton form factor using a symmetric nonrelativistic wave function and obtained results two orders of magnitude below experiment.

Recent advances in quantum chromodynamics have been based on the use of sum rules to estimate the moments of the hadron wave functions, including nonperturbative contributions. The proton wave function evaluated by this method appears to differ dramatically from the asymptotic form. Once a wave function has been found which has the moments predicted by QCD sum rules, the usual perturbative hard scattering formalism can be used to calculate specific properties, such as the proton form factors.

Chernyak and Zhitnitskii [58] have proposed a wave function which satisfies the sum rules and in which about 65% of the momentum of the proton is carried by one of its valence up quarks, with spin directed along the proton spin axis. Using this wave function, they calculate values for $G_p^M$ which have approximately the correct normalisation, within an overall uncertainty of a factor of 2. Gari and Stefanis [59] have proposed an alternative wave function which also satisfies the sum rules, in which the two up quarks share most of the proton momentum. This wave function was chosen to yield neutron form factors in agreement with experiment, and also produces values of $G_p^M$ with approximately the correct normalisation.
Other recent QCD analysis are consistent with the use of an asymmetric wave function such as these.

Once the normalisation of $G^M_p$ is determined, the basic prediction of perturbative QCD can be tested. This prediction is that the evolution of $G^M_p$ with $Q^2$ is given by the running of the strong coupling constant $\alpha_s(Q^2)$, as described in Ref.57. At high momentum transfer, this implies that $Q^2G^M_p$ should decrease with increasing $Q^2$. The rate of decrease is given by the magnitude of the scale parameter. The results of Ref.54 are in agreement with these expectations, as shown in Fig.16.

Unfortunately, at present there is no known theoretical expression which may give values for the electric form factor for proton for all $Q^2 = -t$. The experimental results obtained by Arnold et al [54] along with the previous data [53] are shown in Fig.16 where $Q^2G^E_p = (Q^2G^M_p/\mu_p)$ has been plotted versus $Q^2$. Since all the parametrisations claim to represent the experimental data for $G^E_p$, we expect them to yield the same pattern of values for all $Q^2$. This is not found to be the case. In Fig.16, we have also plotted the results obtained from the expression for the dipole form factor given by $G_n(t) = (1 - t/0.71)^{-2}$ [60], and from the parametrisations of the form factor due to Felst [61], Borkowski et al [62] and Bourrely et al [63]. It is seen that the parametrisation of Bourrely et al [63] is far from the experimental data for large $Q^2$. It agrees with experimental results only up to $Q^2 = 2.5$ (GeV/c)$^2$, then starts deviating from it and becomes more than twice as $Q^2 = 31.3$ (GeV/c)$^2$ is approached. The Borkowski form factor [62] also agrees with experiment only up to $Q^2 = 2.5$ (GeV/c)$^2$ and then starts deviating from the measured values, always remaining substantially below them up to $Q^2 = 31.3$ (GeV/c)$^2$. The Felst form factor [61] for proton agrees with experimental values up to $Q^2 = 15$ (GeV/c)$^2$. Beyond this value, it lies below the experimental data including the recently measured values of Arnold et al [54]. The dipole form factor gives good results, except for the region $10 < Q^2 < 20$ (GeV/c)$^2$, only if old data for $G^E_p$ for large $Q^2$ were taken into consideration. If these data are discarded in favour of the more precise
results obtained recently [54], the dipole form factor does not agree with
experiment beyond \( Q^2 \approx 10 \) (GeV/c\(^2\)). It is interesting to note that all the
parametrisations described above differ substantially from the large \( Q^2 \)
results of Lepage, Brodsky and collaborators [55, 56] and of Chernyak and
Zhitnitrkii [58] which are both based on quantum chromodynamics and are
also shown in Fig.16. The experimental data on the proton electric form
factor have also been parameterised by Saleem et al [64] as

\[
G_p = 0.6405 \exp[4t] + 0.33 \exp[0.85t] + 0.028 \exp[0.22t] \\
+ 0.0015 \exp[0.05t]
\]

This parametrisation of the proton form factor is in very good agreement
with the experimental data for all \(-t\) and is also consistent with the
theoretical curve obtained by Lepage, Brodsky and collaborators [55, 56] by
using perturbative QCD for large \(-t\), as shown in Fig.16. We have used the
electromagnetic form factor one proposed by Saleem et al [64].

It may be mentioned that Samuel and Mariarty [65] have recently
made use of lattice QCD to calculate, inter alia, the electric form factor for
proton for small \(-t\). Their results are consistent with various parametrisations and fit the experimental data well although they are slightly
above the experimental data in the \( Q \approx 0.6 \) GeV/c range. This indicates the
validity of lattice QCD based calculations of proton form factor for small \(-t\).

In their recent work Chou and Yang [9] have proposed \textit{hadronic} form
factor of the proton which is different from the electromagnetic form factor
of the proton. The proposed \textit{hadronic} form factor fits well within the
geometrical model the elastic \( pp \) scattering data from ISR to SPPS collider
energies. The proton matter form factor has the same dipole form as the
electric form factor \( G_e \) except that the radius parameter is now energy-
dependent. The proposed expression for the proton matter distribution is

\[
G_p = (1 + q^2/m^2)^{-2},
\]
where \( m \) is an energy-dependent parameter. For \( pp \) they [9] have fitted the experimental data for the differential cross section at \( \sqrt{s} = 23.5 \) and 546 GeV by taking values of \( m^2 = 0.774 \) and \( 0.573 \) (GeV/c)\(^2\) respectively.

However, no expression is given in ref. 9 to evaluate the value of \( m^2 \) at different energies (they only mention about the linear variation of this parameter with \( \ln s \)). We suggest a simple expression for \( m^2 \) as

\[
\frac{1}{m^2 \sqrt{s}} = 0.58 (\ln s)^{0.434}.
\]

The energy-dependence of the parameter gives \( m^2 = 0.774, 0.573 \) and 0.52 at \( \sqrt{s} = 23.5, 546 \) and 1800 GeV, respectively and is consistent with that given in ref. 9. The same expression is being used at different energies to evaluate differential cross sections of \( p\sigma \) and \( pd \) elastic scattering.

In Fig.17, we have plotted the results obtained from the newly proposed *hadronic* form factor given by \( G_p = (1 + q^2/m^2)^{-2} \) at \( \sqrt{s} = 53, 546 \) and 1800 GeV together the experimental data for the electromagnetic form factor of the proton. As shown in Fig.17 the *hadronic* form factor and electromagnetic form factor differ significantly. The proposed matter form factor of the proton proposed by Chou and Yang [9] has been used successfully to explain the various characteristics of \( p\sigma \) and \( pd \) elastic scattering.
Chapter 4

Predictions of the Geometrical Model
Having discussed the experimental data for nucleus (hadron) - nucleus elastic scattering and the form factors of nucleus/hadrons, we will now undertake theoretical study of these reactions in this chapter. In our study we have included alpha-alpha, proton-alpha, deuteron-deuteron, and proton-deuteron elastic scattering. In order to give a detailed account of these reactions we have subdivided this chapter into four parts: namely, 4.1) pristine Chou-Yang model; 4.2) generalised Chou-Yang model; 4.3) review of theoretical explanations and 4.4) Our predictions.

The Chou-Yang Model

A scattering process is a process in which two free particles are brought sufficiently close so that they interact with one another and give rise to a set of free particles which may or may not be the same as the initial pair. The amplitudes for a scattering process are known as a scattering amplitudes. To describe scattering processes one has to calculate the scattering amplitudes. Therefore the goal of theoretical effort is the determination of amplitudes. In the framework of quantum field theory, quantum electrodynamics (QED) is the highly successful theory of all electromagnetic interactions. Also, the field theory which simultaneously describes electromagnetic and weak interaction processes has been proposed [66,67]. In contrast there does not exist, as yet, a highly successful field theory of all strong interactions. Quantum chromodynamics (QCD) is a candidate for such a theory but mathematical tools are not available to obtain a QCD based comprehensive solution of elastic scattering. Therefore to describe strong interaction processes, we have to rely on phenomenological models of limited utility. The Chou-Yang model is one such model.
4.1) Pristine Chou-Yang Model

In 1968, Chou and Yang [1] proposed a preliminary version of the eikonal model for hadron-hadron elastic scattering with special reference to pp elastic scattering: this is now known as the Chou-Yang model. They employed an eikonal (or opacity or blackness) which was related to the distribution of matter inside the colliding particles. The colliding particles were considered as clusters of particles which pass through each other with attenuation. Elastic scattering then results from the propagation of the attenuated wave function. The scattering amplitude $T(s,t)$ in this model, by neglecting spin, is written as

$$T(s,t) = i \int b \, db \, [1 - \exp(-\Omega(b))] \, J_0(b \sqrt{-t})$$

where $b$ denotes the impact-parameter variable. The central assumption of the model consists in expressing $\Omega(b)$ as:

$$\Omega(b) = K \rho(b)$$

where

$$\rho(b) = \int \sqrt{-t} \, d \sqrt{-t} \, G_p^2(t) \, J_0(b \sqrt{-t}).$$

They assumed that the hadronic matter distribution is proportional to the charge distribution on hadrons, which is represented by $G_p$. Thus $G_p$ in the expression for the amplitude is the hadronic form factor of the interacting particles. The normalization of $T(s,t)$ is such that the differential and total cross sections are given by

$$\frac{d\sigma}{dt} = \pi \, |T(s,t)|^2$$

$$\sigma_t = 4\pi \, \text{Im} \, T(s,t=0).$$
The parameter $K$ was originally treated as a constant, since when this model was proposed, experimental results gave the impression that the total cross section for $pp$ was tending to a constant asymptotic value and the assumption that the energy-independent eikonal was proportional to the Fourier-Bessel transform of the proton form factor appeared to be justified. The differential cross section computations made by Durand and Lipes [68] with

$$G_p = (1 - t/\mu^2)^2, \mu^2 = 1 \text{ (GeV/c)}^2,$$

exhibited two dips, one near $-t = 1.4 \text{ (GeV/c)}^2$ and the other in the vicinity of $-t = 6 \text{ (GeV/c)}^2$. The dip near $-t = 1.4 \text{ (GeV/c)}^2$ has been observed since then, but the second diffraction minimum has not been seen up to $-t = 14 \text{ (GeV/c)}^2$. However, it was later on discovered experimentally that the total cross section rises with energy. The continuous growth of the total cross section through the FNAL and ISR energy ranges makes it imperative to introduce energy dependence in the opacity even at available high energies. For incorporating the $s$ dependence in to this model, several suggestions based on different lines of reasoning have been proposed in literature. In particular, Hayot and Grotchov [2], Henzi et al [69], and Bourrely et al [70] assumed that $\Omega(s, b)$ can be factorized as a product of two functions, one depending upon $s$ alone and the other upon the parameter $b$ only

$$\Omega(b, s) = K(s) \rho(b).$$

The model based on this assumption is known as the factorizable eikonal model. At a particular energy, the value of $K$ is obtained by adjusting it so as to yield the experimental value of the total cross section at that energy. Once $K$ is known, the differential cross section and other characteristics of the process can be calculated.
Chou and Yang repeated [71] the Hayot-Sukhatme calculations using the dipole fit for G_p(t). They have conceded that quantitatively agreement between calculated values and experimental data is not particularly impressive. In fact, these authors did not make any attempt to exhibit comparison of theoretical results and experimental data for differential cross sections at ISR energies which were the highest available energies at that time. However, it was predicted that with an increase of σ_T from 42.5 mb to 60 mb, the ratio σ_em/σ_T should increase from 0.177 to 0.226. This is in qualitative agreement with the now available measurements at ISR and collider energies. The model predicts [72] a very sharp dip around -t = 1 (GeV/c)^2 in contrast with the recently measured dσ/dt data for pp at √s = 546 GeV (σ_T = 61.9 mb) [73] which exhibits a shoulder. The slope parameter was also found to be inconsistent with experiments.

Later on, the model was extended to other elastic reactions. The geometrical picture of Chou and Yang was extended to nucleus-nucleus elastic scattering by Li and Lo [74]. The nucleus is then considered as consisting of a continuous distribution of matter and the boundaries between nucleons are considered to be insignificant. At high energies the nuclei appear each other as discs with vanishing thickness due to Lorentz contraction. The interaction of the nuclei is then the interaction between nucleon matter in discs. Thus virtually there is no difference between nucleon-nucleon and nucleus-nucleus scattering. However, a survey of literature shows [75] that it has been successful only in the diffraction peak region.

4.2) Generalised Chou-Yang Model

In order to explain the experimental results for hadron-nucleus and nucleus-nucleus elastic scattering at all -t, we have to generalise the Chou-Yang model. For that purpose, following Glauber and Velasco [4] we obtain
an expression for the scattering amplitude by using the multiple-diffraction theory. According to this theory, hadrons consist of clusters of particles which, on collision, pass through each other, interacting in pairs i,j and scattering one another with invariant scattering amplitudes $f_i(t)$. By using this concept, it is found [4] that the opaqueness $\Omega(s,b)$ in the expression for the scattering amplitude

$$T(s,t) = i \int b J_0(b \sqrt{-t}) (1-e^{-\alpha x^{b,b}}) db$$

is given by

$$\Omega(s,b) = k(1-i\alpha) \int -t d -t J_0(b \sqrt{-t}) G_A(t)G_B(t) f(t)/f(0).$$

It is interesting to notice that when the scattering of the constituent partons is isotropic, so that $f(t)/f(0)$ is equal to unity and the interactions are considered to be purely absorptive ($\alpha = 0$), then the model based on the multiple diffraction theory reduces to that of Chou and Yang [1], involving only the imaginary part of the scattering amplitude. It is for this reason that the original Chou-Yang model, which tacitly assumes the scattering to be isotropic, is valid in the diffraction-peak region. Since there is no known method for finding an expression for $f(t)/f(0)$, we have to guess its form as a slowly varying function of $t$, so that overall satisfactory results consistent with experiment even for large $-t$ are obtained.

It may be pointed out that the features of the form factors discussed in chapter 3 are also valid for the generalised Chou-Yang model in which the opaqueness $\Omega$ is given by above equation.

4.3) **Review of Theoretical Explanations**

Next we will briefly discuss various theoretical attempts for a piecemeal explanation of the data for these reactions.
a) $aa$ and $pa$ Elastic Scattering

The first attempt to explain the $aa$ and $pa$ elastic scattering was made by Lombard and Tellez-Arenas [33]. They employed the pristine Chou-Yang model, including the contribution of the real part. They have chosen an exponential form of the proton and alpha electromagnetic form factors as

$$G(Q) = (1 + X) e^{-AQ^2} - X e^{-BQ^2} + 2/3 X B Q^2 e^{-BQ^2}$$

where

for proton: $X = 1.767$, $A = 0.1377$ fm, $B = 0.0886$ fm
for alpha: $X = -0.1655$, $A = 0.500$ fm, $B = 0.1736$ fm,

Their choice of the proton form factor is consistent with the experimental measurements only up to $t = 1.6$ (GeV/c)$^2$. However, this restriction of the form factor values up to $t = 1.6$ (GeV/c)$^2$ does not affect the differential cross section results because the experimental data for these reactions is available only up to $t = 0.8$ (GeV/c)$^2$. These authors have observed that for $aa$ and $pa$ elastic scattering, there is a substantial departure from the measured values beyond the dip region. The slope of the differential cross section for both reactions is well produced in the diffraction peak region.

Kamran and Qureshi [76] carried out a study of the $aa$ elastic scattering at $\sqrt{s} = 126$ GeV based on the Chou-Yang model. They predict a multiple dip structure. However, the multiple dip structure predicted by them occurs not due to the nature of the electromagnetic form factor but owing to the oscillations produced when the integrals are not evaluated at sufficiently small intervals of $t$ and $b$ and/or are cut off at low upper limits. In fact, if the two integrals in the model are evaluated in sufficiently small steps and computations are allowed to continue till the integrals converge, significantly different results are obtained in the region lying outside the diffraction peak region. Moreover, the accurate value of the total cross
section obtained by Owen et al [12] has been used to obtain $K$ and then fit the differential cross section data obtained in those experiments [10,11] whose total cross section measurements have been considered unreliable. Actually, in order to explain the differential cross section data for $aa$ elastic scattering at $\sqrt{s} = 126$ GeV as measured by Ambrosio et al [10] and Akesson et al [11] one has to make use of the total cross section values as obtained by these very groups. On the other hand, the accurate total cross section value [12] can be used to make the differential cross section predictions when precise experimental data become available. For example, in the case of measurements of Akesson et al [11], the total cross section is estimated to be $280 \pm 70$ mb. The upper and lower limits of $\sigma_T$ are 350 and 210 mb, respectively. Fazal-e-Aleem et al [77] have obtained the differential cross section curves by using $\sigma_T = 350$ and 210 mb. As expected, the differential cross section in the diffraction peak region corresponds to a total cross section value in between these two extremities. Similar results hold good for the measurements of Ambrosio et al [10]. Thus for $aa$ elastic scattering at $\sqrt{s} = 126$ GeV, the pristine Chou-Yang model gives good results in the diffraction peak region. The model however, fails to give the quantitative explanation in the vicinity of the dip and beyond by using electromagnetic form factor as input.

b) dd and pd Elastic Scattering

Satisfactory explanation of the dd elastic scattering at high energies has been elusive since the first measurements. Fazal-e-Aleem and Shaukat Ali [6] have calculated the scattering amplitude and consequently differential cross section and other characteristic of dd elastic scattering at $\sqrt{s} = 53$ GeV, by using parameterisation proposed by them and that given in Ref. 37 for the charge form factor of deuteron in the opacity integral. The agreement between theory and experiment is quit good in the diffraction
peak region for both the parameterisations. However, in the vicinity of the dip and beyond there is significant departure from experimental data. Theoretical results obtained by using their parameterisation of the deuteron form factor gives two zeros at 0.22 \text{(GeV/c)}^2 and 0.70 \text{(GeV/c)}^2 respectively. The first dip has been predicted at the right position but the value of the differential cross section at the dip is about one third of the experimental value. The differential cross section was also calculated by using the form factor of Prida et al [37]. Once again a multiple dip structure was obtained and the first dip being at 0.42 \text{(GeV/c)}^2. The total cross section was chosen to be the same for both parameterisations. As pointed out earlier, the pristine Chou-Yang model is successful only in the diffraction peak region.

Etim and Satta [78] have attempted to fit the data by an improved representation of the ground state wave function of the deuteron as an admixture of S- and D-waves. The model is inspired by unitarity sum over a specific class of intermediate states. The model is a hybrid in two respects:

i) it combines the multiple scattering approximation of geometrical models, restricted to a "hard" part of the amplitude, with unitarity contributions which are associated with the "soft" part of the amplitude

ii) the complexification of the amplitude is not carried out in an analytic manner but results from separate approximation for the real and imaginary parts. The real and imaginary parts are assumed to be overwhelmingly dominant at high and low momentum transfers, respectively.

The model gives a good fit to the data at high energies by choosing many parameters. Also, as pointed out by them in their paper, the dependence of these parameters on the centre of mass energy is not understandable, a feature shared by geometrical models. It will be interesting to point out that, in the choice of parameters they have chosen
$\sigma_{dd}$ to be 559, 618 and 450 mb at $\sqrt{s} = 4.6.12$ and 63 GeV respectively. These values of $\sigma_{dd}$ seem to be much higher than all extrapolations for dd total cross section. In fact the choice of dd total cross section in their paper far exceeds the measured cross section for $aa$ scattering viz. $\sigma_{+} = 315 \pm 18$ mb. at 126 GeV [12].

Attempts have also been made to explain proton-deuteron elastic scattering on the basis of Chou-Yang model and multiple scattering theory (Glauber theory). Parida and Patel [79] have used the pristine Chou-Yang model [1] to calculate the differential cross section in pd elastic scattering with a view to investigate whether dips and higher-order maxima would appear at very high energies. For this propose they have used the interpolating formulae for the proton and the deuteron form factors which have exploited analyticity properties and yielded good fits to the Fermilab energies in the small $|t|$ region. As in the other cases, their calculations also indicate that the Chou-Yang approximation to the differential cross section, with only the imaginary part of the scattering amplitude, is applicable only in the diffraction peak region. Their calculations for the differential cross sections at higher values of $-t \leq 2$ (GeV)$^2$ show the absence of dips at Fermilab energies. Only a shoulder (and not a dip) appears for $-t = 0.3 \cdot 0.4$ (GeV/c)$^2$ if the real part and/or spin effects are negligible. It is interesting to note that even though no real-part and spin effects have been taken into account, a shoulder continues to appear for energies for which a sharp dip has emerged in other reactions. Although the predictions have been made by Parida and Patel [79] for the $t$ values lying beyond the diffraction peak region, they do not agree with the large $-t$ measurements of Goggi et al [18].

Braun et al [80], using Glauber theory including inelastic correction, have calculated the pd differential cross section at high energy. The predictions have been made by using non-relativistic wave functions of the deuteron including D-wave in the electric and quadrupole form factor at $\sqrt{s} = 53, 63$ GeV. The results agree well with the experimental data [18] up
to \(-t = 1.05\) (GeV/c)² while the measurements have been made up to \(-t = 2.0\) (GeV/c)².

No simultaneous explanation of dd and pd elastic scattering exists so far.

4.4) Our Predictions

We will now compare the predictions of Chou-Yang model based on our calculations with the experimental data. We have further subdivided this part into two sections namely; 4.4.1) predictions of the generalised Chou-Yang model along with the electromagnetic form factors and 4.4.2) predictions of the pristine Chou-Yang model along with the proposed energy dependent hadronic form factors.

4.4.1) Predictions of the generalised Chou-Yang Model along with the Electromagnetic Form Factors

The success of the pristine Chou-Yang model [1] to predict a dip at \(-t = 1.4\) (GeV/c)² in the pp elastic scattering at high energies which was observed later at ISR energies gave a boost to the model. Attempts have since been made to extend the domain of its application to hadron-nucleus and nucleus-nucleus elastic scattering. The Chou and Yang model [1] which was initially proposed for pp elastic scattering at high energies is also quite successful in the diffraction peak region for the hadron-nucleus and nucleus-nucleus elastic scattering.

For \(aa\) and \(pa\) elastic scattering, the pristine Chou-Yang model gives good results in the diffraction peak region [33]. The model however, fails to give the quantitative explanation in the vicinity of the dip and beyond by using electromagnetic form factor as input.
In order to explain the differential cross section beyond the dip region, we make use of the generalised Chou-Yang model [4] as described in section 4.2. In order to take into account the anisotropy of the scattering, we must know the function \( f(t)/f(0) \). As this function can not be derived theoretically at present, we have to guess its form so that after evaluation of the appropriate integral, the computed differential cross section results may agree with the experimental data.

We have tried several functions and by choosing

\[
\frac{f(t)}{f(0)} = (1-at)^b,
\]

the differential cross-section curves for \( \sigma T \) corresponding to the mutually inconsistent experimental results of Akesson et al [11] and Ambresio et al [10] are obtained with \( a = 0.3 \) (GeV/c)^2 and \( b = 1.0 \) for Ref.11 and with \( a = 0.3 \) (GeV/c)^2 and \( b = 10 \) for Ref.10. These curves are shown in Figs.1 and 2 and have been obtained by choosing \( K = 115.0 \) and 114.0 (GeV/c)^2 and \( \sigma = 0.36 \) and 0.45, which yield \( \sigma T = 349.02 \) and 334.58 mb. As pointed out earlier, the measurements of \( \sigma T \) in Refs.10 and 11 are less accurate than those in Ref.12. However, to fit the data on \( d\sigma/dt \) from Refs.10 and 11 we must use the \( \sigma T \) values obtained by these very groups. Now the value of \( \sigma T \) reported in Ref.10 is 250 ± 50 mb, and that reported in Ref.11 is 280 ± 70 mb. Thus we can conclude that the experimental \( \sigma T \) lies between 200 and 350 mb. The theoretical values of \( \sigma T \) relevant to the solid curves in Figs.1 and 2 are consistent with these experimental values.

We have also shown that by the simple choice

\[
\frac{f(t)}{f(0)} = (1-at)^b,
\]

where \( a = 0.1 \) (GeV/c)^2 and \( b = 1.0 \), very good results are obtained for \( \sigma T \) elastic scattering at high energies. The differential cross section results at \( P_L = 100, 150, 250 \) and 300 GeV/c and \( E_{lab} = 301, 393 \) GeV and \( \sqrt{s} = \)
89 GeV, corresponding to \( \sigma_T = 128.7 \pm 0.9, 130.8 \pm 0.8, 131.6 \pm 0.8,
132.0 \pm 0.8, 122.8 \pm 0.7, 125.9 \pm 0.6, \) and \( 130 \pm 20 \text{ mb} \), respectively,
are plotted in Figs. 3, 4a, 4b and 5 along with the experimental data. The
agreement is quite good. The values of \( K \) have been adjusted to 34.6, 35.4,
35.7, 35.8, 32.5, 33.4, and 42.0 (GeV/c)^2, which yield \( \sigma_T = 129.3,
131.6, 132.4, 132.7, 123.4, 126.5 \) and 149.4 mb, respectively.

As expected, attempt to explain dd elastic scattering on the basis of
pristine Chou-Yang model yields a fit in the diffraction peak region [6]. In
order to explain the differential cross section data for dd elastic scattering
at \( \sqrt{s} = 53 \text{ GeV} \) beyond the dip region by using the generalised Chou-Yang
model, we have chosen

\[
f(t)/f(0) = (1 + at^2)^b,
\]

where \( a = 0.02 \) (GeV/c)^2 and \( b = 12.7 \). In order to make calculations at
a particular energy, by using the generalised Chou-Yang model, the total
cross section at that energy should be known. However, the total cross
section at \( \sqrt{s} = 53 \text{ GeV} \) [18] has not yet been measured. We have therefore
made use of the extrapolated differential cross section at \( t = 0 \) to fix the
value of \( K \) which has ultimately been used to predict the total cross section
at this energy. The differential cross section results for \( \sqrt{s} = 53 \text{ GeV} \) [18]
calculated by using the generalised Chou-Yang model along with the
experimental data are shown in Fig. 7. There is very good agreement with
the experimental data up to \( -t = 0.75 \) (GeV/c)^2. The value of \( K \) which fits
the experimental data for dd elastic scattering is 32.8 (GeV/c)^2. The value
of \( a \) is chosen as 0.26 which yields \( \rho = 0.206 \). The ratio of the integrated
to the total cross section \( \sigma_z/\sigma_T \), is 0.17. However, beyond \( -t = 0.75
\) (GeV/c)^2, we do not get a satisfactory agreement.
4.4.2) Predictions of the Pristine Chou-Yang Model along with Proposed Energy Dependent Form Factors

We finally approach the problem from another angle. The Chou-Yang model has been quite successful in reproducing the bulk features of pp, pp, dd, pa, and aa elastic scattering at the ISR energies for not very large values of -t. For large -t, the predictions of the model are not consistent with the experimental data. Moreover, while considering the results of pp elastic scattering at the collider energies [73,81], it has been realised [8] that their conjecture regarding the proportionality of hadronic and electromagnetic form factors of the proton is not strictly valid.

These observations for pp, pp, dd, pa, and aa elastic scattering make it imperative to revise the basic concept underlying the Chou-Yang model. We strongly feel that the basic conjecture of Chou and Yang that the hadronic form factor is proportional to the electromagnetic form factor is only an approximation of the true picture; it is valid for small -t but breaks down when -t exceeds a certain limit. It therefore needs modification.

Keeping in view the inability of pristine model in explaining the data in the entire -t region, Chou and Yang [2] very recently have proposed that the assumption regarding the equivalence of the charge form factor and the hadronic form factor is only an approximation within the framework of their model. They have thus proposed a simple expression for the matter distribution. Similar assumption was made by Saleem et al [8] in explaining the pp, pa, aa elastic scattering. However, unlike the assumption made in Ref.8, Chou and Yang have taken the hadronic form factor energy dependent. By choosing an energy dependent range parameter, they have fitted the pp elastic scattering data at √s = 23.5 and 546 GeV. Their fit to the data at 23.5 GeV is good up to -t = 3.0 (GeV/c)^2. Beyond this value of -t the deviations from the experimental data become significant as the -t value increases. We have used a similar procedure and suggested a
hadronic form factor of the deuteron as dependent on energy which gives very good fit to the data for low as well as high (ISR) energies. The proposed hadronic form factor of deuteron is:

\[ G_d = s^{a+b+s^2} \left( \sum d_i \exp(f_it) \exp(ct^2) \right), \quad i = 1,2,3 \]

where

\[
\begin{align*}
    a &= 0.246 \\
    b &= 0.024 \\
    c &= -0.19 \\
    d_1 &= 0.85 \\
    d_2 &= 0.14 \\
    d_3 &= 0.01 \\
    f_1 &= 20.07 \\
    f_2 &= -0.47 \\
    f_3 &= -1.47
\end{align*}
\]

Now we compare the predictions of the Pristine Chou-Yang [1] model by using energy dependent hadronic form factor with the available data for dd elastic scattering. Fig.6 shows the fit for the differential cross section at \( \sqrt{s} = 6.12 \) GeV [17]. The fit is obtained for \( K = 35 \) (GeV/c)\(^{-2}\) and \( \sigma = 0.05 \). The theoretical predictions of the model are in good agreement with the experimental data. Similarly, a quite good agreement between theory and the experiment is observed at \( \sqrt{s} = 53 \) and 63 GeV [18] as shown in figs.18 and 8, respectively. The values of \( K \) and \( \sigma \) are chosen to be 69.5 (GeV/c)\(^{-2}\) and 0.05 for both 53 and 63 GeV. The choice of the same values of \( K \) and \( \sigma \) at \( \sqrt{s} = 53 \) and 63 GeV suggests that the total cross section at these energies will be consistent within errors.

In order to verify the validity of the proposed hadronic form factor of deuteron we have calculated the differential cross section for pd elastic scattering at \( \sqrt{s} = 53 \) and 63 GeV by using proton hadronic form factor given by \( G_p = (1 + q^2/m^2)^{-2} \) [9] and deuteron hadronic form factor, different from electromagnetic form factor in the pristine Chou-Yang model including the contribution of the real part. We have used the energy dependent deuteron hadronic form factor proposed by us for dd elastic scattering and the corresponding energy dependent parameter values \( m^2 = 0.701 \) and 0.689 (GeV)\(^2\) in the proton hadronic form factor. The predicted differential cross section results along with the experimental data [18] are shown in
Figs. 9 and 10. The overall agreement of the predicted differential cross section with the experimental data is very good. The values of $K$ and $\sigma$ were chosen as 10.0 and 0.05 respectively for both 53 and 63 GeV.

We have then applied the concept of energy depended hadronic form factor different from electromagnetic form factor to $aa$ elastic scattering and again find a good fit to the available experimental data. We now compare the predictions of the pristine Chou-Yang model with the $aa$ elastic differential cross section data at $\sqrt{s} = 126$ GeV. The theoretical differential cross section results along with the experimental data [10, 11] are shown in Figs. 19 and 20. The agreement is very good. The corresponding values of $\sigma$ are 0.35 and 0.30, respectively and the value of $K$ is 115.0 which yield $\sigma_1 = 333.12$ mb and 330.67 mb. It is interesting to note that the value of $K$ in both the cases i.e. for electromagnetic form factor and for hadronic matter form factor, is exactly the same [10]. However the value of $\sigma$ is slightly different. The theoretical differential cross section results obtained by using hadronic matter form factor at the dip positions are better than obtained by using electromagnetic form factor and are consistent with the experimental data [10, 11] as shown in Figs. 19 and 20.

In order to further verify the validity of the proposed hadronic form factor of alpha we have predicted the $pa$ elastic differential cross section at high energies by using proton hadronic form factor (proton matter distribution) proposed by Chou and Yang [9] and alpha hadronic form factor (alpha matter distribution) proposed by us, as input in the pristine Chou-Yang model. We have used the alpha hadronic form factor that gives quite good agreement to the experimental data for $aa$ elastic scattering at $\sqrt{s} = 126$ GeV which is the only available energy for this reaction so far. The proposed alpha hadronic form factor is

$$G_\alpha = 1.03e^{5.5t} + 3.25te^{7.0t} - 0.03e^{0.43t}$$
The differential cross section results at $P_L = 100, 150, 250, 300$ GeV/c and $E_{LAB} = 301, 393$, GeV and $\sqrt{s} = 89$ GeV corresponding to $\sigma_T = 128.7 \pm 0.9, 130.8 \pm 0.8, 131.6 \pm 0.8, 132.0 \pm 0.8, 122.8 \pm 0.7, 125.9 \pm 0.6,$ and $130 \pm 20$ mb respectively are plotted in Figs.21, 22a, 22b and 23 along with the experimental data. The corresponding values of the energy dependent parameter $m^2$ in the proton hadronic form factor are 0.77, 0.74, 0.72, 0.71, 0.70, and 0.66 (GeV/c)$^2$ respectively. The values of $K$ have been adjusted to 35.8, 36.3, 36.5, 36.7, 33.4, 34.4 and 29.0 (GeV/c)$^{-2}$, which yield $\sigma_T = 128.81, 130.39, 131.13, 131.72, 122.90, 125.71$ and $110.93$ mb respectively. The differential cross section curves show that the alpha hadronic form factor is also energy dependent. By considering, the experimental data for $p\alpha$ elastic differential cross section, we have proposed an energy dependent alpha hadronic form factor, which can be used for any energy (at high energy), simply by giving the value of $s$ (energy in the centre of mass frame of reference), as follows:

$$G_a = s^{-0.8 t} (1.03e^{14.29 t} + 3.25te^{14.738 t} - 0.03e^{8.168 t})$$

By using the alpha hadronic form factor for the corresponding energies and the proton hadronic form factor for the corresponding values of the energy dependent parameter $m^2$, we have also calculated the $p\alpha$ elastic differential cross section at the above mentioned energies. The theoretical differential cross section results at $P_L = 100, 150, 250, 300$ GeV/c and $E_{LAB} = 301, 393$ GeV and $\sqrt{s} = 89$ GeV corresponding to $\sigma_T = 128.7 \pm 0.9, 130.8 \pm 0.8$ and $131.6 \pm 0.8, 132.0 \pm 0.8, 122.8 \pm 0.7, 125.9 \pm 0.6,$ and $130 \pm 20$ mb respectively, are plotted in Figs.24, 25a, 25b and 26 along with the experimental data. The agreement is quite good. It is interesting to note that at $\sqrt{s} = 89$ GeV the predicted differential cross section has one dip whereas the predicted differential cross section obtained by using electromagnetic form factor, have two dips. The values of $\alpha$ are chosen as 0.05 for $E_{LAB} = 301, 393$ GeV and 0.1 for $\sqrt{s} = 89$ GeV. The values of $K$
have been adjusted to 33.6, 34.6, 35.0, 35.3, 32.1, 33.3 and 29.0
(GeV/c)^{-2}, which yield \( \sigma_T = 128.17, 130.30, 131.15, 131.74, 122.54,
125.72 \) and 111.94 mb, respectively.

We have also predicted the \( p\alpha \) elastic differential cross section at \( \sigma_T = 125.42 \) mb and 150.54 mb which corresponds to \( \sqrt{s} = 89 \) GeV [10] and
\( \sqrt{s} = 126 \) GeV, respectively. This is shown in Figs.27 and 28. The values
of \( K \) and \( \alpha \) were chosen as 34.0 and 0.1, 43.8 and 0.1, respectively. We
notice that there exist only one dip.
CHAPTER 5

CONCLUSIONS
The analysis of the elastic reactions based on the Chou-Yang model leads to the following conclusion.

1. The Chou-Yang model modified by Hayot and Sukhatme can explain the experimental data on nucleus-nucleus and hadron-nucleus elastic scattering only up to the diffraction peak region.

2. The multiple diffraction theory has been used to generalise the Chou-Yang model. This generalisation makes $K$ complex introducing the real part in the scattering amplitude and introduces an additional factor $f(t)/f(0) = (1-\mathrm{at})^b$, where $a = 0.1 \, (\mathrm{GeV/c})^{-2}$ and $b = 1.0$ for $P\sigma$ elastic scattering and $a = 0.3 \, (\mathrm{GeV/c})^{-2}$ and $b = 1.0, 10$ for $a\sigma$ elastic scattering. The generalised Chou-Yang model is found to yield results consistent with experiment at high energies.

But the data for $d\bar{d}$ elastic scattering at $\sqrt{s} = 53 \, \mathrm{GeV}$ could be explained by choosing the anisotropy function as $(1+at^2)^b$, where $a = 0.02 \, (\mathrm{GeV/c})^{-2}$ and $b = 12.7$, only up to $-t = 0.75 \, (\mathrm{GeV/c})^2$. However, $pd$ elastic scattering at high energy has not been explained beyond the diffraction peak region even by the generalised Chou-Yang model.

3. As a form factor, in general, is related to the distribution of matter in space, the hadronic form factor consequently is related to the distribution of matter inside hadron. Any theory that claims to explain the structure of hadron should be able to give some idea of the hadronic form factor. So far the expression which represents the
exact form of hadronic form factor can not be obtained from the first principles.

4. There has been considerable interest in the form factor of the deuteron as it gives information about the short range nucleon-nucleon interaction and the transition from nucleon to quark degrees of freedom.

5. Furthermore, it has been a question of considerable importance whether or not the electric charge distribution is the same as the nuclear matter distribution. In 1968 Chou and Yang assumed that the matter distribution inside a hadron is proportional to its charge distribution. These form factors of hadrons describe an extended structure which can be represented by spatial distributions for the static quantities. Although at ISR energies the Chou-Yang model has yielded impressive quantitative agreement with experiments, yet as the energy increases deviations from the strict interpretation of the geometrical picture becomes quite noticeable. Also it is valid for small \(-t\) but breaks down when \(-t\) exceeds a certain limit. We strongly feel that the basic conjecture of Chou and Yang that the hadronic form factor is proportional to the electromagnetic form factor is only an approximation of the true picture. We have shown that the nucleus-nucleus and hadron-nucleus elastic scattering can be explained by assuming energy dependent hadronic form factors of deuteron, alpha and proton different from electromagnetic form factors in the pristine Chou-Yang model.

The agreement between the predicted values and the experimental data for nucleus-nucleus and hadron-nucleus elastic scattering at high energies and large momentum transfers is a success of the conjecture that the hadronic form factor is different from the electromagnetic form factor. The physics of strongly
interacting elastic processes in the framework of pristine Chou-Yang model, including the contribution of real part, with energy dependent hadronic form factor appears to be most convincing of all the interpretations given in the thesis. The dependence of hadronic form factors on energy suggests that the hadrons expand as energy increases.

6. The energy dependent form factors also suggest that the radii of deuteron, alpha and proton will be increasing with increase in energy. Also, the increase in opacity implies that these particles become more opaque at higher energies.

We thus conclude that by assuming that hadrons are extended objects and that their matter distribution changes with energy, we can predict the behaviour of nucleus(hadron)-nucleus elastic scattering at high energies and up to large momentum transfers.
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TABLE CAPTIONS

Table 1  \( \sigma_{aa} \) total cross section at \( \sqrt{s} = 126 \) GeV, as well as the values of \( \sigma_{\omega} \) and \( B \) for \( aa \) elastic scattering.

Table 2  \( P\sigma \) total cross sections and the values of \( B \) and \( \rho \) parameters at various momenta and energies.
Figure Captions:

Fig. 1  Differential cross section for $aa$ elastic scattering at $\sqrt{s} = 126$ GeV. The solid curve represents the predictions of the generalised Chou-Yang model. The experimental data points have been taken from Ambrosio et al [10].

Fig. 2  Differential cross section for $aa$ elastic scattering at $\sqrt{s} = 126$ GeV. The solid curve represents the predictions of the generalised Chou-Yang model. The experimental points have been taken from Akesson et al [11].

Fig. 3  Differential cross section for $pa$ elastic scattering at $p_L = 100, 150, 250$ and 300 GeV/c. The solid curves represent the predictions of the generalised Chou-Yang model. The experimental data points have been taken from Burq et al [14].

Fig. 4  Differential cross section for $pa$ elastic scattering at $E_{Lab} = 301$ and 393 GeV. The solid curves represent the predictions of the generalised Chou-Yang model. The experimental data points have been taken from Bujak et al [15].

Fig. 5  Differential cross section for $pa$ elastic scattering at $\sqrt{s} = 89$ GeV. The solid curve represents the predictions of the generalised Chou-Yang model. The experimental data points have been taken from Ambrosio et al [10] and Bell et al [13].

Fig. 6  Differential cross section for $dd$ elastic scattering at $\sqrt{s} = 6.12$ GeV. Hadronic form factor of the deuteron different from its electromagnetic form factor has been used. The solid line represents the predictions of the pristine Chou-Yang model. The experimental points have been taken from Goshaw et al [17].
Fig. 7 Differential cross section for dd elastic scattering at $\sqrt{s} = 53$ GeV. The solid curve corresponds to the predictions of the generalised Chou-Yang model. The experimental data points have been taken from Goggi et al [18].

Fig. 8 Differential cross section for dd elastic scattering at $\sqrt{s} = 63$ GeV. Hadronic form factor of the deuteron different from its electromagnetic form factor has been used. The solid line represents the predictions of the pristine Chou-Yang model. The experimental points have been taken from Goggi et al [18].

Fig. 9 Differential cross section for pd elastic scattering at $\sqrt{s} = 53$ GeV. Hadronic form factors of the proton and deuteron different from their electromagnetic form factors have been used. The solid line represents the predictions of the pristine Chou-Yang model. The experimental points have been taken from Goggi et al [18].

Fig. 10 Differential cross section for pd elastic scattering at $\sqrt{s} = 63$ GeV. Hadronic form factors of the proton and deuteron different from their electromagnetic form factors have been used. The solid line represents the predictions of the pristine Chou-Yang model. The experimental points have been taken from Goggi et al [18].

Fig. 11 The parametrizations of the $a$ form factor by McCarthy et al [32] and Lombard et al [33] plotted against $Q^2$. The experimental data points have been taken from McCarthy et al [32] and Frosch et al [30].

Fig. 12 Comparison of energy dependent hadronic form factors of $a$ based on matter distribution plotted against $Q^2$ at $\sqrt{s} = 47.68, 89$ and $126$ Gev. The experimental data points have been taken from McCarthy et al [32] and Frosch et al [30].
Fig. 13  Comparison of parametrizations of deuteron structure function $|A_t|$ obtained by us and Parida et al [37] plotted against $-t$. The experimental points have been taken from Ref. [36].

Fig. 14  The magnetic form factor of the deuteron plotted against $Q^2$. The experimental points have been taken from Refs. [39-41].

Fig. 15  Comparison of energy dependent hadronic form factors of deuteron based on matter distribution plotted against $-t$ at $\sqrt{s} = 6.12$ and 53 GeV. The experimental points have been taken from Ref. [36].

Fig. 16  Different parameterisations of proton form factor plotted against $Q^2$ after multiplying each one of them with $Q^4$. Experimental data have been taken from Refs. 53 and 54.

Fig. 17  Comparison of energy dependent hadronic form factors of proton based on matter distribution plotted against $Q^2$ after multiplying each one of them with $Q^4$ at $\sqrt{s} = 53$, 546 and 1800 GeV. Experimental data have been taken from Refs. 53 and 54.

Fig. 18  Differential cross section for $dd$ elastic scattering at $\sqrt{s} = 53$ GeV. Hadronic form factor of the deuteron different from its electromagnetic form factor has been used. The solid line represents the predictions of the pristine Chou-Yang model. The experimental points have been taken from Goggi et al [18].

Fig. 19  Differential cross section for $aa$ elastic scattering at $\sqrt{s} = 126$ GeV. Hadronic form factor of the $a$-particle different from its electromagnetic form factor has been used. The solid line represents the predictions of the pristine Chou-Yang model. The experimental data points have been taken from Ambrosio et al [10].
Fig. 20  Differential cross section for \( \alpha \alpha \) elastic scattering at \( \sqrt{s} = 126 \) GeV. Hadronic form factor of the \( \alpha \)-particle different from its electromagnetic form factor has been used. The solid line represents the predictions of the pristine Chou-Yang model. The experimental data points have been taken from Akeesson et al [11].

Fig. 21  Differential cross section for \( p \alpha \) elastic scattering at \( p_L = 100, 150, 250 \) and \( 300 \) GeV/c obtained by using hadronic form factors of the proton at the corresponding energies and of the \( \alpha \)-particle at \( \sqrt{s} = 126 \) GeV being different from their electromagnetic form factors. The solid curves represent the predictions of the pristine Chou-Yang model. The experimental data points have been taken from Burq et al [14].

Fig. 22  Differential cross section for \( p \alpha \) elastic scattering at \( E_{\text{lab}} = 300 \) and \( 393 \) GeV obtained by using hadronic form factors of the proton at the corresponding energies and of the \( \alpha \)-particle at \( \sqrt{s} = 126 \) GeV being different from their electromagnetic form factors. The solid curves represent the predictions of the pristine Chou-Yang model. The experimental data have been taken from Bujak et al [15].

Fig. 23  Differential cross section for \( p \alpha \) elastic scattering at \( \sqrt{s} = 89 \) GeV obtained by using hadronic form factors of the proton at the corresponding energy and of the \( \alpha \)-particle at \( \sqrt{s} = 126 \) GeV being different from their electromagnetic form factors. The solid curve represents the predictions of the pristine Chou-Yang model. The experimental data points have been taken from Ambrosio et al [10] and Bell et al [13].

Fig. 24  Differential cross section for \( p \alpha \) elastic scattering at \( p_L = 100, 150, 250 \) and \( 300 \) GeV obtained by using hadronic form factors of the proton and \( \alpha \)-particle at the corresponding
energies being different from their electromagnetic form factors. The solid curves represent the predictions of the pristine Chou-Yang model. The experimental data points have been taken from Burq et al [14].

Fig. 25 
Differential cross section for $p\alpha$ elastic scattering at $E_{\text{lab}} = 301$ and $393$ GeV obtained by using hadronic form factors of the proton and $\alpha$-particle at the corresponding energies being different from their electromagnetic form factors. The solid curves represent the predictions of the pristine Chou-Yang model. The experimental data points have been taken from Bujak et al [15].

Fig. 26 
Differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV obtained by using hadronic form factors of the proton and $\alpha$-particle at the corresponding energy being different from their electromagnetic form factors. The solid line represents the predictions of the pristine Chou-Yang model. The experimental data points have been taken from Ambrosio et al [10] and Bell et al [13].

Fig. 27 
Predicted differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV corresponding to $\sigma_f = 125$ mb. Calculations have been made by using hadronic form factors of the proton and $\alpha$-particle, different from their electromagnetic form factors in the pristine Chou-Yang model.

Fig. 28 
Predicted differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 126$ corresponding to $\sigma_f = 150$ mb. Calculations have been made by using hadronic form factors of the proton and $\alpha$-particle, different from their electromagnetic form factors in the pristine Chou-Yang model.
### Table 1

<table>
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<th>Reaction</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_T$ (mb)</th>
<th>$\sigma_{el}$ (mb)</th>
<th>$B$ (GeV/c)$^{-2}$</th>
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<td>280 ± 70</td>
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<td>72 ± 6</td>
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<td>250 ± 50</td>
<td>45 ± 15</td>
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<td>100 ± 10</td>
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<td></td>
<td>295 ± 40</td>
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<td>315 ± 18</td>
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Table 2

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<th>$P_L$ GeV/c</th>
<th>$E_L$ GeV</th>
<th>$\sqrt{s}$ GeV</th>
<th>$\sigma_\gamma$ mb</th>
<th>$B$ (GeV/c)$^{-2}$</th>
<th>$\rho$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
<td></td>
<td>128.7 ± 0.9</td>
<td>34.1 ± 0.3</td>
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<td>14</td>
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<tr>
<td>150</td>
<td></td>
<td></td>
<td>130.8 ± 0.8</td>
<td>34.9 ± 0.3</td>
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<tr>
<td>250</td>
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<td>131.6 ± 0.8</td>
<td>34.7 ± 0.2</td>
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<tr>
<td>300</td>
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<td></td>
<td>132.0 ± 0.8</td>
<td>35.1 ± 0.2</td>
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<td>33.4 ± 0.3</td>
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<td>15</td>
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<tr>
<td>393</td>
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<td>34.2 ± 0.4</td>
<td>0.102 ± 0.035</td>
<td>15</td>
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<tr>
<td>89</td>
<td></td>
<td></td>
<td>130 ± 20</td>
<td></td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>
$\alpha \alpha \rightarrow \alpha \alpha$

$\sqrt{s} = 12.6 \text{ GeV}$

Ambrosio et al. [10]

$\frac{d\sigma}{dt} [\text{mb/(GeV/c)}^2]$
$d\sigma/dt \ [\text{mb}/(\text{GeV}/c)^2]$

$\sqrt{s} = 126 \ \text{GeV}$

$\circ$ Akesson et al (11)

Fig. 2
Fig. 3

$p \alpha \rightarrow p \alpha$

Burq et al [14]
Fig. 4 (a)
$p\alpha \rightarrow p\alpha$

$E_{\text{lab}} = 393$ GeV

- Bujak et al. [15]

$\frac{d\sigma}{dt} \left[ \text{mb/(GeV/c)}^2 \right]$ vs $-t \left( \text{GeV/c} \right)^2$

Fig. 4 (b)
\( p\alpha \rightarrow p\alpha \)
\[ \sqrt{s} = 89 \text{ GeV} \]
- Ambrosio et al [10]
- W. Bell et al [13]

\[ \frac{d\sigma}{dt\left(\mu bc^2/\text{GeV}^2\right)} \]

\[-t \left(\text{GeV}/c\right)^2\]

Fig. 5
$\frac{d\sigma}{dt}$ (mb/(GeV/c)^2)

$-t$ (GeV^2)

Fig. 6

$dd \rightarrow dd$

$\sqrt{s} = 6.12$ GeV

Goshaw et al [17]
$\frac{d\sigma}{dt}(\text{GeV/c})^2$

$-t (\text{GeV/c})^2$

$\frac{d\sigma}{dt}(\text{mb})$

$\sqrt{s} = 5.3 \text{ GeV}$

- Goggi et al [18]

Fig. 7
\[ \frac{d\sigma}{dt} (\text{mb/Gev}^2) \]

\[ \sqrt{s} = 6.3 \text{ GeV} \]

- Goggi et al. [18]

\[ -t (\text{GeV}^2) \]

Fig. 8
$p \, d \rightarrow p \, d$

$\sqrt{s} = 5.3$ GeV

Goggi et al. [18]

Fig. 9
$p + d \rightarrow p + d$

$\sqrt{s} = 6.3$ GeV

- Goggi et al. [18]

Fig. 10
Fig. 11
Fig. 12
Fig 13
Fig 14
Fig. 16
$Q^2 G_P (\text{GeV}/c)^4$

- Old data [53]
- Arnold et al [54]

- $\sqrt{s} = 53$ GeV
- $\sqrt{s} = 545$ GeV
- $\sqrt{s} = 1800$ GeV

$Q^2 (\text{GeV}/c)^2$

Fig. 17
Fig 18

$dd \rightarrow dd$

$\sqrt{s} = 53$ GeV

* Goggi et al [18]
\[ \frac{d\sigma}{dt} \left[ \text{mb} / (\text{GeV}/c)^2 \right] \]

\[ \sqrt{s} = 1.26 \text{ GeV} \]

- Ambrosio et al. [10]

Fig. 19
\[ \alpha \alpha \rightarrow \alpha \alpha \]

\[ \sqrt{s} = 126 \text{ GeV} \]

* Akesson et al [11]

Fig. 20
$E_{\text{lab}} = 301 \text{ GeV}$

- Bujak et al [15]

Fig. 22(a)
$\frac{d\sigma}{dt}\mid_{mb}/(\text{GeV}/c)^2$

$p\bar{\kappa} \rightarrow p\kappa$

$E_{\text{lab}} = 393 \text{ GeV}$

Bujak et al. [15]

Fig. 22 (b)
$P \alpha \rightarrow P \alpha$

$\nu s = 89$ GeV

• Ambrosio et al [10]
• W Bell et al [13]

$\frac{d\sigma}{dt} (\mu b c^2 / \text{GeV}^2)$

$-t (\text{GeV}/c)^2$

Fig. 23
Fig. 24

\[ \frac{d\sigma}{dt} [\text{mb}/(\text{GeV/c})^2] \]

- \( -t \text{ (GeV/c)}^2 \)

- \( p\alpha \rightarrow p\alpha \)

- \( 300 \text{ GeV/c} \)
- \( 250 \text{ GeV/c} \)
- \( 150 \text{ GeV/c} \)
- \( 100 \text{ GeV/c} \)

Burq et al [14]
\[ \frac{d\sigma}{dt} \frac{(\text{mb} / (\text{GeV/c})^2)}{t (\text{GeV/c})^2} \]

- \(p \rightarrow p\pi^\pm\)

\(E_{\text{lab}} = 301 \text{ GeV}\)

- Bujak et al [15]

Fig. 25(a)
\[ \frac{d\sigma}{dt} \left[ \frac{mb}{(GeV/c)^2} \right] \]

- \( t (GeV/c)^2 \)

Fig. 25 (b)

- \( E_{lab} = 393 \text{ GeV} \)

- Bujak et al [15]

\[ p\alpha \rightarrow p\alpha \]
$p \alpha \rightarrow p \alpha$

$\sqrt{s} = 89$ GeV

- Ambrosio et al [10]
- W. Bell et al [13]

$\frac{d\sigma}{dt}(\text{mb} \cdot GeV^2)$

$t (GeV/c)^2$

Fig. 26
\( p\alpha \rightarrow p\alpha \)

\[ \sqrt{s} = 89 \text{ GeV} \]

\[ \sigma_T = 125 \]

\[ \frac{d\sigma}{dt} (\text{mb c}^2 / \text{GeV}^2) \]

\[ -t (\text{GeV}/c)^2 \]

Fig 27
The figure shows the differential cross-section $d\sigma/dt$ in microbarns per square centimeter per GeV squared as a function of $-t$ (GeV/c)$^2$. The data is presented for the reaction $p\alpha \rightarrow p\alpha$ with a square root of the center-of-mass energy $\sqrt{s} = 12.6$ GeV and a total cross-section $\sigma_T = 150$. The figure is labeled as Fig. 28.
RESEARCH PAPERS

based on the material
presented in the THESIS
GENERALIZED CHOU-YANG MODEL AND ANALYSIS OF pα AND αα ELASTIC SCATTERING AT HIGH ENERGIES

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(Received 25 October 1988; revised manuscript received 19 April 1989)

Abstract

The various characteristics of pα and αα elastic scattering at high energies are explained by using the generalized Chou-Yang model which takes into account the anisotropy of the scattering process.
I. Introduction

The success of the original Chou-Yang model in predicting a dip at $-t = 1.4 (\text{GeV}/c)^2$ in $pp$ elastic scattering at high energies (which was observed later at ISR energies) gave a boost to the model. Attempts have since been made to extend the domain of its application to hyperon-proton, hadron-nucleus, and nucleus-nucleus elastic scattering at high energies. The first attempt to explain $p\alpha$ and $\alpha\alpha$ elastic scattering by using this model, including the contribution of the real part, was made by Lombard and Tellez-Arenas. They obtained a satisfactory fit to the experimental data up to the first diffraction minimum for $p\alpha$ elastic scattering at an incident energy of 400 GeV. For $\alpha\alpha$ elastic scattering, they noticed that the slope of the differential cross section at $\sqrt{s} = 126$ GeV was well reproduced in the diffraction-peak region. It has been shown that the data for $p\alpha$ elastic scattering can be reproduced by using a modified Chou-Yang conjecture. In this paper, we will show that the $p\alpha$ and $\alpha\alpha$ elastic scattering can be explained by using the generalized Chou-Yang model, which is based on multiple-diffraction-scattering theory.

II. Experimental Measurements

A. $p\alpha$ Elastic Scattering

A study of $p\alpha$ elastic scattering at small angles was made in the WA9 experiment at CERN SPS by Burq et al. in 1981. The differential cross section was measured for $-t$ ranging from 0.008 to 0.05 (GeV/c)$^2$ at momenta from 100 to

![Differential cross section graph](image-url)

FIG. 1. Differential cross section for $p\alpha$ elastic scattering at $p_L = 100, 150, 250,$ and 300 GeV/c. The experimental data points are taken from Burq et al. The solid curves represent the predictions of the model described in the text.
300 GeV/c. The cross sections were normalized absolutely with a precision of 2%, which made it possible to determine, through the optical theorem, the total cross section of the $p\alpha$ interaction. Representative experimental $d\sigma/dt$ data for $p\alpha$ are shown in Fig. 1. The values of the total cross section $\sigma_T$ and the slope parameter $B$ for $p_L = 100$ to 300 GeV/c are given in Table I.

In 1981, Bujak et al. determined the differential cross section for $p\alpha$ elastic scattering for incident laboratory energy from 45 to 400 GeV in the range $0.003 < -t < 0.52 (\text{GeV/c})^2$ by means of the internal gas-jet target technique. Specimen differential-cross-section data at 301 and 393 GeV are shown in Fig. 2. The differential cross section drops 4–5 orders of magnitude in the dip at $-t = 0.22 (\text{GeV/c})^2$ and has a subsequent rise to a secondary maximum at $-t = 0.33 (\text{GeV/c})^2$. The values of the total cross section $\sigma_T$ and the ratio $\rho$ of the real and imaginary parts of the forward scattering amplitude are shown in Table I. Average values of the slope parameter $B$ of the diffraction peak in different $t$ intervals are also given in Table I. The shrinkage in the differential cross section is found to be twice as fast as that in the proton-proton case.

The differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV was measured in 1982 by Ambient et al. This has a slope $B = 41 \pm 21 (\text{GeV/c})^{-2}$ for $0.05 < -t < 0.18 (\text{GeV/c})^2$ and a minimum at $-t = 0.20 \pm 0.02 (\text{GeV/c})^2$. The extrapolation of this slope gives $\sigma_T(p\alpha) = 130 \pm 20 \text{ mb}$ via the optical theorem (assuming that the real part of the forward amplitude is negligible) and $\sigma_{el}(p\alpha) = 20 \pm 4 \text{ mb}$. The numerical values of $p\alpha$ differential cross section are shown in Fig. 3. The values of $\sigma_T$ and $B$ are given in Table I.

Bell et al. have also measured the differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV for $-t$ ranging from 0.2375 to 0.7125 $(\text{GeV/c})^2$. Their results are also shown in Fig. 3.
FIG. 2. Differential cross section for $p\alpha$ elastic scattering at $E_{lab} = 301$ and $393$ GeV. The experimental data points are taken from Bujak et al. The solid curves represent the predictions of the model described in the text.

FIG. 3. Differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV. The experimental data points are taken from Ambrosio et al. and Bell et al. The solid curve represents the predictions of the model described in the text.
The storage of $\alpha$ particles in the CERN ISR enabled Ambrosio et al.\textsuperscript{7} to study $\alpha\alpha$ interaction at unprecedentedly high center-of-mass energies. The first high-energy results, at $\sqrt{s} = 126$ GeV, were published by them\textsuperscript{7} in 1982. The measurements were made in the range $0.05 < -t < 0.8$ (GeV/c)$^2$ with a resolution of $\pm 0.02$ (GeV/c)$^2$. The differential-cross-section data are shown in Fig. 4. The errors are statistical only. The figure exhibits a first minimum at $-t = 0.10 \pm 0.01$ (GeV/c)$^2$ and a second one at $-t = 0.38 \pm 0.02$ (GeV/c)$^2$. The slope $B$ in the range $-t = 0.05$ to $0.07$ (GeV/c)$^2$ is found to be $100 \pm 10$ (GeV/c)$^{-2}$. This is given in Table II. This group\textsuperscript{7} obtained the total and integrated cross-section values as $\sigma_T = 250 \pm 50$ mb and $\sigma_{el} = 45 \pm 15$ mb, which have also been given in Table II. An independent estimate of this cross section, based on the measured luminosity-monitor cross section, is $\sigma_T = 295 \pm 40$ mb. This is also given in Table II.

In 1985, Akesson et al.\textsuperscript{9} also published results, which come from the final ISR measurements of the $\alpha$-particle collisions, at $\sqrt{s} = 126$ GeV, but in the limited range $0.05 < -t < 0.19$ (GeV/c)$^2$. These are shown in Fig. 5. The $t$ range covered, though small, includes the interesting region of the minimum in the differential cross section, and the resolution in $t$ of $\pm 0.005$ (GeV/c)$^2$ represents a significant improvement over previous data. The dip occurs at $-t = 0.098 \pm 0.002$ (GeV/c)$^2$. The differential-cross-section measurements of Ambrosio et al.\textsuperscript{7} and Akesson et al.\textsuperscript{9} are in excellent agreement above $-t = 0.14$ (GeV/c)$^2$, but are not mutually consistent for $-t < 0.14$ (GeV/c)$^2$. It may be noticed, however, that the measurements in Ref. 9 are of superior $t$ resolution. Akesson et al.\textsuperscript{9} also obtained

![FIG. 4. Differential cross section for $\alpha\alpha$ elastic scattering at $\sqrt{s} = 126$ GeV. The experimental data points have been taken from Ambrosio et al.\textsuperscript{7} The solid curve represents the predictions of the model described in the text.](image-url)
TABLE II. $\alpha\alpha$ total cross section at $\sqrt{s} = 126$ GeV, as well as the values of $\sigma_{el}$ and $B$ for $\alpha\alpha$ elastic scattering.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$\sigma_T$ (mb)</th>
<th>$\sigma_{el}$ (mb)</th>
<th>$B$ (GeV/c)$^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$280 \pm 70$</td>
<td></td>
<td>$72 \pm 6$</td>
</tr>
<tr>
<td>7</td>
<td>$250 \pm 50$</td>
<td>$45 \pm 15$</td>
<td>$100 \pm 10$</td>
</tr>
<tr>
<td>7</td>
<td>$295 \pm 40$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$315 \pm 18$</td>
<td>$58 \pm 6$</td>
<td>$87 \pm 4$</td>
</tr>
</tbody>
</table>

a slope $B = 72 \pm 6$ (GeV/c)$^{-2}$ and a total cross section $\sigma_T = 280 \pm 70$ mb, where the error includes both the statistical and scale uncertainties. These are given in Table II. Their result for $\sigma_T$ agrees well with the value $250 \pm 50$ mb obtained earlier by Ambrosio et al. and is given in Table II.

Owen et al. have measured the total interaction cross section for $\alpha\alpha$ scattering at $\sqrt{s} = 126$ GeV. The result obtained for the total cross section, viz., $\sigma_T = 315 \pm 18$ mb, is an improvement on the precision of earlier measurements. The slope $B$ for $-t < 0.07$ (GeV/c)$^2$ is $87 \pm 4$ (GeV/c)$^{-2}$, while the integrated cross section is $\sigma_{el} = 58 \pm 6$ mb. The statistical and systematic errors have been combined in these results. Having measured both the total and the elastic cross sections, they

FIG. 5. Differential cross section for $\alpha\alpha$ elastic scattering at $\sqrt{s} = 126$ GeV. The experimental points have been taken from Akesson et al. The solid curve represents the predictions of the model described in the text.
 estimated the ratio $\sigma_{el}/\sigma_T = 0.184 \pm 0.004$, 

$$B/\sigma_T = 0.276 \pm 0.020 \left(\text{GeV}/c\right)^{-2}/\mu b,$$

$$\langle r^2 \rangle^{1/2} = 2.60 \pm 0.06 \text{ fm},$$

where $r$ is the radius of interaction. The values of $\sigma_T$, $\sigma_{el}$, and $B$ are given in Table II. Bell et al. have also measured the differential cross section for $\alpha\alpha$ elastic scattering at $\sqrt{s} = 126 \text{ GeV}$ for $-t$ from 0.056 to 0.7625 $(\text{GeV}/c)^2$. Since these results are not consistent with other measurements, we have not considered them in this paper.

III. The Chou-Yang Model

In 1968, Chou and Yang proposed a preliminary version of the eikonal model for hadron-hadron elastic scattering with special reference to $\rho\rho$ elastic scattering: this is now known as the Chou-Yang model. They employed an eikonal (or opacity or blackness) which was related to the distribution of matter inside the colliding particles. The colliding protons were considered as clusters of particles which pass through each other with attenuation. Elastic scattering then results from the propagation of the attenuated wave function. By assuming that at high energies the spins of the interacting particles can be neglected and that the hadronic matter distribution is proportional to the charge distribution on protons, the scattering amplitude $T(s, t)$ in this model can be written as

$$T(s, t) = i \int b db (1 - e^{-\Omega(b)}) J_0(b\sqrt{-t}), \quad (1a)$$

where

$$\Omega(b) = K \rho(b)$$

with

$$\rho(b) = \int \sqrt{-t} d\sqrt{-t} G_{\rho}^2(t) J_0(b\sqrt{-t}). \quad (1b)$$
Here $G_p$ is the electromagnetic form factor of the proton and serves as input in the Chou-Yang model. The normalization of $T(s, t)$ is such that the differential and total cross sections are given by

$$\frac{d\sigma}{dt} = \pi |T(s, t)|^2,$$

$$\sigma_T = 4\pi \text{Im} T(s, t = 0).$$

The parameter $K$ was originally treated as a constant, since when this model was proposed, experimental results gave the impression that the total cross section for $pp$ was tending to a constant asymptotic value and the assumption that the energy-independent eikonal was proportional to the Fourier-Bessel transform of the proton form factor appeared to be justified. The differential-cross-section computations made by Durand and Lipes\textsuperscript{11} with

$$G_p = \left(1 - \frac{t}{\mu^2}\right)^{-2}, \quad \mu^2 = 1 \text{ (GeV/c)}^2,$$

exhibited two dips: one near $-t = 1.4 \text{ (GeV/c)}^2$ and the other in the vicinity of $-t = 6 \text{ (GeV/c)}^2$. The dip near $-t = 1.4 \text{ (GeV/c)}^2$ has been observed since then, but the second diffraction minimum has not been seen up to $-t = 14 \text{ (GeV/c)}^2$. However, it was later on discovered experimentally that the total cross section rises with energy. The continuous growth of the total cross section through the FNAL and ISR energy ranges makes it imperative to introduce energy dependence in the opacity even at available high energies. For incorporating the $s$ dependence into this model, several suggestions based on different lines of reasoning have been proposed in literature. In particular, Hayor and Sukhatme,\textsuperscript{12} Henzi et al.,\textsuperscript{13} and Bourrely et al.\textsuperscript{14} assumed that $\Omega(s, b)$ can be factorized as a product of two functions, one depending upon $s$ alone and the other upon the parameter $b$ only:

$$\Omega(b, s) = K(s) \rho(b).$$

The model based on this assumption is known as the factorizable eikonal model. At a particular energy, the value of $K$ is obtained by adjusting it so as to yield the experimental value of the total cross section at that energy. Once $K$ is known, the differential cross section and other characteristics of the process can be calculated. The model can be easily extended to a high-energy collision of two distinct particles.
For low momentum transfer, the alpha form factor has been measured by a number of groups\textsuperscript{15}, an extensive study up to $Q^2 = 20$ fm$^{-2}$ was made by Frosch et al.,\textsuperscript{16} who observed a diffractive minimum and a secondary maximum around $Q^2 = 11.50$ and 18.0 fm$^{-2}$ respectively. The alpha form factor had already been found by Repellin et al.\textsuperscript{17} to deviate from a smooth Gaussian. McCarthy et al.\textsuperscript{18} studied the charge form factor of isotopes of helium and have verified the low-momentum-transfer measurements of Erich et al.\textsuperscript{15} and Frosch et al.\textsuperscript{16} The experimental data on the alpha form factor have been parametrized by McCarthy et al.\textsuperscript{18} as

$$G_{1\alpha} = \left[1 - (a^2 Q^2)^b\right] e^{-\frac{Q^2}{b^2}},$$

where $a = 0.316$ fm and $b = 0.675$ fm, and by Lombard and Tellez-Arenas\textsuperscript{2} as

$$G_{2\alpha} = 0.8345 e^{-0.5Q^2} + 0.1655(1 - 0.1157Q^2)e^{-0.1376Q^2}.$$

Both of these parametrizations agree with the experimental data quite reasonably, except that the parametrization of McCarthy et al.\textsuperscript{18} gives a zero near $Q^2 = 10$ fm$^{-2}$ whereas the experimental data show only a dip in that region. A comparison of these two parametrizations with the data is shown in Fig. 6. We have used the one proposed by McCarthy et al.\textsuperscript{18}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{The parametrizations of the $\alpha$ form factor by McCarthy et al.\textsuperscript{18} and Lombard et al.\textsuperscript{2} plotted against $Q^2$. The experimental data points are taken from McCarthy et al.\textsuperscript{18} and Frosch et al.\textsuperscript{16}}
\end{figure}
It is now known\(^1\) that the original Chou-Yang model\(^1\) does not reproduce the experimental data beyond the diffraction-peak region. In order to explain the experimental results for \(pp\) and \(\bar{p}p\) elastic scattering at all \(-t\), the Chou-Yang model has to be generalized. For that purpose, following Glauber and Velasco,\(^4\) we obtain an expression for the scattering amplitude by using the multiple-diffraction theory. According to this theory, hadrons consist of clusters of particles which, on collision, pass through each other, interacting in pairs \(i, j\) and scattering one another with invariant scattering amplitudes \(f_{ij}(t)\). By using this concept, it is found\(^4\) that the opaqueness \(\Omega(s, b)\) in the expression for the scattering amplitude

\[
T(s, t) = i \int b I_0(b\sqrt{-t})(1 - e^{-i\alpha(b, s)}) \, db
\]  

is given by

\[
\Omega(s, b) = K(1 - i\alpha) \int \sqrt{-t} \, d\sqrt{-t} I_0(b\sqrt{-t}) G_A(t)G_B(t) \frac{f(t)}{f(0)}. \]

It is interesting to notice that when the scattering of the constituent partons is isotropic, so that \(f(t)/f(0)\) is equal to unity and the interactions are considered to be purely absorptive \((\alpha = 0)\), then the model based on the multiple diffraction theory reduces to that of Chou and Yang,\(^1\) involving only the imaginary part of the scattering amplitude. It is for this reason that the original Chou-Yang model, which tacitly assumes the scattering to be isotropic, is valid in the diffraction-peak region. Since there is no known method for finding an expression for \(f(t)/f(0)\), we have to guess its form as a slowly varying function of \(t\), so that overall satisfactory results consistent with experiment even for large \(-t\) are obtained.

We now give a physical picture of hadron-nucleus and nucleus-nucleus elastic scattering in the generalized Chou-Yang model. The colliding particles, according to the multiple diffraction theory, are clusters of objects, called partons, which interact with the target in pairs of high-energy when the wavelengths are very small. It is feasible to describe the interaction between these clusters of partons as involving successive collisions of every parton of one cluster with individual partons making up the other cluster. This interaction depends critically upon the relative positions of these partons, and therefore upon the pair and possibly higher-order correlation functions. The multiple-scattering effect is small compared to single scattering at small momentum transfers, as the peripheral partons dominating the process in this region are not expected to suffer more than one collision before leaving the scattering region. The differential cross section in this case is therefore essentially a consequence of single scattering. Since the original Chou-Yang model
VI. Calculations and Discussion

In this section, we shall show that the generalized Chou-Yang model for $p\alpha$ and $\alpha\alpha$ elastic scattering gives results which are consistent with the experimental data even beyond the dip region.

The data for $p\alpha$ elastic scattering at high energies can be explained by choosing the anisotropy function as $(1 - at)^b$, where $a = 0.1 \text{ (GeV/c)}^{-2}$ and $b = 1.0$. The differential-cross-section results at $p_L = 100, 150, 250, 300 \text{ GeV/c}$ and $E_{\text{lab}} = 301, 393 \text{ GeV}$ and $\sqrt{s} = 89 \text{ GeV}$, corresponding to $\sigma_T = 128.7 \pm 0.9, 130.8 \pm 0.8, 131.6 \pm 0.8, 132.0 \pm 0.8, 125.9 \pm 0.6$, and $130 \pm 20 \text{ mb}$, respectively, are plotted in Figs. 1–3 along with the experimental data. The agreement is quite good. The values of $K$ have been adjusted to $34.6, 35.4, 35.7, 35.8, 32.5, 33.4$, and $42.0 \text{ (GeV/c)}^{-2}$, which yield $\sigma_T = 129.3, 131.6, 132.4, 132.7, 125.8$, and $119.9 \text{ mb}$, respectively.

The first attempt to explain the $\alpha\alpha$ elastic scattering by using the original Chou-Yang model was made by Lombard and Tellez-Arenas.$^2$ They noticed that the slope of the differential cross section at $\sqrt{s} = 126 \text{ GeV}$ was well produced in the diffraction-peak region. Kamran and Qureshi$^2$ have recently carried out a study of $\alpha\alpha$ elastic scattering at $\sqrt{s} = 126 \text{ GeV}$ based on the Chou-Yang model and have deduced a multiple-dip structure in the differential cross section. In fact, this does not occur due to the nature of the electromagnetic form factor but due
to faulty computation of integrals. Firstly, the integrands involve $J_{\alpha}(b \sqrt{-t})$ which oscillates about zero, and the oscillations occur rapidly when $b \sqrt{-t}$ is of appreciable size. Consequently, if the subdivisions of the interval of integration are not small enough, the error introduced in the computation is quite sizable, which renders the results unreliable. Secondly, if the cutoff on the upper limits of $b$ and $V - t$ is made prematurely, a significant contribution to the integrals is left unaccounted for. On either of these scores the computations give wild results.

The differential cross section for $\alpha \alpha$ elastic scattering can be explained quite satisfactorily by using the generalized Chou-Yang model. By choosing

$$\frac{f(t)}{f(0)} = (1 - at)^b,$$

the differential-cross-section curves corresponding to the mutually inconsistent experimental results of Aksesson et al. and Ambroso et al. are obtained with $a = 0.3$ (GeV/c)$^2$ and $b = 1.0$ for Ref. 9, and with $a = 0.3$ (GeV/c)$^2$ and $b = 10$ for Ref. 7. These curves are shown in Figs. 4 and 5 and have been obtained by choosing $K = 115.0$ and $114.0$ (GeV/c)$^2$ and $\alpha = 0.36$ and $0.45$, which yield $\sigma_T = 349.02$ and $334.58$ mb. As pointed out earlier, the measurements of $\sigma_T$ in Refs. 7 and 9 are less accurate than that in Ref. 10. However, to fit the data on $d\sigma/dt$ from Refs. 7 and 9 we must use the $\sigma_T$ values obtained by these very groups. Now the value of $\sigma_T$ reported in Ref. 7 is $250 \pm 50$ mb, and that reported in Ref. 9 is $280 \pm 70$ mb. Thus we can conclude that the experimental $\sigma_T$ lies between 200 and 350 mb. The theoretical values of $\sigma_T$ relevant to the solid curves in Figs. 4 and 5 are consistent with these experimental values.

VII. Conclusion

The generalized Chou-Yang model, which takes into account the anisotropic nature of scattering, can explain the $p \alpha$ and $\alpha \alpha$ elastic scattering at high energies even beyond the diffraction region.

References

DEUTERON-DEUTERON ELASTIC SCATTERING
AT HIGH ENERGIES

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Abstract

It has been shown that the pristine Chou-Yang model can explain the differential cross section for deuteron-deuteron elastic scattering at \( \sqrt{s} = 53 \) GeV in the diffraction peak region. In order to fit the large momentum transfer data, the generalised Chou-Yang model is used.
I. Introduction

In 1978, Goggi et al. [1] reported experimental results on deuteron-deuteron elastic scattering at a CM energy of $\sqrt{s} = 53$ GeV. The data were obtained using the Split Field Magnet detector at the CERN intersecting Storage Rings. The $t$-dependence of the elastic cross section was measured in the range $0.05 < -t < 1.5$ (GeV/c)$^2$. A narrow interference minimum is observed at $-t = (0.179 \pm 0.005)$ (GeV/c)$^2$. The value of $d\sigma/dt$ at the minimum is given by $d\sigma/dt = (37.28 \pm 7.86)$ mb/(GeV/c). The slope parameter value for $0.05 < -t < 0.09$ (GeV/c)$^2$ is $56.8 \pm 0.3$ (GeV/c)$^2$ while for $0.41 < -t < 1.48$ (GeV/c)$^2$, it is $6.3 \pm 0.1$ (GeV/c)$^2$. The differential cross section measurements are shown in Fig.1.

![Differential cross section for dd elastic scattering at $\sqrt{s} = 53$ GeV. The solid curve corresponds to the predictions of the generalised Chou-Yang model. The experimental data points are taken from Goggi et al. [1].](image)

Fig. 1: Differential cross section for dd elastic scattering at $\sqrt{s} = 53$ GeV. The solid curve corresponds to the predictions of the generalised Chou-Yang model. The experimental data points are taken from Goggi et al. [1].
Chou and Yang [2] proposed in 1968 an eikonal model for hadron-hadron elastic scattering at asymptotic energies. This model received a boost when the first dip predicted by it was observed in pp elastic scattering at ISR energies a few years later. A number of attempts have been made since then to apply this model and its generalised form to elastic reactions involving hadrons and nuclei [3]. The model has been used to explain the various characteristics of hadron-hadron and hadron-nucleus and nucleus-nucleus elastic scattering except for the large values of deuteron-deuteron elastic scattering (in the vicinity of dip and beyond). We shall use this generalised model to calculate the characteristics of dd elastic scattering at high energies including the large momentum transfer measurements of the differential cross section. Let us first of all briefly describe the generalised Chou-Yang model.

II. Generalized Chou-Yang Model

The eikonal picture which has theoretical foundations in some areas of physics has been successful in explaining various aspects of elastic scattering at high energies. Chou and Yang [2] first proposed a preliminary version of the eikonal model for hadron-hadron elastic scattering. The model is based on geometrical considerations in which hadrons are treated as extended objects. Elastic scattering then results from the propagation of attenuated wave function. By assuming that at high energies the scattering amplitude is purely imaginary and that the hadronic matter distribution is proportional to the charge distribution on protons, Durand and Lipes [4] studied high energy pp scattering on the basis of this primitive model. Later on, the model was extended to other elastic reactions. However, a survey of literature shows [5] that it has been successful only in the diffraction peak region. The geometrical picture of Chou and Yang was extended to nucleus-nucleus elastic scattering by Li and Lo [6]. The nucleus is then considered as consisting of a continuous distribution of matter and the boundaries between nucleons are considered to be insignificant. At high energies the nuclei appear each other as discs with vanishing thickness due to Lorentz contraction. The interaction of the nuclei is then the interaction between nucleon matter in discs. Thus virtually there is no difference between nucleon-nucleon and nucleus-nucleus scattering.

In order to explain the experimental data beyond the diffraction peak region, Glashow and Veltman [7] have generalised the Chou Yang model. An expression has been obtained for the scattering amplitude in this model which is based on the multiple diffraction theory. According to this theory, hadrons consist of clusters of particles which, on collision, pass through each other, interacting in pairs ij and scattering one another with invariant scattering amplitudes $f_i(t)$. By using this concept to dd elastic scattering, it is found that the scattering amplitude $T(s,t)$ is given by

$$T(s,t) = i \int b \ J_\sigma(b\sqrt{t}) \left[ 1 - \exp(-\Omega(s,b)) \right] \, db$$

where

$$\Omega(s,b) = K(1 - i\alpha) \int \frac{1}{\sqrt{-t}} \, d(-t) \, J_\sigma(b\sqrt{-t}) \frac{f(t)}{f(0)} \, G_e^2(t).$$

(1)
The function $\mathcal{G}(s,b)$ represents principally an opacity effective for clusters passing with a relative impact parameter $b$, but is taken to be complex and thereby includes refractive as well as absorptive effects. $f(t)/f(0)$ is an unknown function which is supposed to take account of the anisotropy of the interacting particles. The parameter $\alpha$ is determined by the ratio of the real and imaginary parts of the scattering amplitude in the forward direction. $G_d(t)$ is the deuteron form factor. The parameter $K$ is adjusted so that the experimental value of the total cross section is obtained. It is interesting to note that when the scattering of the constituent partons is isotropic so that $f(t)/f(0)$ is equal to unity and the interactions are considered to be purely absorptive so that $\alpha = 0$, then the model based on the multiple diffraction theory reduces to that of Chou and Yang [2]. Since this model involves deuteron form factor, we will next describe it briefly.

III. Deuteron Form Factor

Hadronic form factors give us information about the distribution of matter inside hadron. It not only provides us information about the compositeness or elementarity of a hadron but also supply us information about the dynamics of interactions at short distances at asymptotic energies. In the Chou-Yang model this quantity is related to the charge/magnetic form factors of hadrons. These form factors of hadrons describe an extended structure which can be represented by spatial distributions of the static quantities.

The form factors of the deuteron at high momentum transfer have long been of interest for the information they contain on the short range nucleon-nucleon interaction and the transition from nucleon to quark degrees of freedom. Several groups [8] have measured the structure functions $A(t)$ and $B(t)$ involving deuteron form factors. The structure function $A(t)$ which is a combination of charge form factor $G_c(t)$, the magnetic form factor $G_m(t)$ and the quadrupole form factor $G_Q(t)$ is given below:

$$A(t) = G_c^2(t) + \left( t^4/18 M^4 \right) G_Q^2(t) - \left( t^6/6 M^6 \right) \left( 1 - t^2/4M^2 \right) G_m^2(t)$$

where $M$ is the mass of the deuteron. It can be shown that, up to a good approximation, the above equation may be written as

$$G_c(t) \approx \left[ A(t) \right]^{1/3}$$

for $-t < 4M^2 = M^2 \left( \sqrt{\epsilon} - 1 \right)^2$. Extensive data on $A(t)$ are available [8] in the spacelike region extending up to $6 \ (\text{GeV}/c)^2$. Therefore a formula for parametrizing $A(t)$ can be developed. Parida [9], using a modified N/D method where N and D are functions representing anomalous and two point cut contributions, parametrized the deuteron charge form factor as follows:

$$G_c = \left( A(t) \right)^{1/3}$$

where

$$A(t) = c_0 \exp \left( -t/(\epsilon_0 t) \right) \left[ c_1 + t + c_2 t^2 + c_3 t^3 + h(t) + m_0^2/(\pi) \right]$$
This parametrization has quite good agreement with the experimental data [8] in the range $0 \leq |t| \leq 6 \text{ (GeV/c)}^2$. We have ourselves parametrized the experimental data for deuteron form factor in a much simpler form. We obtain the following expression:

$$A(t) = \sum a_i \exp(b_i t)$$

where

- $a_1 = 0.9691$, $a_2 = 0.03$, $a_3 = 0.0009$
- $b_1 = 9.1$, $b_2 = 1.35$, $b_3 = 0.46$

The results of our parametrization of the deuteron form factor are plotted in Fig. 2.

![Graph comparing different parametrizations of deuteron form factor](image)

**Fig. 2:** Comparison of parametrizations of deuteron form factor obtained by us and Prida et al [9] respectively against $Q^2 = -t$. The experimental data points are taken from reference 7.
together with the results of Parida et al [9]. It can be seen in the figure that the results of two parametrizations are in good agreement with the experimental data.

IV. Predictions of the Pristine Chou-Yang Model

By using our parametrization and that given in Ref. 9 for the charge form factor of deuteron in the opacity integral, we have calculated the scattering amplitude and consequently differential cross section and other characteristics of dd elastic scattering at $\sqrt{s} = 53$ GeV. The results for $d\sigma/dt$ data are shown in Fig.3. The agreement between theory and experiment is quite good in the diffraction peak.

![Graph of Differential cross section for dd elastic scattering at $\sqrt{s} = 53$ GeV. The curves correspond to the predictions of the pristine Chou-Yang model by using the parametrization of the form factor in this paper (solid) and that given in Ref. 9 (dotted). The experimental data points are taken from Goggi et al [1].]
region for both the parametrizations. However, in the vicinity of the dip and beyond there is significant departure from experimental data. Theoretical results obtained by our parametrizations of the deuteron form factor gives two zeros at 0.22 (GeV/c)^2 and 0.70 (GeV/c)^2 respectively. The value of K is chosen as 33.0 (GeV/c)^2. The first dip has been predicted at the right position but the value of the differential cross section at the dip is about one third of the experimental value. As regards the results obtained by the form factor used in Prida et al we obtain multiple dips first being at 0.42 (GeV/c)^2. The value of K for this structure is 30.0 (GeV/c)^2. However the total cross section is the same for both parametrizations. As pointed out earlier, the pristine Chou-Yang model is thus successful only in the diffraction peak region. This is because single scattering occurs in this region; beyond that multiple scattering prevails as the hard scattering is occurring in this region. Without taking this into consideration the model remains unsuccessful in the region beyond the dip.

V. Predictions of Generalised Chou-Yang Model

In order to explain the differential cross section beyond the dip region, we make use of the Generalised Chou-Yang model as described in the earlier section. In order to take into account the anisotropy of the scattering, we must know the function f(t)/f(0) in equation 1. As this function can not be derived theoretically at present, we have to guess its form so that after evaluation of the appropriate integral the computed differential cross section results may agree with the experimental data. We have chosen this function as

\[ f(t)/f(0) = (1 + a t^b) \]

where \( a = 0.02 (GeV/c)^2 \) and \( b = 12.7 \). In order to make calculations at a particular energy, by using the generalised Chou-Yang model, the total cross section at that energy should be known. However, the total cross section at \( \sqrt{s} = 53 \) GeV has not yet been measured. We have therefore made use of the extrapolated differential cross section at \( t = 0 \) to fix the value of K which has ultimately been used to predict the total cross section at this energy. The differential cross section results for \( \sqrt{s} = 53 \) GeV calculated by using the generalised Chou-Yang model along with the experimental data are shown in Fig.1. There is very good agreement with the experimental data up to \( \rho = 0.15 (GeV/c)^2 \). The value of \( K \) which fits the experimental data for dd elastic scattering is 32.8 (GeV/c)^2. The value of \( \alpha \) is chosen as 0.26 which yields \( \rho = 0.206 \). The ratio of the integrated to the total cross section \( \sigma_e/\sigma_t \) is 0.17. However beyond \( \rho = 0.75 (GeV/c)^2 \), we do not get a satisfactory agreement.

Thus, although the explanation of high energy elastic scattering on the basis of the pristine Chou-Yang model is confined to the diffraction peak region [3], the generalised Chou-Yang model which takes into account the non-isotropic nature of scattering is expected to explain the high energy dd elastic scattering at a very energy of \( \sqrt{s} = 53 \) GeV. It may further be pointed out that it is for the first time that the explanation of dd elastic scattering in the vicinity of and beyond the dip region has been given.
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CHOU-YANG MODEL AND
HADRONIC FORM FACTOR OF THE DEUTERON

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ABSTRACT

By assuming the hadronic form factor of the deuteron different from the electromagnetic form factor, as has been done by Chou and Yang recently for proton form factor, we fit the data for dd elastic scattering at $\sqrt{s} = 53$ GeV in the entire measured region.

1. Introduction

Satisfactory explanation of the dd elastic scattering at high energies has been elusive since the first measurement at $\sqrt{s} = 53$ GeV [1]. Attempts to explain this reaction on the basis of the Chou-Yang model [2] yielded a fit in the diffraction peak region only. Beyond the dip region the pristine Chou-Yang model gives multi-dip structure which is inconsistent with the experimental measurements. Similarly a straightforward application of the Glauber multiple scattering model does not give a satisfactory fit to the data [3]. In all such models which are based on an analogy with "Fraunhofer diffraction theory", the form factor of the hadron (nucleon) is assumed to be proportional to the electromagnetic form factor. The geometrical picture of the Chou-Yang model in its pristine form fits the data up to the first diffraction minimum. In order to explain the data beyond the diffraction peak region, an effort [4] was made to take in to account the multiple scattering in the hard scattering region. The generalized Chou-Yang model [5], which takes into account the multiple scattering in terms of anisotropy of the scattering, gives the structure around 0.2 (GeV/c)$^2$ and is consistent with the experimental data only up to $-t = .75$ (GeV/c)$^2$. However the model is unable to reproduce the entire structure in dd elastic scattering at $\sqrt{s} = 53$ (GeV) where the measurements have been made up to $-t \approx 1.5$ (GeV/c)$^2$. Thus in the frame work of the
Chou-Yang model with electromagnetic form factor as input, the entire structure in dd elastic scattering can not be explained.

Failure of the geometrical picture to explain the entire structure thus suggests that either the assumption regarding the multiple scattering needs modifications or the equivalence between the electromagnetic form factor and hadronic form factor is only an approximation. In their recent work, Chou and Yang [6] followed the second line of arguments and proposed that the hadronic matter distribution for the proton is energy dependent. They have thus given a good fit to the pp data at ISR and Collider energies. On similar lines, by assuming the hadronic form factor of the deuteron to be energy dependent, we have fitted the data for dd elastic scattering at $\sqrt{s} = 53$ GeV in the entire measured region. The physical significance of the newly proposed form factor is also discussed. Let us first briefly discuss the pristine Chou-Yang model.

2. The Chou-Yang Model

The geometrical picture of the Chou-Yang model [1] was first proposed in 1968. The model employed an eikonal (or opacity or blackness) which was related to the distribution of matter inside the colliding particles. The colliding particles (hadrons or nuclei) were considered as clusters of particles (partons or nucleons) which pass through each other with attenuation. Elastic scattering then result from the propagation of the attenuated wave function. The scattering amplitude $T(s,t)$ in this model, by neglecting spin, is written as

$$T(s,t) = i \int b \, db \, [1-\exp(-\Omega(b))] \, J_0(b \sqrt{-t})$$

where

$$\Omega(b) = K \, \rho(b)$$

with

$$\rho(b) = \int \sqrt{-t} \, d \sqrt{-t} \, G_d^2(t) \, J_0(b \sqrt{-t})$$

They assumed that the hadronic matter distribution is proportional to the charge distribution on hadrons/nuclei, which is represented by $G_p$. Thus $G_d$ in the expression for the amplitude is the hadronic form factor of the interacting particles. Following Ref.6, we have assumed $G_d$ to be energy dependent and given by

$$G_d = s^{-1/4} \sum e^{i \theta} \left( \sum d_i e^{i \theta_i} \right)$$

The values of parameters are given in Table 1.
Table 1

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The normalisation of \( T(s,t) \) is such that the differential and total cross sections are given by

\[
d\sigma/dt = \pi | T(s,t) |^2
\]

\[
\sigma_T = 4\pi \text{Im} \ T(s,t=0).
\]

The parameter \( K \) was originally treated as a constant since when this model was proposed, experimental results gave the impression that the total cross section for pp was tending to a constant asymptotic value and the assumption that the energy independent eikonal was proportional to the Fourier-Bessel transform of the proton form factor appeared to be justified. Later on this was chosen to be energy dependent. The differential cross section computations made for different reactions however show [7] that the model in this form is valid only in the diffraction peak region. The multiple dip structure predicted by this model is not confirmed by the experiment.

Keeping in view the inability of pristine model in explaining the data in the entire \(-t\) region, Chou and Yang very recently have proposed that the assumption regarding the equivalence of the charge form factor and the hadronic form factor is only an approximation within the framework of their model. They have thus proposed a simple expression for the matter distribution. Similar assumption was made by Saleem et al [8] in explaining the \( \bar{p}p \), \( p\alpha \), \( \alpha\alpha \) elastic scattering. However, unlike the assumption made in Ref.8, Chou and Yang have taken the hadronic form factor energy dependent. By choosing an energy dependent range parameter, they have fitted the \( p(\bar{p})p \) elastic scattering data at 23.5 and 546 GeV. Their fit to the data at 23.5 GeV is good up to \(-t \approx 3.0 \ (\text{GeV}/c)^2 \) while the measurements are up to \(-t \approx 7.0 \ (\text{GeV}/c)^2 \). Beyond \(-t \approx 3.0 \ (\text{GeV}/c)^2 \) the deviation from the experimental data becomes significant. We have used a similar procedure and predicted a form factor of the deuteron which gives very good fit to the data at 53 GeV. In our choice, the deuteron form factor is energy dependent.
3. Predictions of the Chou-Yang model

Let us now compare the predictions of our calculations with the available data for dd elastic scattering. Fig. 1 shows the fit for the differential cross section at 53 GeV. The fit is obtained for $K = 69.5 \text{ (GeV/c)}^2$ and $\alpha = 0.05$. The theoretical predictions of the model are in good agreement with the experimental data.

4. Conclusions

There has been considerable interest in the form factor of the deuteron as it gives information about the short range nucleon-nucleon interaction and the transition from nucleon to quark degrees of freedom. Furthermore, it has been a question of considerable importance whether or not the electric charge distribution is the same as the nuclear matter distribution. In 1968 Chou and Yang assumed that the matter distribution inside a hadron is the same as the charge distribution inside a hadron. These form factors of hadrons describe an extended structure which can be represented by spatial distributions for the static quantities. Although at ISR energies the Chou-Yang model has yielded impressive quantitative agreement with experiments, yet as the energy increases, deviations from the strict interpretation of the geometrical picture become quite noticeable. Modifications of the theory are thus necessary. Lombard and Wilkin [9] suggested that there is probably some secondary (energy-dependent) mechanism, which plays an important role for large $t$. Chou and Yang in their recent work take
the proton form factor energy dependent. On similar lines, we assume the hadronic form factor of the deuteron as dependent on energy. This suggests that the hadron are expanding as energy is increasing. We thus conclude that by assuming the hadrons as extended objects and that their matter distribution changes with energy, we can predict the behaviour of dd elastic scattering at high energies and large momentum transfers.

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GEOMETRICAL PICTURE AND  $\alpha$ FORM FACTOR

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GEOMETRICAL PICTURE AND \( \alpha \) FORM FACTOR

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ABSTRACT

We propose the hadronic form factor of the \( \alpha \), in analogy to that of the proton as suggested by Chou and Yang recently. The proposed matter distribution which is energy dependent has been used to fit the experimental data for \( \alpha \alpha \) and \( p\alpha \) elastic scattering at high energy.

1. Introduction

Recently, Chou and Yang have extended the scope of the Geometrical model [1] proposed by them in 1968. Instead of taking the electromagnetic form factor as input, they have proposed [2] the hadronic form factor of the proton and used it in the Geometrical model to fit the elastic scattering data at high energies. Similar attempt to explain the dd elastic scattering [3] has yielded good results at high energies. In analogy to the proton matter form factor, we give an expression for the hadronic form factor of \( \alpha \).

Using the newly proposed \( \alpha \) form factor in the Chou-Yang model, we have fitted the \( \alpha \alpha \) differential cross section at \( \sqrt{s} = 126 \) GeV. The proposed form factor of \( \alpha \) is energy dependent which is consistent with the conclusion drawn by Chou and Yang. In order to further verify the validity of hadronic form factor of \( \alpha \), we have attempted to fit the \( p\alpha \) elastic scattering at various energies and find a good fit to the experimental data.

2. EXPERIMENTAL DATA

The \( \alpha \alpha \) differential cross section at \( \sqrt{s} = 126 \) GeV was measured by Ambrosio et al [4]. The measurements were made in the range 0.05 ( - \( t \) < 0.8 (GeV/c)\(^2\)) with a resolution of \( \pm 0.02 \) (GeV/c)\(^2\). The data exhibits a first minimum at \( -t = 0.10 \pm 0.01 \) (GeV/c)\(^2\) and a second one at -
The results of their measurements are given in Fig. 1. Akesson et al [5] have also published results of their measurements at $\sqrt{s} = 126$ GeV in the limited range $-t = 0.05 - 0.19$ (GeV/c)$^2$ with a significantly improved $t$ resolution. A careful study of the two data show that their measurements differ in the first dip and bump region. Measurements for $p\alpha$ elastic scattering have been carried out by Bujak et al [6] at 45-400 GeV/c, Ambrosio et al [4] and Bell et al [7] at $\sqrt{s} = 89$ GeV. Representative data of these measurements at 301 GeV/c and $\sqrt{s} = 89$ GeV are given in Fig. 2.

### 3. The Chou-Yang Model

The geometrical picture of the Chou-Yang model [1] was first proposed in 1968. The model employed an eikonal (or opacity or blackness) which was related to the distribution of matter inside the colliding particles. The scattering amplitude $T(s,t)$ in this model, by neglecting spin, is given by

$$T(s,t) = i \int b \, \mathrm{d}b \, [1 - \exp(-\Omega(b))] \, J_0(b\sqrt{-t})$$

where

$$\Omega(b) = K \rho(b)$$

with

$$\rho(b) = \int \sqrt{-t} \, \mathrm{d}t \, G_\alpha^2(t) \, J_0(b \sqrt{-t})$$

where the hadronic matter distribution, which was assumed to be proportional to the charge distribution on hadrons/nuclei, is represented by $G_\alpha$. Thus $G_\alpha$ in the expression for the amplitude is the hadronic form factor of the interacting particles. Following Ref. 2, we have assumed alpha form factor to be energy dependent and given by

$$G_\alpha = \frac{s^{0.6}}{1 + 1.65 \cdot 10^{-x} + 3.25 \cdot 10^{-x}} - 0.03 \cdot e^{1.864}$$

(1)

The normalisation of $T(s,t)$ is such that the differential and total cross sections are given by

$$\frac{d\sigma}{dt} = \pi \left| T(s,t) \right|^2$$

$$\sigma_T = 4\pi \mathrm{Im} \, T(s,t = 0)$$

The parameter $K$ was originally treated as a constant since when this model was proposed, experimental results gave the impression that the total cross section for pp was tending to a constant asymptotic value and the assumption that the energy independent eikonal was proportional to the Fourier-Bessel transform of the proton form factor appeared to be justified. Later on this was chosen to be energy dependent. The differential cross section computations made for different reactions however show [8] that the model in this form is valid only in the diffraction
peak region. The multiple dip structure predicted by this model is not confirmed by the experiment.

Keeping in view the inability of pristine model in explaining the data in the entire \( t \) region, Chou and Yang very recently have proposed that the assumption regarding the equivalence of the charge form factor and the hadronic form factor is only an approximation within the framework of their model. They have thus proposed a simple expression for the matter distribution. Similar assumption was made by Saleem et al [9] in explaining the \( \bar{p}p, p\alpha \), \( \alpha\alpha \) elastic scattering. However, unlike the assumption made in Ref.9, Chou and Yang have taken the hadronic form factor energy dependent. By choosing an energy dependent range parameter, they have fitted the \( p(p)p \) elastic scattering data at 23.5 and 546 GeV. Their fit to the data at 23.5 GeV is good up to \( t \approx 3.0 \) (GeV/c)\(^2\) (while the measurements are up to \( t \approx 7.0 \) (GeV/c)\(^2\)). Beyond 3.0 (GeV/c)\(^2\) the deviation from the experimental data becomes significant. We have used a similar procedure and proposed a form factor for the deuteron [3] which gives very good fit to the data at 53 GeV. In this paper we have used the energy dependent alpha form factor given in equation (1).

4. Predictions of the Chou-Yang model

Next we compare the predictions of our calculations with the available data of Refs.4 and 5 for \( \alpha\alpha \) elastic scattering at \( \sqrt{s} = 126 \) GeV. Fig. 1 shows the fit for the differential cross section. The fit is obtained for \( K = 115 \) (GeV/c)\(^{-2}\) while \( \alpha \) has been chosen as 0.035 and 0.03 for Figs. 1a. and 1b. respectively. The theoretical predictions of the model are in good agreement
with the experimental data. The data of Bujak et al [6], Ambrosio et al [4] and Bell et al [7] at 301 GeV/c and 89 GeV respectively, for pp differential cross sections are compared in Fig.2. The predicted results are again in good agreement with the experimental data.

5. Conclusions

It has been a question of considerable importance whether or not the electric charge distribution is the same as the nuclear matter distribution. In 1968, Chou and Yang assumed that the matter distribution inside a hadron is the same as the charge distribution inside a hadron. Lombard and Wilkin [10] have proposed the importance of some secondary (energy-dependent) mechanism, which plays an important role for large t. Chou and Yang [2] also take the proton form factor energy dependent. On similar lines, we have assumed the hadronic form factor of the alpha as dependent on energy.

It will be interesting to point out that various feature of alpha-alpha elastic scattering can also be explained on the basis of the generalized Chou-Yang model [11]. Similar conclusion holds good for pp elastic scattering at ISR, Collider and Tevatron energies. However, the Generalized Chou-Yang model when applied to dd elastic scattering does not give satisfactory results in the entire measured region. The new measurements for pp at 1.8 TeV will be able to throw more light on the concept of energy dependent hadronic form factors.
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Nucleus(Hadron)-Nucleus Elastic Scattering
and Geometrical Picture

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ABSTRACT

A comprehensive explanation of nucleus-nucleus and hadron-nucleus elastic scattering is elucidated since the measurements of these reactions were made. By proposing energy dependent hadronic form factors for deuteron and alpha, in analogy to that of the proton as suggested by Chou and Yang recently, we have fitted all the available data for alpha-alpha and deuteron-deuteron elastic scattering. In order to further verify the validity of the proposed form factor, we have also fitted the data for proton-alpha and proton-deuteron elastic scattering. It is concluded that the hadronic matter is expanding with an increase in energy.
Geometrical (or eikonal) picture has firm theoretical foundations in different areas of physics. In 1968, Chou and Yang extended the scope of the geometrical picture to High Energy Physics. Since then a large number of papers have been written to explain various aspects of hadron-hadron, hadron-nucleus and nucleus-nucleus elastic scattering. Although a piecemeal explanation of the experimental data exists for nucleus(hadron)- nucleus elastic scattering, a comprehensive explanation of these reactions is elusive. In this paper we will attempt to explain the entire data for nucleus(hadron)nucleus elastic scattering at high energies based on recent developments in the Chou-Yang model. In our study we have included alpha-alpha, proton-alpha, deuteron-deuteron and proton-deuteron elastic scattering. As the data for pion-nucleus elastic scattering is in the diffraction peak region and can be explained on the basis of the pristine Chou-Yang model [1], we have not included it in this paper. In order to give a detailed account of these reactions we have subdivided this paper into five parts: namely, (1) introduction; (2) review of the experimental data; (3) review of theoretical models; (4) predictions of Chou-Yang model and (5) conclusions.

1. INTRODUCTION

Geometrical picture, first proposed by Chou and Yang [2] is based on "Fraunhofer Diffraction Theory". The model employed an eikonal (or opacity or blackness) which was related to the distribution of matter inside the colliding particles. Attempts to explain these reactions on the basis of the Chou-Yang model yielded a fit in the diffraction peak region only [3]. Beyond the dip region the pristine Chou-Yang model gives multi-dip structure which is inconsistent with the experimental measurements. In all such models, the form factor of the hadron (nucleon) is assumed to be proportional to the electromagnetic form factor. In order to explain the data beyond the diffraction peak region, an effort was made to take into account the multiple scattering in the hard scattering region. The generalized Chou-Yang model [4], which takes into account the multiple scattering in terms of anisotropy of the scattering, fits the data for αα and pp elastic scattering [5] well but does not give satisfactory results for dd elastic scattering. The model is unable to reproduce the entire
structure in dd elastic scattering at $\sqrt{s} = 53$ (GeV) [6]. Thus in the frame work of the Chou-Yang model with electromagnetic form factor as input, the entire structure in dd elastic scattering can not be explained.

Failure of the geometrical picture to explain the entire structure in dd elastic scattering thus suggests that either the assumption regarding the multiple scattering needs modifications or the equivalence between the electromagnetic form factor and hadronic form factor is only an approximation. In their recent work, Chou and Yang [7] followed the second line of arguments and proposed that the hadronic matter distribution for the proton is energy dependent. They have thus extended the scope of the geometrical model and proposed the hadronic form factor of the proton which was used in the geometrical model to fit the elastic scattering data at high energies. They have given a good fit to the $\bar{p}p$ data at ISR and Collider energies. On similar lines, by assuming the hadronic form factors of deuteron and alpha to be energy dependent, we have fitted the data for dd and $\alpha\alpha$ elastic scattering at all available energies and in the entire measured region. In order to further verify the validity of the proposed hadronic form factors of deuteron and alpha we have also fitted the data for pd and $p\alpha$ elastic scattering at various energies.

2. REVIEW OF EXPERIMENTAL DATA

A series of experiment on alpha-alpha, proton-alpha, proton-deuteron and deuteron-deuteron elastic scattering at high energy have been performed at CERN, Fermilab, Brookhaven, Surpukov and PPA (Princeton-Pennsylvania Accelerator). In the present section we describe these measurements briefly.

a. Elastic differential cross section and other characteristics of Alpha-Alpha scattering

Ambrosio et al [8] first studied $\alpha\alpha$ interaction at CERN-ISR at $\sqrt{s} = 126$ GeV. The measurements were made in the range $0.05<-t<0.8$ (GeV/c)$^2$ with a resolution of $\pm 0.02$ (GeV/c)$^2$. The differential cross section data are shown in Fig.1. The figure exhibits a first minimum at $-t = 0.10 \pm 0.01$ (GeV/c)$^2$ and a second
one at \(-t = 0.38 \pm 0.02 \) (GeV/c)\(^2\). The slope \(B\) in the range \(-t = 0.05\) to \(0.07\) (GeV/c)\(^2\) is found to be \(100 \pm 10\) (GeV/c)\(^{-2}\). The total and integrated cross-section values were measured as \(\sigma_T = 250 \pm 50\) mb and \(\sigma_A = 45 \pm 15\) mb. Another independent estimate, by the same authors, based on the measured luminosity-monitor cross section, is \(\sigma_T = 295 \pm 40\) mb.

Later on, Akesson et al [9] also published results, which come from the final ISR measurements of the \(\alpha\)-particle collisions, at \(\sqrt{s} = 126\) GeV. These measurements are in the limited range \(0.05 < -t < 0.19\) (GeV/c)\(^2\) with a high resolution in \(-t\), which resulted in a significant improvement over previous data [8]. These are shown in Fig.2. The dip occurs at \(-t = 0.098 \pm 0.002\) (GeV/c)\(^2\). The differential cross-section measurements of Ambrosio et al [8] and Akesson et al [9] differ in the dip and bump region. Akesson et al [9] also obtained a slope \(B = 72 \pm 6\) (GeV/c)\(^{-2}\) and a total cross section \(\sigma_T = 280 \pm 70\) mb. Their result for \(\sigma_T\) agree well with the value \(250 \pm 50\) mb obtained earlier by Ambrosio et al [8]. Total cross section, integrated cross section and the slope parameter have also been measured by Qwen et al [10] for \(\alpha\alpha\) scattering at \(\sqrt{s} = 126\) GeV. The results obtained for the total cross section, viz., \(\sigma_T = 315 \pm 18\) mb, the slope \(B\) parameter for \(-t < 0.07\) (GeV/c)\(^2\) viz., \(B = 87 \pm 4\) (GeV/c)\(^{-2}\), the integrated cross section viz., \(\sigma_A = 58 \pm 6\) mb, is an improvement on the precision of earlier measurements.

b. Elastic differential cross section and other characteristics of Proton-Alpha scattering

A study of \(p\alpha\) elastic scattering at small angles was made in the WA9 experiment at CERN SPS by Burq et al [11] in 1981. The differential cross-section was measured for \(-t\) ranging from 0.008 to 0.05 (GeV/c)\(^2\) at momenta from 100 to 300 GeV/c. Experimental \(d\sigma/dt\) data for \(p\alpha\) are shown in Fig.3. The values of the total cross section \(\sigma_T\) and the slope parameter \(B\) for \(p_L = 100, 150, 250\) and 300 GeV/c are \(123.7 \pm 0.7, 130.8 \pm 0.8, 131.6 \pm 0.8, 132.0 \pm 0.9\) mb and \(34.1 \pm 0.3, 34.9 \pm 0.3, 34.7 \pm 0.2, 35.1 \pm 0.2\) (GeV/c)\(^2\) respectively.

Bujak et al [12] determined the differential cross-section
for $p\alpha$ elastic scattering for incident laboratory energy from 45 to 400 GeV in the range $0.003 < -t < 0.52$ (GeV/c)$^2$ by means of the internal gas-jet target technique. The differential cross section data at 301 and 393 GeV are shown in Figs. 4(a) and 4(b). The differential cross section drops 4-5 orders of magnitude in the dip at $-t = 0.22$ (GeV/c)$^2$ and has a subsequent rise to a secondary maximum at $-t = 0.33$ (GeV/c)$^2$. The values of the total cross section $\sigma_t$ and the ratio $\rho$ of the real and imaginary parts of the forward scattering amplitude are $122.8 \pm 0.7$, $125.9 \pm 0.6$ mb and $0.042 \pm 0.03$, $0.102 \pm 0.035$ respectively. The shrinkage in the differential cross-section is found to be twice as fast as that in the proton-proton case.

The differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV was measured by Ambrosio et al [8]. This has a slope $B = 41 \pm 2$ (GeV/c)$^{-2}$ for $0.05 < -t < 0.18$ (GeV/c)$^2$ and a minimum at $-t = 0.20 \pm 0.02$ (GeV/c)$^2$. The extrapolation of this slope gives $\sigma_t = 130 \pm 20$ mb via the optical theorem (assuming that the real part of the forward amplitude is negligible) and $\sigma_d = 20 \pm 4$ mb. The numerical values of $p\alpha$ differential cross section are shown in Fig. 5.

Bell et al [13] have also measured the differential cross section for $p\alpha$ elastic scattering at $\sqrt{s} = 89$ GeV for $-t$ ranging from 0.2375 to 0.7125 (GeV/c)$^2$. Their results are also shown in Fig. 5.

c. Elastic differential cross section and other characteristics of Deuteron-Deuteron Scattering

The differential cross section for dd elastic scattering at 7.9 GeV/c in the range $0.03 < -t < 0.7$ (GeV/c)$^2$ was measured by Goshaw et al [14]. For momentum transfers below 0.2 (GeV/c)$^2$, their results as given in Fig. 6, show a sharp forward peak which falls off approximately as $e^{-t}$. In the region immediately above 0.2 (GeV/c)$^2$ the low statistic of the data indicates only the presence of a break in the slope. The cross section decreases exponentially as $e^{-2t}$ beyond the break out to the limit of $t = 0.8$ (GeV/c)$^2$.

Goggi et al [15] reported experimental results on deuteron-
deuteron elastic scattering at ISR energies, $\sqrt{s} = 53$ and 63 GeV. The $t$-dependence of the elastic cross section was measured in the range from $-t = 0.06 (\text{GeV/c})^2$ to $1.4 (\text{GeV/c})^2$ at $\sqrt{s} = 53$ GeV and from $-t = 0.08 (\text{GeV/c})^2$ to $1.41 (\text{GeV/c})^2$ at $\sqrt{s} = 63$ GeV. A narrow interference minimum is clearly observed at both energies. The exponential slopes below and above the interference region are about 56 (GeV/c)$^{-2}$ and 6.5 (GeV/c)$^{-2}$, respectively. The differential cross section measurements at $\sqrt{s} = 53$ GeV and at $\sqrt{s} = 63$ GeV are shown in Fig.7 and Fig.8, respectively.

d. Elastic differential cross section and other characteristics of Proton-Deuteron Scattering

At Serpukhov, Bartenev et al [16] observed shrinkage of the deuteron diffraction cone with increasing energy while measuring pd elastic scattering up to 70 GeV/c incident momentum.

Later on, at Fermilab, Akimov et al [17] extended these measurements from 50 to 400 GeV/c for incident proton beam momenta in the four-momentum transfer squared region $0.013 < -t < 0.14 (\text{GeV/c})^2$. They observed shrinkage of the diffraction cone with increasing energy. This shrinkage was greater than that observed in pp elastic scattering. The ratio of the elastic to the total cross section was approximately 0.1 and independent of energy above 75 GeV.

Armitage et al [18] published the results of $d\sigma/dt$ measurements for pd elastic scattering at $\sqrt{s} = 53$ GeV. The measurements were made in the momentum transfer squared ($-t$) range 0.15 to 0.42 (GeV/c)$^2$. The data for pd elastic scattering showed a gradually flattening $t$ dependence between $0.15 > -t > 0.42 (\text{GeV/c})^2$. Armitage et al [18] compared the data with the FNAL data of Akimove et al [17] and observed very little energy dependence in the $t$ distribution between $\sqrt{s} = 38$ and $\sqrt{s} = 53$ GeV.

Goggi et al [15] reported experimental results on proton-deuteron elastic scattering measured at ISR energies, $\sqrt{s} = 53$ and 63 GeV. The data covered wide interval of four-momentum transfer i.e. from $-t = 0.05 (\text{GeV/c})^2$ to $-t = 1.65 (\text{GeV/c})^2$ at $\sqrt{s} = 53$ GeV and from $-t = 0.08 (\text{GeV/c})^2$ to $-t = 1.85 (\text{GeV/c})^2$ at $\sqrt{s} = 63$ GeV.
GeV. Exponential fits to the data in limited t intervals yield approximate values of the slopes in the single- and double-scattering regions, about 30 \((\text{GeV/c})^{-2}\) and 5 \((\text{GeV/c})^{-2}\), respectively. They compared lower energy data points (at \(\sqrt{s} = 53\) \(\text{GeV}\)) with Fermilab [17] and ISR [18] results on an expanded t scale. The data match up well with one another. The differential cross section over the entire t range at \(\sqrt{s} = 53\) and \(\sqrt{s} = 63\) \(\text{GeV}\) are shown in Fig.9 and Fig.10, respectively.

3. REVIEW OF THEORETICAL MODELS

a. \(\alpha\alpha\) and \(p\alpha\) Elastic Scattering

The first attempt to explain the \(\alpha\alpha\) and \(p\alpha\) elastic scattering was made by Lombard and Tellez-Arenas [19]. They employed the pristine Chou-Yang model, including the contribution of the real part. They have chosen an exponential form of the proton and alpha electromagnetic form factors. Their choice of the proton form factor is consistent with the experimental measurements only up to \(-t = 1.6\) \((\text{GeV/c})^2\). This facilitates the computation. However, this restriction of the form factor values up to \(-t = 1.6\) \((\text{GeV/c})^2\) does not affect the differential cross section results because the experimental data for these reactions is available only up to \(-t = 1.6\) \((\text{GeV/c})^2\). These authors have observed that for \(\alpha\alpha\) and \(p\alpha\) elastic scattering, there is a substantial departure from the measured values beyond the dip region. The slope of the differential cross section for both reactions is well produced in the diffraction peak region.

Kamran and Qureshi [20] carried out a study of the \(\alpha\alpha\) elastic scattering at \(\sqrt{s} = 126\) \(\text{GeV}\) based on the Chou-Yang model. They predict a multiple dip structure. However, the multiple dip structure predicted by them occurs not due to the nature of the electromagnetic form factor but owing to the oscillations produced when the integrals are not evaluated at sufficiently small intervals of \(-t\) and \(b\) and/or are cut off at low upper limits. In fact, if the two integrals in the model are evaluated in sufficiently small steps and computations are allowed to continue till the integrals converge, significantly different results are obtained in the region lying outside the diffraction
peak region.

We [5] have fitted these reactions in the light of generalised Chou-Yang model. The differential cross section in the diffraction peak region as well as beyond the dip region, for \( \alpha \alpha \) and \( \pi \alpha \) elastic scattering, is explained quite satisfactorily by using the generalized Chou-Yang model. By choosing

\[
f(t)/f(0) = (1-at)^b
\]

the differential cross-section data of Ambresio et al [8] and Akesson et al [9] is fitted with a suitable choice of parameters.

No independent theoretical explanation exists for \( \pi \alpha \) elastic scattering.

**dd Elastic Scattering**

Satisfactory explanation of the dd elastic scattering at high energies has been elusive since the first measurements. As expected, attempt to explain this reaction on the basis of Chou-Yang model yields a fit in the diffraction peak region [6].

In order to explain the dd elastic scattering data beyond the diffraction peak region, an effort was made [6] by taking into account the multiple scattering in hard region. The Generalized Chou-Yang model gives the structure around 0.2 (GeV/c)^2 and is consistent with the experimental data up to \(-t = 0.75\) (GeV/c)^2. The model is, however, unable to reproduce the entire structure in elastic scattering at \(t_o = 53\) GeV where measurements have been made up to \(-t = 1.5\) (GeV/c)^2. Thus in the framework of the Chou-Yang model with electromagnetic form factor as input, the entire structure in dd elastic scattering can not be explained.

Recently, Etim and Satta [21] have attempted to fit the data by an improved representation of the ground state wave function of the deuteron as an admixture of S- and D-waves. The model is inspired by unitarity sum over a specific class of intermediate states. The model is a hybrid in two respects:

i) it combines the multiple scattering approximation of geometrical models, restricted to a "hard" part of the amplitude, with unitarity contributions which are associated with the "soft" part of the amplitude

ii) the complexification of the amplitude is not carried out in an analytic manner but results from separate approximation
for the real and imaginary parts. The real and imaginary parts are assumed to be overwhelmingly dominant at high and low momentum transfers, respectively.

The model gives a good fit to the data at high energies by choosing very many parameters. Also, as pointed out by them in their paper, the dependence of these parameters on the centre of mass energy is not understandable, a feature shared by geometrical models. It will be interesting to point out that, in the choice of parameters they have chosen \( \sigma_m \) to be 559, 618 and 450 mb at \( \sqrt{s} = 4.6.12 \) and 63 GeV respectively. These values of \( \sigma_m \) seems to be much higher than all extrapolations for dd total cross section. In fact the choice of dd total cross section in their paper far exceeds the measured cross section for \( \alpha \) scattering viz. \( \sigma_\alpha = 315 \pm 18 \) mb. at 126 GeV [10].

c) \( pd \) Elastic Scattering

Attempts have also been made to explain proton-deuteron elastic scattering on the basis of Chou-Yang model and multiple scattering theory (Glauber theory). Parida and Patel [22] have used the pristine Chou-Yang model [1] to calculate the differential cross section in \( pd \) elastic scattering with a view to investigate whether dips and higher-order maxima would appear at very high energies. For this propose they have used the interpolating formulas for the proton and the deuteron form factors which have exploited analyticity properties and yielded good fits to the Fermilab energies in the small \( |t| \) region. As in the other cases, their calculations also indicate that the Chou-Yang approximation to the differential cross section, with only the imaginary part of the scattering amplitude, is applicable only in the diffraction peak region. Their calculations for the differential cross sections at higher values of \(-t < 2 \) (GeV)\(^2\) show the absence of dips at Fermilab energies. Only a shoulder (and not a dip) appears for \(-t = 0.3-0.4 \) (GeV/c)\(^2\) if the real part and/or spin effects are negligible. It is interesting to note that even though no real-part and spin effects have been taken into account, a shoulder continues to appear for energies for which a sharp dip has emerged in other reactions. Although the predictions have been made by Parida and Patel [22] for the t
values lying beyond the diffraction peak region, they do not agree with the large \(-t\) measurements of Goggi et al [15].

Braun et al [23], using Glauber theory including inelastic correction, have calculated the pd differential cross section at high energy. The predictions have been made by using non-relativistic wave functions of the deuteron including D-wave in the electric and quadrupole form factor at \(V_s = 53, 63\) GeV. The results agree well with the experimental data [15] up to \(-t = 1.05\) (GeV/c)\(^2\) while the measurements have been made up to \(-t = 2.0\) (GeV/c)\(^2\).

No simultaneous explanation of dd and pd elastic scattering exists so far.

4. Predictions of Chou-Yang model

In this section we will first briefly describe the Chou-Yang model. Later on, the predictions based on our calculations will be compared with the experiment.

The geometrical picture of the Chou-Yang model employs an eikonal which is related to the distribution of matter inside the colliding particles. The scattering amplitude \(T(s,t)\) in this model, by neglecting spin, is given by

\[
T(s,t) = i \int b \, db \left[ \exp(-\Omega(b)) \right] J_0 (b \sqrt{-t})
\]

where

\[
\Omega(b) = K \rho(b)
\]

with

\[
\rho(b) = \int \sqrt{-t} \, d \sqrt{-t} \, G_{hh'}(t) \, J_0 (b \sqrt{-t})
\]

where the hadronic matter distribution, which was assumed to be proportional to the charge distribution on hadrons/nuclei, is represented by \(G_{hh'}\). The normalisation of \(T(s,t)\) is such that the differential and total cross sections are given by

\[
\frac{d\sigma}{dt} = \pi \left| T(s,t) \right|^2
\]

\[
\sigma_t = 4\pi \text{Im} T(s,t=0)
\]

The parameter \(K\) is chosen to be energy dependent.

As pointed out earlier, the model in this form is valid only in the diffraction peak region. The multiple dip structure predicted by this model is not confirmed by the experiments.

Keeping in view the inability of pristine model in explaining the data in the entire \(-t\) region, Chou and Yang very recently [7] have proposed that the assumption regarding the
equivalence of the charge form factor and the hadronic form factor is only an approximation within the framework of their model. They have thus proposed a simple expression for the matter distribution of proton. Similar assumption was made by Saleem et al [24] in explaining the \( \bar{p}p \), \( p\alpha \), \( \alpha\alpha \) elastic scattering. However, unlike the assumption made in Ref.24, Chou and Yang have taken the hadronic form factor energy dependent. By choosing an energy dependent range parameter, they have fitted the \( pp \) elastic scattering data at 23.5 and 546 GeV. Their fit to the data at 23.5 GeV is good up to \( -t = 3.0 \) (GeV/c)\(^2\) (while the measurements are up to \( -t = 7.0 \) (GeV/c)\(^2\)). Beyond 3.0 (GeV/c)\(^2\) the deviation from the experimental data becomes significant. We have used a similar procedure and proposed a form factor for alpha and deuteron which gives very good fit to the data at all measured energies.

The proposed alpha and deuteron hadronic form factors are

\[
\begin{align*}
G_a &= s^{0.81} \left( 1.03e^{-0.23t} + 3.25t e^{1.72t} - 0.03e^{-1.70t} \right) \\
G_d &= s^{0.24} e^{0.24t} \left( \sum a_i e^{b_i t} \right), \quad i = 1, 2, 3
\end{align*}
\]

The values of parameters \( a_1, a_2, a_3 \) and \( b_1, b_2, b_3 \) are 0.85, 0.14, 0.01 and 20.07, -0.47, -1.47, respectively.

The hadronic form factor of proton used in ref.7 is given by

\[
G_p = \left( 1 - \frac{t}{m^2} \right)^{1/2}
\]

However, no expression is given in ref. 7 to evaluate the value of \( m^2 \) at different energies (they only mention about the linear variation of this parameter with \( \ln s \)). We suggest a simple expression for \( m^2 \) as

\[
\frac{1}{(m^2 \sqrt{s})} = 0.58 (\ln s)^{0.31}
\]

This energy-dependence of the parameter gives \( m^2 = 0.774, 0.573 \) and 0.52 at 23.5, 546 and 1800 GeV and is consistent with that given in ref.7. The same expression is being used at different energies to evaluate differential cross sections of \( p\alpha \) and \( pd \) elastic scattering.

We now compare the predictions of our calculations with the available data of Refs.8 and 9 for \( p\alpha \) elastic scattering at \( \sqrt{s} = 126 \) GeV. Figs. 1 and 2 show the fit for the differential cross section. The fit is obtained for \( K = 115 \) (GeV/c)\(^{-2}\) while \( \alpha \) has
been chosen as 0.35 and 0.30 respectively which yields $\sigma_T = 333.12\text{mb}$ and $330.67\text{mb}$ and is consistent with the experimental data. In fact, throughout our calculations the choice of $K$ and $\alpha$ is such that $\sigma_{\text{theor.}}$ and $\rho_{\text{theor.}}$ are consistent with the experimental data. The theoretical predictions of the model for the differential cross section are in good agreement with the experimental data. In order to verify the validity of alpha form factor, we have used it to fit the data for $p\alpha$ elastic scattering. The proton form factor is the same as used in Ref. 7. The data at 100, 150, 250, 300 GeV/c [11] 301 and 393 GeV/c [12] and 89 GeV [8], for $p\alpha$ differential cross sections is compared with the predictions of this model in Figs. 3-6. The predicted results are in good agreement with the experimental data. It will be interesting to point out that unless the hadronic form factor is chosen to be energy dependent, we can not fit the data for $p\alpha$ elastic scattering well.

Next we compare the predictions of our calculations with the available data for $dd$ and $pd$ elastic scattering. Figs. 6-8 show the fit for the differential cross section at 6.12, 53 and 63 GeV. The fit for 6.12 GeV is obtained for $K = 35 \text{ (GeV/c)}^2$ and $\alpha = 0.05$ while at 53 and 63 GeV we have chosen $K = 69.5 \text{ (GeV/c)}^2$ and $\alpha = 0.05$. The theoretical predictions of the model are in good agreement with the experimental data. As in the case of $d\alpha$ and $p\alpha$, we have used the suggested deuteron form factor along with the proton form factor of Jones et al. [Ref. 7] to predict the $pd$ differential cross sections at 53 and 63 GeV. In Figs. 10 and 11 experimental data is compared with the predictions of the model. The agreement is again good. The values of $K$ and $\alpha$ are chosen to be $10 \text{ (GeV/c)}^2$ and 0.05 for both 53 and 63 GeV. The same choice of $K$ and $\alpha$ at 53 and 63 GeV suggests that within errors the total cross sections at these energies will be mutually consistent.

In Fig. 11 we have plotted the proposed form factors for proton, deuteron and alpha along with their charge form factors [25-29]. As is evident from the figure, the results obtained from hadronic form factors are higher than charge form factors. As these form factors are energy dependent, the radii of proton, deuteron and alpha should increase with an increase in energy. This therefore suggests that with in the framework of geometrical picture, the hadronic matter is expanding with an increase in energy.
5. Conclusions

The following observations may be made:

1. As form factor in general is related to the distribution of matter in space, the hadronic form factor consequently is related to the distribution of matter inside hadron. Any theory that claims to explain the structure of matter should be able to give some idea of the hadron form factors. So far the expression which represent the exact form of hadronic form factor can not be obtained from the first principles.

2. There has been considerable interest in the hadronic form factor of lighter nuclei (alpha, deuteron etc.) as it gives information about the short range nucleon-nucleon interaction and the transition from nucleon to quark degrees of freedom.

3. Furthermore, it has been a question of considerable importance whether or not the electric charge distribution is the same as the nuclear matter distribution. In 1968 Chou and Yang assumed that the matter distribution inside a hadron is the same as the charge distribution inside a hadron. These form factors of hadrons describe an extended structure which can be represented by spatial distributions for the static quantities. Although at ISR energies the Chou-Yang model has yielded impressive quantitative agreement with experiments, yet as the energy increases, deviations from the strict interpretation of the geometrical picture become quite noticeable. Modifications of the theory are thus necessary. Lombard and Wilkin [30] suggested that there is probably some secondary (energy-dependent) mechanism, which plays an important role for large t. Chou and Yang in their recent work have suggested that strict interpretation of taking hadronic form factor equivalent to the charge form factor is not valid. They have thus suggested hadronic form factor of the proton and take it to be energy dependent. On similar lines, we assume the hadronic form factor of the alpha and deuteron as dependent on energy. This suggests that the hadrons are expanding with increase in energy.
4. It will be interesting to point out that various features of alpha-alpha elastic scattering can also be explained on the basis of the generalized Chou-Yang model [5]. Similar conclusion holds good for $\bar{p} p$ elastic scattering at ISR, Collider and Tevatron energies. However, the Generalized Chou-Yang model when applied to $d d$ elastic scattering does not give satisfactory results in the entire measured region. The new measurements for $\bar{p} p$ at 1.8 TeV as well as at UNK, LHC and SSC will be able to throw more light on the concept of energy dependent hadronic form factors.

5. The energy dependent form factors also suggest that the radii of proton, deuteron and alpha will be increasing with increase in energy. Also, the increase in opacity implies that these particles become more opaque at higher energies.

We thus conclude that by assuming the hadrons as extended objects and that their matter distribution changes with energy, we can predict the behaviour of nucleus(hadron)nucleus elastic scattering at high energies and large momentum transfers.
REFERENCES

Figure Captions

Fig. 1. Differential cross section for $\alpha$ elastic scattering at $\sqrt{s} = 126$ Gev. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Ambrosio et al [8].

Fig. 2. Differential cross section for $\alpha$ elastic scattering at $\sqrt{s} = 126$ Gev. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Akesson et al [9].

Fig. 3. Differential cross section for $\pi^\pm$ elastic scattering at $p_t \approx 100$ to 300 Gev/c. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Burq et al [11].

Fig. 4. Differential cross section for $\pi^\pm$ elastic scattering at $E_L = 301$ and 393 Gev. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Bujak et al [12].

Fig. 5. Differential cross section for $\pi^\pm$ elastic scattering at $\sqrt{s} = 89$ Gev. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Ambrosio et al [8] and Bell et al [13].

Fig. 6. Differential cross section for $d\bar{d}$ elastic scattering at $\sqrt{s} = 6.12$ Gev. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Goshaw et al [14].

Fig. 7. Differential cross section for $d\bar{d}$ elastic scattering at $\sqrt{s} = 53$ Gev. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Goggi et al [15].

Fig. 8. Differential cross section for $d\bar{d}$ elastic scattering at $\sqrt{s} = 53$ Gev. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Goggi et al [15].

Fig. 9. Differential cross section for $p\bar{d}$ elastic scattering at $\sqrt{s} = 53$ Gev. The solid curves represent predictions of the Chou-Yang model. The experimental points have been taken from Goggi et al [15].

Fig. 10. Differential cross section for $p\bar{d}$ elastic scattering at $\sqrt{s} = 53$ Gev. The solid curves represent
predictions of the Chou-Yang model. The experimental points have been taken from Gnegi et al [15].

Fig.11. Hadronic form factors of proton, deuteron and alpha compared to the respective charge form factors. The experimental points have been taken from [25-29].
Fig. 1

\[ \frac{d\sigma}{dt} \left[ \text{mb/}(\text{GeV/c})^2 \right] \]

- \( t \) (GeV/c)

- \( \sqrt{s} = 126 \text{ GeV} \)

- Ambrosio et al [8]
\[ \alpha \alpha \rightarrow \alpha \alpha \]
\[ \sqrt{s} = 126 \text{ GeV} \]

- Akesson et al [9]

Fig. 2
$d\sigma/dt \ [\text{mb}/(\text{GeV/c})^2]$
$E_{\text{lab}} = 301 \text{ GeV}$

- Bujak et al. [12]

Fig. 4(c)
$p \alpha \rightarrow p \alpha$

$E_{\text{lab}} = 393$ GeV

- Bujak et al [12]

![Graph showing the dependence of $d\sigma/dt$ on $-t (\text{GeV}/c)^2$]
\[ \frac{d\sigma}{dt} \left( \frac{\mu b c}{\text{GeV}} \right)^2 \]

\[ -t \ (\text{GeV}/c)^2 \]

Fig. 5

\[ \sqrt{s} = 89 \ \text{GeV} \]

- Ambrosio et al [8]
- W. Bell et al [13]
$dd \rightarrow dd$

$\sqrt{s} = 6.12$ GeV

Goshaw et al [14]

Fig. 6
\[
\text{Fig. 7}
\]
$d\sigma/dt$ (mb/sr GeV$^2$)

$dd \rightarrow dd$

$\sqrt{s} = 63$ GeV

- Goggi et al [15]

Fig. 8
$p d \rightarrow p d$

$\sqrt{s} = 53 \text{ GeV}$

- Goggi et al [15]

Fig. 9
\( p\bar{d} \rightarrow p\bar{d} \)
\( \sqrt{s} = 6.3 \text{ GeV} \)
- Goggi et al [15]

**Fig. 10**
Fig. 11 (a)
Fig. 11 (c)