Experimental Study of Heat Transfer by Natural Convection through Vertical Cylinders

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A thesis submitted for partial fulfillment of the requirement for the degree of Doctor of Philosophy (Ph.D.) in the Department of Chemical Engineering

By

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January 2012
Declaration

I declare that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other university.

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It is certified that the work in this thesis is carried out and completed under my supervision.

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Dedicated to my family members
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Journal Publications:

I. **M. Arshad**, M. H. Inayat and I. R. Chughtai
   *Heat Transfer through Vertical Cylinder in Stationary Fluid*

II. **M. Arshad**, M. H. Inayat and I. R. Chughtai
    *Experimental Study of Natural Convection Heat Transfer from an Enclosed Assembly of Thin Vertical Cylinders*

Conference Presentation:

I. **M. Arshad**, M. H. Inayat and I. R. Chughtai
   *Heat Transfer through Vertical Cylinder*
   International Conference on Advancements in Process Engineering (ICAPE-2008)
   13-14 October 2008, PIEAS Nilore Islamabad, Pakistan
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NOMENCLATURE

\( A_s \) Surface area, m\(^2\)
\( C_p \) Specific heat capacity, J/Kg\(^\circ\)C
\( d \) Diameter of cylinder, m
\( g \) Gravitational acceleration, m/sec\(^2\)
\( Gr \) Grashof number
\( Gr^* \) Modified Grashof number
\( h \) Heat transfer coefficient, W/m\(^2\)\(^\circ\)C
\( k \) Thermal conductivity, W/m\(^\circ\)C
\( L \) Overall heated length, m
\( Nu \) Nusselt number
\( Pr \) Prandtl number
\( Q \) Rate of heat transfer, Watt
\( q^{''} \) Heat flux \((Q/A_s)\), W/m\(^2\)
\( Ra \) Rayleigh number
\( Ra^* \) Modified Rayleigh number
\( Re \) Reynolds number
\( T_b \) Bulk fluid temperature, \(^\circ\)C
\( TC \) Thermocouple
\( T_f \) Film temperature, \(^\circ\)C
\( T_{in} \) Inlet fluid temperature, \(^\circ\)C
\( T_{out} \) Outlet fluid temperature, \(^\circ\)C
\( T_s \) Surface temperature, \(^\circ\)C
\( T_{\infty} \) Reference Temperature, \(^\circ\)C
\( x \) Local length, m

Subscripts:

\( b \) Bulk values
\( cond \) Conduction
\( conv \) Convection
\( d \) Values on diameter basis
\( e \) Equivalent values
\( FP \) Values for flat plate
\( H \) Values on height basis
\( in \) Inlet values
\( L \) Weighted average on length basis
\( out \) Outlet values
\( rad \) Radiation
\( s \) Values at surface
\( w \) Values at wall
\( x \) Local values

Greek Letters:

\( \delta \) Stefan Boltzmann constant
\( (5.66 \times 10^{-8}) \) W/m\(^2\)K
\( \varepsilon \) Emissivity of the material
\( \nu \) Kinematic viscosity, m\(^2\)/s
\( \infty \) Reference values
\( \Delta \) Difference
\( \ell \) Annular gap
\( \zeta \) Fluids property index
\( \beta \) Thermal expansion coefficient
\( \gamma \) Gamma
\( \mu \) Dynamic viscosity, Kg/m-s
\( \rho \) Density of water, Kg/m\(^3\)
Abstract

Miniature Neutron Source Reactor (MNSR) is a passively cooled system operating in Natural convection heat transfer mode. A typical MNSR located at the premises of PIEAS (Pakistan Institute of Engineering and Applied Sciences) called as Pakistan Atomic Research Reactor-II, PARR-II. From safety analysis point of view the literature lacks any experimental study or information that can be used to predict the outer surface temperatures along the axial length of a fuel rod in the reactor core. Similarly the literature also lacks any information regarding the prediction of the fluid exit bulk temperature as a function of the reactor thermal power. The thermal power range for the operation of this particular MNSR studied ranges from 5.4 to 27 KW.

The current experimental study is a pioneering effort to find a solution to the above mentioned problem. Hence, steady state heat transfer by natural convection was investigated experimentally from an enclosed assembly of thin vertical cylinders at high Grashof numbers. The published literature lacks experimental data regarding such a study in the turbulent boundary layer regime. An enclosed assembly consisted of a 3 x 3 array of vertical cylinders immersed in a large volume tank of water was used. All the cylinders were electrically heated. Various uniform heat fluxes were applied to each cylinder and the surface temperature at different positions along the cylinders were measured.

The experimental results show that the surface temperature increases axially up to a certain length, then decreases due to some extra mixing which increases the heat transfer. However, such a behavior is expected to have little effect if the enclosed
assembly consists of a large number of thin vertical cylinders. A criterion has been proposed for the determination of the onset of turbulent boundary layer in an assembly. The local heat transfer coefficient, local Nusselt number and local modified Rayleigh number have been presented for the experimental data. It has been found that a much better representation of experimental data results, if the Nusselt number is presented as a function of modified Grashof number and Prandtl number separately instead of modified Rayleigh number. This representation includes the effect of different fluids and L/d ratios. Similarly empirical correlations between overall Nusselt number and average modified Rayleigh number have also been developed based on the data of the assembly of cylinders used in the current study. This empirical correlation for an assembly developed in current study is valid for $1.28 \times 10^{12} \leq \text{Ra}_L^* \leq 1.18 \times 10^{13}$. A much better generalized correlation has also been proposed for natural convection heat transfer from a single vertical cylinder in an infinite medium which shows a better fit to all the currently available experimental data. The literature also lacks such a generalized correlation.

The results obtained in this study have been utilized to predict the axial surface temperatures as well as the fluid bulk outlet temperature of PARR-II. Hence the correlations developed from current study are applicable to all thermal power ranges in PARR-II type MNSR reactors or any other assembly of vertical heated thin cylinders.
Chapter 1. Introduction

1.1 Background

Heat transfer is a science in which energy is transferred between two objects as a result of temperature gradient. In thermodynamics, this transferred energy is defined as heat energy. There are three major types of heat transfer modes namely conduction, convection and radiation. Heat transfer by convection is further classified as natural or free convection and forced or advective convection. Heat transfer by natural convection phenomenon is the focus of the present research work.

1.1.1 Criterion for Convection Heat Transfer

The general criterion for the phenomenon of convection heat transfer is the velocity of the fluid. The velocity in natural convection is very small in comparison with the forced convection. The natural convection currents are influenced by the gravitational field. Order of magnitude analysis of the natural convection boundary layer equations indicates the criterion for determining the convection mode[1]. The criterion equation is:

$$\frac{Gr}{Re^2} > 10$$

(1.1)

Where $Gr$ is the Grashof number and $Re$ is the Reynolds number. This criterion shows that the velocity plays a vital role in distinguishing natural convection from forced convection phenomena. For very small fluid velocities, Reynolds number is also very small and hence this criterion becomes true for natural convection phenomenon.

1.1.2 Application of Natural Convection:

Heat transfer by natural convection phenomenon is the scope of this research work. Natural or free convection is present both in nature and man-made engineering systems. Oceanic currents, wind formation overseas and in the rising plume of hot air from fire are some of the naturally occurring examples of natural convection. While the solar ponds, building heating system, formation of microstructures during cooling of molten
metals, steam heated coils and electric immersion heaters in process vessels, heat loss from process piping, HVAC system, cooling of electronic components, heat removal from spent nuclear fuel bundles and cooling of nuclear reactor after loss of coolant accident etc are examples of some engineering applications.

1.1.3 Significance of Natural Convection:

The fluid motion, in the phenomenon of natural convection, is not generated by any external source (like a pump, fan etc.) except only by the density gradient developed by the heating or cooling process. In natural convection, fluid surrounding the heated surface becomes less dense and rises. The cold fluid in the surroundings then moves to replace it, gain heat and process continues to move the fluid, forming convection currents. Buoyancy is the driving force for natural convection. The denser layer exerts a buoyancy force on lighter layer of fluid in a direction opposite to gravity. The lighter layer moves under the influence of buoyancy force and fluid attains some velocity. The inertial and buoyancy forces are balanced by the opposing viscous force. The dimensionless combination of these forces is known as Grashof number ($Gr$) which characterizes the natural convection. The Grashof number is defined as:

$$Gr = \frac{g \beta x^3 (T_s - T_b)}{\nu^2}$$

(1.2)

Where, $T_s$ is the temperature of fluid at hot surface, $T_b$ is the bulk temperature of surrounding fluid, $g$ is the acceleration due to gravity, $\beta$ is the coefficient of volume expansion, $x$ is the characteristic length scale and $\nu$ is the kinematic viscosity of fluid. Grashof number is used as a heat transfer parameter for natural convection and is correlated with Nusselt number. $Gr^*$ is known as modified Grashof number [1], when uniform surface heat flux, $q''$, from a solid surface is known instead of $T_s$. The modified Grashof number is defined as:

$$Gr^* = \frac{g \beta x^4 q''}{k \nu^2}$$

(1.3)
Rayleigh number \( (Ra) \) is another important dimensionless group for natural convection heat transfer which is obtained as the product of Grashof number \( (Gr) \) and Prandtl number \( (Pr) \) and is defined as:

\[
Ra = Gr \cdot Pr = \frac{\rho g \beta C_p x^3 (T_s - T_b)}{k \nu} \tag{1.4}
\]

Where \( C_p \) is the specific heat at constant pressure, \( k \) is thermal conductivity and \( \rho \) is the density of the fluid. Similarly the modified Rayleigh number \( (Ra^*) \) is defined as:

\[
Ra^* = Gr^* \cdot Pr = \frac{\rho g \beta C_p x^4 q''}{k^2 \nu} \tag{1.5}
\]

The Grashof numbers characterize natural convection flow whether it is laminar or turbulent in the same way as the Reynolds number in forced convection. For example natural convection over single vertical cylinder is laminar for Grashof number \(< 7 \times 10^9 \) \cite{2} and is turbulent if above this value of \( Gr \). The general correlation for natural convection \cite{1} is:

\[
Nu = A \left( Gr \cdot Pr \right)^n = A \left( Ra \right)^n \tag{1.6}
\]

The correlations (between Rayleigh and Nusselt numbers) for natural convection have been developed by different researchers for various geometric and thermal conditions, mostly in laminar region. The turbulent regime for natural convection heat transfer is yet to be investigated.

Heat transfer through the fluid layer is due to some conduction where the fluid layer is motionless and some convection where the motion is involved. The Nusselt number is obtained by taking the ratio of convection to conduction heat transfer. Therefore this dimensionless number describes the enhancement of heat transfer through a fluid layer in terms of convection relative to the conduction across the same fluid layer.

\[
Nu_L = \frac{hL}{k} \tag{1.7}
\]
A fluid adjacent to the hot surface gains some heat and a flow establishes in layers. The layer formed in this way is called thermal boundary layer. The thickness of the thermal boundary layer increases in the direction of fluid flow. While flowing over a solid surface the velocity and thermal boundary layers are developing simultaneously. The fluid velocity has a strong effect on temperature profile and will have strongly effect the convection heat transfer. The dimensionless parameter for velocity and thermal boundary layer is described by Prandtl number (momentum to heat dissipation ratio).

\[
Pr = \frac{C_p \mu}{k}
\]  

(1.8)

The flow of fluid will be stream line in laminar thermal boundary layer and turbulent in case of highly disorder due to increase in temperature fluctuations and velocity in boundary layer. The flow of fluid fluctuates before it converted to completely turbulent called the transition boundary layer.

As the fluctuations enhance the heat and momentum transfer between the fluid particles, therefore, the convection heat transfer is increased due to turbulence and the boundary layer is enlarged. When the friction and heat transfer coefficients become maximum the boundary layer becomes fully turbulent and breaks up. As the boundary layer breaks up, the resistance to heat transfer is decreased and rate of heat transfer is increased.

1.2 Basis of Research:

In this research work, the reactor core of Pakistan Atomic Research Reactor-II (PARR-II) has been taken as the basis of experiments. PARR-II is a tank in pool type reactor having a design similar to the Miniature Neutron Source Reactor (MNSR) operating in China Institute of Atomic Energy. Natural convection is the only mode of heat transfer through the reactor core of MNSR type reactors during operation. The fluid temperatures are only measured at the inlet and outlet of the reactor core. There is no provision to measure the temperatures along the surface of fuel rods. It is therefore necessary to accurately predict the surface temperatures from the safety of fuel and analysis point of view. The prediction of the outer surface temperature of fuel rods of the
reactor core along its length and correspondingly the fluid exit bulk temperature are very important from reactor safety point of view. One of the major motivations of this study was to experimentally obtain such data which would then be transformed into a correlation. Such a correlation could then be used to predict the surface temperature of fuel rods as well as fluid bulk exit temperature. The knowledge of surface temperature for a particular set of operating conditions could then be used to develop a complete temperature profile within the fuel rods of such research reactors. To achieve this goal of prediction of surface temperatures an exit bulk temperature an experimental assembly was designed which should have a dynamic similarly with a typical MNSR. One such reactor is available in the premises of PIEAS called PARR-II which is the best example for assembly of vertical cylinders.

PARR-II is situated at Pakistan Institute of Engineering and Applied Sciences for research purposes. Highly enriched uranium is used as a fuel in this reactor, with light water as a coolant and moderator. The fuel is in the form of cylindrical pin. The reactor core consists of 344 fuel pins forming an assembly of vertical cylinders. Coolant enters and leaves the core through a narrow orifice at lower and upper side of reactor core. The only mechanism to remove the fission heat is natural convection during normal or accidental conditions. Hence, a deeper understanding of heat transfer by natural convection phenomenon is required for the safety aspects of this reactor. Computational studies by Chughtai et al [3] and Basit et al [4] are available for such kind of reactor, however, an experimental study for similar assemblies is still lacking. The study of temperature distribution inside the assembly of cylinders is the major objective of this experimental study. Empirical correlations are also required for an assembly of vertical cylinders to characterize the natural convection phenomenon.

1.3 Problem Statement

In industry, the axis-symmetric objects are commonly encountered. Natural convection phenomenon has been investigated by different researchers for various axis-symmetric geometries like pipe, heater, rods, coils, wires and plates etc. These geometries can be oriented horizontal, vertical or inclined at some angle. These
orientations of the geometries show different nature of natural convection phenomenon. Natural convection from horizontal cylinders has been investigated analytically as well as numerically by making the solution easier; the problem can be specified in two dimensions. The same is the case of natural convection from vertical cylinders. Natural convection from single vertical cylinder has been investigated experimentally by number of researchers. All these researchers have mainly focused on laminar boundary layer while turbulent region was investigated by a few of them. The scattered data are available in literature for both laminar and turbulent boundary layer regime as presented by Popiel [2]. So the investigation is required to comprehend and find the general correlations.

Cebeci [5] categorized the vertical cylinder into two type i.e. short (thick) & long (slender or thin) cylinders. According to his results, short cylinders have boundary layer thickness much thinner than the radius of the cylinder. Popiel [2] has provided the criteria for thin (long) vertical cylinder as:

$$ Gr_e^{0.25} \left( \frac{d}{L} \right) \leq 11.474 + \left( \frac{48.92}{Pr^{0.5}} \right) - \left( \frac{0.006085}{Pr^2} \right) $$

(1.9)

Natural convection trends of the short vertical cylinders are similar to that of flat plate and their correlations are applicable without any significant error, however, the long cylinders have considerable curvature effects, hence, further analysis are required to understand this phenomenon. In the present study long (thin) vertical cylinders are investigated.

The major objective of this research work is to investigate natural convection phenomenon of heat transfer from an assembly of vertical cylinders. Each cylinder has to be verified for its mechanical integrity before being installed in the assembly. Hence experiments were performed on each cylinder. This served two purposes:

i). Verified the integrity of the cylinders

ii). Verified the data obtained for single cylinder studies with that of available literature and developed a general correlation
Natural convection phenomenon from single vertical cylinder has been investigated by many researchers however very limited information is available for an assembly of vertical cylinders. Heat transfer by natural convection from a single vertical cylinder and from a vertical cylinder in an assembly is different due to the presence of other cylinders heat effects in the assembly. Computationally it is difficult to do the three dimensional analysis of heat transfer by natural convection phenomenon from a vertical cylinder in an assembly because more than one cylinder could not be represented efficiently in either Cartesian or cylindrical coordinate system. Due to this difficulty, a little literature is found for the computational solution of natural convection from assemblies of thermally interacting vertical cylinders. Some simplified numerical work is available for nuclear fuel storage point of view which is different from thermal hydraulics of operational reactor like MNSR. Three experimental studies of Keyhani et al [6], Haldar et al [7] and Isahai et al [8] are available in literature but these are limited to spent fuel storage point of view and are not fully applicable to an operational reactor.

The reactor core of PARR-II is enclosed in beryllium reflectors with restricted inlet and outlet for flow of liquid. This reactor has no instrumentation inside the core, hence the flow and temperature profiles are not known yet. The available thermal hydraulic model for PARR-II is a one dimensional lump model. Similarly the transient behavior of vertical cylinder has not been modeled to visualize the local effects within or around the cylinder. Therefore, the study of heat transfer by natural convection phenomenon from an enclosed assembly is required on the basis of PARR-II.

The above mentioned problems have been analyzed numerically by Chughtai et al [9] solving the governing differential equations in two dimension using Computational Fluid Dynamics (CFD) code Fluent based upon Finite Volume Method (FVM). Basit et al [4] have also computationally solved this problem by computer simulation COSINAC code.

A similar experimental setup can be used to solve the above mentioned problems as used in above computational studies. For this purpose an assembly of nine electrically heated cylinders is prepared dynamically similar to PARR-II. Natural convection heat transfer from each cylinder of assembly prior to its fitting in assembly is experimentally
investigated and verified by comparing with previous published data. The qualitative flow visualization is also done to observe the flow patterns inside the assembly. Empirical correlations have been suggested for single vertical cylinder as well as for an assembly of cylinders by performing different experiments on this experimental setup.

1.4 Research Objective

The major objective of this research work is to experimentally determine the thermal hydraulics behavior of tank in pool type reactor. The experiments have been done on the experimental set up, prepared to understand the heat transfer by natural convection in an assembly of heated cylinders. A point wise description of research objectives is as follows:

1. To obtain the surface temperature distributions at various heat flux values for single vertical cylinder in an infinite medium experimentally at steady state

2. To compare the results of natural convection phenomenon through single vertical cylinder with the previously published data

3. To obtain empirical correlations between Nusselt number and modified Rayleigh number for single vertical cylinder in an infinite medium

4. To obtain the surface temperature distributions at different heat flux values for vertical cylinders at different positions in an assembly experimentally

5. To identify the circumstances at where the boundary layer becomes turbulent in assembly of vertical cylinders

6. To develop the Nusselt number correlations for vertical cylinders at different positions in an assembly

7. To compare the overall Nusselt numbers obtained from the data of single vertical cylinder with a vertical cylinder in an assembly

8. To compare the experimental results of an assembly data with the previous data
9. To visualize the velocity distribution qualitatively at the surface of cylinders using tracer particles in an assembly of vertical cylinders

10. To identify the convection of central cylinder in comparison with its surrounding cylinders in an assembly

11. To develop some suitable Nusselt number correlations for an assembly of vertical cylinders

12. To develop a correlation for the prediction of local surface temperatures along the cylinders in an assembly

13. To predict the surface temperatures on fuel pins of PARR-II assembly

14. To predict the bulk outlet temperature from the reactor core of PARR-II to validate this research

**Summary:**

This chapter deals with the general introduction of present study. The natural convection phenomenon of heat transfer along with the cooling mechanism of tank in pool type reactor, the problem statement and the main objective of this research work has been briefly discussed here.
Chapter 2. Literature Review

Heat transfer by natural convection phenomenon has been investigated by different researchers for various geometries and different boundary conditions. Literature review related to the thermal hydraulics of tank in pool type reactor may be classified into two categories like a review of related studies in the area of heat transfer by natural convection phenomena and a survey of available literature related to similar kind of reactors. A further classification of the literature on heat transfer by natural convection phenomenon can be made on the basis of geometry or method of study adopted by researchers. The geometry may be classified as flat and curved surfaces, single as well as assembly of these objects, in an enclosure or infinite medium with some orientation like horizontal, vertical or at some angle. Heat transfer by natural convection phenomenon from these geometries has been investigated by different methods like numerical, analytical and experimental. Laminar boundary layer has been investigated extensively however the turbulent regime is required the investigation. Previously related published literature is reviewed below for different geometries.

2.1 Natural Convection from Flat Surfaces

2.1.1 Flat Plate

Flat plate is a simple geometry to study the natural convection heat transfer. A free convection boundary layer is formed by heating the plate. At the surface of plate the velocity is zero due to the no slip boundary condition; it increases to some maximum value and then decreases to zero where the boundary layer ends. Initially the boundary layer developed is laminar. At certain distance from the leading edge, turbulent eddies are formed due to the temperature difference between the surface and the surroundings which changes the fluid properties, hence a transition to turbulent layer begins which may become fully turbulent after some distance.

The analysis of natural convection heat transfer on a vertical flat plate is the simplest case which can be solved mathematically and it has served to introduce the new
dimensionless variable, the Grashof number, which is important in all natural convection problems. The Grashof number has a similar role in natural convection as that role played by Reynolds number in forced convection system. Grashof number is used as a criterion for the transition to turbulent boundary layer flow for natural convection. Critical Grashof number observed by Eckert and Soehngen [10] is approximately $4 \times 10^8$ for natural convection on vertical flat plate using air. Later on, Grashof numbers of $10^8$ to $10^9$ are observed as turbulent boundary layer for different fluids. The stability and transition of natural convection boundary layers have been surveyed by Gebhart et al [11-13]. For turbulent natural convection it is difficult to predict velocity and temperature profiles analytically, therefore the experimental measurements are more reliable to obtain relations for heat transfer. In natural convection problems, the velocities are so small that they are very difficult to measure experimentally. Despite of this difficulty, the velocities have been measured by different techniques like hydrogen-bubble technique [14], hot-wire anemometry [15] quartz-fiber anemometers and laser anemometry [16] etc. among these techniques the laser anemometry is non invasive and more reliable as it does not affect the flow. The temperatures have been measured by Zehnder-Mach interferometer. Interferometric studies have been done by different researchers [10, 17, 18] and discussed the isotherms obtained on vertical flat plate with air. These isotherms provide the information about the behavior of boundary layer either laminar or turbulent. A number of researchers have investigated theoretical and empirical aspects of natural convection problems. An extensive discussion is given by Gebhart et al [11].

A number of researchers have developed correlation for natural convection. A general empirical relation for natural convection heat transfer found in literature is given in the following equation;

$$Nu_f = A(Gr_f Pr_f)^n = A(Ra)^n$$

(2.1)

Where, $f$ in the subscript indicates that the properties are calculated at film temperature which is calculated as;

$$T_f = \frac{T_s + T_b}{2}$$

(2.2)
Where, $T_s$ is the surface temperature and $T_b$ is the fluid bulk temperature.

Linhui et al [19] experimentally studied the laminar natural convection phenomena from a vertical plate with discrete heat sources. They found the constants of the general correlation between Nusselt and Rayleigh numbers, as a quadratic equation in terms of $d / L$ and computed the values of $A$ and $n$ of equation (2.1).

Steady state natural convection from a vertical flat plate at constant temperature in an infinite medium is the simplest case to study. This problem has been studied analytically to formulate the dimensionless equation by solving the governing differential equations. Some researchers have used the constant density to solve the momentum balance equation, except the body force term. This assumption is known as “Boussinesq’s approximation”. Jaluria [20] used this technique to correlate Nusselt number over an isothermal vertical flat plate. The correlation (for $Ra < 10^9$) is given as:

$$Nu_x = 0.39(Gr_x Pr)^{1/4}$$  \hspace{1cm} (2.3)

Where, $Nu_x$ is the local Nusselt number at distance ‘$x$’ from lower edge of flat plate and $Gr_x$ is the Grashof number at distance ‘$x$’. The product of Prandtl number and Grashof number is known as Rayleigh number ($Ra=Gr.Pr$) which is used in natural convection correlations extensively.

Mostly researchers have studied either or both of the cases, isothermal and constant heat flux surfaces, for natural convection heat transfer from vertical surfaces. For the isothermal vertical flat plate, the correlation for the average Nusselt number as a function of Rayleigh number has been correlated by Eckert [21]. The correlation (for $Ra_L < 10^9$) is given as:

$$Nu_L = 0.508(Ra_L Pr/(0.953 + Pr))^{1/4}$$ \hspace{1cm} (2.4)

Where, $Nu_L$ and $Ra_L$ are the average Nusselt and Rayleigh numbers respectively.

For constant heat flux vertical surfaces, Bird et al [22] have developed the correlation ($Ra_L < 10^9$) as:
\[ q'' = C(k / L)(T_w - T_\infty)(Ra_L^*)^{1/4} \]  \hspace{1cm} (2.5)

where,

\( L = \) Height of the Plate

\( k = \) Thermal conductivity of the fluid

\( T_w = \) Wall Temperature

\( T_\infty = \) Bulk Temperature

\( Ra_L^* = \) Modified Rayleigh number = \( Gr_L^* \cdot Pr \)

\( Gr_L^* = \) Modified Grashof number

and \( C \) is the function of Prandtl number (Pr)

Besides the cases of vertical flat plates in infinite medium there are a large number of articles found in the literature solving natural convection heat transfer problems in enclosed geometries using computational or experimental techniques. The computational studies in 2-D & 3-D for a large number of enclosed flat plate cases are available in literature explaining the steady state and transient conditions for natural convection heat transfer. Ozisik [23] have proposed the empirical correlations of overall Nusselt number for parallel plates as:

\[ Nu = \max \{Nu_1, Nu_2, Nu_3\} \]  \hspace{1cm} (2.6)

where,

\[ Nu_1 = 0.0605Ra_d^{1/3} \]  \hspace{1cm} (2.7)

\[ Nu_2 = \left[ 1 + \left( 0.104 Ra_d^{0.293} / (1 + (6310 / Ra_d)^{1.36})\right) \right]^{1/3} \]  \hspace{1cm} (2.8)

\[ Nu_3 = 0.242(Ra_d d / L)^{0.272} \]  \hspace{1cm} (2.9)

where,
L = Height of Plates

d = separation between the plates

Ra_d = Rayleigh number based on d and the temperature difference between the plates

Nu_d = Overall Nusselt number based on d

Natural convection phenomenon between two horizontal flat plates is different from the natural convection phenomenon between two vertical flat plates. The heat transfer surface is perpendicular to the direction of buoyancy force in case of horizontal plates which may restrict the movement of fluid while the fluid moves up along the surface in case of vertical plates as buoyancy force and heat transfer surface are parallel. Tanda [24] have investigated natural convection phenomena from a staggered array of horizontal plates and found that heat transfer performance of middle and upper plates were affected by changing the position of the plate. They found that the convective interactions among the plates were identified by examining the heat transfer coefficient. Ozisik [23] has presented the empirical correlations for overall Nusselt number, of which some are described here:

**Semi-infinite horizontal plate with heated surface facing upward:**

**Uniform Wall Heat Flux:**

\[ Nu = 0.13Ra^{*\frac{1}{3}} \quad (Ra^* < 2 \times 10^9) \]  
\[ Nu = 0.16Ra^{*\frac{1}{3}} \quad (5 \times 10^8 < Ra^* < 10^{11}) \]

where \( Ra^* \) is the modified Rayleigh number

**Constant Wall Temperature:**

\[ Nu = 0.54Ra^{\frac{1}{4}} \quad (10^5 < Ra < 2 \times 10^7) \]  
\[ Nu = 0.14Ra^{\frac{1}{3}} \quad (2 \times 10^7 < Ra < 3 \times 10^{10}) \]
Semi-infinite horizontal plate with heated surface facing downwards:

**Uniform Wall Heat Flux:**

\[ Nu = 0.58Ra^{1/5} \quad (10^6 < Ra < 2 \times 10^9) \]  \hspace{1cm} (2.14)

**Constant Wall Temperature:**

\[ Nu = 0.27Ra^{1/4} \quad (3 \times 10^5 < Ra < 3 \times 10^{10}) \]  \hspace{1cm} (2.15)

In the above equations \( Nu \) and \( Ra \) are based on characteristic length \( L \) and the physical properties are evaluated at mean temperature defined as;

\[ T_m = T_w - 0.25(T_w - T_\infty) \]  \hspace{1cm} (2.16)

where,

\( T_w \) = Temperature at the surface of plate

\( T_\infty \) = Bulk Temperature

The correlations for different orientations of flat plates have been presented by Jaluria [20] and Ozisik [23]. Radziemska et al [25] have compared their correlation with the correlations of Nusselt number for different geometries of plate like rectangular, square & round with inclination like horizontal, vertical or at some angle \( \theta \). They also determined the contour of static temperature and velocity distributions using FLUENT/UNS code and estimated by qualitative flow visualizations using interferometry and images of particle track.

The experimental techniques for the measurement of temperature have been used by different researchers to investigate the natural convection phenomenon of heat transfer study. Nagendra et al [26], Keyhani et al [27], Hussein et al [28], Jarall et al [29], Popiel et al [30] and Isahai et al [8] have used thermocouples for surface temperature measurements while Naylor and Tarasuk [31, 32] used the optical method of laser
interferometry. Flow visualization technique is used to find stream line patterns by Ho et al [33]. The velocity measurement by tracer particles was investigated by Eichhorn [34].

Naylor and Tarasuk [31, 32] have investigated the natural convection phenomenon by numerical as well as experimental techniques using a vertical channel divided by a heat generating wall. They used, the finite element method based upon CFD code FIDAP to solve the Navier-Stokes equations and the laser interferometry, an experimental technique to observe the isotherm patterns. They also observed the ‘Chimney Effect’, because the average Nusselt numbers in their findings were twice in comparison with a flat plate in infinite medium. They also compared their numerical and experimental results and found that the average Nusselt numbers trend is similar for both techniques but the experimental values of Nusselt number were 10% lower than the numerical predictions.

Elpidorou et al [35] have also studied the ‘Chimney Effect’ using the similar geometry as that of Naylor et al[31, 32]. Elpidorou et al [35] used the heat source on one of the surrounding walls while Naylor et al [31, 32] used the heat source in the center of two walls. Elpidorou et al[35] have found that the rate of heat transfer increases at low Grashof numbers ($Gr < 10^4$) due to the chimney effect. At high Grashof numbers, they found that the rate of heat transfer from enclosed heat source is less than the isolated heat source. This study is only numerical and requires the experimental verification.

2.2 Cylindrical surfaces

The heat transfer by natural convection phenomenon from cylindrical geometries has a significant importance, therefore, it has been extensively studied in near past. Natural convection from cylindrical surfaces has been studied mostly for two different cases of their orientation, vertical and horizontal cylinders. The mechanism of fluid flow and heat transfer by natural convection is different for the both cases due to their orientation with respect to gravity.
2.2.1 Natural Convection from Horizontal Cylinders

2.2.1.1 Horizontal Cylinder in an Infinite Medium:

The heat transfer by natural convection phenomenon over a horizontal cylinder in an infinite medium has been studied numerically using two dimension polar coordinate system. In this case, the thermal boundary layer starts developing from the lower portion of the horizontal cylinder. The thermal boundary layer, at the lower portion of cylinder has very small thickness which increases with the cylinder wall forming almost concentric shape. An analytical solution for natural convection heat transfer from lower portion of a horizontal cylinder has been presented by Jaluria [20]. The suggested expression for local Nusselt number is:

$$\textit{Nu}_x = 0.63 [1 - 0.0349(x/D)^2] (Ra_x \Pr)^{1/4} \quad \text{ (for } x < D)$$

where,

\begin{align*}
x &= \text{Distance along the surface from lower end of the cylinder} \\
D &= \text{Diameter of cylinder} \\
\textit{Nu}_x & \text{ & } Ra_x \text{ are local Nusselt and Rayleigh numbers based on } x
\end{align*}

The above expression is valid for laminar natural convection heat transfer on the lower end of the horizontal cylinder only for $x \leq D$. Jaluria [20] also suggested an analytical solution for average Nusselt number based on the diameter of the horizontal cylinder. The expression is given as:

$$\textit{Nu}_D = C(\Pr)Ra_D^{1/4}$$

where,

\begin{align*}
Ra_D &= \text{Rayleigh number based on diameter} \\
C(\Pr) & \text{ is a function of Prandtl number, given in Table 2-1:}
\end{align*}
<table>
<thead>
<tr>
<th>Pr</th>
<th>0.7</th>
<th>1.0</th>
<th>10</th>
<th>100</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(Pr)</td>
<td>0.436</td>
<td>0.456</td>
<td>0.520</td>
<td>0.523</td>
<td>0.523</td>
</tr>
</tbody>
</table>

The general correlation between Nusselt and Rayleigh numbers is:

$$Nu = A(Ra)^n$$ (2.19)

Morgan [36] have predicted the constants of above correlation over a wide range of Rayleigh numbers for natural convection phenomena over horizontal cylinders, the constants A and n are given in following Table 2-2.

<table>
<thead>
<tr>
<th>Ra</th>
<th>A</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{10} - 10^{-2}$</td>
<td>0.675</td>
<td>0.058</td>
</tr>
<tr>
<td>$10^{-2} - 10^{2}$</td>
<td>1.02</td>
<td>0.148</td>
</tr>
<tr>
<td>$10^{2} - 10^{4}$</td>
<td>0.85</td>
<td>0.188</td>
</tr>
<tr>
<td>$10^{4} - 10^{7}$</td>
<td>0.48</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$10^{7} - 10^{12}$</td>
<td>0.125</td>
<td>$\frac{1}{3}$</td>
</tr>
</tbody>
</table>

Churchill and Chu [37] have experimentally studied the natural convection over a horizontal cylinder and also predicted a more complicated expression over a wider range of Rayleigh number ($10^{5} - 10^{12}$). Their expression is given as:

$$\overline{Nu}^{\frac{1}{2}} = 0.60 + 0.387 \left\{ \frac{Gr Pr}{[1 + (0.559 / Pr)^{9^{16}/16^{9}}]} \right\}^{1/6}$$ (2.20)

They have also predicted a simpler equation for laminar region (Rayleigh number range $10^{-6} - 10^{9}$), the expression is given as:
\[ Nu_d = 0.36 + \frac{0.518(Gr_d \text{ Pr})^{1/4}}{[1 + (0.559 / \text{ Pr})^{9/16}]^{4/9}} \] (2.21)

McAdams [38] have predicted the Nusselt numbers for a wide range of Rayleigh number. The correlation given by Morgan [36] is very much similar with McAdams [38]. Hyman has presented the equation for the heat transfer from horizontal cylinders to liquid metals. The expression is given as:

\[ Nu_d = 0.53(Gr_d \text{ Pr}^2)^{1/4} \] (2.22)

2.2.1.2 Horizontal Cylinder in Porous Medium:

Two dimensional case has been adopted in most of the studies to investigate natural convection about an infinitely long horizontal cylinders embedded in porous medium. Hardee [39] presented the following relation for Nusselt number using integral method.

\[ Nu = 0.465 Ra^{1/2} \] (2.23)

where \( Ra \) is modified Rayleigh number

Merkin [40] investigated the isothermal horizontal cylinder for porous layer adjacent to axisymmetric and two dimensional bodies of arbitrary shape. They solved the governing differential equations using curvilinear coordinates and boundary layer approximation. The gravitational force normal to the surface and viscous dissipation effects are neglected in their investigations. By using the similar solution technique Fand et al. [41] derived following expression for the average Nusselt number based on the Darcy’s law with slip flow.

\[ Nu = 0.565 Ra^{1/2} \] (2.24)

The above two equations are almost similar with a slightly different magnitude only. Bejan et al [42] have studied natural convection in both Darcy and non-Darcy regions and obtained an expression for Nusselt number based on isothermal vertical wall in contact with a porous medium given as:
\[ Nu = 0.494(Ra_\infty)_y^{1/4} \]  
(2.25)

where \( y \) is the vertical Cartesian coordinate and \((Ra_\infty)_y\) is a new Rayleigh number defined as:

\[ (Ra_\infty)_y = \frac{g \beta y^2 \Delta T}{C_2 \alpha^2} \]  
(2.26)

Fand et al [41] have carried out good experimental work and reported the results for natural convection from a horizontal cylinder in porous media saturated by Silicon oil or water. They used spherical particles of soda lime glass of different diameters around the tested cylinder. The effective conductivity of the porous medium is defined as:

\[ k = \varepsilon k_f + (1-\varepsilon)k_s \]  
(2.27)

Where \( k_s \) and \( k_f \) are thermal conductivity of glass-spheres and saturated fluid respectively calculated at mean film temperature. It was assumed that Darcy flow occurs at low Rayleigh numbers while Forchheimer flow occurs at high Rayleigh numbers. Two regions based on flow, defined for different Reynolds numbers are given as:

Darcy flow region: \( 1 \times 10^{-05} < (Re)_{max} \leq 3 \)

Forchheimer flow region: \( 3 < (Re)_{max} \leq 100 \)

Fand et al [41] presented the following expression for Nusselt number in Darcy flow region.

\[ Nu = 0.679 Ra^{0.646} Pr^{-0.126} \]  
(2.28)

Bhat et al[43] used Fluent 6.0 CFD package to create a two dimensional model and investigated the steady state numerical solution for different values of constant heat flux from the horizontal cylinder in infinite medium. They found different behavior of two sub regions i.e. Darcy flow and Forchheimer flow and
presented separate equations for each region. They found the following equation for Nusselt number in Darcy flow region and found slight difference while comparing it with Fand et al. [41] above equation.

\[ Nu = 0.901Ra^{0.497}Pr^{-0.126} \]  

(2.29)

The computational results are slightly lower than the experimental, may be due to the wall effect which were not considered in Bhat et al [43] computational model.

Saada et al [44] investigated heat transfer by natural convection about a horizontal cylinder with porous coating by using mathematical model introducing non-Darcian effects. They found that the heat transfer is reduced by decrease of Rayleigh number which enhances the non-Darcian effect. According to them, at Darcy number greater than \(10^6\) the critical thickness of insulation obtained should be the sum of conduction and convection instead of conduction alone. However, there is a region which is bounded by a Darcy number limit depending on the porous coating thickness where the conduction theory is applicable for thermal insulation design.

### 2.2.1.3 Horizontal Cylinder in Enclosures:

The natural convection from horizontal cylinder in enclosure has been investigated frequently in near past like the flat plate in enclosure. Ozisik [23] has predicted the constants A and n used in generalized correlation for Nusselt number. He has given the constant values as follows for Prandtl number ranges from 1 to 5000.

\[ C = 0.11, \quad n = 0.29 \quad \text{for } 6 \times 10^3 \leq Ra < 10^6 \]

\[ C = 0.40, \quad n = 0.20 \quad \text{for } 10^6 \leq Ra \leq 10^8 \]

Due to the complex nature of heat transfer in enclosures, the above equations are not very much accurate. Therefore, the research work in this area is being done to find a better solution for these geometries.
Ghaddar [45] has simulated the uniformly heated cylinder in a large isothermal enclosure to study natural convection phenomenon and suggested the following correlation.

\[ \text{Nu} = 1.81Ra^{0.207} \]  

(2.30)

This purely numerical study was not compared with experimental data. Hsiao et al [46] numerically solved two-dimensional transient natural convection over heated cylinder. The cylinder, they used, was embedded in a porous medium. They found the vorticity, stream functions and temperature using a body fitted coordinates by solving the governing differential equations. The average Nusselt numbers, they found, were in good agreement with the experimental data for non-uniform porosity. They concluded that the time to reach steady state increases as the Rayleigh number is decreased. Thermal dispersion effect in porous medium natural convection is small at low Rayleigh numbers.

Sadeghipour et al [47] investigated the laminar natural convection from an isothermal horizontal cylinder confined between vertical walls by using both experimental and numerical techniques. They correlated the numerical and experimental equations which gave the variation of Nusselt number in terms of geometrical and physical variables for each H/D (height of confining wall to diameter ratio) and found that the Nusselt number is increased by increasing H/D.

2.2.1.4 Multiple Horizontal Cylinders:

To increase the surface area of heat transfer, thermally interacting multiple horizontal cylinders have been investigated by different researchers. Most of the computational analysis for these types of geometries has same major problems i.e. numerical errors and converging difficulties. A mismatch between the physical boundaries and the mesh or the grid singularity in the region of high gradients can lead to such problems. Park and Chang [48] investigated numerically the laminar natural convection from single horizontal cylinder and compared with literature for its reliability. They used two similar horizontal cylinders with different spacing to find out the velocity profiles and local & overall Nusselt numbers for
Rayleigh number ranges from $10^4$ to $10^5$. They found a reduction in heat transfer due to preheating of the incident buoyant plume of upper cylinder. The lower cylinder remained uninfluenced even at close spacing between cylinders. They also concluded that the upper cylinder faced the flow from lower cylinder hence, forced convection like situation is present on the upper cylinder.

Chouikh et al [49] investigated experimentally the natural convection flow around vertical array of two heated horizontal cylinders. Effects of cylinders spacing and Rayleigh number on heat transfer are investigated. Heat transfer observed from the bottom cylinder was similar to that of a single cylinder. The top cylinder showed the reduced Nusselt numbers at small spacing while augmented Nusselt numbers at large spacing. Ghaddar [45] solved the governing equation for a single cylinder in rectangular enclosure using finite element method. Two or more cylinders can be investigated on similar approach.

Ho et al [33] have investigated natural convection between two horizontal cylinders in a circular adiabatic enclosure using finite difference method and verified their results with flow visualization. The gap (between two cylinders), their orientation with respect to gravity and Rayleigh number have a strong effect on the heat transfer characteristics and fluid flow. If the gap between cylinders is increased or its inclination with the gravity direction is increased, the heat exchange rate was found to be decreased. They found that the heat transfer mechanism was controlled by the re-circulating fluid around the hot and cold cylinders. They also carried out the flow visualization and compared the flow patterns which were in good agreement.

Ho et al [33], Hsiao et al [46] and Park & Chang [48] have used the two dimensional body fitted coordinates in their investigations. Hsiao et al [46] have used this method to represent a single cylinder in porous medium while Ho et al [33] and Park & Chang [48] have used this to represent two cylinders. They have improved the method to accurately handle the high shear region near walls of cylinder using composite grid. They have used closely spaced cylindrical grid
near walls of cylinder and non-orthogonal curvilinear grid elsewhere in the domain based on the method of Thompson et al [50].

Schneider and Straub [51] have simulated the laminar natural convection in a cylindrical enclosure with different end temperatures. The computer code based on finite difference method, they developed, can handle three dimensional transient problems. They found the critical Rayleigh number for the onset of Rayleigh-Benard convection for various values of L/D ratio and inclination angle \( \alpha \), the axis of cylinder with the gravity. They also validated their results with experimental results and found good agreement. They suggested the following correlation for Nusselt number.

\[
Nu = 1 + 0.052 \left( \frac{L/D - 0.01}{0.9} \right)^{\frac{1}{2}} \left[ (\sin \alpha)^{\frac{1}{3}} + 1.3 (L/D)^{\frac{1}{3}} (1 + \cos \alpha)^{\frac{1}{3}} \right] + \\
+ 0.42 \left[ Ra_{D}^{\frac{1}{4}} - 5000 \right] \left[ 2 \left( \cos \left( \frac{\alpha}{2} \right)^{\frac{1}{2}} \right) - \cos \left( \frac{\alpha}{2} \right) \right]^{4}
\]

(2.31)

The above correlation is valid for following ranges:

\[ 4000 \leq Ra_{D} \leq 80000, \ 0 \leq \alpha \leq 180, \ \frac{1}{2} \leq L/D \leq 2 \text{ and } Pr \geq 0.7 \]

Rao and Wang [52] have investigated natural convection in a vertical porous enclosure of horizontal cylinders with internal heat source. They have found the effective schemes for different ranges of Rayleigh number. They also found that there is no significant effect of curvature for aspect ratio of 0.2 to 8 which was proved by comparing it with rectangular enclosure results.

**2.2.2 Natural Convection from Vertical Cylinders**

Natural convection heat transfer from vertical cylinders has many engineering applications such as heat loss from process piping, HVAC system, cooling of electronic components, steam heating coils and electric immersion heaters in process vessels, heat
removal from spent nuclear fuel bundles and cooling of nuclear reactor core after LOCA (Loss of Coolant Accident) etc.

Natural convection heat transfer from vertical cylinders and a flat vertical plate differs significantly due to the curvature effects [2]. Cebeci [5] categorized the vertical cylinder into two type i.e. short (thick) & long (slender or thin) cylinders. According to his results, short cylinders have boundary layer thickness much thinner than the radius of cylinder. Popiel [2] has provided the criteria for thin (long) vertical cylinder as:

\[
Gr_L^{0.25} \left( \frac{d}{L} \right) \leq 11.474 + \left( \frac{48.92}{Pr^{0.5}} \right) - \left( \frac{0.006085}{Pr^2} \right) \]  

(2.32)

Natural convection trends of the short vertical cylinders are similar to that of flat plate and their correlations are applicable without any significant error, however, the long cylinders have considerable curvature effects, hence, further analysis are required to understand this phenomenon. In present study thin vertical cylinders are investigated.

Natural convection heat transfer from a single vertical cylinder has been investigated extensively both experimentally and numerically. Popiel [2] reviewed these studies of natural convection along vertical cylinder. In his study natural convection phenomenon in laminar, transition and turbulent boundary layer regimes were discussed and compared the results of different researchers. Popiel performed some experiments to identify the transition region over the copper pipe polished with nickel. Popiel used two qualitative techniques for the detection of transition region.

1. Temperature Fluctuation: Popiel moved the tip of thermocouple vertically along the pipe with 2 mm gap from surface. Popiel observed the strong fluctuation of temperature and named the regions as:

- Transition region where temperature fluctuations started
- Turbulent region where temperature fluctuations have higher amplitude
**ii). Smoke Visualization:** A simple Indian type smoke stick was used to roughly visualize the air movement in boundary layer at surface of pipe. Three regions laminar, transition and turbulent were observed by smoke layer disturbance. After long observations on transition region, Popiel concluded the critical Grashof number \((Gr_{x-cr} = 4.9 \times 10^9)\) and \((x/D)_{cr} \sim 50\) for turbulent region. In Popiel comparison of natural convection phenomenon investigated by different researchers over vertical cylinder below this critical Grashof number was done. Some researchers have extended their numerical results to investigate the natural convection over vertical cylinder beyond this Grashof number which can not reliable as the flow regime changes. The turbulent region is yet to be investigated experimentally.

Most of the researchers have used different geometric configurations in their investigations; commonly used are discussed here.

**2.2.2.1 Single Vertical Cylinder:**

Natural convection heat transfer from a single vertical cylinder has been investigated extensively both experimentally and numerically. Popiel [2] reviewed these studies of natural convection along vertical cylinder. In his study of natural convection the laminar, transition and turbulent boundary layer regimes were categorized. He compared the results of different researchers. Natural convection phenomenon has been investigated by different researchers over vertical cylinder in laminar regime. Some researchers have extended their numerical results to investigate natural convection over vertical cylinder beyond the critical Grashof number which are not reliable. A few of research have been done in turbulent regime and the scattered data is available in literature. Thus the turbulent region is to be further investigated experimentally.

**Categories of Vertical Cylinders:**

The natural convection heat transfer through vertical cylinders in various categories has been studied by Nagendra et al [26]. They experimentally studied the natural convection phenomenon over a wide range of dimensionless parameter \(Ra_L/D\) and suggested the following correlations for three types of cylinders i.e.
wires, long cylinders and short cylinders. According to him, the criteria for wires is \( Ra_D \frac{D}{L} < 0.05 \), for long cylinders \( 0.05 \leq Ra_D \frac{D}{L} \leq 10^4 \) and for short cylinder \( Ra_D \frac{D}{L} \geq 10^4 \).

i. Wires:

\[
Nu_D = 1.3 \left( Ra_D \frac{D}{L} \right)^{0.05} \quad \text{for} \quad Ra_D \frac{D}{L} < 0.05 \quad (2.33)
\]

ii. Long Cylinders

\[
Nu_D = 1.3 \left( Ra_D \frac{D}{L} \right)^{0.16} \quad \text{for} \quad 0.05 \leq Ra_D \frac{D}{L} \leq 10^4 \quad (2.34)
\]

iii. Short Cylinders

\[
Nu_D = 0.57 \left( Ra_D \frac{D}{L} \right)^{0.25} \quad \text{for} \quad Ra_D \frac{D}{L} \geq 10^4 \quad (2.35)
\]

**Cases Commonly Under Investigation:**

The two cases mostly found in literature are:

- Constant wall temperature or isothermal case
- Uniform heat flux on the wall of cylinder.

The above correlations are suggested while investigating isothermal cylinders in infinite medium. The same authors [53] have studied numerically the second case of uniform heat flux on the wall of vertical cylinder. They investigated the natural convection over vertical cylinder by transforming the governing equation into two parameter system and presented the following correlations.
i. Wires:

\[ Nu_D = 0.93 \left( Ra_D \frac{D}{L} \right)^{0.05} \quad \text{for} \quad Ra_D \frac{D}{L} < 0.05 \quad (2.36) \]

ii. Long Cylinders

\[ Nu_D = 1.37 \left( Ra_D \frac{D}{L} \right)^{0.16} \quad \text{for} \quad 0.05 \leq Ra_D \frac{D}{L} \leq 10^4 \quad (2.37) \]

iii. Short Cylinders

\[ Nu_D = 0.6 \left( Ra_D \frac{D}{L} \right)^{0.25} \quad \text{for} \quad Ra_D \frac{D}{L} \geq 10^4 \quad (2.38) \]

There are many instances of practice found where the surface temperature is not constant or the heat flux is the function of local axial length. Lee et al. [54] have presented the method for the solution of governing equations for vertical cylinders and needles in the dimensionless form for variable surface temperature. They calculated the wall temperature at different axial locations using equation \( T_s = T_\infty + ax \) and presented the correlation for local Nusselt number. They also compared their results with previous numerical study of Fujii et al. [55] and found good agreement for larger diameter to length ratios, however, for smaller diameter to length ratios the Nusselt number were underestimated. Chung et al. [56] gave the numerical solution for buoyant convection in a vertical cylinder with azimuthally varying sidewall temperature using the SIMPLE algorithm. Chung and Hyun [57] considered the flow of incompressible Boussinesq-fluid and studied the effect of circumferential variation of sidewall temperature on buoyant convection in a vertical cylinder. They observed that the flow in the interior weakens as the gradient of temperature is intensified azimuthally. Surma Devi et al. [58] analyzed the power law variation in the wall temperature along vertical cylinders and needles for large curvature parameters.

Heat flux can be defined as a function of axial position over a vertical cylinder in some cases e.g. natural convection from nuclear fuel rod. Heckle et al. [59] numerically
solved this type of problem. They have presented a general dimensionless form of governing equations using the heat flux as a power function of axial distance, i.e. \( q(x) = ax^n \). Their numerical results have not been validated or compared with any previous numerical studies or any experimental work. The local heat transfer coefficient they found increases by increasing the curvature effect. Different researchers have used different criteria for this limiting curvature. Ozisik [23] presented the following correlation for curvature parameter:

\[
\frac{(L / D)}{\sqrt[4]{Gr_L}} < 0.025
\]

In above equation if the curvature parameter is less than 0.025, the correlation for the flat plate can be used for the vertical cylinder.

Popiel et al [60] investigated the free convection heat transfer from the side walls of a square cylinder in air and established a correlation for the boundary layer curvature effects on the average heat transfer coefficient. This correlation differs from the flat plate data because of the local transversal curvature effects produced by the cube sharp vertical edges. Hussein et al [61, 62] experimentally studied the natural convection phenomenon effected by different entry restrictions configuration for uniformly heated vertical circular pipes. Jarall et al [29] experimentally investigated the natural convection from electrically heated vertical cylinders immersed in air. They used three test sections of different slenderness ratios d/L. They applied the uniform heat flux to the vertical cylinder and measured the temperature to obtain an empirical correlation equation relating the local Nusselt number to modified Rayleigh number and the position to cylinder diameter. They studied only the laminar region over the tested cylinder and did not study the length where the temperature decreases due to turbulence. This section of the cylinder is considered in the present study [63, 64] and overall natural convection phenomenon is investigated.

Fujii et al [65] investigated the natural convection phenomenon from the outer surface of a vertical cylinder using water and oil (Pr = 2.4—300). Their test cylinder was located in a pipe of 385 mm in diameter and 1420 mm long assembled from 20 segments
heated electrically (both isothermal wall and uniform heat flux). They found that the transition region starts where the instabilities and vortex street layer appears and the turbulent region where the wall vortices develop & disintegrate and a rapid increase in convective heat transfer coefficient is observed due to intensive mixing at the wall, forming finally a fully turbulent boundary layer. Their results concerning transition region give only a qualitative picture which cannot be used to pin point the transition and turbulent region limits.

Buchlin [66, 67] investigated the transition section over vertical cylinder with constant heat flux but were not able to determine the location where transition region starts. Al-Arabi and Khamis [68] investigated the heat transfer transition on a vertical cylinder in air (Pr = 0.71) heated by condensing steam (isothermal wall) and found critical Rayleigh number = 2.7 x 10^9 and critical Grashof number = 3.8 x 10^9.

Kimura et al [69] investigated the free convection from vertical cylinder (x/D < 11) in water (Pr = 5.4) in transition region having constant heat flux at wall. They found the critical modified Rayleigh number Ra_{x-cr}^* = 4.5 x 10^{12} and critical modified Grashof number Gr_{x-cr}^* = 3.47 x 10^9 which is very close to the critical Grashof number for vertical flat plate. Experiments of Kimura revealed also that the value of heat flux does not affect the critical modified Rayleigh number.

Popiel [2] identified the free convection transition region using copper pipe with polished nickel as studied by Bober [70]. They used two qualitative techniques (Temperature fluctuation and Smoke visualization) to detect the transition region. After long observations, they found the critical Grashof number Gr_{x-cr} = 4 x 10^9.

Popiel [2] has listed the correlations which are given here for comparison in Table 2-3 and Table 2-4. The results of laminar boundary layer numerical analysis of Lee et al [54] and Cebeci [5] has been experimentally validated by Popiel et al [30] and precisely represent following correlation.

\[ Nu_H = A Ra_{H}^n \]  \hspace{1cm} (2.40)

where
\[ A = 0.519 + 0.03454 \left( \frac{H}{D} \right) + 0.0006772 \left( \frac{H}{D} \right)^2 + 8.855 \times 10^{-6} \left( \frac{H}{D} \right)^3 \]  
\[ (2.41) \]

\[ n = 0.25 - 0.00253 \left( \frac{H}{D} \right) + 1.152 \times 10^{-5} \left( \frac{H}{D} \right)^2 \]  
\[ (2.42) \]

Above equation is valid for Isothermal cylinder wall, \( Pr = 0.71 \), \( \left( \frac{H}{D} \right) = 1 \) to 60 and \( Ra_H = 10^8 \) to 1.1\( \times 10^9 \). Bober [70] found that the boundary layer approximations are not valid for low Rayleigh numbers (\( Ra_H < 10^4 \)), hence, the numerical results are not applicable. In laminar region following equations are also applicable.

Cebeci numerical correlation for \( Pr = 0.72 \) has the form

\[ \frac{Nu_H}{Nu_{H-FF}} = 1 + 0.3 \left[ 32^{0.5} Gr_H^{-0.25} \left( \frac{H}{D} \right) \right]^{-0.909} \]  
\[ (2.43) \]

A general correlation based on the data of Cebeci [5] valid in the laminar region with Prandtl number range \( Pr = 0.01 \) to 100 and developed by Popiel et al [30] has the form

\[ \frac{Nu_H}{Nu_{H-FF}} = 1 + B \left[ 32^{2} Gr_H^{-0.25} \left( \frac{H}{D} \right) \right]^{C} \]  
\[ (2.44) \]

where

\[ B = 0.0571322 + 0.20305 Pr^{-0.43} \]  
\[ (2.45) \]

\[ C = 0.9165 - 0.0043 Pr^{0.5} + 0.01333 \ln Pr + 0.0004809 / Pr \]  
\[ (2.46) \]

The results obtained from above equation deviate approximately ±2% from the numerical correlation of Cebeci.

Popiel [2] has also compared the natural convection phenomenon through vertical cylinders investigated by different researchers. This comparison shows that the most of the research work done is in laminar region i.e. \( Ra_H < 4 \times 10^9 \). Limited but scattered
research is available for turbulent region. McAdams [38] suggested the following approximate equation for average heat transfer in turbulent regime:

\[ Nu_H = 0.13Ra^{\frac{1}{5}}_H \]  \hspace{1cm} (2.47)

Eigenson [71] have provided the empirical correlation in this region

\[ Nu_H = 0.148Ra^{\frac{1}{3}}_H \]  \hspace{1cm} (2.48)

The above two equations (2.47) & (2.48) are valid for \( 4 \times 10^9 \leq Ra_H \leq 2.5 \times 10^{10} \) and gives very close results with the Bober [70] of undisturbed situation.

For the disturbed free convection Bober [70] and Kedzierski [72] presented the following correlation:

\[ Nu_H = 0.582x10^{-5}Ra^{0.675}_H \]  \hspace{1cm} (2.49)

The experimental data of Al-Arabi [68] in turbulent region is higher as that of his data in laminar region which is due to some disturbance. The experiments in turbulent region are very much difficult because a slight movement can disturb the sensitive boundary layer and the results show a wide spread. In the present research work, the turbulent region has been focused and the empirical correlations to describe the phenomenon of natural convection heat transfer are presented.

\section*{2.2.2.2 Multiple Vertical Cylinders:}

Natural convection heat transfer around a single cylinder and around a cylinder in assembly of cylinders is different because the heat transfer around the cylinder in assembly is affected by the heat transfer of other cylinders present around it [73]. Natural convection from assembly of vertical cylinders has been studied by a few researchers only. Numerically this problem can be solved by taking axisymmetric flow, two or three-dimensional analysis and body fitted coordinates to represent the geometry. Three-dimensional analysis in this area has not been presented yet because of its difficult nature of work. However, a few simplified two-dimensional numerical analysis and
experimental studies can be found in literature. However, only the laminar region has been investigated in these studies.

Adlam [74] numerically investigated the stream functions, vortices and temperature inside a cavity where there are vertical internal bodies. They used two-dimensional Cartesian coordinates to represent the rectangular shaped vertical bodies and presented the contours of stream functions to show flow patterns. However no comparison has been made with experimental data yet. Davis and Perona [75] performed experiments on two tube bundles and found that the local Nusselt number show a strong parametric dependence on pitch to diameter ratio and weak dependence on dimensionless flow rate. Rogers and Yao [76] numerically investigated natural convection in a vertical annulus and suggested that the natural convection from assembly of vertical cylinders can be approximated by a heat generating cylinder placed in and adiabatic cylindrical enclosure.

Butterworth [77] suggested that the flow through assemblies of cylinders can be approximated by considering the cylinders in porous media. Using this approach McCann[78] numerically simulated natural convection in vertical rods bundle. He investigated various assemblies of spent fuel rods considering anisotropic porous media and used momentum equation empirically modified by the so called Darcy’s Law. He compared his numerical results with experimental data and found his results in good agreement. Natural convection in an open ended vertical concentric cylinders has been studied numerically by Hadjadj et al [79]. They used SIMPLE algorithm to solve the governing equations and presented the temperature & velocity profiles and stream functions. They found that the change in aspect ratio have no influence on the flow structure and isotherm patterns. Their results show that stream functions increase with aspect ratio and the average & local Nusselt numbers increase with increasing aspect ratio, Prandtl & Rayleigh numbers. They also presented the correlation for local and average Nusselt numbers. A comparison between the present numerical results and available numerical and experimental data has also been made. The agreement between these results is found to be in good again but this research is limited to laminar region only (up to $Ra_H < 10^5$).
Isahai and Hattori [8] numerically studied the natural convection phenomenon with few experiments. They investigated the bundle of nineteen vertical cylinders with different triangular pitch for modified Grashof number \(Gr^*\) ranges from 10 to \(10^8\). They used the boundary fitted coordinates system in their investigations. This study has also been performed in the laminar region.

Keyhani et al [6] have experimentally investigated free convection in different configurations of rod bundles. They have studied the natural convection heat transfer over 3 x 3 and 5 x 5 rod bundles enclosed in a cooled cylindrical canister and used helium, air and water as a coolant. They have found that the radiation heat transfer between the heated rod and the cooled canister plays an important role at low Prandtl numbers. They have compared their results of annulus [27] with the 3 x 3 & 5 x 5 rods bundles and found almost similar results. Hence they proposed an equivalent annulus model for enclosed rod bundles. They have presented correlations in conduction and boundary layer regimes. They have defined these regimes [27] as:

(i). Conduction Regime:

\[
\frac{Ra}{H} \leq 400 \quad \text{for } H > 5 \text{ and Pr} = 1
\]  
\[
\frac{Ra}{H} \leq 1000 \quad \text{for } H = 1
\]  

(ii). Boundary Layer Regime:

\[
\frac{Ra}{H} \geq 3000 \quad \text{for } H \leq 5 \text{ and Pr} = 1
\]  
\[
\frac{Ra}{H} \geq 8000 \quad \text{for } H = 1
\]  

For an adequate description of experimental results of annulus, 3 x 3 & 5 x 5 rods bundles [6], they have considered the effects of aspect ratio \(H\), diametric ratio \(K\), pitch to diameter ratio \(P/d\) and number of rows in square array \(N\) of rods bundle. They have
correlated the results of three geometries in conduction and boundary layer regimes separately with geometric effects and presented as:

(i). Conduction Regime:
\[
Nu = 0.797 K0.505 H0.052 \left( \frac{P}{d} \right)^{0.045 N+0.541} Ra_G^{0.077}
\] (2.54)

(ii). Boundary Layer Regime:
\[
Nu = 0.188 K0.442 H0.238 \left( \frac{P}{d} \right)^{0.045 N+0.541} Ra_G^{0.322}
\] (2.55)

where

Diametric ratio of inner & outer cylinders of annulus; \( K = D/d \)

Aspect ratio; \( H = L/G \)

Characteristic length or Annular Gap; \( G = (D-Nd)/2 \)

Length of cylinder \( L \), diameter of cylinder \( d \), number of cylinder rows \( N \) &
diameter of annulus \( D \)

Keyhani et al [6] study is limited to spent fuel storage point of view and not applicable to operating reactor like MNSR. Two following researchers have studied the assembly of vertical cylinders numerically for MNSR type reactor but no experimental data found yet. Hence, the study of natural convection through assembly at higher Rayleigh numbers is still required as for as that is required for single cylinder.

Chughtai et al [3] presented the correlation for Nusselt number based on numerical results. They used the Computational Fluid Dynamic (CFD) technique in their study and presented the velocity & temperature profiles. They compared their results with available limited experimental data of tank-in-pool type reactor. Basit et al [4]
numerically investigated natural convection heat transfer from an assembly of vertical cylinders of PARR-II. They solved the two-dimensional axisymmetric case with COSINAC computer code and presented the temperature & velocity profiles and Nusselt number variations at different heat flux values. Both the researchers, Basit et al [4] & Chughtai et al [3], compared their numerical results with steady state operating parameters of PARR-II. As per the complexity of PARR-II temperature of coolant can be measured at inlet and outlet positions only. The temperature profiles at the surface of cylinders in assembly are still required to be measured experimentally and compared with the numerical results of the above mentioned researchers.

A review of the literature shows that very limited experimental data is available for such a configuration. Moreover for the operational MNSR only the inlet and exit fluid bulk temperature are measured and reported against power. No information regarding the outer surface temperature of the fuel rods is available.

Hence to achieve the objective a comparison of the current experimental study with the previous studies available in the literature was carried out. This is done by following the procedure given below.

- Comparison of the local / average Nusselt number with the local / average Grashof / Rayleigh numbers.
- Suggesting the true representation of Nusselt as a function of Grashof and Prandtl numbers.
- Proposing a procedure to predict the surface temperature as a function of fluid properties, heat flux and inlet temperature.
- Propose a correlation to predict the bulk liquid exit temperature as a function of heat flux, fluid properties and fluid inlet bulk temperature.
- Compare the predicted bulk exit temperature for the geometry & operating conditions of MNSR and compare it with the actual data reported and with the computational prediction of Chughtai et al [3].
Summary:

The literature regarding the investigation of natural convection phenomenon has been thoroughly reviewed. On summarizing the above literature review, some areas can be focused where the research work is required:

- Most of the researchers have investigated the natural convection over single vertical cylinders in laminar region only. Very few researchers have studied the turbulent region. Hence, a single vertical cylinder is required to be investigated experimentally for high Rayleigh numbers.

- A little literature has been found for assembly of vertical cylinders and the studies are performed at low Rayleigh numbers. Hence, the assembly of vertical cylinders is required to be investigated experimentally at both low and higher Rayleigh numbers in the laminar and turbulent regions respectively.

- The surface temperature profiles, heat transfer coefficient and Nusselt number along the single as well as assembly of vertical cylinders at steady state are required to be investigated at higher Rayleigh numbers.

- Flow visualization at the surface of vertical cylinders is also required to be investigated to observe the flow behavior in an assembly.

- The empirical correlations for single and assembly of vertical cylinders are required to present the natural convection phenomenon at higher Rayleigh numbers.

An experimental setup was prepared to investigate the natural convection heat transfer in the present study.
<table>
<thead>
<tr>
<th>Authors</th>
<th>D, mm</th>
<th>H/D</th>
<th>Heat transfer surface</th>
<th>Test fluid, Pr number</th>
<th>Range of $Ra_H$</th>
<th>Correlation equation</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Koch</td>
<td>14–100</td>
<td>20–152</td>
<td>—</td>
<td>Air Pr = 0.71</td>
<td>$10^9 &lt; Ra_H D/H &lt; 10^{11}$</td>
<td>$Nu_H = 0.0086 (Ra_H D/H)^{0.23} or Nu_H = 0.0086 0.23 (H/D)^{0.23}$</td>
<td>Experiments correlated by Monger</td>
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<td>Brebbia</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>Air, gases</td>
<td>$10^4 &lt; Ta &lt; 10^6$</td>
<td>$Nu_H = 2.35 \exp \left(2.8 \frac{Nu_H}{Pr} \right) \frac{Pr}{Pr_c}$</td>
<td>Correlation based on the results of simple</td>
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<td>Correlation based on the results of simple</td>
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<td>Le Fevre and BFs</td>
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<td>$Nu_H = 4 \frac{Ra_H}{Pr} \left(1 + 0.105 \frac{Pr}{Pr_c} \right) \frac{Pr}{Pr_c}$</td>
<td>Results of integral method</td>
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<tr>
<td>Porschen and Vehera</td>
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<td>$Nu_H = 4 \frac{Ra_H}{Pr} \left(1 + 0.105 \frac{Pr}{Pr_c} \right) \frac{Pr}{Pr_c}$</td>
<td>Results of integral method</td>
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<tr>
<td>LeFanu et al.</td>
<td>—</td>
<td>—</td>
<td>$q_w = const$</td>
<td>—</td>
<td>$Pr = 0.72–100$</td>
<td>$Nu_H = 0.545 \frac{Pr}{Pr_c}$ $Nu_H = 0.545 \frac{Pr}{Pr_c}$ $Nu_H = 0.545 \frac{Pr}{Pr_c}$</td>
<td>Results of integral method</td>
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<td>Results of integral method</td>
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<td>Yang</td>
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<td>$q_w = const$</td>
<td>$Pr &lt; 10^9$</td>
<td>Complete range of $Ra_H$</td>
<td>$Nu_H = 0.54 + 0.870 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$ $Nu_H = 0.54 + 0.870 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$</td>
<td>Empirical general equation</td>
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<td>Empirical general equation</td>
</tr>
<tr>
<td>Kinners et al.</td>
<td>10–60</td>
<td>8.5–10.9</td>
<td>$q_w = const$</td>
<td>Water Pr = 3.3</td>
<td>$Gr_x &lt; 3 \times 10^6$</td>
<td>$Nu_H = 0.93 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$ $Nu_H = 0.93 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$</td>
<td>Experimental data</td>
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<tr>
<td>Nagendra et al.</td>
<td>0.5–8</td>
<td>95–3050</td>
<td>$q_w = const$</td>
<td>Water Pr = 0.72</td>
<td>$Gr_x &lt; 2 \times 10^6$ $Gr_x &lt; 2 \times 10^6$</td>
<td>$Nu_H = 1.37 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$ $Nu_H = 1.37 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$</td>
<td>Experimental data</td>
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<td>Gallo</td>
<td>—</td>
<td>—</td>
<td>$T_w = const$</td>
<td>Air Pr = 0.73</td>
<td>$Pr = 6$</td>
<td>$Nu_H = 1 + 0.870 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$ $Nu_H = 1 + 0.870 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$</td>
<td>Experimental data</td>
</tr>
<tr>
<td>Akrab and Khanzad</td>
<td>1.275–5</td>
<td>300–6000 mm</td>
<td>$T_w = const$ (steam condensation)</td>
<td>Air Pr = 0.71</td>
<td>$1.08 \times 10^6 &lt; Gr_x &lt; 6.9 \times 10^6$ Lam: $9.38 \times 10^5 \leq Nu_H \leq 2.7 \times 10^6$ Turb: $2.7 \times 10^6 &lt; $ $1.08 \times 10^6 &lt; Gr_x &lt; 6.9 \times 10^6$ Lam: $9.38 \times 10^5 \leq Nu_H \leq 2.7 \times 10^6$ Turb: $2.7 \times 10^6 &lt; $</td>
<td>$Nu_H = 2.96 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$ $Nu_H = 2.96 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$</td>
<td>Experimental data</td>
</tr>
<tr>
<td>Lee et al.</td>
<td>—</td>
<td>—</td>
<td>$T_w = const$</td>
<td>$Pr = 0.3–100$</td>
<td>$Nu_H = 0.23 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$</td>
<td>$Nu_H = 0.23 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$</td>
<td>Correlation, boundary layer analysis</td>
</tr>
<tr>
<td>Popp et al.</td>
<td>10–80</td>
<td>1–60</td>
<td>$T_w = const$</td>
<td>Air Pr = 0.71</td>
<td>$10^8 &lt; 1.1 \times 10^9$</td>
<td>$Nu_H = 3.5104 \frac{Ra_H}{Pr} \left(1 + 0.40 \frac{Pr}{Pr_c} \right)$</td>
<td>Correlation, boundary layer analysis</td>
</tr>
</tbody>
</table>

Table 2-3 Comparison of Correlation Equations for Vertical Cylinders taken from Popiel [2]: $Nu_H = f(Ra_H, Pr, H/D)$
Table 2-4 Comparison of Correlation Equations for Vertical Cylinder taken from Popiel[2]: \( \text{Nu}_H = f(\text{Ra}_H, \text{Pr}) \)

<table>
<thead>
<tr>
<th>Authors</th>
<th>D, mm</th>
<th>H/D</th>
<th>Heat transfer surface</th>
<th>Test fluid, Pr number</th>
<th>Range of ( \text{Ra}_H )</th>
<th>Correlation equation</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Griffith and Davis</td>
<td>174</td>
<td>0.87–15.2</td>
<td>—</td>
<td>Air</td>
<td>( 10^7-10^9 ) ( 10^7-10^9 )</td>
<td>( \text{Nu}_H = 0.67 \text{Ra}_H^{0.25} ) ( \text{Nu}_H = 0.078 \text{Ra}_H^{0.557} )</td>
<td>Experiments correlated by Morgan</td>
</tr>
<tr>
<td>King</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>Air, liquids</td>
<td>( 10^6-2.5 \times 10^6 )</td>
<td>( \text{Nu}_H = 0.55 \text{Ra}_H^{0.25} )</td>
<td>Experiments correlated by Morgan</td>
</tr>
<tr>
<td>Jakob and Linde</td>
<td>55</td>
<td>4.3</td>
<td>—</td>
<td>Air, Pr = 0.71</td>
<td>( 10^4-10^6 )</td>
<td>( \text{Nu}_H = 0.55 \text{Ra}_H^{0.25} )</td>
<td>Experimental, see Morgan</td>
</tr>
<tr>
<td>Bigeminen</td>
<td>2.4–58</td>
<td>50–140</td>
<td>—</td>
<td>Air, Pr = 0.71</td>
<td>( 10^6-10^{10} )</td>
<td>( \text{Nu}_H = 0.48 \text{Ra}_H^{0.25} ) ( \text{Nu}_H = 0.14 \text{Ra}_H^{0.353} )</td>
<td>Experimental, see Morgan</td>
</tr>
<tr>
<td>Touloukian et al.</td>
<td>69.7</td>
<td>2.2–15.2</td>
<td>—</td>
<td>Water, ethylene glycol</td>
<td>( 2 \times 10^8-4 \times 10^9 )</td>
<td>( \text{Nu}_H = 0.72 \text{Ra}_H^{0.28} ) ( \text{Nu}_H = 0.674 \text{Ra}_H^{0.129} )</td>
<td>Experimental, see Morgan</td>
</tr>
<tr>
<td>McAdams</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>( 10^3-10^8 ) ( \text{Gr}_H &gt; 10^9 )</td>
<td>( \text{Nu}_H = 0.59 \text{Ra}_H^{0.25} ) ( \text{Nu}_H = 0.12 \text{Ra}_H^{0.383} )</td>
<td>Correlations. See Morgan or Keith</td>
</tr>
<tr>
<td>Cane</td>
<td>47–76.2</td>
<td>8–12.7</td>
<td>—</td>
<td>Air, Pr = 0.71</td>
<td>( 2 \times 10^6-2 \times 10^8 )</td>
<td>( \text{Nu}_H = 0.52 \text{Ra}_H^{0.28} )</td>
<td>Experiments correlated by Morgan</td>
</tr>
<tr>
<td>Haseirin and Kalish</td>
<td>25.4</td>
<td>3.12</td>
<td>—</td>
<td>Air, fluoro-carbon gas</td>
<td>( 10^6-10^{10} )</td>
<td>( \text{Nu}_H = 0.48 \text{Ra}_H^{0.25} )</td>
<td>Experiments correlated by Morgan</td>
</tr>
<tr>
<td>Dottersmane</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>Dielectric ( \text{Pr} = 0.72 )</td>
<td>( 8 \times 10^4-4 \times 10^7 )</td>
<td>( \text{Nu}_H = 0.56 \text{Ra}_H^{0.25} )</td>
<td>Experimental, see Morgan</td>
</tr>
<tr>
<td>Keith</td>
<td>—</td>
<td>—</td>
<td>( \text{Re}_H = \text{const} )</td>
<td>—</td>
<td>( 10^{-10^9} ) ( 10^{-10^9} )</td>
<td>( \text{Nu}_H = 0.41 \text{Ra}_H^{0.25} ) ( \text{Nu}_H = 0.25 \text{Ra}_H^{0.29} )</td>
<td>—</td>
</tr>
<tr>
<td>Jarall and Campos</td>
<td>16, 33.5, 48.4</td>
<td>47.5, 58.7, 101.2</td>
<td>( \text{Re}_H = \text{const} )</td>
<td>Air, Pr = 0.71</td>
<td>( \text{Re}_H = \text{Ra}_H \text{Nu}_H \leq 2 \times 10^{12} )</td>
<td>( \text{Nu}_H = 1.25 \text{Ra}_H^{0.25} ) ( \text{Nu}_H = 0.30 \text{Ra}_H^{0.016} )</td>
<td>Experimental: electrically heated pipe. Results higher than theoretical data of Metzbuch and Lee et al.</td>
</tr>
<tr>
<td>Bober</td>
<td>15, 18.3, 20</td>
<td>—</td>
<td>( \text{Re}_H = \text{const} )</td>
<td>Air, Pr = 0.71</td>
<td>( 5 \times 10^6-2.5 \times 10^{10} ) ( 10^{10}-10^{12} )</td>
<td>( \text{Nu}_H = 0.13 \text{Ra}_H^{0.25} ), undisturbed ( \text{Nu}_H = 5.82 \times 10^{-5} \text{Ra}_H^{0.75} ), disturbed</td>
<td>Experimental. Buoyancy technique: hot water flow in vertical pipe.</td>
</tr>
</tbody>
</table>
Chapter 3. Experimental Setup

Experimental techniques have been used to understand the behaviors of physical processes and to develop the basic empirical correlations. Although the computers and numerical techniques has been used extensively as an alternative of physical experiments. However there are some disadvantages of numerical techniques despite of their advantages.

Advantages of Numerical Techniques are:

i. Lowest cost
ii. Flexibility in geometry and parameters
iii. Easily derive the integral quantities
iv. Complete information in results like velocity and temperature profiles at each and every point

Disadvantages of Numerical Techniques are:

i. Less accurate results
ii. Assumptions deviate the results

Due to the above disadvantages of numerical techniques, the experiments are always required to increase the accuracy level for the solution of a problem. Moreover the experimental techniques provide the actual behavior rather than the assumption based results of numerical techniques.

3.1 Goals& Design Basis of Experimental Set up

The goal of the experiments was to validate the numerical simulation of natural convection heat transfer in an assembly of vertical cylinders presented by Chughtai et al.[3]. They validated their study with PARR-II (tank in pool type) reactor. This reactor has no instrumentation inside the core, so the flow and temperature profiles are not known yet. The available thermal hydraulic model for PARR-II is one dimensional lump model and the transient behavior of vertical cylinder has not been modeled to visualize local effects within or around the cylinder. Therefore, the study of heat transfer by natural convection phenomenon from an enclosed assembly is required.
The physical design of apparatus has a vital role in the experimental analysis. The goal of design was to fabricate the apparatus that have configurations and boundary conditions as close as possible to those used in the computational work. An attempt was made to fabricate the experimental facility indigenously. The schematic diagram of experimental facility is illustrated in Figure 3-1.

### 3.1.1 Design & Fabrication of Cylinder

The most important part of the experimental setup was to design and fabricate the heat generating cylinders with surface temperature measurement, cylinders configurations and their assembling. The basis of the design was the dynamic similarity of experimental setup with the PARR-II reactor. Three parameters were selected to calculate the length, diameter and operating limits.

**i). L/d & P/d Ratios:** The length to diameter ratio (L/d) of PARR-II is 41.82 and pitch to diameter ratio (P/d) is 1.99. In designing of cylinder it is ensured that these ratios remain as close as possible.

**ii). Power:** Wattage per rod of PARR-II was taken as basis of experimental set up. At normal power of reactor (27 KW), heat energy produced in one fuel pin is around 80 W. So the experimental set up was designed to provide 0—100 Watt energy per rod.

**iii).Grashof Number:** Grashof number was selected slightly higher than its value at normal power of PARR-II to calculate the length of cylinder using following equation.

\[
L = \left[ \frac{v^2 kGr^*}{g \beta q^*} \right]^{1/4}
\]  
(3.1)

The length obtained from above equation was used to optimize the diameter of the cylinder to make L/d ratio closest to above mentioned value. A 12.7 mm diameter cylinder was selected. A heating length of 570 mm calculated from equation (3.1) was used. The L/d ratio of the cylinder is 44.88 which is quite close to the value of PARR-II. However a total length of cylinder taken is 609.6 mm (2 ft) by adding some non heating length on both ends of the cylinder. Hence Stainless steel tube of ½ inches diameter and 16 BWG was used as cylinder of the experimental setup.
For temperature measurements six axial equidistant locations on cylinder were selected where thermocouples were installed from inside at the surface of cylinder. At the same position two thermocouples were installed at diametrically opposite side to avoid perturbation effects. Hence a total of twelve thermocouples were installed at the surface of the cylinder to measure the surface temperature at six positions.

A uniform tubular heater of 400 watts, 6 mm diameter and same length as that of cylinder was prepared and adjusted at the center of cylinder. The gap between the heater and the cylinder inner wall was filled with a paste of MgO. The cylinder was allowed to dry for few days in same vertical position as the MgO paste was filled and then assembled for experimental analysis. MgO was selected due to its high thermal conductivity. The schematic diagram & actual picture of the SS cylinder is shown in Figure 3-2.

3.1.2 Assembling of Cylinders

This study is composed of two parts:

a). Single Cylinder Analysis

b). Assembly of Cylinders Analysis

In single cylinder analysis only one cylinder is placed vertically in infinite medium to study the phenomenon of natural convection as shown in Figure 3-3. While for assembly of cylinders analysis, nine similar cylinders are prepared. These cylinders are assembled in the following way to achieve as close as possible the similarity with PARR-II assembly. The assembly of cylinders is shown in Figure 3-4.

- The assembly of cylinders was enclosed in a 5 inches diameter hollow Perspex cylinder.

- The Perspex cylinder has inlet slits at the bottom for water flow in and exit slits at the top to flow out. Exit slit is a little bit larger than the inlet slit similar to PARR-II. In PARR-II the exit orifice is larger than the inlet orifice.
- Cylinders were fixed at the top and bottom using two circular tube sheets so that the pitch remained same throughout the length of cylinders.
- Four tie rods were used to strengthen the assembly and to fix the distance & orientation of top & bottom tube sheets.
- The cylinders assembly was vertically immersed in a large volume water tank.

3.2 Details of the Experimental Set up

The experimental apparatus was designed to have a heat generation section along with temperature measurement at different points to investigate natural convection heat transfer phenomenon as discussed above. A schematic diagram of the experimental setup is illustrated in Figure 3-1. The experimental setup consists of the following parts:

1. Stainless Steel Cylinder
2. Assembly of Stainless Steel Cylinders
3. T-type Thermocouples
4. Data Acquisition System
5. Personal Computer
6. Variable Voltage Power Supply
7. Voltmeter and Ammeter
8. Water Tank

The experimental set up has been fabricated by assembling different parts. The major parts are described in detail as given below.

3.2.1 S.S. Cylinder

The cylinders used in assembly were made of stainless steel (S.S.) tube. The engineering & schematic diagram of Stainless Steel Cylinders is shown in Figure 3-2. Each cylinder has an outer diameter of 12.7 mm and a total length of 609 mm. Out of this total length, 570 mm is the active heated length.
T-type thermocouple (copper constantan, welded tip, 1.5 mm outer diameter including fiber glass insulation) was embedded into the cylinder wall from inner surface and adjusted to keep its tip at outer surface of cylinder wall. At diametrically opposite position a thermocouple was also embedded in the same way in order to reduce the perturbation effects of temperature measurement. Similarly additional thermocouples were embedded in the cylinder at six different equidistance axial positions.

A tubular heater of the length of cylinder was installed at the cylindrical axis of the S.S. cylinder which acted as the heat generating source. Its total length is 609 mm and heat generating length is 570 mm. The heater installed has maximum power of 400 Watts, so a controlled power was complementary. The gap between the cylinder wall and the tubular heater was filled with Magnesium Oxide (MgO) as it has high thermal conductivity.

All the thermocouples were calibrated before and after their installation. The electric and thermocouple wires were enclosed in a 6 mm diameter plastic pipe and the silicon sealant was filled at the junctions to avoid leakage of water to the wires.

### 3.2.2 Assembly of S.S. Cylinders

The cylinders were assembled vertically in a 3x3 array of square pitch. The pitch to diameter ratio (P/d) of PARR-II is 1.99. In the experimental set up P/d ratio was adjusted to 2 and the gap between the two successive rods became equal to the diameter of a cylinder as in PARR-II. Two circular (top and bottom) tube sheets, having the holes to fix the tie rods and cylinders, were used to keep the same configuration of cylinders in assembly. Tie rods are solid steel rods to strengthen the assembly and to maintain equidistance between upper and lower tube sheets. Four tie rods were used at the corners of tube sheets. The couplings were used to fix the cylinders in both top and bottom tube sheets. The assembly was enclosed in a 5 inches diameter Perspex cylinder which has inlet slits at the bottom and exit slits at the top. The assembly is immersed in a water tank and aligned vertically in the direction of gravitational force. The cold water enters from the inlet slits and due to the density difference moves up and leaves from the exit slits. The pictures of assembly are shown in Figure 3-4.
3.2.3 Configuration of S.S. Cylinders in Assembly

The nomenclature of the cylinders in the assembly is shown in Figure 3-5 clearly showing the symmetric positions of the cylinders around the central cylinder 11 and top view of engineering diagram. For the measurement of the surface temperatures three cylinders (11, 22 & 33) in the assembly were selected. The remaining cylinders at the symmetric positions were assumed to be at same surface temperatures. T-type thermocouples were installed in these three cylinders only. However, every cylinder has the heating element.

3.2.4 Temperature Measurement

Surface temperatures were measured with thermocouples installed at six different axial positions on each of the three selected cylinders. The thermocouples were also installed at the inlet slit near the bottom tube sheet, the exit slit near the top tube sheet and in the bulk fluid, to measure the inlet, outlet and bulk temperatures of the fluid respectively. Specifications of thermocouples are given in appendices.

3.2.5 Data Acquisition System

Data acquisition system was used to convert the analog signals of thermocouples to digital signals in order to interface with personal computer. T-type miniature thermocouple connectors were used to connect the thermocouple wires to the data acquisition card. The detail of data acquisition and thermocouple connectors are given in appendices.

3.2.6 Variable Voltage Power Supply

Variable voltage power supply was used to set the various power levels for experiments. For constant surface heat flux was adjusted by voltage regulation. The detail of variable voltage power supply (variac) is given in appendices.

3.3 Qualitative Flow Visualization

The cylinders assembly used in these experiments was enclosed in a Perspex cylinder having entry and exit slits at the bottom and top respectively as discussed above. Whole of the assembly is submerged in a large tank full of water. For the
qualitative flow visualization two Perspex cylinders with one closed end were placed adjacent to the assembly on opposite sides of the assembly as shown in Figure 3-6. A light source was inserted in one cylinder while a high speed camera was adjusted inside the other Perspex cylinder to capture the video (40 frames per second) of the flow pattern inside the assembly. A dye and tracer (low density poly ethylene) particles were used to qualitatively visualize the flow patterns. The tracer particles were injected at the bottom and then the particles movement was observed along the surface of cylinders inside the assembly for 0.763 < x/L < 1. The focus of visualization was to observe the flow behavior near the surface of cylinder.

3.4 Heat Transfer Model

Total heat transfer can be calculated as:

\[ Q = Q_{\text{cond}} + Q_{\text{conv}} + Q_{\text{rad}} \]  

(3.2)

where \( Q_{\text{cond}} \) is conduction, \( Q_{\text{conv}} \) is convection and \( Q_{\text{rad}} \) is radiation and \( Q \) is total heat transfer.

3.4.1 Conduction Heat Transfer

Heat transfer by conduction can be calculated by Fourier law:

\[ Q_{\text{cond}} = kA \frac{dT}{dx} \]  

(3.3)

where \( k \) is thermal conductivity of material, \( A \) is the area of heat transfer and \( dT/dx \) is the temperature gradient or slope of temperature profile. In this case, the heat transfer by conduction can take place at three points, as discussed below:

a) Conduction from heat source to the surface of S.S. cylinder:

Negligible losses were observed while calculating heat conducted from heat source (heater embedded at the center of cylinder) to the surface of S.S. cylinder.

b) Conduction to the bottom brass coupling (non heated length):
As the temperature difference at the bottom of assembly is zero, hence no heat is conducted to the bottom brass coupling.

c) Conduction to the top brass coupling (non heated length):

Heat conduction to the top brass coupling is calculated by using the slope of measured temperature profile. Heat transferred from the heated part to this non heated part is only 0.6% of total heat at maximum heat flux.

As around 0.6% of the total heat is transferred by conduction, hence it is neglected.

### 3.4.2 Radiation Heat Transfer

Heat transfer by radiation from the surface of S.S. cylinder can be calculated as:

\[
Q_{rad} = \varepsilon A \delta (T_s^4 - T_b^4)
\]

where \(\varepsilon\) is the emissivity of weathered stainless steel, \(\delta\) is the Stefan Boltzmann constant \((5.66 \times 10^{-8} \text{ W/m}^2\text{K})\), \(T_s\) is the average surface temperature and \(T_b\) is the bulk temperature in Kelvin scale. The heat transfer by radiation was calculated at average temperatures (the whole rod is assumed to be at an average temperature) and found less than 0.65% of total heat transfer, hence neglected.

### 3.4.3 Convection Heat Transfer

It was found that almost 99% of total heat is transferred to the fluid by convection, hence, by neglecting conduction and radiation heat transfer; the equation (3.2) can be simplified as:

\[
Q = Q_{conv} = h A (T_s - T_b)
\]

where \(h\) is the heat transfer coefficient \((\text{W/m}^2\text{K})\).

### 3.4.4 Maximum Experimental Error

The sum of the error values obtained in conduction & radiation heat transfer (calculated above) is the maximum possible error expected in experiments. The
maximum qualitative error of around 1.25% can be expected in a single cylinder due to the above reasons.

### 3.5 Procedure for Solving Convection Problem

The procedure to solve the convection problems is as follows:

1. Specify the fluid and determine its properties
2. Specify the geometry of the problem i.e. the fluid is inside or outside the tube or flow across or along the length of tube etc
3. Specify the problem according to fluid movement and decide that the convection is either Natural or Forced. If there is no force added to move the fluid and movement exists due to heated surface, Natural convection may be presumed. If there is a force added for the movement of fluid then Forced convection can be assumed. If very small force is present with heated surface to establish velocities it may be the combination of Natural and Forced convection.
4. Film temperature can be determined by taking the average of bulk and surface temperatures. The properties of the fluid should be determined using film temperature.
5. Now the flow regime can be determined by calculating dimensionless numbers. Reynolds number can determine the flow situation of forced convection while the product of Grashof and Prandtl number for Natural convection. Geometry of the problem must be considered in this step carefully.
6. Select the appropriate correlation for finding the heat transfer coefficient.
7. Determine convection heat transfer for the problem either Forced or Natural convection using following equation:

\[ Q_{\text{conv}} = hA_s(T_s - T_b) \]  

(3.6)
3.6 Calculation Procedure

The physical properties of fluid have been calculated by the equation [38]

\[ PP = A + BT_f + CT_f^2 + DT_f^3 \]  

(3.7)

where A, B, C & D are thermodynamic constants and are given in Table 3-1 and Table 3-2, \( T_f \) is the absolute film temperature (°K) and PP is the physical property (Cp, \( \rho \), \( \mu \) & k) of the fluid.

Thermal expansion coefficient (\( \beta \)) of water can be calculated as:

\[ \beta = \frac{1}{\rho} \frac{d\rho}{dT} = \frac{1}{\rho} \left( A + 2BT_f + 3CT_f^2 \right) \]  

(3.8)

where A, B and C are given in Table 3-1 and \( \rho \) (density of water) as calculated by equation (3.7).

**Table 3-1: Thermodynamic constants for physical properties of water [3]**

<table>
<thead>
<tr>
<th>Thermodynamic Constants →</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Heat (C( _p )) ( (\text{J} / \text{Kg} \text{ K}) )</td>
<td>5021.28</td>
<td>-5.4571</td>
<td>8.842 x 10(^{-3} )</td>
<td>---</td>
</tr>
<tr>
<td>Density (( \rho )) ( (\text{Kg} / \text{m}^3) )</td>
<td>223.127</td>
<td>6.7678</td>
<td>-1.8538 x 10(^{-2} )</td>
<td>1.522 x 10(^{-5} )</td>
</tr>
<tr>
<td>Viscosity (( \mu )) ( (\text{Kg} / \text{m sec}) )</td>
<td>6.3225 x 10(^{-2} )</td>
<td>5.1485 x 10(^{-4} )</td>
<td>1.415 x 10(^{-6} )</td>
<td>-1.306 x 10(^{-9} )</td>
</tr>
<tr>
<td>Thermal Conductivity (k) ( (\text{W} / \text{m K}) )</td>
<td>-5.084 x 10(^{1} )</td>
<td>6 x 10(^{3} )</td>
<td>-7.565 x 10(^{6} )</td>
<td>---</td>
</tr>
<tr>
<td>Thermal Expansion Coefficient (( \beta )) ( (1 / \text{K}) )</td>
<td>6.7678</td>
<td>-1.8538 x 10(^{-2} )</td>
<td>1.522 x 10(^{-5} )</td>
<td>---</td>
</tr>
</tbody>
</table>
Table 3-2: Thermodynamic constants for physical properties of air [3]

<table>
<thead>
<tr>
<th>Thermodynamic Constants</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Heat (C(_p)) (J/Kg K)</td>
<td>1024.06</td>
<td>-0.175768</td>
<td>3.70976 x 10(^{-4})</td>
</tr>
<tr>
<td>Viscosity ((\mu)) (Kg/m sec)</td>
<td>2.12075 x 10(^{-6})</td>
<td>6.24755 x 10(^{-8})</td>
<td>-2.6162 x 10(^{-11})</td>
</tr>
<tr>
<td>Thermal Conductivity (k) (W/m K)</td>
<td>2.317 x 10(^{-3})</td>
<td>8.7113 x 10(^{-5})</td>
<td>-2.55188 x 10(^{-8})</td>
</tr>
<tr>
<td>Density ((\rho)) =(PM/RT_f) (Kg/m(^3))</td>
<td></td>
<td></td>
<td>P = 101325 N/m(^2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M = 28.966 Kg/Kg mole</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R = 8314.4J/Kg mole (^{\circ})K</td>
</tr>
<tr>
<td>Thermal Expansion Coefficient ((\beta)) (1/K) = 1/T(_f)</td>
<td>-----</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The fluid properties are calculated at mean film temperature (average of surface and bulk temperature), Cengel [80]

\[
T_{fc} = \frac{T_s + T_b}{2}
\]  
(3.9)

The local Nusselt number has been calculated as

\[
Nu_s = \frac{h_sL}{k}
\]  
(3.10)

where

the local heat transfer coefficient has been calculated as
\[ h_x = \frac{q''}{\Delta T_x} \]  

(3.11)

where \( \Delta T_x = T_{x_s} - T_b \) and \( T_{x_s} \) is the local surface temperature

The local modified Grashof & local modified Rayleigh numbers have been calculated as

\[ Gr_x^* = \frac{g \beta_x x q''}{v^2 k} \]  

(3.12)

\[ Ra_x^* = Gr_x^* \cdot Pr \]  

(3.13)

The overall Nusselt number has been calculated as

\[ Nu_L = \frac{\overline{h_x} L}{k} \]  

(3.14)

where average heat transfer coefficient has been calculated as

\[ \overline{h_x} = \frac{\sum_{i=1}^{6} h_{x_i} \Delta x_i}{L} \]  

(3.15)

The average values of surface and mean film temperatures have been calculated as:

\[ \overline{T_{x_s}} = \frac{\sum_{i=1}^{6} T_{x_i} \Delta x_i}{L} \]  

(3.16)

\[ \overline{T_f} = \frac{T_{x_s} + T_b}{2} \]  

(3.17)

The average modified Grashof and Rayleigh numbers have been calculated as
\[
\overline{Gr_L} = \frac{gL^4 \beta q''}{\nu^2 k}
\]

(3.18)

\[
\overline{Ra_L} = \overline{Gr_L}.Pr
\]

(3.19)

All the physical properties (\(C_p, \rho, \beta, \mu\) and \(k\)) of water were evaluated at average film temperature using thermodynamic constants.
Figure 3-1 Experimental Setup for Assembly of Vertical Cylinders (a) Schematic diagram (b) Picture of Actual Experimental Setup
S.S. Cylinder

Heater

Thermocouple

MgO

Coupling
Figure 3-2 Stainless Steel Single Cylinder (a) Engg Drawing of SS Cylinder (not scaled) (b) Schematic Diagram showing the Inter construction (c) Picture of S.S. Cylinder
Figure 3-3 Schematic diagram of Experimental Setup for Single Vertical Cylinder in an Infinite Medium
a). Assembly of SS Cylinders

b). Vertically Hanged Assembly in Water Tank
c). Upper Portion of Assembly

d). Lower Portion of Assembly

Figure 3-4 a, b, c & d Assembly of Vertical Cylinders
Figure 3-5 (a) Configuration of S.S. Cylinders in Assembly (b) Engg Drawing (Top View)
Figure 3-6 Flow Visualization Setup for Assembly of Vertical Cylinders
Chapter 4. Natural Convection from Single Vertical Cylinder

4.1 Introduction

Heat transfer by natural convection phenomenon has many engineering applications to encounter heating and cooling purposes. Heat transfer and fluid flow mechanisms in natural convection are different for dissimilar geometrical orientation with respect to gravity i.e. horizontal, inclined or vertical etc. Natural convection from vertical cylinder has significant curvature effects to differentiate it from other geometrical surfaces. Flat surfaces and horizontal cylinders have been investigated by a number of different researchers. The vertical cylinders have been examined for the natural convection phenomenon by different researchers but a general consensus has not yet been built.

Cebeci [5] categorized the vertical cylinder into two types i.e. short (thick) & long (slender or thin) cylinders. According to his results, short cylinders have boundary layer thickness much thinner than the radius of cylinder. Popiel [2] has provided the criteria for thin(long) vertical cylinder as:

\[
Gr_{L}^{0.25} \left( \frac{d}{L} \right) \leq 11.474 + \left( \frac{48.92}{Pr^{0.5}} \right) - \left( \frac{0.006085}{Pr^{2}} \right)
\]  

Natural convection trends of the short vertical cylinders are similar to that of flat plate and their correlations are applicable without any significant error, however, the long cylinders have considerable curvature effects, hence, further analysis are required to understand this phenomenon. In the present study thin vertical cylinders have been investigated.

The major objective of this research work is to investigate natural convection phenomenon of heat transfer from an assembly of vertical cylinders. Each cylinder has to be verified for its mechanical integrity before being installed in the assembly. Hence experiments were performed on each cylinder. This served two purposes:
i). It verified the integrity of the cylinders
ii). It verified the data obtained for single cylinder studies with that of available literature and developed a general correlation

For this purpose, an electrical heated cylinder (explained in Chapter 3) was hung vertically in a large volume water tank. The experiments were performed at different power levels to generate a uniform heat flux. Thermocouples reading were recorded on a personnel computer using a Data Acquisition System. Convection currents developed on the surface of heated cylinder causing the fluid to flow upward due to the density difference. The surface to ambient fluid temperature difference was obtained axially along the cylinder for each location and dimensionless correlations were developed. Natural convection phenomenon is characterized by a dimensionless combination of buoyancy & viscous forces, known as Grashof number. Rayleigh number is another dimensionless group which has been studied with variation of Nusselt number to characterize the natural convection. At the end a general correlation is suggested and compared with previous experimental studies.

4.2 Results and Discussions

A known heat flux was applied to a single heat generating vertical cylinder in an infinite medium. The temperature at the surface of cylinder increased with time and attained a constant value after some time. An increase in surface temperatures was observed for some time showing the transient behavior. The region, where the temperature becomes constant over a long period is called the steady state. Figure 4-1 shows the steady state surface temperatures history at different axial positions along the cylinder. (a, b, c & d) figures show the surface temperature history at different heat flux values. The surface temperature profiles show similar trends however their magnitudes are proportional to the heat flux. The bulk temperature remained constant throughout the experiment for each power level. The legend of the temperatures lines on these figures is explained as:

<table>
<thead>
<tr>
<th>Legend</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk Temp</td>
<td>fluid bulk temperature in large volume tank</td>
</tr>
<tr>
<td>TC-ii</td>
<td>position of temperature measurement along the cylinder</td>
</tr>
</tbody>
</table>

where
TC  thermocouple

ii position of thermocouple from bottom of cylinder

The surface and bulk temperature difference variations at steady state are shown in Figure 4-2. The temperature difference profiles at different but uniform heat flux values are presented in this figure as a function of axial distances along the cylinder. This figure shows that the surface temperature initially increases along the axial distance up to a certain position and then decreases [61, 65]. The boundary layer development is responsible for this variation in surface temperatures. The thickness of the boundary layer is zero at the bottom of cylinder. The decrease of local heat transfer coefficient occurred due to the development of boundary layer and hence the surface temperature increases along the cylinder axially. Generally, the temperature along the vertical cylinder should increases up to the top, however, a different behavior was observed in these experiments. The surface temperature near the top of cylinder decreases in similar manner as presented by Hussein et al. [61] and Fujii et al [65].

The physical properties of water are a function of temperature. An increase in thermal conductivity due to temperature decrease offers less resistance to heat flow and similarly flow of water faces less resistance due to the decrease in viscosity. Due to these changes in the physical properties of water, the movement of cold fluid towards the rod in radial direction takes place and combines with hot fluid rising in the boundary layer. This behavior should become more and more pronounced as the axial distance increased. As the temperature of the cylinder increases axially thus increasing the fluid rising velocity. This creates a sort of increased suction for the surrounding fluid in the radial direction. This inward radial flow of fluid causes a mixing of cold radial fluid with hot rising fluid. Hence, the local heat transfer in this portion is increased resulting in a decrease in the surface temperature. The recorded surface temperatures are used in the computation of the local heat transfer coefficient at different axial length of the cylinder using following equation.

\[ h_s = \frac{q''}{(T_{ss} - T_b)} \]  \hspace{1cm} (4.2)

Figure 4-3 shows the local heat transfer coefficient profiles at steady state. The surface temperature rapidly rises from bottom as the axial length is increased. This is due to a relatively high local heat transfer coefficients at the lower side, as the heat
transfer coefficient gradually decreases in the region where the boundary layer development takes place. Once the boundary layer is developed, some mixing takes place between the cold fluid in bulk and hot fluid in the boundary layer. As a result the heat transfer is increased and decrease in surface temperature is observed after this position. The temperature drop is due to the mixing of hot rising fluid with the cold fluid entrains from the radial direction and this become more prominent at the thermocouple number five (TC-05) where the surface temperature is maximum. At maximum temperature density difference is higher. Hence maximum possibility of cold fluid entrained here and beyond this region surface temperature decreases and heat transfer coefficient increases.

### 4.2.1 Dimensionless Correlations

The correlations were obtained in terms of Nusselt and Rayleigh numbers. The heat transfer from the surface of vertical cylinder has combined effects of diameter and length [26, 29]. Length is mostly used as characteristic linear dimensions for vertical surfaces [2]. Hence, the cylinder length ($L$) has been taken as the characteristic linear dimension in Nusselt and Grashof numbers calculations. The general correlation for Nusselt and Rayleigh numbers based on length [2] is

$$ Nu_L = A \cdot Ra_L^n $$  \hspace{1cm} (4.3)

The modified Rayleigh numbers ($Ra_L^*$) are used for dimensionless correlations where the uniform heat flux case is under investigation [65]. The same is used in the present study as the experiments were performed on the vertical cylinders having uniform heat flux.

For local values at the surface of cylinder, $x$ was taken as the local characteristic linear dimension. The physical properties of water were calculated at local film temperatures using equation (3.7) & (3.8) and Table 3-1. Similarly the physical properties of air were calculated using Table 3-2.

The local Nusselt and local modified Rayleigh numbers for water were calculated from equation (3.10) and (3.13). The data for $Nu_x$ and $Ra_x^*$ taken at different uniform heat flux values (835—4400 W/m²) is plotted in Figure 4-4. The
dimensionless correlation of local Nusselt and local modified Rayleigh numbers was obtained by regression analysis and given by:

\[ \text{Nu}_x = 0.682 \text{Ra}_x^{0.19} \]  \hspace{1cm} (4.4)

This local dimensionless correlation (4.4) is valid for the range \( 4.71 \times 10^7 \leq \text{Ra}_x \leq 1.91 \times 10^{13} \).

For overall dimensionless correlation, the weighted average heat transfer coefficients \( \overline{h}_L \), surface temperatures \( \overline{T}_{st} \) and film temperatures \( \overline{T}_{fl} \) were calculated using equations (3.15), (3.16) and (3.17) respectively. The physical properties were calculated at the averaged film temperatures \( \overline{T}_{fl} \). The overall Nusselt and modified Rayleigh numbers were calculated by using equations (3.14) and (3.19). Overall Nusselt numbers are plotted against overall modified Rayleigh numbers as shown in Figure 4-5. The dimensionless equation obtained from regression analysis is given as:

\[ \overline{\text{Nu}}_L = 0.893 \overline{\text{Ra}}^*_{L}^{0.19} \]  \hspace{1cm} (4.5)

This overall dimensionless correlation(4.5) is valid for the range \( 3.4 \times 10^{12} \leq \overline{\text{Ra}}^*_{L} \leq 2.41 \times 10^{13} \).

### 4.2.2 Comparison with Previous Studies

The difference of the surface and fluid bulk temperature initially increases up to a certain length and then decreases as shown in Figure 4-2. This behavior is similar to the finding of Fujii et al[65] hence, the trends of the present set of data for surface temperatures is validated.

Nagendra et al. [26] presented his Nusselt number data based on the diameter of cylinder. Hence the present study dimensionless numbers were also calculated on the basis of cylinder diameter \( d \) as dimensional characteristic length and compared with his experimental data. The Figure 4-6 shows that the present study is in good agreement with the previous one.
The local dimensionless numbers ($Nu_x \text{ vs } Ra_x^*$) were also compared with the previously reported data.

Figure 4-7 shows the comparison of present study for both air and water with the data of previous numerical study of Chughtai et al [3], experimental studies of Jarall et al. [29], Kimura et al. [69] and Fujii et al [65]. The figure shows the scattered data. It indicates that some analysis is required so that the data should be converged to give a better presentation. In order to investigate the reason of this scattering, different fluids are compared separately as shown in Figure 4-8 and Figure 4-9. The scattered data is due to the change in various parameters and fluid they used. All the researchers have studied different cylinder length and diameter under different conditions. Mostly air and water are used as cooling medium, however, Fujii et al [65] reported the data for some oils also.

Figure 4-8 shows comparison of present study air data with Jarall et al [29]. Present data with air as coolant medium is very close to Jarall data at L/d 38.8 hence validated. Jarall studied for different L/d ratios and found three different but parallel trends. This figure shows clear effects of L/d ratios. So it has been concluded that there is an effect of geometric parameter which may cause the scattering.

Figure 4-9 shows comparison of different studies with water. The effect of geometric parameters is also observed for water. The figure shows that the data of Kimura et al [69] for different L/d ratios shows different Nusselt numbers for the same Rayleigh numbers. Hence the effect of geometric parameters is confirmed. Hence the L/d ratios effects should be considered for better presentation of data.

Length, diameter and fluid used as coolant are the major contributors for this scattering of data. Now the data of different fluids should be compared to find out the reason of scattering.

The data of two different fluids (spindle oil & mobiltherm oil) have also been studied by Fujii et al [65] as shown and compared in Figure 4-9. The whole data of these two fluids can be observed within the data of water. This observation shows that all the boundary layer of same phase fluids have similar thermodynamic behavior.
The present data for air and water is compared in Figure 4-10. This figure shows a different behavior. For the same Rayleigh number air shows the higher Nusselt numbers than water. This shows the clear dependence of fluid as well as the geometric parameters. Hence it has been concluded that the behavior of different fluids is different and it should be minimized.

From above discussion, it is now very clear that the different fluids and the different geometric parameters of cylinder are the major contributor for scattering the data. In order to minimize this scattering, the dimensionless term was required to multiply with local Nusselt and modified Rayleigh numbers. To minimize the effects of geometric parameters it is suggested that the local dimensionless number should be updated by with the local length to diameter of cylinder. Behavior of different fluids can be incorporated by taking the viscosity ratio \( \frac{\mu}{\mu_s} \) which is discussed in subsequent section and presented in Figure 4-11.

### 4.2.3 General Dimensionless Correlation

As Rayleigh number is the product of Prandtl & Grashof numbers so the correction factor of Prandtl number is not much effective. It is the viscosity of the fluid which has an additional contribution to converge the data. Hence the correction factor of viscosity ratio is used to develop a general correlation. In analyzing the scattering of data in

Figure 4-7 to Figure 4-10 it is found that the following parameters also affect the local Nusselt number and modified Rayleigh numbers.

i) Ratio of viscosity of fluid in bulk to that of at surface \( \frac{\mu}{\mu_s} \)

ii) Diameter to length ratio \( \frac{x}{d} \)

For measurement of viscosity, the surface temperatures should be known but the researchers have not provided the surface temperature profiles. Since the surface temperature data is not available, therefore the comparison of all the studies mentioned in
Figure 4-7 is not possible. For this purpose only two different studies Fujii & Jarall data was available. The suggested parameters were calculated for both of these studies and present study of air as well as water and compared in Figure 4-11(a). The data has been converged by using the above parameters which can be observed by comparing Figure 4-7 and Figure 4-11(a). Hence a new dimensionless correlation, based on the data obtained from the present studies, has been developed.

L/d ratio effects as well as the effect of different fluids are taken into account as shown in Figure 4-11(a) and the following general correlation has been developed for all fluids and geometric parameters.

\[ Nu_x = 0.45 Ra_x^{0.22} \left( \frac{d}{x} \right)^{0.073} \left( \frac{\mu_b}{\mu_{sc}} \right)^{-1} \]  
\( (4.6) \)

The error band of this correlation is ±10% as shown in Figure 4-11(b) and it is valid for the range \( 4.71 \times 10^7 \leq Ra_x^* \leq 1.91 \times 10^{13} \).
Figures:

For Figure 4-1 TC means thermocouple and numbering is from bottom to top of vertical cylinder.

(a) Surface Temperatures at 835 W/m²

(b) Surface Temperatures at 1695 W/m²
(c) Surface Temperatures at 2770 W/m²

(d) Surface Temperatures at 4400 W/m²

Figure 4-1(a, b, c & d) Surface temperature variations w.r. to time at different positions along a vertical cylinder at specific heat flux using water as coolant medium
Figure 4-2 Axial temperature difference profiles at different uniform heat flux for a single cylinder using water as coolant medium.

Figure 4-3 Axial local heat transfer coefficient profile at different uniform heat flux for a single cylinder using water as coolant medium.
Figure 4-4 Present study local Nusselt number for a single vertical cylinder in infinite medium using water as coolant medium

Figure 4-5 Present Study overall Nusselt number for a single vertical cylinder in an infinite medium using water as coolant medium
Figure 4-6 Comparison of present study with Nagendra et al [26] for a single vertical cylinder

Figure 4-7 Comparison of local Nusselt number with previous studies for a single vertical cylinder
Figure 4-8 Comparison of local Nusselt number using air as coolant medium for a single vertical cylinder

Figure 4-9 Comparison of local Nusselt number using water as a coolant medium for a single vertical cylinder
Figure 4-10 Present studies comparison of local Nusselt number using air and water as coolant medium for a single vertical cylinder in an infinite medium.
Figure 4-11(a) Correlation for local Nusselt number with local Rayleigh number for a single cylinder (b) Prediction of four experimental data sets of different researchers within ±10% error band
Chapter 5. Natural Convection from an Assembly of Vertical Cylinders

5.1 Introduction

Natural convection phenomenon of heat transfer from a single hot body is quite different from an assembly of hot bodies in an infinite medium. This phenomenon of natural convection through an assembly of hot bodies plays a vital role in various natural and engineering situations of heating and cooling processes.

Natural convection is the only mode of heat transfer through the reactor core of MNSR type reactors during operation. The prediction of the outer surface temperature of fuel rods of the reactor core along its length and correspondingly the fluid exit bulk temperature are very important from reactor safety point of view. One of the major motivations of this study was to experimentally obtain such data which would then be transformed into a correlation. Such a correlation could be used to predict the surface temperature of fuel rods as well as fluid bulk exit temperature. The knowledge of surface temperature for a particular set of operating conditions could then be used to develop a complete temperature profile within the fuel rods of such research reactors. To achieve this goal of prediction of surface temperatures an exit bulk fluid temperature an experimental assembly was designed which has a dynamic similarity with a typical MNSR. On such reactor is available in the premises of PIEAS called PARR-II which is the best example for assembly of vertical cylinders.

The experimental setup was designed on the basis of L/d & P/d ratios and overall Grashof numbers of research reactor PARR-II. However in PARR-II the fluid temperatures are only measured at the inlet and outlet of the reactor core. There is no provision to measure the temperatures along the surface of fuel rods. It is therefore necessary to accurately predict the surface temperatures from the safety of fuel and analysis point of view. A computational model for this purpose was presented by Chughtai et al [3] but its experimental verification was required. No such experimental study is available in literature.
Natural convection from single vertical cylinder has been investigated for the verification of individual heating rods before installing them in an assembly as described in Chapter 4. An assembly of 3 x 3 square array of vertical cylinders was prepared. Due to symmetric configuration, only three cylinders were selected to measure the surface temperatures at different axial positions while the surface temperature profile of remaining cylinders was assumed to be same at each symmetric position. Experiments were carried out to measure the surface temperature variations at different uniform heat flux values. The heat transfer coefficients were calculated and fluid bulk outlet temperatures from the assembly were measured and their correlations were presented. The dimensionless correlations for Nusselt numbers were also found out to characterize the natural convection at steady state. The correlations obtained were compared with the available data for assemblies & were also applied to the available computational model of PARR-II. The correlations obtained were also applied to the actual PARR-II assembly to predict its surface temperature and fluid bulk outlet temperature. The bulk outlet temperatures obtained are close to the experimental exit temperatures from assembly of PARR-II at different power levels. The results of present study are found to be in good agreement with the available data.

5.2 Results and Discussions

The natural convection heat transfer data are presented here for an assembly of vertical cylinders. The data were obtained experimentally by performing the experiments at various input power levels applied to the cylinders varying from 20 to 100 W/cylinder corresponding to a heat flux range of 880 to 4470 W/m$^2$. A uniform heat flux is used to measure the surface temperatures for a single set of data. The experiment for a particular set of conditions was performed twice to find a clear and actual representation.

In these experiments the heated length of the cylinder is taken as the characteristic length in the definition of overall Nusselt, average heat transfer coefficient & modified Grashof numbers. The average modified Grashof number range covered in these experiments is $3.59 \times 10^{10} < \mbox{Gr}_L < 3.95 \times 10^{11}$. The surface temperatures were measured for three cylinders at locations 11, 22 and 33. The surface temperature profiles for the remaining cylinders at locations 21, 23 and 24 would be
similar to the cylinder at location 22 and cylinders at locations 31, 32 and 34, similar to the location 33 due to symmetry respectively.

The variations in surface and water inlet & outlet temperatures with time are shown in Figure 5-1 to Figure 5-9 at different heat flux values. Once the power was switched on, the surface temperature increased and this increasing behavior continued for about 10 minutes. Hence one can call this time as transition time. Steady state surface temperature values were observed at each position on cylinder after almost 10 minutes. The steady state values after 700 seconds (cylinders were switched on after 1st 100 second) are shown in Figure 5-1 to Figure 5-9. The legend of the temperatures lines on these figures is explained as:

Inlet fluid bulk inlet temperature to the assembly  
Outlet fluid bulk outlet temperature from the assembly  
TC-ij axial position of a thermocouple measuring the surface temperature for a specific cylinder

where

TC thermocouple  
i cylinder number (1 for cylinder 11, 2 for 22 & 3 for 33 respectively)  
j position of thermocouple from bottom of cylinder

Figure 5-1 to Figure 5-9 elaborate the behavior of fluid at the surface of cylinders. The surface temperatures recorded at steady state behave as straight smooth line at some thermocouples where as they fluctuate around a mean point at the other locations. The smooth line temperature behavior show a laminar boundary layer as the flow is not disturbed at these points. The fluctuations of temperatures around mean position shows that the boundary layer is now turbulent. This behavior is however different from the single cylinder at same power levels. Therefore it can be concluded that the surrounding heated cylinders extend an effect on the thermal boundary layer developed at the inner cylinder. This conclusion can be verified by observing the temperature changes at central cylinder. For this purpose, a heat flux 4470 W/m² was provided to all eight cylinders surrounding the central cylinder while the central cylinder was kept at zero heat flux. The surface temperatures for this case are shown in Figure 5-10. The surface temperature of the cylinder 11 shows the increasing trend. However this increase is smaller in lower half of the cylinder while rapid increase is
observed in upper half of the cylinder. This rapid increase in upper half is due to the turbulence and the boundary layers effects of surrounding cylinders. This shows that the lower half of the cylinder is slightly affected while the upper half receives the significant thermal disturbances from surrounding cylinders. This effect was observed at maximum heat flux of present experimental ranges however slightly lower effects will be seen at lower heat flux.

In analyzing the above conclusion if the gaps between the cylinders are reduced, the boundary layers effects may be started from bottom point of cylinder.

The behavior of temperature fluctuation on the surface of cylinders are qualitatively examined from Figure 5-1 to Figure 5-14 to identify the physical location along the axial direction where turbulence in boundary layer in an assembly is initiated. This information is not available in the literature. If the location where boundary layer became turbulent is identified then one can calculate the corresponding Grashof and Rayleigh numbers. Hence the limiting values of these dimensionless numbers can be found.

The behavior of surface temperatures in Figure 5-1 is smooth at TC-11, 12, 13 & 14 while fluctuations are present at TC-15 & 16. Same fluctuating behavior is observed at TC-15 & 16 in Figure 5-2 to Figure 5-9. At these two points the boundary layer is turbulent throughout the experimental ranges of present study. TC-11, 12, 13 & 14 are the positions where the smooth and fluctuating lines are observed at different flux are discussed one by one in flowing paragraphs.

Figure 5-11(a) shows the comparison of surface temperatures at different heat flux at thermocouple position TC-14 on the cylinder 11. Surface temperature has a smooth line at 880 W/m² and the slight fluctuations are started at 1330 W/m² but the boundary layer is neither laminar not completely turbulent. Hence, it may be called the transition boundary layer. As the significant fluctuations in surface temperature have been observed at 1800 W/m², therefore it can be deduced that the boundary layer has become completely turbulent here. The steepness of temperature profile slope between 1330 to 2220 W/m² shown in Figure 5-12 at TC-14 (X = 0.335 m) also verify the turbulent behavior at this thermocouple position.
Figure 5-11(b), (c) & d shows the comparison of surface temperatures at different heat flux at thermocouple position TC-13, TC-12 & TC-11 on the cylinder 11 respectively. Surface temperature has a smooth line at 880 W/m². The slight transitions are observed at 1330, 1800 & 2220 W/m² but the boundary layer has not yet become completely turbulent here. The significant fluctuation in surface temperature has been observed at 2640 W/m², therefore the boundary layer has become completely turbulent here. The steepness of temperature profile slope between 2220 to 3100 W/m² shown in Figure 5-12 at TC-13, TC-12 & TC-11 (x = 0.335, 0.235 & 0.135 m) also verify the turbulent behavior at these thermocouple position.

On the basis of above discussion the Grashof number ranges for initiation of turbulence at different thermocouple positions on cylinder are given in Table 5-1.

<table>
<thead>
<tr>
<th>Gr*L</th>
<th>Ra*L</th>
<th>q&quot; (W/m²)</th>
<th>TC Position</th>
<th>Gr*x</th>
<th>Gr<em>d</em></th>
<th>T*x</th>
<th>Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.59E+10</td>
<td>2.74E+11</td>
<td>880</td>
<td>11</td>
<td>2.08E+06</td>
<td>3.61E+04</td>
<td>Smooth</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>4.92E+08</td>
<td>3.85E+04</td>
<td>Smooth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>4.67E+09</td>
<td>3.98E+04</td>
<td>Smooth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>2.00E+10</td>
<td>4.12E+04</td>
<td>Smooth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>6.17E+10</td>
<td>4.48E+04</td>
<td>Turbulent</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1.39E+11</td>
<td>4.41E+04</td>
<td>Turbulent</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.78E+10</td>
<td>4.37E+11</td>
<td>1330</td>
<td>11</td>
<td>3.23E+06</td>
<td>5.59E+04</td>
<td>Transition</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>7.64E+08</td>
<td>5.99E+04</td>
<td>Transition</td>
<td></td>
<td></td>
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</tr>
<tr>
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<td>3.23E+10</td>
<td>6.67E+04</td>
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</tr>
<tr>
<td>15</td>
<td>9.97E+10</td>
<td>7.24E+04</td>
<td>Turbulent</td>
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<td></td>
</tr>
<tr>
<td>16</td>
<td>2.23E+11</td>
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<td>8.96E+10</td>
<td>6.56E+11</td>
<td>1800</td>
<td>11</td>
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<td>12</td>
<td>1.18E+09</td>
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<tr>
<td>13</td>
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</tr>
<tr>
<td>15</td>
<td>1.54E+11</td>
<td>1.12E+05</td>
<td>Turbulent</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>3.47E+11</td>
<td>1.10E+05</td>
<td>Turbulent</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.31E+11</td>
<td>9.17E+11</td>
<td>2220</td>
<td>11</td>
<td>7.10E+06</td>
<td>1.23E+05</td>
<td>Transition</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.72E+09</td>
<td>1.35E+05</td>
<td>Transition</td>
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<tr>
<td>13</td>
<td>1.68E+10</td>
<td>1.44E+05</td>
<td>Transition</td>
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</tr>
<tr>
<td>14</td>
<td>7.40E+10</td>
<td>1.53E+05</td>
<td>Turbulent</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>15</td>
<td>2.28E+11</td>
<td>1.66E+05</td>
<td>Turbulent</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>16</td>
<td>5.00E+11</td>
<td>1.59E+05</td>
<td>Turbulent</td>
<td></td>
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</tr>
<tr>
<td>1.72E+11</td>
<td>1.18E+12</td>
<td>2640</td>
<td>11</td>
<td>9.11E+06</td>
<td>1.58E+05</td>
<td>Turbulent</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>2.23E+09</td>
<td>1.75E+05</td>
<td>Turbulent</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>13</td>
<td>2.20E+10</td>
<td>1.88E+05</td>
<td>Turbulent</td>
<td></td>
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</tr>
<tr>
<td>14</td>
<td>9.87E+10</td>
<td>2.04E+05</td>
<td>Turbulent</td>
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<td></td>
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<tr>
<td>15</td>
<td>3.03E+11</td>
<td>2.20E+05</td>
<td>Turbulent</td>
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<tr>
<td>16</td>
<td>6.50E+11</td>
<td>2.06E+05</td>
<td>Turbulent</td>
<td></td>
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</tbody>
</table>
From Table 5-1 the range of local modified Grashof number for turbulence can be defined. If we take the local length \((x)\) of cylinder as characteristic length in the definition of Grashof number, there is a different range for each position to define the turbulence. However on the basis of diameter of cylinder \((d)\) turbulence criteria can be clearly defined as:

TC-14 turbulence range is \(1.02 \times 10^5 < Gr_d^* < 1.53 \times 10^5\)

TC-13 turbulence range is \(1.44 \times 10^5 < Gr_d^* < 1.88 \times 10^5\)

TC-12 turbulence range is \(1.35 \times 10^5 < Gr_d^* < 1.75 \times 10^5\)

TC-11 turbulence range is \(1.58 \times 10^5 < Gr_d^* < 1.85 \times 10^5\)

Hence, it can be concluded that the boundary layer become completely turbulent around \(Gr_d^* > 1.53 \times 10^5\). For the overall assembly, Figure 5-11 & Table 5-1 shows that the whole boundary layer along the cylinder will become turbulent around \(Gr_L^* > 1.31 \times 10^{11}\).

The difference between the surface and inlet temperatures for the vertical cylinders in an assembly at steady state are shown in Figure 5-12, Figure 5-13 and Figure 5-14. The temperature difference profile of central cylinder 11 at different heat flux values is shown in Figure 5-12. The temperature difference profile of cylinders at positions 21, 22, 23 and 24 are similar due to symmetry and shown in Figure 5-13. The temperature difference profile of cylinders at positions 31, 32, 33 and 34 are similar due to symmetry and shown in Figure 5-14. These figures show that the temperature difference increases along the axial length and after a certain length it decreases. Correspondingly the heat transfer coefficient is higher and decreases. This decrease in heat transfer coefficient is due to the thermal boundary layer development and hence results in an increase in the surface temperatures. The increase in heat transfer is observed at the last point shown in Figure 5-15. This increase is due to the extra turbulence and some back mixing of cold fluid at the top. As the heat transfer is increased at this point the decrease in temperature is observed. The fluid coming from bottom, attaining some velocity due to density difference in the boundary layer, some mixing takes place at the top due to the entrainment of relatively cold fluid from sides and as a result there may be an increase in turbulence in boundary layer in this region.
The temperature difference profiles for three selected cylinders are similar however the difference is observed in magnitude only. The comparison of temperature difference for three cylinders 11, 22 and 33 at different heat flux values (2220 W/m$^2$, 3100 W/m$^2$ and 4470 W/m$^2$) are shown in Figure 5-16. The figure shows that the temperature differences are almost same at lower half of the cylinder and at the top where the temperatures decrease, justifying our conclusion that some degree of extra turbulence and back mixing is present at the top and bottom regions while the variation is observed along the length with maximum variation around 0.435 m (76% of length) from bottom the of the cylinder. This variation increases as the heat flux increases which can be observed at 0.435 m from Figure 5-16. This variation shows that the temperature difference values are higher for cylinder 33 than cylinder 22 and similarly it is higher for cylinder 22 than cylinder 11.

The relatively low temperature is observed for the cylinder 11 because it is surrounded by eight heated cylinders. The effect of these eight heated cylinders results in an increased turbulence in the area surrounding the cylinder 11 thus inducing increased velocity at the surface of cylinder 11 as compared to the surface of cylinder 22. This has also been observed in the qualitative flow visualization. Similarly at the surface of cylinder 33, the effects of surrounding cylinders are lesser than cylinder 22, hence resulting in higher temperatures.

The trends of the temperature difference distribution are same as that for single cylinder, however the magnitudes are different. The surface temperatures of the cylinder increases along the height and after attaining a maximum value it decreases due to the mixing of hot and cold fluid as described above and also in the flow visualization.

From the above discussion it can be concluded that the temperatures of cylinders at different positions in an assembly are different. Keyhani et al [6] have not seen this effect in case of water. They reported their data for lower heat flux values of 290, 420 & 648 W/m$^2$ which are much smaller than the heat flux used in the experiments of present study. Keyhani et al [6] investigated the 3x3 assembly of P/d = 3 with water and the least count of temperature measurement devices was 0.2°C. Moreover they affixed the thermocouples externally at the surface of cylinders which should have disturbed the boundary layer development to some extent at the surface.
of the cylinder. All above reasons combined together point out to the reason why they did not observe any significant temperature difference at different locations in case of water. However, they presented different temperature profiles for different position of cylinders in an assembly using air as coolant medium. The surface temperatures are increased due to low heat transfer coefficients of air. For the larger range of surface temperatures they got some difference at different positions of cylinders in assembly by using air.

Keyhani et al [6] showed the higher temperatures at central cylinder in an assembly using Helium and Air as a coolant medium. However in present study the surface temperatures at central cylinder are lower than its surrounding cylinders. In flow visualization, the qualitative velocity at central cylinder was observed higher than its surrounding cylinders showing the higher heat transfer at central cylinder. Hence the surface temperatures would be smaller at central cylinder as all cylinders are at same heat flux.

5.3 Flow Visualization:

The cylinders assembly used in these experiments was enclosed in a Perspex cylinder having entry and exit slits at the bottom and top respectively. Whole of the assembly is submerged in a large tank full of water. For the qualitative flow visualization two Perspex cylinders with one closed end were placed adjacent to the assembly on opposite sides of the assembly. A light source was inserted in one cylinder while a high speed camera was adjusted inside the other Perspex cylinder to capture the video (40 frames per second) of the flow pattern inside the assembly. A dye and tracer (low density poly ethylene) particles were used to qualitatively visualize the flow patterns. The tracer particles were injected at the bottom and then the particles movement was observed along the surface of cylinders inside the assembly for $0.763 < x/L < 1$. The focus of visualization was to observe the flow behavior near the surface of cylinder and to find out the reason for decrease in temperature in this region.

For different frames the distance traveled by tracer particle was calculated at 4470 W/m². The average linear velocity of the particles near the surface of central cylinder 11 for $0.763 < x/L < 1$ was around 1.25 cm/sec. The movement of particles at
the surface of cylinder 11 was observed to be faster than at the surface of cylinder 22 and similarly particles move faster at the surface of cylinder 22 than at the surface of cylinder 33. The tracer particles moved towards the upper tube sheet and got stuck there due to their irregular shapes. Moreover the exit slit was 25 mm below the upper tube sheet due to similarity with PARR-II. The dye was injected inside the exit slit which traveled and dispersed outside the exit slit. However, a very small downward dispersion of dye was observed near the inside wall of outer Perspex cylinder. The hot fluid moved as a layer to the upper tube sheet and then returned back to exit slit with small amount of liquid moved down as per observance of dye dispersion. Hence the mixing with cold fluid occurred in this region. Due to entrainment the cold fluid moved towards the boundary layer and in the result extra turbulence was created at the top of the cylinder which explains the decrease in temperature in the upper area. The boundary layer was observed at the surface of S.S. cylinder which moves in the upward direction with a wavy pattern. Four frames are provided in Figure 5-17 to illustrate the turbulent boundary layer phenomena. White portion in the figure shows the liquid and the black shows of cylinders.

Keyhani et al [6] visualized the flow inside the assembly and observed a low speed downward flow between the cylinders. In flow visualization of present study no such flow was observed between the cylinders however some flow was observed at outer Perspex cylinder inside wall. The only reason is that the pitch to diameter ratio of Keyhani assembly is 3 which is greater than the present study assembly.

There was an interaction present between the boundary layers of adjacent cylinders in the present study of flow visualization due to which the fluid inside the assembly was moving faster in only upward direction. This effect was not observed by Keyhani et al [6].

There was an extra mixing observed at the upper region and its effects travelled to the central rod by entrainment due to small assembly (9 cylinders). However in assemblies having large number of vertical heated rods such as PARR-II (having 344 fuel pins), this effect would be present in the outer one or two rings of vertical cylinders, thus the fluid around the inner vertical cylinders would not be affected leading to a higher temperature. The temperature drop at the end of cylinder observed in the current study is due to this mixing; however, the surface temperatures
of our assembly would be affected to a lesser extent if the number of vertical heated cylinders in the assembly increases.

### 5.4 Dimensionless Correlations:

The surface temperatures at local length were used to calculate the local heat transfer coefficient \( (h_x) \), local Nusselt number \( (Nu_x) \), local modified Grashof number \( (Gr^*_x) \) and local modified Rayleigh number \( (Ra^*_x) \). The data for local Nusselt number and local modified Rayleigh number for cylinders 11, 22 and 33 in an assembly are shown in Figure 5-18. The figure shows that the local Nusselt number is a function of local modified Rayleigh number and the correlations for each cylinder obtained are as follows. These correlations are valid for the local modified Rayleigh number range \( 1.65 \times 10^7 \leq Ra^*_x \leq 9.5 \times 10^{12} \).

- **Cylinder 11** \( Nu_x = 2.056Ra^*_x^{0.16} \) (5.1)
- **Cylinder 22** \( Nu_x = 1.968Ra^*_x^{0.16} \) (5.2)
- **Cylinder 33** \( Nu_x = 1.874Ra^*_x^{0.16} \) (5.3)

Figure 5-19 shows the relation of overall Nusselt number \( (Nu_L) \) and average modified Rayleigh number \( (Ra^*_L) \) for a single as well as for an assembly of vertical cylinders. The increase in heat transfer by natural convection is observed at the central cylinder 11 having higher Nusselt numbers as compared with cylinder 22 and the cylinder 22 has higher as compared with cylinder 33. The correlations for three cylinders are as follows. These correlations are valid for average modified Rayleigh number range \( 1.28 \times 10^{12} \leq Ra^*_L \leq 1.18 \times 10^{13} \).

- **Cylinder 11** \( Nu_L = 0.325Ra^*_L^{0.24} \) (5.4)
- **Cylinder 22** \( Nu_L = 0.230Ra^*_L^{0.25} \) (5.5)
- **Cylinder 33** \( Nu_L = 0.200Ra^*_L^{0.25} \) (5.6)
The cylinders at symmetric positions as mentioned in previous paragraphs have the same respective correlations. It can be clearly observed from Figure 5-19 that the overall Nusselt number for the assembly of vertical cylinders is about 30% higher than that of the single cylinder in an infinite medium. The ratio of Nusselt number for assembly to single cylinder is well above unity which is due to the so-called chimney effect.

5.5 Comparison with Previous Experimental Studies:

The two main objectives of this study were (1) to predict the outer surface temperatures of fuel rods of a typical MNSR (2) to predict the exit fluid bulk temperature. Both these temperatures are a function of the fluid properties, fluid inlet temperature and heat flux.

A review of the literature shows that very limited experimental data is available for such a configuration. Moreover for the operational MNSR only the inlet and exit fluid bulk temperature are measured and reported against power. No information regarding the outer surface temperature of the fuel rods is available.

Hence to achieve the objective a comparison of the current experimental study with the previous studies available in the literature was carried out. This is done by following the procedure given below.

1. Comparison of the local / average Nusselt number with the local/average Grashof / Rayleigh numbers.

2. Suggesting the true representation of Nusselt as a function of Grashof and Prandtl numbers.

3. Propose a procedure to predict the surface temperature as a function of fluid properties, heat flux and inlet temperature.

4. Propose a correlation to predict the bulk liquid exit temperature as a function of heat flux, fluid properties and fluid inlet bulk temperature.

5. Compare the predicted bulk exit temperature for the geometry & operating conditions of MNSR and compare it with the actual data reported and with the computational prediction of Chughtai et al[3].
5.5.1 Comparison of *Nu* vs *Gr* / *Ra* Numbers

Natural convection from an assembly of vertical cylinders has been studied by a few researchers. The results have been presented in different ways in a few studies available in the literature. Following basis have been used as the characteristic length by different researchers:

i)  *d* (diameter of the cylinder)  

ii) *de* (equivalent diameter)  

iii) *ℓ* (annular gap)  

iv) *x* (length of the cylinder)

The present study data have been compared with using all these characteristic lengths to obtain some logical comparison with the published data can be made. Efforts have also been made to specify which characteristic length should give a true representation of the data.

Figure 5-20 shows the comparison of overall Nusselt number on the basis of cylinder diameter. The present study data is well above the ranges studied by Haldar et al & Keyhani et al [6]. It has been observed from figure that the Nusselt number initial increases rapidly and then its slope is decreased due to turbulence.

Figure 5-21 and Figure 5-22 shows the comparison of Nusselt number on the basis of equivalent diameter. Haldar et al [7] correlated *Nu* for *Gr* *de* up to 10^6 and if his data is extrapolated to the present study ranges it will be in reasonable agreement. Isahai et al [8] presented the local Nusselt number with *Gr* *de*(*de/x*) and showed that the rate of increase of Nusselt number decreases for *Gr* *de*(*de/x*) > 3x10^6 while in the present study no such effect was observed. The Nusselt number followed the previous slope and increases.

Figure 5-23 shows the comparison with Isahai et al [8] for overall Nusselt numbers on the basis of cylinder length. They concluded that the average Nusselt number has some constant values at *Gr* *de*(*de/L*). However no such trend was observed in the present study which shows an increasing trend. They placed thermocouples in only one cylinder and used air as a coolant. They assumed that the effect of surrounding cylinders is similar. However they did not show any verification.
of this assumption. In the present study distinct profiles for different positions of cylinder in the assembly were observed as discussed & presented earlier. This observation of distinct profile also supported by Keyhani et al [6]. The present study ranges for annular characteristic length are well above the ranges of Keyhani et al [6] and if their data is extrapolated then the present study is in good agreement as shown in Figure 5-24.

Figure 5-25 shows the local Nusselt number comparison of present data with the two numerical studies and the data calculated from limited available data set for Keyhani et al [6].

5.5.2 A Better Representation of Data

On analyzing the above comparison it has been suggested that the active length or height of cylinder is the true characteristic length to present the data for assembly. It provides maximum information for cylinders. From the application point of view, axial information is more useful than the radial which support the suggestion to select length as characteristic length.

For comparison no experimental data was found in terms of length of cylinder. Surface temperatures are also not available in the literature except one data set for air found in the Keyhani study. Temperature distribution for air in 3 x 3 cylinder assembly of Keyhani et al [6] has been evaluated for central cylinder and local dimensionless numbers are calculated as per the procedure adopted in the present study. Now their experimental data can be compared with the present study data for the central cylinder.

In the comparison of the local Nusselt number with the local modified Rayleigh number as shown in Figure 5-25 (a). The figure shows a good agreement with other data but if we predict the data of other studies using present experimental data, the scattering is observed and the error band is around ±24% which is shown in Figure 5-25 (b). In order to find out the reason of this scattering, the Prandtl number effects were analyzed separately. It was found that the modified Rayleigh number (in a combined form of modified Grashof and Prandtl number) may not be a true representation. However the Grashof and Prandtl numbers have their separate effects. The Prandtl number effect was also observed by Keyhani et al [6] at different
operating conditions. Hence for assemblies the Nusselt number should be represented as a function of Grashof and Prandtl number separately.

\[ Nu = A(Gr^m \text{ Pr}^n) \]  (5.7)

where A, m & n are constants

On comparison of the two experimental studies as shown in Figure 5-26 (a) following correlation for local Nusselt number as a function of local modified Grashof & Prandtl has been suggested.

\[ Nu_x = 1.92Gr_x^{0.168} \text{ Pr}^{0.11} \]  (5.8)

This correlation has a 98% fit with experimental data and valid for the local modified Grashof number range 2.08 x 10^6 \leq Gr_x \leq 1.48 x 10^{12}. The error band of experimental and predicted Nusselt number is now reduced to about \( \pm 10\% \) as shown in Figure 5-26 (b). The data of Chughtai et al [3] computational study is also predicted within error band of \( \pm 15\% \) but due to some constraints in their model.

### 5.6 Proposes Solution of the Main Problem

Major applications of natural convection from vertical cylinders assembly are heat exchanger, spent fuel storage and operational nuclear reactors. Examples of naturally cooled operational reactors are Multi-Application Small Light Water Reactor (MASLWR) and Miniature Neutron Source Reactor (MNSR) etc. The experimental set up has been designed on the basis of operational MNSR reactor, PARR-II. From thermal hydraulics point of view, the temperatures are only measured at the inlet and outlet of core. No information is known about the surface temperature of the fuel rods. Hence, it is necessary to provide the surface temperatures profile from reactor safety and analysis point of view which is given below.

PARR-II has 344 fuel pins fixed in a hexagonal configuration (detailed information is given in appendices). A typical fuel pin in reactor core is surrounded by six fuel pins, except the last ring of the reactor core. The heat effects on typical fuel pin within the assembly except the outer ring fuel pins receive effect from surrounding fuel pins. These effects enhance the heat transfer at the inner fuel pin by increasing the fluid velocity near the surface of fuel pin. All the fuel pins have same behavior due to surrounded fuel pins except the last ring. In present study it has been
observed that the temperatures of surrounding position cylinders are slightly higher, however, in the case of PARR-II this effect is minimized as large number of fuel pins are in the inner circles and the contribution of the outer ring fuel pins is thus minimized. Moreover the gap between the beryllium reflector & the outer ring of fuel pins is small. Hence it can be safely assumed that there would be no significant difference in the temperature of the outer ring and central fuel pin. Therefore all the fuel pins in reactor should show the similar temperature profile. Using this concept, only central cylinder profiles are used to predict the surface temperatures of fuel pins and outlet temperatures of PARR-II assembly.

5.6.1 Local Surface Temperature ($T_{sx}$)

Axial surface temperatures of cylinder in an assembly depend on the inlet temperature, heat flux and the change in fluid properties axially along the cylinder. Heat flux and the inlet temperature are the known parameters while the fluid properties are required to be evaluated. The physical properties of the fluid are calculated at local film temperatures but the local surface temperatures are required to be evaluated. Hence it is necessary to predict a relation to evaluate the combined effect of change in properties as a function of known parameters i.e. heat flux ($q''$), local length ($x$) and inlet temperature ($T_{in}$).

As we have correlated local Nusselt number as a function of local Grashof and Prandtl numbers. By simplifying the equation (5.8) local heat transfer will be:

$$h_x = 2.818 \left( \frac{q^0.168}{x^0.328} \right) \left( \frac{k^0.722 C_p^0.11 \rho^0.336 \beta^0.168}{\mu^0.226} \right)$$ (5.9)

The physical properties multiplying on right hand side of equation (5.9) are named as local property index ($Z_{sx}$) and are given as:

$$Z_{sx} = \left( \frac{k^0.722 C_p^0.11 \rho^0.336 \beta^0.168}{\mu^0.226} \right)$$ (5.10)

The present experimental data set is used for the prediction of local property index. The surface temperatures are varying along the cylinder axially due to the changes in the physical properties, heat flux, inlet temperature and distance from bottom of cylinder. Hence the property index $Z_{sx}$ should be correlated in terms of heat
flux, inlet temperature and distance from bottom of cylinder. By regression analysis of
the experimental data following correlation has been obtained.

\[
\varphi_{sx} = \left(12.568 - 0.00118q^{0.15} + 0.3987T_{in} + 0.00111q^{0.15}x\right)^{1.55}
\] (5.11)

where \(Pr\) (Prandtl number) of fluid used as a coolant medium and water are
calculated at the inlet temperature. This equation is valid for both air and water.

Local property index correlation is compared with the experimental value at
heat flux of 3950 W/m² in Figure 5-27. \(\varphi_{sx}\) predicted are compared in Figure 5-28 and
found that the predicted \(\varphi_{sx}\) error band for the previous experimental study of Keyhani
et al [6] and computational study of Chughtai et al [3] is within ±5%.

Local heat transfer coefficients are calculated by equation (5.9) & (5.11) and
\(T_{sx}\) is predicted from following equation:

\[
T_{sx} = T_{in} + \left(\frac{q^*}{h_x}\right)
\] (5.12)

\(T_{sx}\) predicted in this way lie within a ±10% error band when compare with the
experimental data of current study and Keyhani et al [6] as well as with the
computational study of Chughtai et al [3] as shown in Figure 5-29. Keyhani studied
for lower heat flux than current study. The surface temperatures predicted for central
cylinder of Keyhani’s assembly are in good agreement, shown in Figure 5-30. The
slight differences are due to the different geometric parameters and different fluids
used. Current study and Chughtai predicted surface temperatures are compared at
same thermal power (5.4 KW) of PARR-II and found the lesser temperatures as
shown in Figure 5-31. The lower prediction for same assembly is at same condition is
analyzed as given below.

As per operating experience of PARR-II, 10% energy loss has been observed
at full power. All the remaining 90% energy is not transferred through surface. Some
part of the energy is directly added to the coolant.

In the nuclear reactor the distribution of fission energy released and deposition
at different positions in reactor is given by El-Wakil [81] as shown in Table 5-2.
According to this distribution 2.52% energy is directly added to moderator or coolant.
inside the core. Moreover, some energy is transferred indirectly to moderator from structural materials. Therefore one can safely assume that about 4% of total thermal energy is directly added to coolant uniformly throughout the reactor core. This energy does not pass through the fuel rod outer surface as heat flux \( (q''') \). However it is taken by the water. This energy dissipation in the water acts as a volumetric source. This would then raise the bulk temperature of the water to a certain degree uniformly throughout the reactor coolant inside. This uniform rise of water temperature cannot contribute to the natural convection heat transfer. Hence this energy should not be considered as the one which should generate or modify the Grashof number or results in the corresponding natural convection heat transfer. Hence the correlations developed in the current study cannot be applied by considering the thermal power of the reactor. Therefore it is proposed that only 96% of this energy flows to the coolant as surface heat flux \( (q'''') \) and should be used in the correlation. However the remaining 4% of power should be considered as a volumetric energy generation source for a time of 4 minutes. This is the transient time for the current MNSR to come to a steady state [82]. Hence the total energy added to the coolant, for this transition period, results in a volumetric temperature rise which should be added to temperature rise occurring through heat flux \( (q''') \) being calculated by the suggested correlations.

<table>
<thead>
<tr>
<th>Type</th>
<th>Process</th>
<th>Percentage of Reactor Energy</th>
<th>Approximate Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fission</td>
<td>Instantaneous Energy</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Kinetic energy of fission fragments</td>
<td>80.50</td>
<td>very short</td>
</tr>
<tr>
<td></td>
<td>Kinetic energy of new born fast neutrons</td>
<td>2.50</td>
<td>medium</td>
</tr>
<tr>
<td></td>
<td>γ energy released at time of fission</td>
<td>2.50</td>
<td>long</td>
</tr>
<tr>
<td>Delayed Energy</td>
<td>Kinetic energy of delayed neutrons</td>
<td>0.02</td>
<td>medium</td>
</tr>
<tr>
<td></td>
<td>β-decay energy of fission products</td>
<td>3.00</td>
<td>short</td>
</tr>
<tr>
<td></td>
<td>Neutrinos associated with β decay</td>
<td>5.00</td>
<td>non recoverable</td>
</tr>
<tr>
<td></td>
<td>γ energy of fission products</td>
<td>3.00</td>
<td>long</td>
</tr>
<tr>
<td>(n, γ) due to excess neutron</td>
<td>Instantaneous and Delayed Energy</td>
<td>Non fission reactions due to excess neutrons plus β and gamma decay energy (n, γ)</td>
<td>3.50</td>
</tr>
</tbody>
</table>

Table 5-2 Approximate Distribution of Fission Energy by El-Wakil [81]
\[ T_{sx} = (T_{in})_{volumetric} + \left( \frac{q^*}{h_i}_{at96\%Power} \right) \]  

(5.13)

where

\[ (T_{in})_{volumetric} = T_{in} + \left( \frac{4\%Power}{mC_p} \right)_{for\,steady\,state\,time} \]  

(5.14)

Now the surface temperatures are calculated as per above discussion and compared with Chughtai predicted temperatures and found in good agreement (±4%) as shown in Figure 5-32. Hence the predicted assumption and equations are valid. The surface temperature distribution for whole range of power (5.4 to 27 KW) is given in Figure 5-33.

5.6.2 Assembly Outlet Temperature \((T_{out})\)

The measured outlet temperature of present experimental set up at different heat flux is shown in Figure 5-34. The outlet temperature of assembly depends upon heat flux, average property index \(\mathcal{Z}_L\) and inlet temperatures. The correlation obtained for \(T_{out}\) of assembly is given as:

\[ T_{out} = -0.787 + 0.000574q^* + 0.877T_{in} + 0.261\mathcal{Z}_L \]  

(5.15)

where

\(q^*\) Heat flux (W/m²)
\(T_{in}\) Inlet temperature (°C)
\(\mathcal{Z}_L\) Weighted average of property index given as:

\[ \mathcal{Z}_L = \sum_{x=0}^{x=L} \Delta x \mathcal{Z}_sx \]  

(5.16)

Present study experimental set up has an electrical heating source and no extra energy is available in the assembly hence the entire heat crossing through surface will be observed in the fluid bulk outlet temperature of the assembly. However in nuclear reactors total temperature rise is equal to the surface heat plus the heat added as volumetric source as discussed in previous section. Therefore the heat flux should be calculated on 96% of total thermal power of PARR-II for its outlet temperature.
prediction by the suggested empirical correlation. The bulk temperatures are calculated at 4% of the reactor thermal power being added to the inlet temperatures of the liquid available inside the core for 4 minutes (time to achieve steady state) [82].

Outlet temperatures for PARR-II thus predicted for the thermal power equivalent to PARR-II and compared with actual PARR-II bulk fluid outlet temperatures are shown in Figure 5-35. The comparison shows that the predicted outlet temperatures are in close proximity to that of PARR-II.

This empirical correlation is valid for the range of averaged modified Grashof number 1 x 10^{11} to 1.8 x 10^{12} which is similar to PARR-II as suggested by Chughtai et al [3] and calculated in the present study. This was the primary criterion in the design of the current assembly on the basis of dynamic similarity. The averaged modified Grashof numbers correspond to the heat flux 3950 to 19500 W/m². Hence the prediction for PARR-II power ranges is valid. The outlet temperature comparison with respect of average Grashof number is shown in Figure 5-36. The predicted outlet temperatures from equation (5.15) for PARR-II are almost in ±4% error band. The reason of this error band is discussed below.

The gaps between fuel pins of PARR-II are smaller than present experimental assembly. The boundary layers interaction would be started at a much lower part of the adjacent fuel pins of PARR-II. However in present study it was observed that such interaction was present in the upper region only. Due to this reason the PARR-II bulk outlet temperatures might be higher.
**Figures:**
The legend of the temperatures lines on Figure 5-1 to Figure 5-9 is explained as:

- **Inlet** fluid bulk inlet temperature to the assembly
- **Outlet** fluid bulk outlet temperature from the assembly
- **TC-ij** axial position of a thermocouple measuring the surface temperature for a specific cylinder

where:
- TC thermocouple
- i cylinder number (1 for cylinder 11, 2 for 22 & 3 for 33 respectively)
- j position of thermocouple from bottom of cylinder

![Diagram (a)](image-a)

![Diagram (b)](image-b)
Figure 5-1(a, b & c) Surface temperatures at six axial positions with inlet and outlet temperatures of an assembly for cylinders 11, 22 & 33 at 880 W/m²

Figure 5-2 Surface temperatures at six axial positions with inlet and outlet temperatures of cylinder 11 in assembly at 1330 W/m²
Figure 5-3 Surface temperatures at six axial positions with inlet and outlet temperatures of cylinder 11 in assembly at 1800 W/m²

Figure 5-4 Surface temperatures at six axial positions with inlet and outlet temperatures of cylinder 11 in assembly at 2220 W/m²
Figure 5-5 Surface temperatures at six axial positions with inlet and outlet temperatures of cylinder 11 in assembly at 2640 W/m$^2$.

Figure 5-6 Surface temperatures at six axial positions with inlet and outlet temperatures of cylinder 11 in assembly at 3100 W/m$^2$. 
Figure 5-7 Surface temperatures at six axial positions with inlet and outlet temperatures of cylinder 11 in assembly at 3530 W/m$^2$

Figure 5-8 Surface temperatures at six axial positions with inlet and outlet temperatures of cylinder 11 in assembly at 3950 W/m$^2$
Figure 5-9 Surface temperatures at six axial positions with inlet and outlet temperatures of cylinder 11 in assembly at 4470 W/m²

Figure 5-10 Surrounding effect of heated cylinders on central cylinder in assembly at 4470 W/m²
Figure 5-11 Surface temperatures at axial positions (a) TC-14, (b) TC-13, (c) TC-12 & (d) TC-11 cylinder 11 in assembly at different heat flux values
Figure 5-12 Axial temperature difference profiles at different uniform heat flux for cylinder 11 in an assembly

Figure 5-13 Axial temperature difference profiles at different uniform heat flux for Cylinder 22 in an assembly
Figure 5-14 Axial temperature difference profiles at different uniform heat flux for Cylinder 33 in an assembly

Figure 5-15 Local heat transfer coefficients ($h_x$) varying axially at different uniform heat flux for Cylinder 11 in an assembly
Figure 5-16 Temperature difference profiles for three cylinders (11, 22 & 33) in an assembly of cylinders at different uniform heat flux values.

Figure 5-17 Qualitative flow visualization: Movement of boundary layer at the surface of S.S. cylinder (White area shows water & Black shows the cylinder)
Figure 5-18 Local Nusselt number for three cylinders at different positions in an assembly of cylinders

Figure 5-19 Comparison of overall Nusselt number for single cylinder with each cylinder at three symmetric positions in an assembly of cylinders
Figure 5-20 Comparison of overall Nusselt number on the basis of cylinder diameter with cylinder 11 in an assembly of cylinders
Figure 5-21 Comparison of overall Nusselt number on the basis of equivalent diameter with cylinder 11 in an assembly of cylinders

Figure 5-22 Comparison of local Nusselt number of Isahai et al with cylinder 11 in an assembly
Figure 5-23 Comparison of overall Nusselt number of Isahai et al with cylinder 11 in an assembly

Figure 5-24 Comparison with Keyhani data of Nusselt number on the basis of annular length with cylinder 11 in an assembly
Figure 5-25(a) Comparison of local Nusselt number with cylinder 11 in an assembly (b) ±24% Error band for the prediction of data
Figure 5-26(a) Effect of Prandtl number on local Nusselt number in an assembly
(b) ±10% Error band of correlation
Figure 5-27 Local property index $\zeta_{sx}$ for experimental data at 3950 W/m$^2$

Figure 5-28 Local property index $\zeta_{sx}$ prediction with experimental data with Chughtai & Keyhani study within ±5% error band
Figure 5-29 Local surface temperatures $T_{sx}$ predicted from property index $Z_{sx}$ correlation fits with the actual data within $\pm 10\%$ band.

Figure 5-30 Comparison of local surface temperatures $T_{sx}$ predicted for Keyhani assembly with Keyhani experimental data at 648 W/m$^2$. 
Figure 5-31 Comparison of local surface temperatures $T_{sx}$ predicted for PARR-II with Chughtai prediction at 5.4 KW

Figure 5-32 Comparison of local surface temperatures $T_{sx}$ predicted for PARR-II with Chughtai prediction at 5.4 KW after correction of fission energy distribution
Figure 5-33 Predicted Surface Temperature Distribution for PARR-II Fuel Pin

Figure 5-34 Experimental bulk outlet temperature $T_{out}$ of an assembly used for present study at different $T_{in}$
Figure 5-35 Comparison of Predicted $T_{\text{out}}$ for PARR-II at Thermal Powers

Figure 5-36 Comparison of Predicted $T_{\text{out}}$ for PARR-II at $Gr_L^*$
Chapter 6. Conclusions and Future Recommendations

6.1 Conclusions

The steady state heat transfer by natural convection phenomenon has been investigated experimentally both for a single vertical cylinder in an infinite medium and vertical cylinders in an assembly. The main conclusions drawn from the present experimental study are:

1. Common presentation of local Nusselt number with local modified number is not valid for all fluids and geometric parameters of single vertical cylinder.

2. The actual presentation of local Nusselt and modified Rayleigh numbers with dimensionless parameters is presented and correlated as equation (4.6).

3. The turbulence in boundary layer at the surface of cylinder in assembly starts for local modified Grashof number $Gr_d^* > 1.53 \times 10^5$. The assembly of vertical cylinders become turbulent at averaged modified Grashof number $\overline{Gr}_L^* > 1.31 \times 10^{11}$.

4. The turbulence is enhanced in assembly as compared with single cylinder.

5. The surface temperatures axially along the vertical cylinder in an infinite medium and in an assembly increases up to a certain height and then decreases. However the magnitudes of surface temperatures in assembly are higher than single cylinder at same power levels.

6. The surface temperatures at central cylinder observed slightly lower than its surrounding cylinders in an assembly.

7. The higher velocity observed in the boundary layer at central cylinder rather than its surrounding cylinders in an assembly. The average velocity at central rod was 1.25 cm/sec.
8. Nusselt numbers observed higher for central cylinder than its surrounding cylinders in an assembly. Due to more cooling the central rod is safer in an assembly.

9. Chimney effects are observed in assembly.

10. The gap between the cylinders has more significance than pitch to diameter ratio (P/d).

11. The cylinder length is the actual characteristic length in the definition of Nusselt and Grashof numbers rather than the diameter.

12. Nusselt number in assemblies is the function of Grashof and Prandtl numbers separately rather than their product form.

13. The surface temperatures at the surface of fuel pins in an operational MNSR type nuclear reactor are predicted.

14. The outlet temperatures of MNSR type nuclear reactor are predicted on the basis of present experimental setup and found in good agreement.

15. The correlations for local and averaged Nusselt numbers are developed for single as well as assembly of vertical cylinders.

6.2 Future Recommendations

The whole area of natural convection could not be studied due to limited time and some limitations in experimental conditions. Many dimensions of this area still required some more investigations. Some of them are:

1. The gaps between the cylinders in an assembly are to be optimized to see the boundary layer interaction effects.

2. Quantitative flow visualization is required for better understanding of flow behavior inside the assemblies.

3. Velocities at the surface of each cylinder in assembly and over all flow developed by the assembly are required to measure.

4. Transient data analysis for assemblies as well as single cylinder is required.
References


C. Bhat and D. Garg, 2007, *Study of Heat Transfer by Natural Convection from a Horizontal Cylinder Embedded in a Porous Medium*, Purdue University, West Lafayette, IN 47907, USA


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J. M. Buchlin and M. Peelman, 1996, Natural Convection along a Vertical Slender Cylinder, 2nd European Thermal Sciences and 14th UIT National Heat Transfer Conference,


Appendices

Appendix 1. Data Logger

The TC-08 USB thermocouple data logger was used to record the temperature data using thermocouples. The TC-08 interfaces up to 8 thermocouples of any type to a PC, and up to 8 TC-08s can be connected to a PC.
Appendix 2. Thermocouples

Thermocouples are the most popular temperature sensors. They are cheap, interchangeable, have standard connectors and can measure a wide range of temperatures. The main limitation is accuracy, system errors of less than 1°C can be difficult to achieve.

While choosing a thermocouple consideration should be given to the thermocouple type, insulation and probe construction. All of these will have an effect on the measurable temperature range, accuracy and reliability of the readings. Note that thermocouples with low sensitivity (B, R and S) have a correspondingly lower resolution.

<table>
<thead>
<tr>
<th>Thermocouple type</th>
<th>Overall range °C</th>
<th>0.1°C resolution</th>
<th>0.025°C resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>20 to 1820</td>
<td>150 to 1820</td>
<td>600 to 1820</td>
</tr>
<tr>
<td>E</td>
<td>-270 to 910</td>
<td>-270 to 910</td>
<td>-260 to 910</td>
</tr>
<tr>
<td>J</td>
<td>-210 to 1200</td>
<td>-210 to 1200</td>
<td>-210 to 1200</td>
</tr>
<tr>
<td>K</td>
<td>-270 to 1370</td>
<td>-270 to 1370</td>
<td>-250 to 1370</td>
</tr>
<tr>
<td>N</td>
<td>-270 to 1300</td>
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<tr>
<td>R</td>
<td>-50 to 1760</td>
<td>-50 to 1760</td>
<td>20 to 1760</td>
</tr>
<tr>
<td>S</td>
<td>-50 to 1760</td>
<td>-50 to 1760</td>
<td>20 to 1760</td>
</tr>
<tr>
<td>T</td>
<td>-270 to 400</td>
<td><strong>-270 to 400</strong></td>
<td>-250 to 400</td>
</tr>
</tbody>
</table>

T-type thermocouple was used in experimental setup some specifications are:

- Fibre glass insulated
- Wire diameter is 0.315 mm each
- Overall diameter 1.5 mm
- Positive leg: Copper
- Negative Leg: Constantan
Appendex 3. Variac

Variable voltage power supply (variac) is used in experiments. By adjusting the voltage the current was measured across single cylinder and shown here in figure. And the circuit board for on/off control and to measure voltage and current across each heated cylinder.
Appendix 4. Magnesium Oxide

Magnesia or magnesium oxide MgO is an alkaline earth metal oxide and available in white powder form. There are few dense engineering ceramics of the structural type made from pure magnesia. However there is a wide range of refractory and electrical applications where magnesia is firmly established. It was used in our experimental setup due to its high thermal conductivity.

The properties of major interest are as follows:

- Good Refractoriness
- Good Corrosion Resistance e.g. Sodium Hydroxide, Fe, Co, Ni
- High Thermal Conductivity (42 W/m°K)
- Low Electrical Conductivity
- Transparency to Infrared

General Application:

Magnesia powder is widely used as a filling for electrical heating elements for applications in contact with air or liquids such as electric cooker rings, storage heaters, washing machines, and diesel engine glow plugs. Fused magnesia has the ideal combination of electrical resistance and thermal conductivity. The MgO forms a layer between the element and the outer sheath.
Appendix 5.  PARR-II

The PARR-II Reactor is an indigenously designed and constructed reactor owned by the Pakistan Atomic Energy Commission. The PARR-II Reactor's design is similar to Miniature neutron source reactor (MNSR) and SLOWPOKE reactor. The PARR-II Reactor had gone critical and began operating on January 21, 1974. The PARR-II Reactor is a type-in-pool reactor with a rated power of 27-30 KW. The reactor is design to utilize the High Enrich Uranium (HEU) fuel. The demineralized light water is used as a coolant moderator and the reactor core is reflected by metallic Be4. A PARR-II consists of a core reactor, control rod, and nuclear reflectors, and it is enclosed in a water-tight cylindrical Al13 alloy vessel. The nuclear reactor core is an under-moderated array with $^1\text{H}$ to $^{235}\text{U}$ ratio of temperature of 20°C and provides a strong Negative temperature coefficient & thermal volume coefficients of reactivity.

![Schematic diagram of PARR-II](image)

![Arrangement of fuel pins in PARR-II](image)
Coolant Flow Path of PARR-II

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
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<tr>
<td>Enrichment in U-235</td>
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<tr>
<td>Maximum excess reactivity</td>
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<tr>
<td>Thermal neutron flux at 27 kW</td>
<td>$1.0 \times 10^{12} \text{ #/cm}^2\text{-s}$</td>
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<tr>
<td>Moderator and Coolant</td>
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<td>Control Rod:</td>
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<tr>
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<td>Fission Chamber</td>
</tr>
<tr>
<td>Temperature</td>
<td>k-type thermocouples</td>
</tr>
</tbody>
</table>

PARR-II Reactor Description