PEDAGOGICAL ADAPTIVE HYPERMEDIA BASED SYSTEM FOR MATHEMATICS AT SECONDARY LEVEL

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بسم الله الرحمن الرحيم
Dedicated to my father

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ABSTRACT

Hypermedia systems give freedom of navigation to the users but majority of them give same view to all of them. In this regard, adaptive hypermedia provides solution to this problem through personalisation. E-learning can also be made effective by adapting the needs and background of learners. PAHMS (pedagogical adaptive hypermedia based educational system for mathematics at secondary level) provides adaptive presentation of content and adaptive navigation by adapting the preferences and educational background of learners. It also provides the facility of adaptive peer’s searching. In this way, it reduces the cognitive overload of learners and prevent them being lost in hyperspace of the system. This thesis describes the development of PAHMS.

This system provides pedagogy of mathematics following curriculum designed by Ministry of Education, Pakistan to both in-service and student teachers. Pedagogy comprises of application of famous teaching methods in different branches of mathematics with examples. It also describes complete procedures along with merits and demerits of these methods.

E-learning objects including instructional, collaborative and assessment objects for the pedagogy have been developed as well.

Effectiveness of PAHMS has been ascertained through survey research conducted from the teachers of mathematics at secondary level. Majority of them gave positive response about the system.
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Fawad Baig
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CHAPTER 1

INTRODUCTION

This chapter presents the research agenda of the thesis. Section 1.1 focuses on the importance of e-learning and qualities of pedagogical web based systems. Section 1.2 describes the statement of the problem. Section 1.3 focuses on the objective of this research. Section 1.4 explains the research questions. Section 1.5 defines the scope of the research. Section 1.6 describes the significance of the study. Section 1.7 explains the basic assumptions for this research. Section 1.8 defines the key terms used in this thesis. Section 1.9 provides the outline of the thesis.

1.1 Background

We are passing through information age. Now it is very important that right information at right time through economical way and in efficient manner should reach us. That is why, most of the institutions are trying to adopt the way of e-learning nowadays. In this regard, tools of information technology (IT) can play a vital role in the shape of information resources on web, communication technologies like audio, video conferencing, virtual reality, simulations and database management systems (Hung, Ten, & Chen, 2003). E-learning has changed the whole educational scenario by utilising all of these tools of IT (Hamilton & Zimmerman, 2002).

E-learning is a broader concept and it is existed in different modes. Online education is one of them in which web is used as a communication tool (Hegngi,
At this moment a lot of web based educational systems are running but still something is lacking that is personalisation or customization which means that systems should adapt learner’s characteristics and guide them according to their educational background and previous academic performance. In this regard, adaptive hypermedia can solve these problems through user modelling (De Bra, 1999), adaptive navigation (Brusilovsky, 2003a; Brusilovsky, 2004a) and adaptation presentation of content (Brusilovsky, 2001). If such adaptive features are added to educational hypermedia based applications then these systems become adaptive hypermedia based educational systems which are more learner centred.

Adaptive hypermedia based educational system is one of the different types of intelligent tutoring system (ITS) (Murray, 1999). Most of the online educational systems present same content to all learners without considering their educational background and preferences. In this way mechanism of “one-size-fits-all” makes the learning process more difficult and boring because every learner has to spend more time to dig out the relevant material (Brusilovsky & Maybury, 2002). On the other hand, the aim of adaptive hypermedia (AH) based educational systems is to reduce cognitive overload and decrease the time, learner spends in finding specific information. These systems build a profile of the learner (user model) and use this knowledge to simplify the browsing experience and guide the learner through an information space (domain) to find the right information.

E-learning can be fifty to sixty percent faster than classroom instruction (Brandon-Hall, 2004), so how much it is important to adopt this mode of instruction, we can easily guess. Martindale, Cates and Qian (2003) classified educational websites in thirteen categories in which they also discussed “instructional” and “teacher resource” web sites. Instructional websites must include learning objectives,
teaching strategies, educational content and assessment tools. Similarly, teacher resource online systems must provide teaching standards, pedagogical content, lesson plans and helping material. If we talk about pedagogical web based system then it may contain some characteristics of instructional sites and some of teacher resource websites.

Pedagogical web based system should have some qualities (Giguere & Minotti, 2003).

- The content of pedagogical websites should be learner centred.
- Appropriate tools for learning activities like discussion forums and announcement board should be part of this.
- Learning objectives should be clearly defined.
- Educational material should be easily accessible.
- Educational material should be available in different formats.
- Learners should have facility to share their views and material easily.
- Training process should not be much lengthy.
- Any expert should monitor and facilitate the learning process.
- Online technical help should be provided.
- Online assessment through tests, assignments and discussions should be done.

These guidelines have much importance but still online teachers’ training is difficult and there are always some possibilities of non-completion of such type of online training programs which are not providing what the teachers expect and what they are really interested in. So the workload of teachers is increased and it becomes very difficult for them to manage online learning (Kearsley & Blomeyer, 2004). If we
add the element of adaptation in the educational environment then it can be made more effective and more learners centred.

1.2 Statement of the Problem

Pakistan is the sixth largest country in the world by population. There is huge number of teachers as well. Moreover, Pakistan is a developing country and there is lack of resources. It is very difficult to train teachers to enrich teaching force through economics of education. Science education plays a dynamic role in the progress of a nation, especially in the area of technology and economics. Mathematics is the mother of science and without proper base of this subject, it is impossible for students to study Statistics, Physics and Computer Science at post secondary level. On the other hand, many teachers face difficulty to teach mathematics as a difficult subject. Hence, there is a great need to train teachers about the pedagogy of mathematics at secondary level. Sometimes, it also becomes difficult to train teachers according to their own needs and particular academic background. Unfortunately, there is no online training system working at national or provincial level in Pakistan through which we can train teachers in more personalised way at any time at any place without involving huge amount of financial resources.

1.3 Objective

The objective of the study is two fold.

a) To develop a pedagogical adaptive hypermedia based system for mathematics at secondary level.

b) To enrich the pedagogy of mathematics at secondary level in Pakistani context.
1.4 Research Questions

This research provides answers to the following questions.

1) What are the methods and techniques required to achieve adaptation in hypermedia based educational and training systems?

2) What types of authoring frameworks or tools are available to develop adaptive hypermedia based educational and training systems?

3) What would be the e-learning objects as learning material or content?

4) What are the best instructional methodologies to teach mathematics at secondary level in Pakistani context?

1.5 Scope

All the teachers of mathematics of public and private secondary schools in Lahore city, Pakistan.

1.6 Significance of the Study

As pedagogical adaptive hypermedia based system is web based hence it is useful because of the following reasons:

- Such type of system is more economical because there is no need to hire employees to train teachers.
- This adaptive system provides flexible learning environment in which any teacher can learn at any time at any place.
- At a time, thousands of teachers can be trained in a personalised environment with efficient manner.
- This system can help a teacher after training as well if he wants to refresh his pedagogical knowledge about any particular area of mathematics.
• This system provides online pedagogical leadership in the area of mathematics in Pakistan.

Because of the above mentioned reasons, not only teachers and school administrators can be benefited but NGOs, PBOs, CBOs and policy makers can also gain benefits who are working in the field of education and teachers training.

1.7 Basic Assumptions

These were the following assumptions in this study.

• Majority of the teachers know the importance of web based teaching and learning.

• Most of the schools are providing better facilities of computer labs and internet to their teachers.

• Many teachers are familiar with online teachers’ training programs.

• Web based teaching is fastest, economical and convenient way to train teachers than traditional classroom based instruction.

1.8 Definition of Key Terms

Pedagogy: Pedagogy is “the art or science of teaching” (Agnes, 2003, p. 475).

Adaptive Hypermedia based System: Adaptive hypermedia based system “structures information in such way that is possible to read its documents in a no linear order and which is able to adjust the presented information to certain user features; facilitating the navigation and comprehension of the offend material” (Medina-Medina, Garcia-Cabrera, Torres-Carbonell & Parets-Llorca, 2002, p. 34)

Mathematics: “Mathematics is the science and study of quantity, structure, space, and change” (Wikipedia, The Free Encyclopedia, 2009a, para. 1).
Secondary Level: The level of education where students of class 9 and 10 are being taught.

1.9 Outline of the Thesis

The main focus of this thesis is to describe the process of development of pedagogical adaptive hypermedia based system for mathematics at secondary level (PAHMS). Initial chapters are related to the previous work related to adaptive hypermedia and later chapters are related to the development and evaluation of the system.

Chapter 2 focuses on the background of adaptive hypermedia and role of user modelling in adaptation. This chapter also describes two classifications of adaptation: adaptive presentation and adaptive navigation. It also describes application areas of adaptive hypermedia. At the end of this chapter, reference models used for implementation of adaptive hypermedia systems are described.

Chapter 3 focuses on e-learning and adaptive hypermedia based educational systems (AHES). It provides architecture of an online learning environment. It also describes the features of already existing AHESs. It discusses some authoring tools or frameworks for AHESs as well.

Chapter 4 describes the methodology adopted for this research including development process of pedagogical adaptive hypermedia based system for mathematics at secondary level (PAHMS), strategy, population, sample, development of research tool, validation and pilot testing of tool, procedure for collecting data and application of statistical test on the data.

Chapter 5 describes the adaptive learning environment and implementation of PAHMS. E-learning objects for pedagogy of mathematics are also described in this chapter.
Chapter 6 focuses on the pedagogy of mathematics at secondary level in Pakistani context. It describes application of teaching methods in different branches of mathematics by giving relevant examples from the curriculum designed by Ministry of Education, Pakistan.

Chapter 7 focuses on analysis of data collected from teachers through survey research to check the effectiveness of PAHMS.

Chapter 8 provides summary of the thesis, research findings, conclusions and recommendations for future work.
CHAPTER 2

REVIEW OF ADAPTIVE HYPERMEDIA

This chapter presents the background of adaptive hypermedia. Section 2.1 differentiates between hypertext and hypermedia. Section 2.2 describes personalisation and difference between adaptable, adaptive and dynamic hypermedia. Section 2.3 briefly describes adaptive hypermedia systems, their history, advantages and drawbacks. Section 2.4 describes domain model as a part of adaptive hypermedia systems. Section 2.5 gives an overview of user model and its types. Section 2.6 describes two classifications of adaptation: adaptive presentation and adaptive navigation. It also describes methods and techniques for adaptive hypermedia systems. Section 2.7 focuses on application areas of adaptive hypermedia systems. Section 2.8 gives an overview of reference models used for implementing adaptive hypermedia systems. Section 2.9 summarises the background of adaptive hypermedia and describes the answer to research Question 1 given in chapter 1.

2.1 Hypertext and Hypermedia

The idea of hypertext was evolved from “MEMEX”, an electro-mechanical device described by Vannevar Bush (1945). In that machine, a person could insert microfilms upon which number of writing and books are stored. It could be displayed at any time. Bush also described the links among these documents as associative “trails”. Theodor Holm (Ted) Nelson got inspired by this idea in 1960s and he
initiated first hypertext project “XANADU” (Conklin, 1987). Afterwards, he coined the terms “hypertext” and “hypermedia” in 1965 (Nelson, 1965).

In 1967, Andries van Dam (1988) made first hypertext Editing System (HES) and then he made FRESS (File Retrieval and Editing System) with the help of Ted Nelson at Brown University, USA in 1968. 1960s Douglas Engelbart also started his project NLS (oN Line System) at Stanford Research Institute, USA which was completed in December 1968 and then for the first time he showed hypertext interface to the common people (Engelbart, 1988).

Hypertext is an extension of normal text with non-linear behaviour. In text, information is always in a linear form but hypertext is consisted of a set of nodes linked with each other in a non-linear manner and each node is actually a page having some textual information. Nelson (1981) defined hypertext as “...non-sequential writing-text that branches and allows choices to the reader, best read at an interactive screen... this is a series of chunks connected by links which offer the reader different pathways” (p. 2).

Hypermedia is an enhanced form of hypertext and a combination of multiple forms of media. It has structure just like of hypertext but the nodes can be made up of different kinds of media like text, audio, video and graphics etc. Lowe and Hall (1999) defined hypermedia as “an application which uses associative relationships among information contained within multiple media data for the purpose of facilitating access to, and manipulation of, the information encapsulated by the data” (p. 32).
2.2 Personalisation

As for as hypermedia or web based systems are concerned, majority of them provide same view to all the users but every person has different educational, social and cultural background. That is why, everybody needs more personalised view of any particular hypermedia system in terms of presentation and content. Blom (2000) defines personalisation as “a process that changes the functionality, interface, information content, or distinctiveness of a system to increase its personal relevance to an individual” (p. 313).

Personalisation is more helpful to maintain good one to one relationship with a user or customer especially in E-business context. Personalisation is also known as customisation, individualisation (Riemer & Totz, 2003), and adaptation.

Personalised hypermedia exists in different shapes. According to De Bra (1999), there are three kinds of it: *adaptable hypermedia, adaptive hypermedia and dynamic hypermedia.*

- In *adaptable hypermedia*, user is given some options which are stored in user profile to modify the system’s presentation or functionality according to his preferences (Fischer, 2001).
- In *adaptive hypermedia*, user profile is maintained and updated continuously according to the user’s behaviour, so presentation of the system is changed accordingly by adapting the new requirements and the knowledge level of the user.
- In *dynamic hypermedia*, system does not depend on predefined settings by the user related to presentation or functionality. It monitors the user’s actions
continuously and presents the content and functionality accordingly at runtime.

Discussion on personalisation can be found in (Sunikka & Bragge, 2008; Rossi, Schwabe, Danculovic, & Miaton, 2002; Pretschner & Gauch, 1999; Greenberg & Witten, 1985).

2.3 Adaptive Hypermedia Systems (AHSs)

As classic hypermedia systems provide static view of web pages to all users who have different kind of preferences and background. So it leads towards a lack of interest. On the other hand AHSs adapt to the preferences of user and provide personalised views to everybody by maintaining a user model. “Adaptive hypermedia systems build a model of the goals, preferences and knowledge of each individual user, and use this model throughout the interaction with the user, in order to adapt to the needs of that user” (Brusilovsky, 2001, p. 87).

2.3.1 History of Adaptive Hypermedia

In 1985, Greenberg and Witten (1985) explained the first successful adaptive hypermedia system but this area was properly focused in early 1990s. Before this, research was continued on user modelling. In 1990, Böcker, Hohl and Schwab (1990) explained individualised hypertext. Then different researchers started research on adaptive hypermedia and its applications (De La Passardiere & Dufresne, 1992; Kaplan, Fenwick, & Chen, 1993; Brusilovsky, 1994; Pérez, Gtiérrey, & Lopistéguy, 1995).

Revolution in this field came in 1996 when Brusilovsk (1996a) discussed the methods and techniques for AHS in very comprehensive way. Then first book on
AHSs titled “Adaptive Hypertext and Hypermedia” was published (Brusilovsky, Kobsa, & Vassileva, 1998). Finally, the first conference on AHS (Brusilovsky & Maybury, 2002) titled “International Conference on Adaptive Hypermedia and Adaptive Web-based Systems” was held in August 2000 at Trento, Italy. Since 1985, a lot of AHSs have been developed in different areas because increase in the use of internet requires more adaptivity for its diverse users.

2.3.2 Advantages and Drawbacks of AHSs

As Adaptive hypermedia is more individualised to the users of a system, so AHSs has some potential advantages:

- In AHS, any user can navigate the system in more fastest way so operational speed is raised. Hence human computer interaction is improved (Fischer, 2001).
- Quite relevant information is presented to the individuals in desired format and medium (De Bra, 2000).
- In huge hypermedia systems, there is a chance that user may lost in hyperspace but in AHS appropriate hyperlinks are presented by adapting user’s needs and preferences so user can navigate easily (Brusilovsky, 1994).

Adaptive hypermedia has some drawbacks as well:

- The development of AHSs is quite complex and time consuming so these systems are expensive as well (Koch, 2000).
- Appropriate guidance is provided to the users in AHS which is based on prerequisite relationships between concepts of domain. If system’s author does any mistake in defining these relations then user may be guided in wrong direction.
• If any web page of the system is visited again then the content and presentation of that page may look different which can cause confusion for some users (de Bra, 2000).

2.4 Domain Model in AHSs

Every AHS is designed for one particular domain. This domain is consisted of small or bigger pieces of information which are called concepts or fragments. Each concept represents a topic, description about on particular thing or an object. The set of domain concepts or fragments is called domain model (Brusilovsky, 1996b).

There are three kinds of concepts. They may be atomic concepts if they are small units of information. A group of some relevant concepts is called page. If concepts are bigger then they are called abstract concepts (De Bra, Brusilovsky, & Houben, 1999). All concepts are linked with each other through concept relationships. Navigational paths are existed between hypertext links and concept relationships can be used for defining proper desirable navigation paths (De Bra, 1999).

2.5 User Model and Its Role in Adaptation

User model is stored on the server side instead of client’s side. It can be managed on client side with the help of cookies but there is a risk in this kind of act because user can change PC anytime and cookies can’t be transferred from one computer to another. Moreover, a cookie is so small in size. Maximum size of a cookie is 4KB so it is impossible to store user’s related data in a cookie (De Bra, 1999). According to Kobsa, Koenemann and Pohl (1999), three kinds of data related to user may be
considered for adaptation in user model: *User data, Usage data* and *Environment data*.

**User data:** It is mainly consisted of personal user’s characteristics (Medina-Medina et al., 2002). Such kind of data is usually provided by the user directly in the beginning of his interaction with the system.

- **Demographic Data:** It is basically the personal information about user like name, address, contact number, age and sex etc.

- **User’s Knowledge:** Every AHS is related to one particular domain. So, prior user’s knowledge about those domain concepts is also gathered when user starts interaction with the system.

- **User’s Skills:** All users do not possess equal capabilities to use computers and internet operations so storing information about personal skills also helps in better adaptation.

- **User’s Preferences:** AHS provide facility to the users to enter data related to their preferences so the layout and theme of the system is changed according to their needs. In some systems, users can also select language to read the content.

- **User’s Interests:** This feature is usually provided by the adaptive information retrieval systems. In these systems, users point out their long term as well as short term interests for searching appropriate information. System adapts to these interests for filtering information according to their needs (Brusilovsky, 2001).
• **User’s Goals:** These are associated with the context in which user interacts with the system so basically they define that what user really wants to achieve by using the system (Wu, 2002).

• **User’s Individual Traits:** These are the combination of some personal characteristics of a user which help to maintain his or her individuality. These can not be changed like background or knowledge of a user. Traits can be related to the personality of a user or his or her learning behaviour (Brusilovsky, 2001).

**Usage Data:** This data describes that how a user interacts with AHS. User’s actions are continuously observed and stored in user model.

• **Observable Usage:** In this kind of usage, system observes different activities of a user for proper adaptation to fulfil the needs of that user. For example, observing selection of web pages by user about certain information or concept may help in adapting user’s interests.

• **Usage Regularities:** Sometimes data about observable usage is not enough so this is processed to generate more results for better adaptation of user’s preferences and habits. For example, it can be stored that how many times a user goes for chatting with other users of the system which may help to adapt the preferences related to user’s social activities.

**Environment Data:** Every user has a particular environment. In this regard, some AHSs also consider user’s platform and his or her location because both of them affect the user’s interaction with the system.

• **User’s Platform:** It mainly refers to hardware, software and bandwidth speed. There is possibility that a user has a machine with slow processing
speed. In this case user can only read text material instead of downloading heavy video files or video chatting. Similarly, a user may have old browser version. In this case, AHS has to adapt user’s platform for providing content in appropriate media format (Brusilovsky, 2001).

- **User’s Location:** Some AHSs may adapt the characteristics of user’s location. According to the data related to user’s geographical locale, system can suggest different options to the user for presenting content or overall presentation theme of a system.

### 2.5.1 Types of User Model

According to the internal structure of user model, we mainly distinguish it in three types: overlay model, stereotyped model and Bayesian model.

**Overlay Model:** User model is usually based on the concepts provided in the domain model of AHS because the major functionality of user model is to store and maintain the record of the level of user’s knowledge about each concept in the domain model. In this kind of model, record can be maintained as Boolean values (0 for known concepts and 1 for not known concepts), qualitative values (poor or good or best) or quantitative values (probability of user’s knowledge about one particular concept) (Brusilovsky, 1996b; Koch, 2000). Hence user’s knowledge is stored as a group of pairs “Domain concept - Value”. Other characteristics of a user like goals and preferences can also be stored in this shape. This model is not only quite simple to implement but the value of user’s knowledge about each concept can also identified easily (Brusilovsky, 1994).

**Stereotyped Model:** Users can be divided in different groups according to their characteristics, for example, they can be novice or expert in one particular domain.
So in this case, we can define two stereotypes: novice and expert. There can be number of stereotypes according to the situation. In stereotype model, users are first distinguished in different stereotypes where a group of pairs “Domain concept – Value” is defined for each stereotype. So automatically, user’s knowledge is stored in user model in terms of his or her stereotype (Brusilovsky, 1994; Brusilovsky, 1996b).

**Bayesian Model:** It is a graphical representation of user’s knowledge, goals and preferences. In this model, a Bayesian network is developed which is a directed, acyclic graph. In this graph, nodes represent the values of domain concepts and probabilistic relationships among them are represented by the links. Bayesian network estimates the user’s knowledge by observing his or her interaction with the system and then updates the model continuously, for example, if an expert user fails to understand one particular domain concept then Bayesian network will only update the estimation of that user by measuring the level of his or her understanding the concept rather than changing his or her classification from expert to novice. In this way, it manages the uncertainty and keeps record of gray area between “failed” and “not failed” (Henze & Nejdl, 1999).

### 2.6 Methods and Techniques for AHSs

According to Brusilovsky (1996a), hypermedia adaptation can be classified in two different ways: Content level adaptation and link level adaptation. First one is called adaptive presentation and second one is called adaptive navigation support. Some methods and techniques are used for both of them.
2.6.1 Adaptive Presentation

When presentation of web pages of a system is altered by adding or deleting content according to the user’s goals, preferences and background then it is called adaptive presentation, for example, if someone is novice in one field then some basic concepts will be shown to that user first. Then he or she would be guided towards more difficult content later. On the other hand, advanced concepts will be shown directly to the experts (Medina-Medina, Garcia-Cabrera, Torres-Carbonell, & Parets-Llorca, 2002; European Centre for the Development of Vocational Training, 2003). In adaptive hypermedia systems, content is not only available in textual form but it can be available in multiple formats, so the presentation of both text and multimedia is adapted (Brusilovsky, 1996a).


- **Additional Explanations:** If someone is novice in any field then additional information about domain concepts is shown to him or her otherwise that information may not be presented to the experts.

- **Prerequisite Explanations:** Before presenting a domain concept to the user, prerequisite concepts are first shown to him or her according to the user’s knowledge and background.

- **Comparative Explanations:** If current domain concept which is being presented to the user is related to already known concepts then a comparison or relationship between them is shown to the user.

- **Explanation Variants:** Explanation of one particular concept is not suitable for all kinds of users, so different explanations of that concept are stored by
the system and then presented to the users according to their needs and preferences. In this regard, length and medium to present a concept can also be changed (De Bra, Brusilovsky, & Houben, 1999).

- **Sorting:** Different small portions of information about a domain concept are stored according to the level of user’s knowledge and only relevant portion among them is shown to the user. Order to present the portions of information can also be changed if all of them are needed to show (De Bra, Brusilovsky & Houben, 1999).

**Techniques for Adaptive Presentation:** There are four techniques to implement methods for adaptive presentation (Brusilovsky, 1996a).

- **Conditional Text:** Every domain concept is divided into small portions of text and then each portion is shown to the user according to the condition related to the user’s preferences and level of knowledge. This is very simple technique to implement all kinds of methods for adaptive presentation except sorting.

- **Stretcttext:** During reading content on web pages, some words or terms need more explanation so when user clicks on them, they get stretched in the shape of small text boxes in which their small description is appeared instead of jumping towards another page because of hyperlink (De Bra, Brusilovsky & Houben, 1999). Additional explanation method is implemented by this technique.

- **Fragment and Page Variants:** Variants of some web pages or their small portions are stored in the system. This technique is used to implement explanation variants method.

- **Frame based Technique:** Information about one particular domain concept is shown in one frame. In each frame, different variants of that concept are
stored in different slots then only relevant slots are shown to the user according to his or her background or preferences.

2.6.2 Adaptive Navigation

This kind of adaptation helps user to navigate within the AHS by adapting hyperlinks presentation based on information stored in user model. It prevents user to lost in hyperspace.


- **Global Guidance:** This method is used when there is a goal of the user to search specific kind of information within the hyperspace of the system. This method guides user at each step that which are the next possible links to follow.

- **Local Guidance:** During navigation, system suggests that next most appropriate link from the current page.

- **Global Orientation Support:** This provides the whole map of the system to the user to find out his or her exact location in the complete hyperspace.

- **Local Orientation Support:** This method helps user to find out his or her location in the system by showing all links accessible from the current page or hiding irrelevant links from the user.

- **Managing Personalised Views:** Users are given options to organize their own personalised views of the system in which links are appeared or disappeared according to their preferences. Overall theme and language of the system can also be changed by the user.

- **Direct Guidance:** This technique is used to implement global and local guidance method by suggesting the best “next” link to follow from the current web page.

- **Link Sorting:** Links are sorted at the basis of user’s knowledge and goals for searching information. List of links of most relevant hyper documents to the current web page are appeared in-front of user. This technique is used to implement global and local guidance method.

- **Link Hiding:** Links to not relevant hyper documents are hidden and they are appeared as “normal text” instead of as “hot words”. Web pages are considered as irrelevant because of two reasons. First, if they are not relevant to the current user’s preferences and second, if user is unable to understand the information contained by that page (Eklund, & Brusilovsky, 1998). Local and global orientation methods can be implemented by this technique. Link hiding can be applied as “link removal” by simply removing irrelevant links from the list of the links. Another form of this technique is “link disabling”. In this form, irrelevant links become inactive so there is no effect when user clicks on them (De Bra, 1999).

- **Link Annotation:** Annotation of links helps users to understand the current status of the links behind them. In the simplest form it is used to distinguish different levels of user’s knowledge about domain concepts like “not known”, “known” and “well known” etc. Another form of annotation is used to show that how relevant links are, for example, “very relevant”, “relevant” and “less
relevant”. Ng, Hall, Maier and Armstrong (2001) described a third form of link annotation which is history based. In this form, links are annotated according to the history of visiting web pages by the user. User has to concentrate more on the web pages which have been visited for less time as compare to the required time. Required time to visit a page is based on the length and difficulty level of the content contained by page.

Annotation of links is implemented in textual or visual forms, for example, with the help of different icons, colours, font sizes (Tsandilas & Schraefel, 2003) or font types. Usually, bullets of traffic light colours are used to apply annotation on links, where red bullets can represent the links which are ready to be visited or which are more relevant. Green bullets can represent the links which are not ready to be visited or less relevant and the links to the pages which are already known to the user can be represented by yellow bullets.

Annotation of links is better technique as compare to link hiding to implement local and global orientation methods because link hiding can be used to represent only two states of a link, for example, whether it is ready to be visited or not ready to be visited. On the other hand, annotation of links can be used to distinguish more than two states of a link (Brusilovsky, 1996b).

- **Link Generation:** This technique is also called link augmentation. In this technique, web pages containing information about different domain concepts are considered in spatial context of user’s browsing history and his or her preferences. All context based links to the web pages are stored in a Linkbase (database of links) and these links are augmented in the current web page to the existing set of links according to their context (Bailey, El-Beltagy, & Hall, 2001).
2.7 Applications of Adaptive Hypermedia

Brusilovsky (1996a, 2001) classified application areas of AHS in five categories: *educational hypermedia systems, on-line information systems, on-line help systems, information retrieval hypermedia systems and institutional information systems.*

2.7.1 Educational Hypermedia Systems

Usually, educational hypermedia systems provide the same information in a same manner to all kinds of users, so it becomes boring for expert users because they have to browse the hyperspace from scratch. Similarly, it becomes difficult for novices, if they are presented advanced kind of information in the beginning of the online course. On the other hand adaptive educational hypermedia systems (AEHS) maintain user model and provide the course material according to their previous knowledge, background, goals and preferences.

These systems also maintain the relationship between domain concepts. So they are able to suggest to users that whether links to next web pages are ready to be visited or not according to their user model (De Bra, 2002). Some examples of AEHS are ELM-ART (Brusilovsky, Schwarz, & Weber, 1996a), JointZone (Ng, Hall, Maier, & Armstrong, 2002), AES-CS (Triantafillou, Pomporstis, & Georgiadou, 2002) and ACTIVEMATH (Melis & Siekmann, 2004).

2.7.2 On-line Information Systems

These systems provide reference access to information. In these systems, each web page contains information about one domain concept and some references to related concepts. Adaptive on-line information systems provide the information about one concept to the users according to their level of knowledge. According to Brusilovsk
(2001) this application area includes electronic encyclopaedias, information Kiosks, virtual museums, handheld guides, e-commerce systems and performance support systems.

Electronic encyclopaedias provide information about all kinds of subjects or one specific branch of knowledge. Adaptive online encyclopaedias continuously observe the browsing activities of user and then suggest most relevant links according to his or her goals and preferences. PEBA-II (Milosavljevic, 1997) is an example of such encyclopaedias.

Information Kiosks provide information about one particular area, city or an industry, for example, city related kiosk can provide information about its important places and transportation etc. Adaptive information kiosks adapt the user’s interests, background and knowledge. Examples of such systems are AVANTI (Fink, Kobsa, & Nill, 1998) and GUIDE (Cheverst, Davies, & Mitchell, 2002).

Virtual museums provide environments through which users can visit virtual or virtually presented real museums. When these systems become adaptive then they provide view considering visitor’s preferences and already visited objects by him or her. LISTEN (Zimmermann & Lorenz, 2008), ILEX (Oberlander, O’Donell, Mellish, & Knott, 1998) and Marble Museum (Paterno & Mancini, 1999) are the examples of such museums.

Handheld guides are also related to virtual museums. Adaptive guides trace the user’s navigation and then help to determine his or her exact location and behaviour in virtual environments or in physical museums by using handheld computer devices. HYPERAUDIO (Not, Petrelli, Sarini, Stock, Strapparava, & Zancanaro, 1998), HIPSC (Oppermann & Specht, 1999) and ARCHIE (Luyten,
Gabriëls, Teunkens, Robert, Coninx, & Manshoven, 2007) are some examples of handheld guides.

E-commerce systems are used for different kinds of commerce related activities like e-marketing, online advertisement, online transaction and supply chain management. Usually it is referred to online stores. Browsing in such systems is not a main activity, so adaptive e-commerce systems provide personalised presentation of the catalogues to help the customers in finding out the products according to their needs in more fastest way. Moreover, only those banners are appeared on such systems which are relevant to the customer’s preferences and background. The examples of these systems are TELLIM (Joerding, 1999) and SETA (Ardissono & Goy, 2000).

Performance support systems are basically expert systems, in which knowledge of one particular domain is feeded, gathered from human experts. They can be related to agriculture, medicine and business etc. Adaptive performance support systems behave intelligently and provide solutions according to the user’s context and goals. Such systems include MMA (Francisco-Revilla & Shipman, 2000) and ADAPTS (Brusilovsky & Cooper, 2002).

### 2.7.3 On-line Help Systems

On-line help systems are quite similar to on-line information systems. These systems do not work independently but these are attached with different computer applications and provide help to the users about different operations of those applications. Adaptive on-line help systems provide information considering the context of the user’s work in the computer application (Brusilovsky, 2004c). Examples of these systems are Lisp-Critic (Fischer, Mastaglio, Reeves, & Rieman,
1990), EPIAM (De Rosis, De Caralis, & Pizzutilo, 1993) and ORIMUHS (Encarnação & Stoev, 1999).

2.7.4 Information Retrieval Hypermedia Systems

These systems help users to find out the required information content within the hyperspace of information retrieval (IR) hypermedia. Because of huge hyperspace of such systems, adaptation can be beneficial by providing guidance at individual level. Adaptive IR hypermedia systems can be divided into four groups (Brusilovsky, 2001): search oriented adaptive IR hypermedia systems, browsing oriented adaptive IR hypermedia systems, systems for managing personalized views and adaptive information services.

Search oriented adaptive IR hypermedia systems provide links to hyperdocuments according to the query of user but unlike common search engines they consider preferences, short term and long term goals and interests of user (Sugiyama, Hatano & Yoshikawa, 2004). Examples of these systems are SmartGuide (Gates, Lawhead, & Wilkins, 1998) and Syskill and Webert (Marinilli, Micarelli, & Sciarrone, 1999).

Browsing oriented adaptive IR hypermedia systems help users to find out their required hyperdocuments through adaptive navigation support. Adaptive guidance system presents the most relevant link or set of links to the user according to his or her goals and preferences such as WebWatcher (Joachims, Freitag, & Mitchell, 1997) and Personal WebWatcher (Mladenic, 1996). Adaptive annotation systems provide annotating links to users through which they can distinguish most relevant and less relevant links such as Syskill and Webert (Pazzani, Muramatsu & Billsus, 1996) and IfWeb (Asnicar & Tasso, 1997). Adaptive recommendation systems keep track of
browsing of a user within hyperspace and then recommend links to the hyper
documents by deducing goals and preferences from this browsing history (Syed &
Andritsos, 2007). Examples of these systems are SurfLen (Fu, Budzik, & Hammond,
2000) and SiteIF (Magnini & Strapparava, 2004).

As IR systems are used to find out relevant hyper documents, Systems for
managing personalized views help to organize this content. They can be divided in
further two groups: personalized site views and bookmark organizers. Examples of
personalized site view include My Yahoo! and My MSN. Similarly, examples of
bookmark organizers are PowerBookmarks (Li, Vu, Agrawal, Hara, & Takano, 1999)
and Siteseeer (Rucker & Polano, 1997).

Adaptive information services are based on artificial intelligent agents and
work with communities. They observe activities of users within a community and
learn about pool of hyper documents. Then they provide different services for
filtering and searching documents according to the user’s short term and long term
goals and preferences such as FAB (Balabanovic & Shoham, 1997) and ELEI
(Billsus, Pazzani, & Chen, 2000).

2.7.5 Institutional Information Systems

These systems provide information related to work in one institution, for example,
airport, bank or hospital. These systems are quite different from IR hypermedia and
on-line information systems because employees of an institution interact with only
small portion of the system related to their working area rather than with the whole
hyperspace. Each employee is provided personalized access to his or her relevant
portion of the system. Such systems like Hynecosum (Vassileva, 1996) also try to
prevent users being lost in hyperspace.
2.8 Reference Models for Adaptive Hypermedia Systems

Hypermedia engineering is not a simple task because it includes so many processes for developing front-end & back-end of a hypermedia application. That is why, some reference models are used to define the components and functionalities of a hypermedia application at conceptual level. After that, application can be developed in any programming environment.

Hypertext Abstract Machine (HAM) was the first model presented by Campbell and Goodman (1988). Afterwards some more and better reference models including The Dexter model (Halasz & Schwartz, 1990), The Trellis model (Furuta & Stotts, 1990) and A Formal Model of Hypertext (Lange, 1990) were presented at a workshop held in the National Institute of Standards and Technology, Gaithersburg, USA in 1990. But these models did not fulfil the requirements of object oriented software engineering that is why it led to object oriented reference models including the Tower model (De Bra, Houben, & Kornatzky, 1992), the Hypermedia Design Model (HDM) (Garzotto, Paolini, & Schwabe, 1993) and the Object-Oriented Hypermedia Design Model (OOHDM) (Schwabe & Rossi, 1995). To implement the adaptation, reference models for AHSs were also presented like AHAM, XHAM and Munich Reference Model.

2.8.1 Adaptive Hypermedia Application Model (AHAM)

AHAM was the first reference model for adaptive hypermedia systems which was based on the Dexter Reference Model. It was basically designed for developing educational adaptive hypermedia systems. According to Wu (2002), it divides adaptive hypermedia application in three layers just like the Dexter model as shown in Figure 2.1.
Figure 2.1. Three layered model of AHAM (Wu, 2002, p. 41).

The content and structure of all components or nodes are described in the within-component layer. The main layer is the storage layer which describes the main structure of the application consisted of components or nodes and links between them. So the complete hierarchy of nodes is presented here. Anchoring provides the interface between storage layer and within-component layer. Anchors basically identify the data elements within components or nodes which help to describe hyperlinks during development of hypermedia application.

AHAM further divides the storage layer into three models: domain model, user model and adaptation model. In domain model, concepts and relationships between them are defined. In AHAM, concepts can be atomic or composite. Small piece of information or fragment can be an atomic concept. If a concept has further sub-concepts then it is called composite concept. The simplest relationship between concepts is called hypertext link. In AHAM, other kinds of relationships are also considered like prerequisite. If a concept “A” is prerequisite for concept “B” then it means that the user should visit concept “A” before “B”. There can be the inhibitor relationship. If concept “A” inhibits “B” then it means that after visiting the concept
“A” there is no need to visit the concept “B”. Another kind of relationship is called part-of concept. If the concept “B” is part-of “A” then the user can not understand the concept “A” fully unless the user read the concept “B” (Wu, De Kort, & De Bra, 2001).

In AHAM, user related information is stored in overlay user model for which a table structure is used. In that structure attribute values related to domain model concepts are stored as shown in Figure 2.2.

| concept name (uid) | Knowledge value | read | ...
|-------------------|-----------------|------|------
| Xanadu            | well learned    | true | ...
| KMS               | learned         | true | ...
| WWW-page1         | well learned    | true | ...
| WWW-page2         | not known       | false| ...
| WWW               | learned         | false| ...
| ...               | ...             | ...  | ...

**Figure 2.2.** Example of overlay user model (Wu, Houben, & De Bra, 2000, p. 93).

Figure 2.2 shows that first two concepts and WWW-page1 are read, Xanadu and WWW-page1 are well learnt but KMS is not fully learnt by the user. On the other hand WWW-page2 and concept WWW are not read and user is not familiar with WWW-page2 as well but he or she is already familiar with the concept WWW. Similarly, attribute values for preferences and objectives of the user can also be stored (Wu, Houben, & De Bra, 2000).

Adaptation model describes how adaptation is done in AHS including content level adaptation and link level presentation. This model is consisted of adaptation rules through which user model is updated regarding domain model concepts and user’s goals and preferences. In AHAM, there are two kinds of rules used for
describing adaptation model. First kinds of rules are general or default rules through which adaptation based on relationships between domain concepts is defined. AHAM also provides facility to authors to define their own specific rules related to some specific concepts. For describing adaptation model, AHAM-CA language is used through which condition action (CA) rules are defined. When condition of a particular rule becomes true then action associated with it is triggered (Wu, 2002).

At the level of presentation specification, features of user interface of AHS are defined and then in the last layer run-time layer, it is defined that how the user instantiates the components of a system. Whenever the user interacts with the system, a session is created for keeping track of his or her activities. It is also defined here that how hypertext and multimedia objects are presented to the user.

2.8.2 XML-based Adaptive Hypermedia Model (XAHM)

XAHM helps us to describe adaptive hypermedia application at logical level including its contents and their adaptation. All domain concepts of the application are described through weighted directed graphs and pages comprises of these concepts are basically XML documents (Cannataro & Pugliese, 2001a).

Adaptation Space: For implementing adaptation process in adaptive hypermedia application, XAHM focuses on three aspects: user’s needs and preferences, user’s social background and technology used by him or her. Some needs and preferences are asked by the system when user starts interaction with it and some of them are assessed by the system by observing the activities of the user. On the basis of user’s browsing activities, system allocates the most relevant group to the user. In this way, XAHM supports stereotype user model in which different profiles are maintained.
User’s social background includes user’s language and his or her location related aspects. Technology aspects are also adapted that how much internet speed is being utilized by the user and how better machine, he or she is using. User’s preferences and social background affect the content and link level adaptation, where as the technological aspects affects the presentation of content, for example, size and resolution of images and video clips (Cannataro, Cuzzocrea, & Pugliese, 2002).

**The Layered Data Model:** In XAHM, domain model is implemented as the *layered data model* which is consisted of some levels (Cannataro & Pugliese, 2001a, 2001b). At the lowest level, domain model is consisted of information fragments (IF) or atomic concepts. These information fragments can be represented by text files, images or audio and video clips.

*Presentation descriptions (PDs)* are the XML documents which are comprised of information fragments or concepts which are described by XML metadata. Metadata helps in the process of adaptation that which fragments or concepts would be presented to the user through presentation unit or final page at run time. Each information fragment or concept can be existed in more than one form, for example, an image can be stored with different resolutions or a text file can be existed in different languages. Then according to the metadata, image of particular resolution or text file in particular language would be presented to the user.

*Elementary abstract concepts (EAC)* are the bigger information units which are consisted of one or more than one presentation descriptions. To learn an abstract concept, a user must learn number of elementary concepts. At the highest level, application domain is existed which is consisted of number of elementary abstract concepts. These concepts are represented by weighted directed graphs and links between them are shown by arcs.
2.8.3 Munich Reference Model

Munich reference model (Koch, 2000) is the object oriented approach for designing adaptive hypermedia applications. It is based on Unified Modelling Language (UML) and Object Constraint Language (OCL). Architecture of applications designed using Munich reference model is similar to three layered Dexter model (Koch & Wirsing, 2001; Koch & Wirsing, 2002) as shown in Figure 2.3.

![Figure 2.3. Architecture of Munich Reference Model (Koch, 2000, p. 74).](image)

*Run time layer* manages sessions of user’s interaction with application. It also describes the presentation of components to the users. *Storage layer* is responsible for describing the whole data of application including domain concepts and user related information. Storage layer consists of *domain meta model, user meta model* and *adaptation meta model*.

*Domain meta model* describes the structure of all nodes or components of an application and relationship between them. These components contain all the domain
concepts which are described by class \textit{Domain} and class \textit{Components}. These components are retrieved, accessed or constructed by the operations including \textit{resolver}, \textit{accessor} and \textit{constructor}. \textit{User meta model} stores all the data related to the users including their goals, preferences and background. It is modelled by class \textit{UserManager}. Users’ related data is initialised, updated and evaluated with the help of operations including \textit{initilizer}, \textit{updater} and \textit{evaluator}. \textit{Adaptation Meta Model} is responsible for adaptation process in the application. It describes all the rules through which adaptation is achieved that how the content would be adapted and presented to the user. This model is defined by two classes \textit{Adaptation} and \textit{Rule}. The last one, \textit{Within Component Layer} describes the content of all nodes or components of application.

\textbf{2.9 Summary}

Hypermedia provides navigational freedom to the users on the web but sometimes it becomes very difficult for them to find out the relevant information from the given hypermedia space according to their priorities and background. In this regard, personalisation can resolve such kind of problem. There are three kinds of personalised hypermedia: adaptive hypermedia, adaptable hypermedia and dynamic hypermedia.

Adaptive hypermedia system (AHS) maintains user model to store preferences, options and background information of the user. User model helps AHS to provide content and layout according to the needs of a user. In this way, AHS prevents user being lost in hyperspace. There is also a domain model in AHS which contains content in the shape of concepts related to one particular domain. AHS
extracts concepts or fragments of information from domain model at the basis of user model.

Overlay model stores level of knowledge of a user about each concept as a group of pairs “Domain concepts – value”. Other needs and preferences of a user are also stored in the same manner. In stereotyped model, users are divided in different groups according to their characteristics. For example, there may be two stereotypes like novice and expert. Then goals and preferences of each user are saved as a group of pairs “Domain concept – value” defined for relevant stereotype. Bayesian model is based on Bayesian network which estimates the user’s level of knowledge by observing his or her interaction with AHS and then updates itself continuously.

Adaptation can be classified in two ways: Adaptive presentation and adaptive navigation. Adaptive presentation is related to the content which is presented to the user according to his or her needs and preferences. On the other hand, adaptive navigation works at link level. It helps users to navigate within the hyperspace of AHS by adapting presentation of hyperlinks. In answer to research Question 1, different methods and techniques used to implement adaptation in AHSs including adaptive hypermedia based educational systems have been reviewed. Methods for adaptive presentation include additional explanations, prerequisite explanations, comparative explanations, explanation variants and sorting. Techniques to implement these methods are conditional text, stretchtext, fragment and page variants and frame based technique. Methods for adaptive navigation include global guidance, local guidance, global orientation, local orientation and managing personalised view. Techniques to implement these techniques are direct guidance, link sorting, link hiding, link annotation and link generation.
Application areas of adaptive hypermedia (Brusilovsky, 1996a, 2001) include *educational hypermedia systems, on-line information systems, on-line help systems, information retrieval hypermedia systems and institutional hypermedia systems.*

Development of AHSs is not an easy task because it involves so many processes. That is why, different reference models for AHSs have also been designed in which AHAM (Adaptive Hypermedia Application Model), XAHM (XML-based Adaptive Hypermedia Model) and Munich Reference Model are included. Following procedures defined in these models, AHS can be developed using any programming language.
CHAPTER 3

ADAPTIVE HYPERMEDIA BASED EDUCATIONAL SYSTEMS

This chapter focuses on e-learning and adaptive hypermedia based educational systems (AHESs). Section 3.1 gives overview of e-learning, architecture of e-learning environment and its advantages. Section 3.2 describes AHESs and features of already existing systems including ELM-ART and JointZone. Section 3.3 discusses frameworks for the development of AHESs and gives comparison between them. Section 3.4 summarises the review of AHESs and describes the answer to Question 2 given in chapter 1.

3.1 E-learning

Socrates, the most ancient teacher to whom we are familiar believed that the best way to deliver knowledge is face to face teaching which he called “dialect” (Brandle, 1998). No doubt, this way of teaching is still most effective but with the passage of time it did not remain convenient for everybody because of drastic changes in the life, for example, if someone had to leave for another country or he or she was busy in business or family related activities then it was very difficult to continue studies. Then the concept of distance education was evolved in the eighteenth century when for the first time on March 20, 1728, Caleb Phillips gave an advertisement in Boston Gazette for learning shorthand at weekly basis through mail (Battenberg, 1971).
Universities and different colleges started their distance courses in nineteenth century when postal service was developed. The University of London (2009) became pioneer in this field in 1858. Visual material like motion pictures for distance education was begun to use in 1910s. Radio was started to use for this purpose between 1910 and 1920. Afterwards, TV came at horizon and then University of Iowa launched first educational television station in 1932 (Stevens-Long, & Crowell, 2002).

The use of electronic devices like radio, television and video caste players for educational purposes was the preliminary mode of e-learning. The revolution came in e-learning with the use of computers in 1960s (Harasim et al. 1995). First software “Coursewriter 1” for developing educational material was made in 1960 (Suppes & Macken, 1978). E-learning became more attractive and effective with the advent of internet in 1969 (Krol & Hoffman, 1993).

E-learning exists in two different shapes: synchronous and asynchronous. In synchronous e-learning, learners interact with other learners and instructors at the same time. It is possible through audio and video conferencing or chatting. In this way, not only a learner can ask instructor anything else but instructor can also monitor the performance of all learners. But in this mode, learning does not remain learner centred because learner can’t learn at his or her own pace. On the other hand, asynchronous e-learning is not simultaneous. Learner interacts with other learners and instructor whenever he or she gets free generally through e-mail. In this way, learner gains knowledge at his or her own pace. But there is a disadvantage that sometimes learner goes in isolation because there is no live conversation between learner and instructor (Hum & Ladouceur, 2001). Therefore combining synchronous and asynchronous technologies can make e-learning process more effective.
According to Moore (1989), three kinds of interactions are necessary during distance education. First one is interaction between learner and instructor for proper conversation and feedback. Second one is interaction between learner and course material which should be more convenient and interactive. Third kind of interaction is between all learners of a particular course. These three kinds of interactions are implemented in a better way in the mode of e-learning as compare to conventional mode of distance learning. With the help of these three kinds of interactions, e-learning provides virtual learning environment in which virtual class is replaced with conventional classroom but there should be each and everything just like real classroom.

3.1.1 Architecture of E-learning Environment

Architecture of e-learning environment or virtual class (Bučko, et al., 2005) can be divided into four parts: educational resources, evaluation, supplement environment and collaboration environment.

Educational Resources: These resources include course content comprises of learning objects like text pages, tutorials, simulations, audio and video clips etc. along with reference material. All topics of a course are presented in a proper order and according to the prescribed date and time. Reference material can be accessed by some hyperlinks provided by instructor or through properly designed digital library (Sun Microsystems, 2002; McCray & Gallagher, 2001) within the e-learning system.

Evaluation: Learner’s educational performance is measured by some evaluation tools like online tests and assignments. Both of them are launched according to the prescribed date and time. In return, learner’s marks and grades are also mentioned online.
**Supplement Environment:** For educational activities, learner requires some supporting tools within virtual environment. Online documents manager helps learner to manage his or her online educational material. Learner can also share such kind of material with other learners. Instructor can place messages related to course or upcoming activities on announcement board. Moreover instructor can place answers of frequently asked questions asked by learners on a separate FAQ portion. In the part of learners list, learner can find out information about other learners.

**Collaboration Environment:** This provides some communication tools to a learner to interact with instructor and other learners. Discussion forum provides a place where learners can leave their messages or opinions about one particular discussion topic. Chatting provides text based communication facility to all learners. Audio and video conferencing provide better communication environment but in developing areas where internet speed is not much better, such tools are not remained effective. E-learning facility provides passive mode of communication.

### 3.1.2 Advantages of E-learning

E-learning has changed all the parameters of distance education. Learning in a distance mode was a dull and dry activity but now e-learning is fast, convenient, self-paced and interactive. According to Hodges and Saba (2002), advantages of e-learning are not only for the learner but for the instructor and institute as well.

For the learner, e-learning is quite fast because he or she can learn at home or office and no need to travel towards college or university, so it not only saves time but travelling cost as well. E-learning can be adaptive and learner can also skip that study material which is already known to him or her (Hum & Ladouceur, 2001). E-learning also provide collaborative environment which is quite good in social point of view,
because learner gets opportunity to interact with learners belong to different communities and cultures (De Verneil & Berge, 2000).

For the instructor, study material for e-learning is prepared once and then used by so many times and by so many learners. Even that material can be shared easily with other instructors all over the world or within a specific community of instructors. Within e-learning, spectrum of interaction level of instructor with learners also becomes wider because instructor can communicate with huge number of learners at once. He or she can develop personal relationships with the learners as well.

E-learning is also beneficial at the institution level in terms of cost of delivering education. First of all, there is no need of huge buildings and classrooms. In this way, administrative kinds of activities are also reduced. Time to deliver education is reduced as well and institution can earn more by educating thousands of learners at once (Hum & Ladouceur, 2001).

### 3.2 Adaptive Hypermedia Based Educational Systems (AHESs)

E-learning can be more learners centred if adaptation is added to it because adaptive hypermedia helps in providing education according to the preferences and academic background of a learner. The origin of adaptive hypermedia based education is intelligent tutoring systems. The primary purpose of an intelligent tutoring system (ITS) was to provide problem solving support to the learners. With the advent of World Wide Web, research on online ITS was started. Then AHESs came into existence. These systems are also one of the types of ITS. These systems maintain student or learner model in which all the relevant information and preferences of a learner is saved for providing adaptation facility (Brusilovsky, 2000). Along with all facilities of learning, AHESs also provide adaptive educational material, adaptive
collaboration and adaptive testing. ELM-ART (Brusilovsky, Schwarz, & Weber, 1996b) and PT (Kay & Kummerfeld, 1997) are pioneers in this field. After them, we have so many other examples of AHES like JointZone (Ng, Hall, Maier, & Armstrong, 2002), AES-CS (Triantafillou, Pomportsis, & Georgiadou, 2002) and ActiveMath (Melis & Siekmann, 2004).

3.2.1 Episodic Learner Model – Adaptive Remote Tutor (ELM-ART)

ELM-ART (Brusilovsky, Schwarz, & Weber, 1996b; Weber & Brusilovsky, 2001) is an adaptive hypermedia based system for learning LISP programming language. It is an intelligent system because it intelligently analyses the solutions of different kinds of problems and provides help to solve them. It is basically an online version of ELM-PE (Weber & Möllenberg, 1994). It was developed using Common LISP HTTP server. Its overall structure is like adaptive electronic textbook which provided the base for the development of two authoring tools InterBook and NetCoach.

Domain model of ELM-ART consists of conceptual and procedural knowledge related to LISP programming language. Conceptual knowledge is declarative and presented in the shape of lessons and their subsections through web pages. Procedural knowledge is used for reasoning process to solve the problems related to LISP.

ELM-ART support multi layered overlay learner model. First layer describes that which pages explaining concepts are visited by a learner. Second layer stores data about those pages or tests related to one particular concept which are learned or passed successfully. Third layer deals with inferred concepts which are automatically considered as known. These concepts are advanced but marked as already learnt by the learner. Fourth layer describes those concepts which are already known by the
learner. In ELM-ART, a learner can check his or her educational performance in learner model. Figure 3.1 shows the learner model in which learner has facility to modify the status about concepts that whether they have been already known or not.

**Figure 3.1.** Learner model in ELM-ART.

ELM-ART supports different kinds of adaptation features including *direct guidance, link generation, link sorting and link annotation*. For *link annotation*, ELM-ART uses coloured balls attached with links. Description of link annotation is given in the Figure 3.2.

**Figure 3.2.** Description of link annotation in ELM-ART.
Learner is also given options to change the layout of the system. All the preferences are stored in the learner model. These preferences are shown in Figure 3.3.

![Options](image)

**Figure 3.3.** Individual preferences setting in ELM-ART.

ELM-ART offers different kinds of tests for assessment of learners. For practising programming functions of LISP, ELM-ART presents different programming problems to the learners. Then the solutions of these exercises are diagnosed by the system as well as shown in Figure 3.4.

![Figure 3.4](image)

**Figure 3.4.** Diagnosed solution of an exercise about calculating area of square.
Different kinds of communication tool are also offered by the system including messaging, chatting and discussion lists. Figure 3.5 shows the messaging window through which a learner can send message to his or her tutor.

![Messaging window for communication with tutor.](image)

**Figure 3.5.** Messaging window for communication with tutor.

If learner enters a web page which is not ready to be visited or if he or she has some problem to understand a concept then system provides prerequisite based help to him or her as well.

### 3.2.2 JointZone

JointZone (Ng et al., 2002) is an adaptive and interactive online educational system of rheumatology for medical learners. The project of this system was funded by Arthritis research campaign, UK and it was developed by Intelligence, Agent, Multimedia group, University of Southampton, UK.

It is just like an electronic textbook. Its domain model only consists of information related to rheumatology which is stored in XML files. Each file represents a web page. There are different images, videos and animations as well for clearing the concepts. Glossary is also the part of system for finding out the details of so many terms related to rheumatology.
User model of JointZone stores information related to the knowledge level of learner about domain. When learners login to the system for the first time then they are asked about their previous knowledge about rheumatology then they are assigned groups like beginning, intermediate or advanced level. Browsing history of each learner is also traced by the system for adaptive presentation. If learner spends an optimal time on one page then *effort index* of that page would be high otherwise that would be low. Optimal time is depended on the reading speed of learner which is measured by the *reading speed test*. This test is presented to the learner by the system at his or her first visit. At the basis of *effort index*, history based annotation of links is provided in which effort signs of high (H), medium (M) or low (L) are indicated with links. This *effort index* also helps to develop reading gauge through which learners can judge their educational performance.

JointZone provides interactive study with the help of case studies which are presented to the learners according to their levels. At the end of each case study, proper feedback is also given to learners (Maiera, Armstrongb, Halla, & Nga, 2005) as shown in Figure 3.6.

**Figure 3.6.** Feedback of a case study at beginner level.
JointZone also provides two different features of personalisation: **personalised site map** and **personalised topic map**. Personalised site map gives the whole picture of the performance of a learner. Figure 3.7 shows all the portions of system and number of read and unread documents by the learner.

![Personalised Site Map](image)

**Figure 3.7.** Personalised site map in JointZone.

Personalised topic map shows the concepts related to one particular topic selected by learner as short term goal. These concepts are shown with the sign of **effort index** as shown in Figure 3.8.

![Personalised Topic Map](image)

**Figure 3.8.** Personalised topic map of particular reading topic selected by the learner.
3.3 Frameworks for the Development of AHESs

Development of AHES is not an easy task because it involves so many processes including development of e-learning tools like discussion forums, chatting, emailing and digital library etc. Moreover, educationists who need to develop such kind of systems are not familiar with computer programming languages. It is a very difficult task for computer specialists as well to take care of all adaptation processes, user modelling and e-learning tools. Hence, some frameworks or authoring tools are available which provide shell for the development of AHES. These tools are very helpful for teachers to develop online adaptive courses quickly. If someone has background of web related programming languages then he or she can develop web pages manually in these tools. We have analysed the adaptation features and authoring environments of four authoring tools including AHA! (De Bra & Calvi, 1998), InterBook (Brusilovsky, Eklund, & Schwarz, 1998), NetCoach (Weber, Kuhl, & Weibelzahl, 2001) and COFALE (Chieu, 2007).

3.3.1 The Adaptive Hypermedia Architecture (AHA!)

The development of AHA! (De Bra & Calvi, 1998) was started in 1996 at Eindhoven University of Technology and its first version 1.0 was released in 2000 (De Bra, Aerts, Houben, & Wu, 2000). Then its second version 2.0 was released (De Bra, Aerts, Smits, & Stash, 1999) and now AHA! 3.0 (De Bra et al., 2003) is being used. It is an open source authoring tool which consists of Java servlets. Figure 3.9 shows the architecture of AHA!
Java servlets are activated when web pages are requested by the user. AHA! engine presents pages to the user from local or external servers. These pages are generated from combined domain and adaptation model. These are adapted as well. Along with this, user model or learner model is updated with the help of event-condition-action rules. All data related to domain, adaptation and learner models is stored as XML files or in a single MySQL database. AHA! runs on Tomcat server which is also open source.

AHA! supports overlay learner model. For each concept in domain model there is a concept attribute and Boolean attribute value in learner model. Whenever learner visits a concept then attribute value of that concept is changed, for example, whether a learner knows that concept or not. At the start of a course designed by AHA!, values of all attributes are “false”. There is another concept “Personal” in learner model which has also some attributes through which learners’ preferences and characteristics are described (De Bra, Stash, & Smits, 2004a).

**Course Authoring:** AHA! provides some authoring tools for course development which are not difficult in use (De Bra, Smits, & Stash, 2006; De Bra, Stash, Smits,
Romero, & Ventura, 2007). It provides *Graphical Author tool* for creating concepts and relationship between them as shown in Figure 3.10.

![Graphical Author tool window in AHA! (De Bra, Smits, & Stash, 2006).](image)

**Figure 3.10.** *Graphical Author tool* window in AHA! (De Bra, Smits, & Stash, 2006).

Conceptual structure of the application or course is saved as XML file. This structure can be made in any other tool as well which can generate XML files. The *Graphical Author tool* uses this conceptual structure for generating adaptation rules. Information content of the course can be made using any other authoring tool which can generate files of HTML, XHTML or XML. AHA! does not offer tool for creating information content. It only uses handlers for processing files of different formats. Along with this AHA! also supports images and media files of different formats. Usually, authors only use *Graphical Author tool* but if anyone wants to implement adaptation rules by himself or herself at low level then *Concept Editor* is also available.
For transferring pages of information content and other media files from author’s personal computer to AHA! server, the Application Management tool is used as shown in Figure 3.11.

![Application Management Tool](image)

**Figure 3.11.** Application Management Tool in AHA! (De Bra, Smits, & Stash, 2006).

It can also be used to activate Graphical Author tool and Concept Editor. For learner’s evaluation AHA! offers Test Editor for developing randomised or sequential adaptive multiple choice tests. AHA! also supports different layouts and skins. Multiple frame course can be developed through it in which every concept can be presented with different layouts.

**Adaptation Support:** Adaptation process in AHA! is based on conceptual structure saved in combined domain & adaptation model and information stored in learner model. AHA! supports both adaptive presentation and adaptive navigation (De Bra, Stash, & Smits, 2004b; De Bra, Smits, & Stash, 2006)

AHA! supports only two adaptive presentation techniques: conditional text and fragment variants. In advance, adaptivity of inclusion of external objects can also be implemented in the course. Another presentation style of fragments called
dimming fragments can be implemented as well through which either fragments get emphasised or dimmed according to the learner model.

In AHA!, adaptive navigation techniques can also implemented including link sorting, link hiding, link annotation and direct guidance.

### 3.3.2 InterBook

InterBook is used to develop adaptive online educational courses in the shape of textbooks. Its architecture is based on textbook ELM-ART, one of the pioneer adaptive systems. It was developed using CL-HTTP, Common LISP HTTP server. CL-HTTP is used to develop online intelligent systems (Brusilovsky, Eklund, & Schwarz, 1998). InterBook is only available for Apple Macintosh systems.

Domain model of an electronic text book developed by InterBook consists of all the educational concepts described by author. These concepts can be shown by two main windows provided by InterBook: Glossary Window and Textbook Window (Brusilovsky, Schwarz, & Weber, 1996). Glossary window is divided into two parts. Upper portion shows the glossary of concepts. Lower portion shows the pages in which required concept can be found along with the list of those concepts for which required concept is prerequisite as shown in Figure 3.12.
**Textbook Window** is designed to show the detail of required concept. It also shows the list of relevant concepts. Upper portion of this window shows the current position of the concept in the whole textbook. It is also used to open **Search Window**, **Help Window**, **Glossary Window** and **Content Window** as shown in Figure 3.13.

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**Figure 3.12.** Production details in *Glossary Window* (Brusilovsky, Eklund, & Schwarz, 1998, p. 296).

**Figure 3.13.** Detail of “production rules” and its position in textbook (Brusilovsky, Eklund, & Schwarz, 1998, p. 297).
InterBook supports overlay learner model for adaptive navigation support. When a learner gets registered in electronic textbook then he or she is asked to select experience level with online systems then the interface layout is changed according to this selection.

**Course authoring:** There are few steps to develop electronic textbook through InterBook (Eklund, Brusilovsky, & Schwarz, 1997). First of all, an electronic textbook is written in Microsoft Word file then it is converted into InterBook format. Textbook should be written in such a way that it can be recognised by InterBook. For this purpose, titles or subtitles in textbook should have text style like “Header 1”, or Header 2” etc. In this way, InterBook can easily develop adaptive annotation in the file. Then file is saved as RTF format which is converted to HTML file using any conversion tool. After this, extension of HTML file is changed to “*.inter”. Now file is recognisable by InterBook. The textbook material presented to the learners is based on learner model, learners’ level of knowledge about online educational systems and fragments extracted from original HTML files.

**Adaptation Support:** InterBook provides facilities to implement both adaptive presentation and adaptive navigation (Eklund, Brusilovsky, & Schwarz, 1997). For implementing adaptive navigation in the shape of local and global guidance, it provides direct guidance facility. It also supports adaptive annotation as shown in Figure 3.14.
As for as adaptive presentation is concerned, InterBook helps to implement prerequisite explanations in the shape of prerequisite-based help. If a learner has some problem to understand one particular domain concept then he or she must has problem in its prerequisite concepts so prerequisite-based help provides links of all those prerequisite concepts to learner (Brusilovsky, Eklund, & Schwarz, 1998).

3.3.3 NetCoach

NetCoach (Weber, Kuhl, & Weibelzahl, 2001) is based on Common LISP HTTP server. It is available for Microsoft Windows, Apple Macintosh and Linux. It is also used to develop online adaptive courses without knowing programming concepts (Brusilovsky, 2003b). In NetCoach, domain model consists of all the concepts related to a course. There are two types of these concepts: prerequisites and inferences.

Prerequisites are those concepts which are necessary to be learnt before reading one particular concept. On the other hand, inferences are those concepts which can be skipped to reach that specific concept (Henze, 2003).
NetCoach supports multi layered overlay model (Weber, 1999). In this model, first layer stores information that whether a learner has visited one particular concept. Status of different tests or exercises related to that concept is described by second layer that whether these tests has been passed successfully or not. Information about that concept as prerequisite or inference of any other concept is stored in third layer. Last layer stores information that whether learner is already familiar with the concept or not.

**Course Authoring:** NetCoach provides some editors to develop adaptive course (Weber, Kuhl, & Weibelzahl, 2001). In *Concept Editor*, author can describe all the concepts of a course and their relationship with each other as *prerequisites* or *inferences* in the shape of chapters as shown in Figure 3.15.

These concepts can be written in simple text form and then can be converted in HTML form in NetCoach. Already developed HTML files, images or flash animations can be uploaded to NetCoach. *Test Editor* helps author to design tests.
consist of closed or open ended questioners. System evaluates the result automatically.

In NetCoach, an author of a course has facility to register some tutors for facilitating learners regarding course. These tutors monitor performance of learners through online-interface. Tutors can add or remove learners and provide access privileges to them. They also manage different learners’ groups through which learners exchange their ideas and educational material.

**Adaptation support:** NetCoach supports both adaptive presentation and adaptive navigation (Weber, Kuhl, & Weibelzahl, 2001) as shown in Figure 3.16.

![Figure 3.16. Curriculum sequencing and link annotation in the adaptive environment developed by NetCoach (Weber, Kuhl, & Weibelzahl, 2001, p. 230).](image)

Adaptive presentation can be achieved by setting learning goals. If a learner wants to read only introductory portion of a course then only introductory concepts will be presented to him or her.

**Communication support:** Courses designed through NetCoach provide both synchronous and asynchronous communication facilities like email and chatting. Learners can not only interact with other learners but tutors as well. Tutors can provide help through help window. For open debate on something especial, tutors
place discussion topics in discussion list. Then learners give their opinion about that topic which can be read by everybody. Tutors can also send important messages to all learners. Learners receive it during studying the course or when they get login to it (Weber, Kuhl, & Weibelzahl, 2001).

3.3.4 Cognitive Flexibility in Adaptive Learning Environment (COFALE)

COFALE (Chieu, 2007) is an open source adaptive learning content management system (LCMS) which is based on ATutor (Adaptive Technology Resource Centre, 2009). COFALE is used to develop adaptive educational systems providing cognitive flexibility. It has been designed using PHP!. All data related to domain, adaptation and learners is stored in MySQL database. It runs on open source Apache server.

**Course Authoring:** COFALE provides some authoring tools for adaptive online course development. First of all, it provides *hypermedia editor* for development of HTML web pages through which domain content is shown to the learners. Using hypermedia editor, author can develop web pages in either plain text or HTML editor mode. Already developed web pages can also be uploaded. Different kinds of images can also be imported to COFALE. It supports stereotyped learner model because different modes are defined according to different groups of learners like novices or experts then they are assigned to each learner. Figure 3.17 shows *Learner Model Manager* through which models can be defined.
COFALE also provides Enrolment manager tool as shown in figure. This tool is used to cancel the enrolment of learners. Model of any learner can also be changed with it. For learners’ assessment, COFALE also provide Test manager tool for designing open and closed ended tests. Course author can also develop glossary consisting of terms related to domain. He or she can add relevant external resources like reference books or websites as well with the help of Resource manager.

COFALE supports both adaptive navigation and adaptive presentation. It provides direct guidance along with local or global view of content. Learner can also change the layout of the system by changing the options given for personal preferences.

With the help of My Tracker, learners can check their progress that how many pages he or she has visited and for how much time. My Test tool provides information that which tests have been cleared with how much marks.

**Communication Support:** COFALE also provide adaptive communication support because learner is given facility to find out peers according to his or her model for
interaction. Different communication tools are part of COFALE like discussion forums, chat, email and instant messaging facility to online learners as shown in Figure 3.18.

![Chat facility provided by COFALE.](image)

**Figure 3.18.** Chat facility provided by COFALE.

Author can create different forums for learners to share their views with the help of *Add Forum* tool.

### 3.3.5 Comparison of Frameworks

There are so many frameworks or authoring tools like KBS-Hyperbook (Nejdl & Wolpers, 1998), Multibook (Fischer & Steinmetz, 2000), WEAR (Moundridou & Virvou, 2001), MOT (Cristea & De Mooij, 2003) and ACCT (Dagger, Wade & Conlan, 2005) but we had selected AHA!, InterBook, NetCoach and COFALE because they are providing better features than others. After reviewing the features of these tools, we compare them with each other as shown in Table 3.1.
Table 3.1. Comparison of frameworks for the development of AHESs.

<table>
<thead>
<tr>
<th>Features of System</th>
<th>AHA!</th>
<th>InterBook</th>
<th>NetCoach</th>
<th>COFALE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platform Independent</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Adaptive Presentation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Additional explanations</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Prerequisite explanations</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Comparative explanations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explanation variants</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sorting</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adaptive Navigation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global guidance</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Local guidance</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Global orientation support</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Local orientation support</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Managing personalised view</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Communication Support</td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

Table 3.1 shows that InterBook does not support adaptive presentation as compared to AHA!, NetCoach and COFALE. It also not gives any communication facility. It is platform dependent as well. AHA! and NetCoach are platform independent and they also support both adaptive presentation and adaptive navigation. On the hand, AHA! does not provide communication facilities and NetCoach supports only one feature of adaptive presentation. So COFALE seems quite better because it supports maximum features. It is open source as well. After this comparison, we can easily conclude that COFALE is better option for the development of pedagogical adaptive hypermedia based educational system for mathematics at secondary level.
3.4 Summary

The best way of education is face to face teaching and learning but it did not remain convenient for everybody because of drastic changes in life. In this regard, distance education played a vital role but revolution came with e-learning. It replaced the conventional classroom based education with virtual learning environment. The whole mechanism of virtual learning should cover all the aspects of conventional mode of education. That is why, the architecture of e-learning can be divided into four parts: learning resources, assessment, supporting tools and collaborative learning environment. Learning resources include course content and reference material in multiple formats. Assessment can be done through online tests and assignments. Supporting tools include online documents manager, announcement board and online tracking of educational performance. Collaborative learning environment deals with communication tools like e-mail, chatting, audio & video conferencing and discussion forums.

E-learning can be learners centred if it becomes adaptive. AHES maintains learner or learner model to provide adaptive presentation and navigation to a learner. These adaptation features of two AHESs including ELM-ART and JointZone have also been reviewed.

Authoring of AHES is a lengthy and difficult process because it involves implementation of adaptation processes and learner model along with development of communication tools and digital library etc. To make this process shorter and easier, different authoring tools or frameworks are available. In answer to Question 2, features of four authoring tools including AHA!, InterBook, NetCoach and COFALE have been reviewed. They have been compared with each other and then finally
COFALE has been selected for the development of pedagogical adaptive hypermedia based system for mathematics at secondary level (PAHMS).
CHAPTER 4

METHODOLOGY

This chapter describes the methodology adopted for this research. Section 4.1 focuses on the procedure for the development of PAHMS. Section 4.2 gives strategy for survey research to ascertain the effectiveness of the system. Section 4.3 briefly describes population of the research. Section 4.4 describes a convenient sample of both in-service and student teachers. Section 4.5 explains the development of questionnaire as a research tool. Section 4.6 describes the validity and reliability of the research tool. Section 4.7 provides procedure to conduct survey research. Section 4.8 briefly explains the application of statistical test. Section 4.9 summarises the methodology adopted for this research.

4.1 Procedure for the Development of PAHMS

This research was done in two phases.

i) Development of pedagogical adaptive hypermedia based system for mathematics at secondary level (PAHMS).

ii) Survey research to ascertain the effectiveness of the system.

PAHMS is an adaptive hypermedia based educational system for the teachers of mathematics at secondary level in Pakistan. It provides pedagogy of mathematics according to the new syllabus with the support of adaptation. For the development of PAHMS, researcher adopted R&D research design. First of all, literature related to
adaptation and different techniques and methods used to implement it was reviewed. Then features of already existing adaptive educational systems were also observed. Afterwards, some authoring tools were compared with each other at the basis of features provided by them and finally an open source tool COFALE (see chapter 3) was selected for the development of PAHMS. Features of PAHMS and its implementation can be read in chapter 5.

The domain of PAHMS is pedagogy of mathematics at secondary level so it was also developed according to the new syllabus designed by the Ministry of Education (2002), Islamabad, Pakistan. It includes application of famous teaching methods in different branches of mathematics (see chapter 6). Different learning objects like images, graphs and tables for the pedagogy were also designed. To assess the performance of the learners, online tests were also designed as the part of PAHMS (see appendix A).

4.2 Strategy for Survey Research

To ascertain the effectiveness of PAHMS, an online survey was conducted from different teachers of mathematics at secondary level in Lahore city, Pakistan. PAHMS provides adaptive learning environment to both experienced teachers and novices. That is why two types of teachers were selected for survey: In-service teachers and student teachers.

4.3 Population

All the teachers of mathematics at secondary level and student teachers enrolled in B.Ed. and M.Ed. with specialisation in mathematics education in Lahore city.
4.4 Sampling

A convenient sample of hundred teachers of mathematics at secondary level was selected for survey. Fifty out of them were in-service and teaching in different schools of Lahore. The names of those schools are given below:

- Saint Anthony’s High School, Lawrence road, Lahore.
- Queen Mary College, Durand road, Lahore.
- Govt. Central Model High School, Rattigan road, Lahore.
- Govt. Central Model School, Lower Mall, Lahore.
- Govt. Muslim Model High School, Urdu Bazar, Lahore.
- Govt. Practising Girls High School, Township, Lahore.

Rest of the fifty were student teachers who were enrolled in B.Ed. in University of Education, Lower Mall campus, Lahore.

4.5 Research Tool

To ascertain the effectiveness of PAHMS, researcher developed a questionnaire consisting of twenty five statements on five point Likert scale to get the response of the teachers after using PAHMS. This scale consists of five options including strongly agree, agree, undecided, disagree and strongly disagree against each statement (see appendix B). Each statement of the questionnaire is related to one particular feature of PAHMS. These statements were further divided into three clusters. Statements 1 to 10 were related to the learning environment of PAHMS.
Statements 11 to 20 were related to the adaptation features of the system and statements 21 to 25 were related to the pedagogy provided by PAHMS.

4.6 Validation and Pilot Testing

Face validity of the questionnaire was checked by five experienced teachers of pedagogy. Construct validity was assured considering specifications of pedagogical online system described by Giguere and Minotti (2003) and taxonomy of adaptive hypermedia technologies described by Brusilovsky (2001). Pilot test of questionnaire was conducted from 10 teachers of mathematics to make it reliable and valid. These teachers are teaching at Saint Anthony’s High School, Lawrence road, Lahore and EAST (Educational Assessment System & Training) Lower Mall, Lahore. After pilot testing relatively less effective questions were removed from questionnaire. Cronbach’s Alpha value of questionnaire is 0.823.

4.7 Procedure for Survey Research

The online survey research was administered by the researcher himself. He kept in contact with all the teachers through e-mail, discussion forums and chatting available in PAHMS. Then questionnaire was sent to all of them through e-mail and get back in printed form through mail with the school stamps on it.

4.8 Application of Statistical Test

Data were collected from the teachers to check the effectiveness of PAHMS then analysed after applying “one-sample t-test” by computing mean response values for each statement of the questionnaire, their clusters and all of them collectively.
4.9 Summary

R&D approach has been adopted for the development of PAHMS. It has been developed using the framework of COFALE, an open source authoring tool. PAHMS provides pedagogy of mathematics at secondary level which has also been developed. To ascertain the effectiveness of the system, survey was conducted from hundred teachers through a questionnaire designed at five point Likert scale. Validity and reliability of the questionnaire were also checked. Both in-service and student teachers of Lahore city were selected for the survey and then data collected through survey research were analysed after applying “one-sample t-test”.
CHAPTER 5

PEDAGOGICAL ADAPTIVE HYPERMEDIA BASED SYSTEM FOR MATHEMATICS AT SECONDARY LEVEL (PAHMS)

This chapter focuses on the adaptive learning environment of PAHMS and development of learning objects for pedagogy of mathematics. Section 5.1 gives brief introduction of PAHMS and tools provided by it to the learners and instructor. Section 5.2 describes adaptive environment of PAHMS including adaptive presentation, adaptive navigation and adaptive peers’ searching. Section 5.3 focuses on learning objects used in the system including instructional, collaborative and assessment objects. Section 5.4 summarises the development of PAHMS and describes the answer to Question 3 given in chapter 1.

5.1 PAHMS

PAHMS (www.pahms.pk) is an adaptive learning system that provides pedagogy of mathematics at secondary level. PAHMS has been developed using an open source framework of COFALE (Chieu, 2007). COFALE is a free learning content management system (LCMS), through which online courses can be designed for all kinds of domains. PHP has been used to develop COFALE. It supports MySQL 3.23.x for database creation and uses Apache 1.3.x as a web server. It is distributed under the General Public License of GNU. According to GNU, free softwares can be
shared or changed by anybody according to his or her needs. PAHMS has been
developed by incorporating pedagogy and defining relationship between pedagogical
content for adaptation in the framework of COFALE. Technical review of COFALE
can be seen in appendix D. Pedagogy of mathematics has also been developed
following the curriculum designed by Ministry of Education, Islamabad, Pakistan (see
chapter 6). Comparison of PAHMS with an existing adaptive e-learning system
GRAPPLE Tutorial can be seen in appendix E.

5.1.1 Login and Registration

PAHMS is open for everybody. It can be accessed through just login to the system as
shown Figure 5.1.

![Login window of PAHMS.](Figure 5.1)

Every new learner has to register by entering some particulars about him or
her as shown in Figure 5.2.
Three kinds of users may access this system: learner, instructor and administrator. Learner can only access the pedagogical content and learner tools until he or she is given privileges to manipulate the learning environment by the instructor. In our case, learners are teacher including in-service or student teachers. Instructor and administrator logins have been made at the time of installation of COFALE. Instructor has the right to design the course including lessons, tests, glossary and other learning objects using instructor tools. Administrator has all kinds of privileges. He or she can manipulate the learning environment of COFALE.

Welcome page provides brief introduction of PAHMS and some basic instructions to the learners. System has very simple user interface. That is why, it is not difficult for anybody to understand the learning environment.

5.1.2 Tools Provided by PAHMS

PAHMS provides three kinds of tools: student tools, instructor tools and administrator tools. Student tools are helpful for learners in which communication and assessment tools are included. Some facilitating tools like My Tracker and Search are also part of
these tools. Through *My Tracker*, a learner can observe his or her educational performance. Through online search, learner can find any specific pedagogical content. Online *Help* is also available to understand the use of different tools provide by PAHMS.

Instructor tools are available for instructor to develop lessons, tests, glossary, discussion forums, announcements and learner models. Instructor also defines relationships of content with learners’ models to implement adaptation in PAHMS. He or she also manages enrolment of learners and monitor their educational performance.

Different options are available to administrator as well. Administrator has the full authority to manipulate the whole learning process. The status of a learner and instructor can also be changed by him or her. Administrator has rights to customise the environment as well.

### 5.2 Adaptive Environment of PAHMS

PAHMS supports both adaptive presentation and adaptive navigation. The whole adaptation process revolves around *Learners’ Mental Model* (user model). PAHMS provides pedagogy to two kinds of teachers. At the basis of these two kinds, two learners’ models have been developed: *In-Service Teacher* and *Student Teacher*. These models can be re-edited. Moreover, other learners’ models can also be developed using *Learner Model Manager* in instructor tools (see Figure 5.3).
Figure 5.3. Defining **Student Teacher** model.

Every teacher is assigned a model and then pedagogical content is shown to him or her according to that model. **Student Teacher** is a default model but if someone is in-service then he or she can change the model to **In-Service Teacher** through an option of **My Mental Models** in learner tools as shown in Figure 5.4.

Figure 5.4. Changing model from **Student Teacher** to **In-Service Teacher**.

Both adaptive presentation and adaptive navigation are based on these models.
5.2.1 Adaptive Presentation in PAHMS

PAHMS supports two methods for adaptive presentation including *additional explanations* and *prerequisite explanations* with the help of *fragment and page variants* technique. As far as *additional explanations* are concerned, all the lessons about teaching methods have been developed in two forms. Lessons in detailed form are shown to the student teachers because they have not much knowledge of pedagogy. On the other hand, in-service teachers are shown the same lessons in brief form.

*Prerequisite explanations* are given through the option of *Related Topics* in the system (see Figure 5.5). This option shows the links of prerequisite concepts to the current webpage along with those links which are compulsory to be visited afterwards.

![Figure 5.5. Related Topics showing “pedagogy of mathematics” as prerequisite to heuristic method of teaching.](image)
5.2.2 Adaptive Navigation in PAHMS

PAHMS supports all methods for adaptive navigation including global guidance, local guidance, global orientation support, local orientation support, managing personalised views with the help of direct guidance and link sorting techniques.

Global guidance is supported with the help of Global Menu and Contents option in the system (see Figure 5.6). Global Menu provides the list of all web links that can be accessible from current lesson. Contents are shown at the top of each lesson showing the next most relevant links that can be followed.

Local guidance is supported by indicating the next most appropriate link from the current web pages. Figure 5.6 shows the local guidance with the help of Next option at the top of the web page indicating “application of deductive method in sets” as the most appropriate link after visiting the lesson of deductive method.

Local orientation support is provided with the help of Local Menu (see Figure 5.6) through which a learner can find his or her location with reference to most
relevant links from the current webpage. Option of *Related Topics* is also helpful in this regard.

*Global orientation support* is provided with the help of *Site-map* through which a learner can easily find out his or her location in the whole learning space as shown in Figure 5.7.

![Site-map](image)

**Figure 5.7.** *Site-map* indicating the position of the learner who is reading the lesson of project method.

Learners can also manage personalised view by changing the layout of the system according to their needs as shown in Figure 5.8.
5.2.3 Adaptive Peers Searching

PAHMS provides adaptive peers (learners) searching facility for collaborative study with the help of Peers’ Learning Hyperspace option in learner tools (see Figure 5.9). A learner can search all other learners enrolled in PAHMS. He or she can search more appropriate learners related to his or her mental or learner model.
The constraints for learners’ models can be defined in Learner Model Manager by the instructor as shown in Figure 5.10.

At this moment, in-service teachers can search other in-service teachers only as appropriate peers in PAHMS. The same case is with the student teachers.

5.3 Learning Objects and their Implementation

A learning object is a re-usable piece of information or content package that fulfils the requirements of one specific learning objective. Learning objects include instructional, collaboration and assessment objects (ASTD & SmartForce, 2002).

5.3.1 Instructional Objects of PAHMS

These objects provide pedagogical content to learners including lessons, web resources, glossary and document management tools.

Lessons: Application of teaching methods in mathematics is explained by different lessons. Each lesson exists in two forms. These lessons are shown in detail form to the student teachers and for in-service teachers; they have been designed in brief
form. *Edit Content* option is available for instructor to design lessons (see Figure 5.11). Already designed lessons by the instructor can also be imported.

![Figure 5.11. Designing lesson of lecture method.](image)

Different properties of all lessons have been defined here like release date, keywords and its relationship with other topics as shown in Figure 5.12.

![Figure 5.12. Defining the keywords and relationship of lecture method with other topics.](image)
Defining relationship of a lesson with other topics helps in adaptation process. After defining relationships of all lessons, they have also been associated with relevant models. Lessons in detailed form have been associated with Student Teacher model and lessons in short form have been associated with In-Service Teacher model (see Figure 5.13). The whole process to define relationships is required to implement adaptive presentation of content in PAHMS.

Figure 5.13. Associating appropriate content with the relevant model.

Different images, tables and graphs have also been designed for pedagogy of mathematics. These learning objects are part of the lessons as shown in Figure 5.14.

Figure 5.14. Geometry related diagrams as learning objects.
**Web Resources:** These are quite helpful for learners to get further knowledge about pedagogy (see appendix C). At the end of each lesson, these resources are available as external web links as shown in Figure 5.15.

![External Web Links](image)

**Figure 5.15.** External web links for the lesson of deductive teaching method.

Along with web links, references of books and research papers are also given at the end of each lesson.

**Glossary:** It consists of explanation of different terms related to teaching methods and mathematics as shown in Figure 5.16.

![Glossary](image)

**Figure 5.16.** Definitions of important terms in glossary.
**Document Management Tools:** In such tools, *My Own Content, My Own Concept Map* and *My Own Summary* are included. Learners are provided these tools to manage educational material or notes by themselves. These tools are similar in nature but separately available for the convenience of learners.

### 5.3.2 Collaboration Objects of PAHMS

These objects include different communication tools like discussion forums, inbox and chat as shown in Figure 5.17.

![Figure 5.17. Communication tools for collaborative study.](image)

Discussion forums can be generated by the instructor only and then learners can debate on the topics provided by him or her. Instructor can put both general and specific questions for the convenience of learners. They can use those questions in forums. Some questions are designed specifically related to pedagogy using the option of *Discussion Questions* as shown in Figure 5.18.
5.3.3 Assessment Objects of PAHMS

Assessment of learners is done through online tests in PAHMS (see Figure 5.19). Tests have been designed for all teaching methods (see appendix A).

Figure 5.18. Questions designed by instructor related to pedagogy.

Announcement is another communication tool provided to the instructor only. It is used to convey some important messages to all learners. Announcements are shown at homepage, whenever some one login to the system.

Figure 5.19. Online tests related to pedagogy of mathematics in PAHMS.
After finishing test, result is appeared at once. Instructor can design or edit both open and closed ended tests with the help of Test Manager.

5.4 Summary

PAHMS is an adaptive hypermedia based educational system that provides pedagogy of mathematics to both in-service and student teachers according to their educational background. An open source tool COAFEL has been used to develop this system. PAHMS supports both adaptive presentation and adaptive navigation. It provides adaptive peers searching facility as well. In answer to research Question 3, different learning objects for pedagogy of mathematics have been designed including instructional, collaboration and assessment objects. Instructional objects include lessons and glossary. Collaborative objects include communication tools which are provided by the framework of COFALE. Assessment objects are used to assess the educational performance of the learners. They include online tests related to all teaching methods. These objects can be edited by instructional tools provided to the instructor.
CHAPTER 6

PEDAGOGY OF MATHEMATICS AT SECONDARY LEVEL

This chapter focuses on pedagogy of mathematics at secondary level in Pakistani context. Section 6.1 provides overview of pedagogy and theories related to it. Section 6.2 describes professional standards for teacher in Pakistan. Section 6.3 focuses on the aims of teaching mathematics. Section 6.4 describes pedagogy of mathematics at secondary level according to the curriculum designed by Ministry of Education, Pakistan. Sections from 6.5 to 6.13 describe application of different teaching methods in mathematics including lecture method, inductive method, deductive method, heuristic / discovery method, analytic method, synthetic method, problem solving method, laboratory method and project method. Section 6.14 summarises the pedagogy of mathematics at secondary level and discusses the answer of research Question 4 given in chapter 1.

6.1 Pedagogy

It is referred to as “the art or science of teaching” (Agnes, 2003, p. 475). This word came from a Greek word “παιδαγωγός” or “paidagogos” (Wikipedia, The Free Encyclopedia, 2009b) which is the combination of “pais” means child and “ago” that means “to lead”. The literal meaning of “paidagogos” is “child leader” (Forlines, 2001, p. 244). In modern days, the word pedagogy is used as an alternate of teaching
or educating the children. Along with this, another word andragogy is used which means teaching adults (New World Encyclopedia, 2009).

Pedagogy basically refers to the teaching strategies or methods. Every teacher should adopt proper teaching method according to the situation, learning environment and educational background of students. Pedagogy also refers to the interaction between teacher and students which is very important to encourage the students and for giving feedback about their educational performance (Siraj-Blatchford, Sylva, Muttock, Gilden, & Bell, 2002).

Pedagogical strategies or methods are based on psychological learning theories. These theories help to understand the learning process and interest level of students. That is why, all teaching methods are designed according to these theories which include behaviourism, cognitivism and constructivism (New World Encyclopedia, 2009).

**Behaviourism:** This theory is based on conditioned behaviour. It was described for the first time by famous psychologist John Broadus Watson (1913). Then major developments in this theory were made by Burrhus Frederic Skinner, when he related it with pedagogy and his new philosophy was called “operant conditioning”. According to him, behaviours of learners can be changed or reshaped with the help of positive and negative reinforcement (Skinner, 1974).

**Cognitivism:** This theory is based on cognition or the mental process to think, learn and remember (Meadows, 1993). It is comprised of two concepts: “schema”, an internal structure in which information is stored and the process to memorise that information at short or long term basis. When new information enters the schema then it is compared with already existing information. Schema is updated or modified because of that information. If the information is meaningful then it is memorised
easily for long term basis. In this way, cognitive theory deals with learning in terms of processing and storing the information. Jean Piaget, a famous psychologist contributed a lot to develop this theory in 1920s but his work was recognised in late 1960s. Then cognitivism was replaced with behaviourism but still cognitive theorists gave much importance to reinforcement (Mergel, 1998).

**Constructivism:** Bartlett (1932) was the first psychologist who gave the idea of constructivism. According to this theory, learners construct their own knowledge by utilising previously learnt concepts. In collaborative learning environments, they share the ideas and information and enrich their knowledge base. Teacher only facilitates the learning process. Many teaching methods like heuristic method, laboratory method and project method are based on constructivism.

### 6.2 Professional Standards for Teachers in Pakistan

In all countries, ministries of education or independent private organisations develop and maintain professional standards for teachers to enhance the educational performance. All teachers are bound to follow these standards to achieve better results in teaching and learning in their national and cultural context. These standards enable teachers to apply pedagogical strategies or methods in better way. Training and Development Agency for Schools (2007) has developed the standards for the professional development of teachers of England. Similarly, National Board for Professional Teaching Standards (2002), an independent private organisation has developed standards for American schools but these standards are not applicable to all Americans schools because there are so many other organisations working in this field. In Pakistan, these standards at government level has been designed by ministry of education for all the teachers who are working in government schools or those
private schools who are affiliated with government secondary or higher secondary boards. There are the following national professional standards for Pakistani teachers (Ministry of Education, 2009).

**Subject matter knowledge:** All the teachers should have thorough knowledge about the subject or content of curriculum. It is also very necessary to familiar with latest developments related to the subject. They should also help students to acquire and learn knowledge. Students should also be taught that how the basic concepts of the subject can be applied in real life scenarios and how these concepts are related with other disciplines of education.

**Human growth and development:** Teachers should know about each and every student that how he or she gains and constructs the knowledge. All the students should be encouraged and treated equally. Different kinds of thinking and learning styles should be appreciated and try to promote creativity among students. Teachers should also encourage social interaction and cooperative learning among them.

**Knowledge of ethical values/social life skills:** Ethical values include justice, equality and tolerance about which every teacher should be familiar. They should also be familiarised with the importance of peace all over the world and the dialogue which is the only solution of any kind of conflict. Students should be taught without any discrimination of race, gender, faith, cast or class. Different kinds of societies should be made inside institutes where students can openly express their views about ethical values and teachers should try to guide them that how they can practise these values in their social life.

**Instructional planning and strategies:** First of all, the aims and objectives to teach curriculum must be cleared. It should also be known that what kind of relevant material is available and how can new material be prepared. Teachers should have
also full command on instructional strategies or methods which must be adopted according to the learning environment and the nature of syllabus. Classroom management is another important aspect for all the teachers through which classroom discipline can be maintained.

**Assessment:** Teachers should know that how can students’ learning be assessed with the help of open and closed ended tests and assignments to improve the teaching and learning process. To give feedback at right time to students and their families is also very important. Along with this, encouraging students improves overall performance of them. Moreover, teachers should keep changing their plans and teaching methods according to the assessment results for improving the learning process. Students should also be enabled to judge their own progress.

**Learning environment:** Teachers should try to create such environment in which students may interact with each other in a good manner and cooperative learning can be promoted. Competitive environment should be developed as well for improving students’ performance. Utilising the time by the teachers in a proper way is also very important to make the learning environment affective and interesting.

**Effective communication and proficient use of information communication technologies:** Way of expression and presentation skills play vital role in teaching. All the teachers should practise to speak and write clearly. Pronunciation of every teacher should be correct and he or she should have full command on speaking and writing local language, Urdu and English. Use of information communication technologies became also important to make the learning process fast and affective. All teachers should be able to use computer properly to show relevant images, animations and videos to the students. They can also use it for the preparation of tests, assignments and papers for students’ assessment.
**Collaboration and partnerships:** Parents and guardians of students should also be considered as home teachers, so that all teachers must be in contact with parents regarding educational progress of their children. Parents should be informed about strengths and weaknesses of their children and they should also be guided that how they can help in the learning of children.

Professionals can also be helpful for teaching and learning process. It is the responsibility of teachers to remain in contact with their subject related professionals who can be invited on special request to share their ideas and experiences in seminars or workshops organised by school administration. These seminars or workshops can be organised in collaboration with different NGOs or societies working in educational sector. Teachers should try to develop good relationships with them.

**Continuous professional development and code of conduct:** All the teachers should give proper attention to maintain their personal portfolios if they want to excel in the field of teaching. Professional growth of teachers is directly associated with teaching and learning. For this purpose, they should keep exchanging ideas with their colleagues and professionals and try to engage themselves in educational research. To maintain the peaceful educational environment inside the institutes or schools, they should also strictly follow code of conduct and behave in the limits of professional ethics.

### 6.3 Teaching of Mathematics

The word mathematics came from a Greek word “μάθημα” or “máthēma” which means *science* or *study*. “Mathematics is the science and study of quantity, structure, space, and change” (Wikipedia, The Free Encyclopedia, 2009a, para. 1). Mathematicians try to examine patterns of numbers, shapes, reasoning, motion and
chance to generalise new formulas or methods in different branches of mathematics (Devlin, 2004).

Before teaching mathematics, every teacher should be informed well about the educational values of it. Then it is also very important to keep the motivational level of students high. Otherwise students lose interest in mathematics (Butler & Wren, 1965). Students can be motivated by highlighting the importance of mathematics that how it is necessary to learn other subjects. Moreover, students can avail good employment opportunities in their future life because of applicability of mathematics in so many fields (Rani, 2007).

It should also be clear to the teachers that what they have to achieve through teaching of mathematics. There are the following aims of teaching mathematics to the students (Cornelius, 1982; Sidhu, 1995).

- To develop reasoning in thinking process of the students.
- To enable students to do different kinds of calculations for daily life problems.
- To make them creative by developing analytical and discovering abilities in them.
- To enable them to learn other subjects of science or general science.
- To prepare them for higher studies.
- To develop scientific approach in them to understand the realities of life on the base of logic.
- To enable them to find out the similar patterns in one particular activity or phenomenon for generalising the results from them.
- To prepare them for all those fields of life in which mathematics is applicable.
- To develop the ability in them to think and concentrate on one particular thing.
6.4 Pedagogy of Mathematics at Secondary Level in Pakistan

Curriculum of mathematics at secondary level has been developed for grade 9 and 10 by Ministry of Education, Government of Pakistan. Pakistan has four provinces: Punjab, Sindh, Balochistan and North West Frontier Post (N.W.F.P). Each province has its own textbook board which publishes textbooks following this curriculum. Textbooks for secondary and higher secondary level are taught in all government and private schools in Pakistan except those schools which are affiliated with boards of the United Kingdom. National curriculum for mathematics at secondary level (Ministry of Education, 2002) has been divided into different small units in which the benchmarks are clearly mentioned. These units are designed for different branches of mathematics like number & operations, algebra, geometry, information handling and trigonometry.

Pedagogy for national curriculum of mathematics is consisted of application of teaching methods in mathematics. Teaching methods include lecture method, inductive method, deductive method, heuristic / discovery method, analytic method, synthetic method, laboratory method, problem solving method and project method.

6.5 Lecture Method

This is a method to deliver the knowledge through speech. Teacher speaks and students listen to him or her. This is the most important teaching method because if the teacher adopts any other way of teaching then lecture would be remained part of that. In this method, teacher behaves like an active participant and students are at receiving end most of the time. That is why; it is a teacher centred approach. This is also referred to as direct instruction, training model (Joyce, Weil, & Shower, 1992), active teaching (Good, Grouws, & Ebmeier, 1983) and explicit instruction
(Rosenshine & Stevens, 1986). Lecture method is not only used for teaching concepts about something but for giving training of complex skills and procedures as well.

### 6.5.1 Procedure of Lecture Method

According to Arends (1994), there are three stages in lecture method: Planning the lecture, delivering the lecture and post instructional tasks.

**a) Planning the Lecture:** It includes selection of concepts to be taught, defining the concepts and analysis of them. Usually, the main source of concepts is the textbook from where these concepts are selected and arranged according to daily lesson plans. Teacher only focuses on the concepts of one particular lesson plan.

After selecting the concepts, teacher focuses on the appropriate definitions of those concepts. Sometimes, appropriate definitions are not given in the text books, so teacher should also consult other reference books. Defining a concept includes the identification of correct name of the concept and all the attributes associated with it. Then the comprehensive definition of one particular concept can be finalised (Merrill & Tennyson, 1977). Then the analysis of a concept is started in which teacher finds out the relevant examples which should be easy and related to the previous knowledge of the students. Charts and figures can also be selected as visual examples for complicated concepts.

**b) Delivering the Lecture:** Teacher should start the lecture by telling the main objectives of a lesson. These objectives can be written in short form on the corner of black or white board. Then it should also be told that how these objectives are related to the previous concepts and where they will be used in next lessons. Teaching a concept also requires thorough knowledge of that concept and proper rehearsal to deliver the lecture.
Teacher should also keep in mind that how student can practise the concepts and procedures. During the lecture, teacher can assign short duration practices to the students under his or her guidance. In the case of teaching complex concepts, teacher should assign the practice divided into small segments. Such kind of practice is more beneficial in refining previously learnt concepts.

During the lecture, it is very necessary to provide feedback to the students after the practice. That should be in time and specific. It is also necessary to praise the students while they are learning new concepts. In the case of providing negative feedback, teacher should also guide that how a student can adopt the correct procedure. Teacher should make sure that students are focusing at the procedure not at the result. Along with this, students should also be taught in such a way that they can assess the performance themselves.

c) Post Instructional Tasks: These tasks include giving independent practice to the students and testing their understanding about the concepts. Independent practice includes homework or assignments. Their feedback should also be given within appropriate time to the students.

It is also very necessary to assess the understanding level of the students about concepts through different kinds of tests after finishing a lesson because if students fail to pass those tests then it would be difficult to move towards next advance lessons.

6.5.2 Usage of Lecture Method

Lecture method can be used for the following purposes (Singh, 2007).

- To introduce new concepts or chapters.
- To motivate the students.
• To provide important instructions about new concepts or lesson.
• To conclude the lesson.
• To give answers to the queries.
• To deliver the experiences from real life.
• To provide some important knowledge or information which is not given in the textbook.
• To give historical background of one particular concept.
• To address the huge number of students in the classroom.

6.5.3 Guidelines for Delivering Effective Lecture

There are the following guidelines for making the lecture effective (Singh, 2007; SRA/McGraw-Hill, 2008).

• The concepts should be analyzed carefully before lecture to avoid any hurdle in presenting them.
• Divide a concept in small segments, so that students can understand them easily.
• Important points and definitions should be written clearly on the board.
• Most suitable examples should be given to the students.
• Teachers should keep asking questions so that students remain active.
• Always multiple examples should be given.
• Correlate the previous concepts with new ones.
• Tests should be taken continuously to assess the performance.
• Teacher should speak clearly.
• Speed to deliver the lecture should not be too slow or too fast.
• To make the lecture interesting, teacher can use audio-visual material.
• At the end of the lecture, that should be summarised.

6.5.4 Merits and Demerits of Lecture Method

Lecture method has some merits and demerits as well.

Merits: Lecture method has the following merits (Sidhu, 1995; Sellers, Roberts, Giovanetto, Friedrich, & Hammargren, 2007).

• This is the most convenient and easy method.
• This is the fastest way to deliver knowledge so when the syllabus is so heavy then it becomes necessary.
• When the strength of the class is very high then this is the only solution.
• This is so economical as well because there is no equipment involved in it and only one teacher can teach so many students.
• This is very helpful to introduce the new concepts.
• This can be used to raise the interest level of the students.

Demerits: Lecture method has the following demerits (Singh, 2007; Sellers et al., 2007).

• This is a teacher centred approach so students cannot play an active role.
• This method does not develop reasoning and thinking ability in the students.
• Sometimes, this method becomes boring because there is no activity involved in it.
• In this method, teacher-student relationship is not developed to much extend.
• This method is useful in higher classes only.
• Teacher needs to enhance writing and communication skills.
6.5.5 Application of Lecture Method in Secondary Mathematics

As no practical work is involved in this method, so it can only be used to clarify the basic concepts of each unit given in the textbooks. That is why, it is applicable to teach all branches of mathematics including sets, logarithms, algebra, matrices, statistics, geometry and trigonometry. Mathematical problems related to these branches of mathematics can not be solved by this method but the procedures and methods to solve them can be explained by this. The relevance and applicability of these branches to real life and their historical perspective can also be explained by this method.

6.6 Inductive Method

This method is also called the scientific method in which we proceed from known to unknown, specific to general and from example to rule or formula. In this method, induction is used which means students are presented some similar examples related to one particular domain. Then students try to establish a formula, rule, law or principal by observing those similar examples. If that generalised result is true for those similar examples then it would also be true for all such examples (Sidhu, 1995).

6.6.1 Procedure of Inductive Method

According to (Singh, 2007; Landmark, 2005) there are four steps to reach one particular formula, rule or law through inductive method: Presenting the similar examples, observation, generalization and verification of the result.

a) Presenting the Similar Examples: Here the teacher presents some similar examples or questions before students then students get solution of those examples.
b) **Observation:** After solving those examples, students are asked to find out the common elements among those examples.

c) **Generalisation:** With the help of common elements among similar examples, students try to generalise the result in the shape of one particular rule, formula, law, principal or method.

d) **Verification of Result:** After generalising the rule or formula, students are helped to verify the result with some more examples or any other method.

### 6.6.2 Merits and Demerits of Inductive Method

This method has also some merits and demerits (Neubert & Binko, 1992; Sekhar, 2006).

**Merits:** This method has the following merits.

- This method is useful to introduce a new concept because a formula or rule associated with that concept is also established at that stage.
- Students who like the inductive approach can infer the more complicated rules or formulas (Felder, 1993).
- This is a student centred approach because students play active role in it.
- As the students establish the laws and principles by themselves so this gives them confidence.
- This method helps to motivate the students to think logically and make the learning environment more interesting.
- This is based on reasoning and experimentation.
- This is quite suitable for primary and secondary level classes.
- Students easily remember the laws or principles which they prove by themselves.
**Demerits:** This method has some demerits as well.

- This method is quite time consuming and laborious as well.
- To establish a law or principle is not the complete process of learning. Students have to practise a lot to understand the concept fully.
- Sometimes a formula or rule proved by this method with the help of some examples does not applicable in some other similar cases.
- Only experienced teachers can use this method in a right way.
- This method does not help in developing problem solving ability in the students.

6.6.3 Application of Inductive Method in Secondary Mathematics

Inductive method is only used to establish laws, principals, formulas and methods instead of solving mathematical problems. Therefore it can be used in all branches of mathematics but at the secondary level, establishing laws or formulas is only involved in algebra, matrices and to some extant geometry.

6.6.4 Application of Inductive Method in Sets

**Problem:** Prove the following associative property of union of sets.

\[(A \cup B) \cup C = A \cup (B \cup C)\]

**Solution:**

a) **Presenting the similar examples:** Teacher will present some examples of similar nature to prove this property.

**Example (i):** If \(A = \{1,2,4,8\}, B = \{2,4,6\}\) and \(C = \{3,4,5,6,7\}\)

Then show that \((A \cup B) \cup C = A \cup (B \cup C)\)

\[
A \cup B = \{1,2,4,8\} \cup \{2,4,6\}
\]
\[(A \cup B) \cup C = \{1,2,4,6,8\} \cup \{3,4,5,6,7\}\]
\[(A \cup B) \cup C = \{1,2,3,4,5,6,7,8\} \quad \ldots \quad (1)\]
\[B \cup C = \{2,4,6\} \cup \{3,4,5,6,7\}\]
\[= \{2,3,4,5,6,7\}\]
\[A \cup (B \cup C) = \{1,2,4,8\} \cup \{2,3,4,5,6,7\}\]
\[= \{1,2,3,4,5,6,7,8\} \quad \ldots \quad (2)\]

Thus from (1) and (2), students can conclude that
\[(A \cup B) \cup C = A \cup (B \cup C)\]

**Example (ii):** If \(A = \{a,b,d\} , B = \{c,e,f,h\} \) and \(C = \{g,i\}\)

Then show that \((A \cup B) \cup C = A \cup (B \cup C)\)

\[A \cup B = \{a,b,d\} \cup \{c,e,f,h\}\]
\[= \{a,b,c,d,e,f,h\}\]
\[(A \cup B) \cup C = \{a,b,c,d,e,f,h\} \cup \{g,i\}\]
\[(A \cup B) \cup C = \{a,b,c,d,e,f,g,h,i\} \quad \ldots \quad (1)\]
\[B \cup C = \{c,e,f,h\} \cup \{g,i\}\]
\[= \{c,e,f,g,h,i\}\]
\[A \cup (B \cup C) = \{a,b,d\} \cup \{c,e,f,g,h,i\}\]
\[= \{a,b,c,d,e,f,g,h,i\} \quad \ldots \quad (2)\]

Thus from (1) and (2), students can conclude that
\[(A \cup B) \cup C = A \cup (B \cup C)\]

**Example (iii):** If \(A = \{1,3,5,9\} , B = \{2,4,6,8,10\} \) and \(C = \{7,11\}\)

Then show that \((A \cup B) \cup C = A \cup (B \cup C)\)

\[A \cup B = \{1,3,5,9\} \cup \{2,4,6,8,10\}\]
\[= \{1,2,3,4,5,6,8,9,10\}\]
\[(A \cup B) \cup C = \{1,2,3,4,5,6,8,9,10\} \cup \{7,11\}\]

\[(A \cup B) \cup C = \{1,2,3,4,5,6,7,8,9,10,11\} \quad \text{………(1)}\]

\[B \cup C = \{2,4,6,8,10\} \cup \{7,11\}\]

\[= \{2,4,6,7,8,10,11\}\]

\[A \cup (B \cup C) = \{1,3,5,9\} \cup \{2,4,6,7,8,10,11\}\]

\[A \cup (B \cup C) = \{1,2,3,4,5,6,7,8,9,10,11\} \quad \text{………(2)}\]

Thus from (1) and (2), students can conclude that

\[(A \cup B) \cup C = A \cup (B \cup C)\]

**b) Observation:** Students will observe all the above examples then they can easily find out that the result of all examples is same.

**c) Generalisation:** On the basis of commonness of the examples, students can generalise that the associative property of union of sets is true that is

\[(A \cup B) \cup C = A \cup (B \cup C)\]

**d) Verification of the result:** Verification of this property can be done through Venn diagram. For example, there are three sets A, B and C which are mutually overlapping then they can be represented with the help of Venn diagram as shown in Figure 6.1.
Venn diagram for $(B \cup C)$ is shown in Figure 6.2.

![Figure 6.2. Lined region represents $(B \cup C)$.

Venn diagram for $A \cup (B \cup C)$ is shown in Figure 6.3.

![Figure 6.3. Lined region represents $A \cup (B \cup C)$.

Now Venn diagram for $(A \cup B)$ is shown in Figure 6.4.

![Figure 6.4. Lined region represents $(A \cup B)$.

Similarly $(A \cup B) \cup C$ can also be represented with the help of Venn diagram as shown in the Figure 6.5.
Figure 6.5. Lined region represents \((A \cup B) \cup C\).

As the diagrams for \(A \cup (B \cup C)\) and \((A \cup B) \cup C\) are similar then students can easily conclude that \((A \cup B) \cup C = A \cup (B \cup C)\). Similarly students can also prove other properties and laws of sets including

- Commutative property of union of sets
- Commutative property of intersection of sets
- Associative property of intersection of sets
- Distributive property of union over intersection of sets
- Distributive property of intersection over union of sets
- De Morgan’s laws

6.6.5 Application of Inductive Method in Algebra

**Problem:** Prove the following formula

\[(x + a) (x + b) = x^2 + (a + b)x + ab\]

**Solution:**

a) **Presenting the similar examples:** Teacher will present some examples of similar nature to prove the formula.

Example (i) \((x + 3) (x + 4) = x^2 + 7x +12\)

Example (ii) \((x + 3) (x – 4) = x^2 - x -12\)
Example (iii) \((x - 3) (x - 4) = x^2 - 7x + 12\)

Example (iv) \((x - 3) (x + 4) = x^2 + x - 12\)

**b) Observation:** Student will observe all examples given above and draw a table for concluding a result from these examples as given below in Table 6.1.

**Table 6.1.** Analysis of similar algebraic expressions.

<table>
<thead>
<tr>
<th>Examples</th>
<th>Multiplicands of expressions</th>
<th>Parts within results of multiplications</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variables</td>
<td>Integers</td>
</tr>
<tr>
<td>(i)</td>
<td>(X)</td>
<td>3, 4</td>
</tr>
<tr>
<td>(ii)</td>
<td>(X)</td>
<td>3, -4</td>
</tr>
<tr>
<td>(iii)</td>
<td>(X)</td>
<td>-3, -4</td>
</tr>
<tr>
<td>(iv)</td>
<td>(X)</td>
<td>-3, 4</td>
</tr>
</tbody>
</table>

**c) Generalisation:** Students will generalise the formula from the commonness of the examples, that is

\((x + a) (x + b) = x^2 + (a + b)x + ab\)

**d) Verification of the result:** For the verification of the proof, student can take another example that is

\((x + 5) (x + 6) = x^2 + 11x + 30\)

It can be written like this \((x + 5) (x +6) = x^2 + (5 + 6) x + (5) (6)\)

Hence students can verify that the following formula is absolutely right

\((x + a) (x + b) = x^2 + (a + b)x + ab\)

Similarly students can also prove other formulas of algebra like

\((a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca\)

\((a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 -3ab(a -b) – b^3\)

\((a - b)(a^2 + ab + b^2) = a^3 - b^3\)
6.6.6 Application of Inductive Method in Matrices

Problem: Prove the following associative property of addition of matrices

\[(A + B) + C = A + (B + C)\]

Solution:

a) Presenting the similar examples: Teacher will present some examples of similar nature.

Example (i): If \[A = \begin{pmatrix} 3 & 4 \\ 0 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} -4 & 2 \\ 6 & -3 \end{pmatrix}\]

Then show that \[(A + B) + C = A + (B + C)\]

\[A + B = \begin{pmatrix} 3 & 4 \\ 0 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 2 & 11 \end{pmatrix}\]

\[(A + B) + C = \begin{pmatrix} 4 & 6 \\ 2 & 11 \end{pmatrix} + \begin{pmatrix} -4 & 2 \\ 6 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ 8 & 8 \end{pmatrix}\] \[\text{.........(1)}\]

\[B + C = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} -4 & 2 \\ 6 & -3 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ 8 & 2 \end{pmatrix}\]

\[A + (B + C) = \begin{pmatrix} 3 & 4 \\ 0 & 6 \end{pmatrix} + \begin{pmatrix} -3 & 4 \\ 8 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 8 \\ 8 & 8 \end{pmatrix}\] \[\text{.........(2)}\]

From (1) and (2), students can conclude that \[(A + B) + C = A + (B + C)\]

Example (ii): If \[A = \begin{pmatrix} -3 & 2 \\ 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 2 & -2 \\ 4 & 5 \end{pmatrix}\]

Then show that \[(A + B) + C = A + (B + C)\]
\[ A + B = \begin{pmatrix} -3 & 2 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 2 & 2 \end{pmatrix} \]

\[ (A + B) + C = \begin{pmatrix} -2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 6 & 7 \end{pmatrix} \] ...........(1)

\[ B + C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 4 & 6 \end{pmatrix} \]

\[ A + (B + C) = \begin{pmatrix} -3 & 2 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 6 & 7 \end{pmatrix} \] ...........(2)

From (1) and (2), students can conclude that

\[(A + B) + C = A + (B + C)\]

**Example (iii):** If \[A = \begin{pmatrix} 0 & -4 \\ 6 & 7 \end{pmatrix}, \ B = \begin{pmatrix} -2 & 1 \\ 1 & 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}\]

Then show that \((A + B) + C = A + (B + C)\)

\[ A + B = \begin{pmatrix} 0 & -4 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} -2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -3 \\ 7 & 8 \end{pmatrix} \]

\[ (A + B) + C = \begin{pmatrix} -2 & -3 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 5 & 9 \end{pmatrix} \] ............(1)

\[ B + C = \begin{pmatrix} -2 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ -1 & 2 \end{pmatrix} \]

\[ A + (B + C) = \begin{pmatrix} 0 & -4 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 5 & 9 \end{pmatrix} \] ............(2)

From (1) and (2), students can conclude that

\[(A + B) + C = A + (B + C)\]

**b) Observation:** If students observe all the above examples, they can easily find out that the result of all examples is same.
c) **Generalisation:** On the basis of commonness of the examples, students can generalise that associative property of addition of matrices is true that is

\[(A + B) + C = A + (B + C)\]

d) **Verification of the result:** For the verification of this property, students can consider three matrices A, B and C which consist of scalar values. Dimension of these matrices should be same because it is prerequisite for addition.

Let A = the set of values of \(X_{i,j}\) (where “i” is the number of rows and “j” is the number of columns)

B = the set of values \(Y_{i,j}\)

C = the set of values \(Z_{i,j}\)

\[A + (B + C) = \text{the set } X_{i,j} + (Y_{i,j} + Z_{i,j})\]

\[(A + B) + C = \text{the set } (X_{i,j} + Y_{i,j}) + Z_{i,j}\]

since \(X_{i,j}\), \(Y_{i,j}\) and \(Z_{i,j}\) are scalar, therefore

\[X_{i,j} + (Y_{i,j} + Z_{i,j}) = (X_{i,j} + Y_{i,j}) + Z_{i,j}\]

Or \( (A + B) + C = A + (B + C)\)

Similarly students can prove other properties of matrices including

- Commutative property of addition of matrices.
- Associative property of matrices with respect to multiplication.
- Distributive property of matrices with respect to multiplication.

### 6.6.7 Application of Inductive Method in Basic Geometry

**Problem:** Prove that the sum of the measures of the three angles of a triangle is \(180^\circ\).

**Solution:**

a) **Presenting the similar examples:** Teacher will present three different triangles
as shown in Figure 6.6.

![Figure 6.6. Three triangles \( \triangle ABC \), \( \triangle XYZ \) and \( \triangle PQR \) for students’ observation.](image)

**b) Observation:** Students will calculate the angles of each triangle separately and get the sum of them as shown in Table 6.2.

**Table 6.2.** Sum of the angles of triangles \( \triangle ABC \), \( \triangle XYZ \) and \( \triangle PQR \).

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Values of angles of a triangle</th>
<th>Sum of the angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>( m&lt; A = 60 ) ( m&lt; B = 60 ) ( m&lt; C = 60 )</td>
<td>( 60 + 60 + 60 = 180 )</td>
</tr>
<tr>
<td>(ii)</td>
<td>( m&lt; X = 90 ) ( m&lt; Y = 65 ) ( m&lt; Z = 25 )</td>
<td>( 90 + 65 + 25 = 180 )</td>
</tr>
<tr>
<td>(iii)</td>
<td>( m&lt; P = 30 ) ( m&lt; Q = 30 ) ( m&lt; R = 120 )</td>
<td>( 30 + 30 + 120 = 180 )</td>
</tr>
</tbody>
</table>

Now students can easily observe that the sum of angles is same.

c) **Generalisation:** As the sum of angles of all triangles is same which is 180\(^0\), then students can easily conclude that the sum of the measures of the three angles of a triangle is always 180\(^0\).

d) **Verification of the result:** For the verification of this problem, students can take another triangle like \( \triangle GHI \) as shown in Figure 6.7.

![Figure 6.7. Triangle \( \triangle GHI \) for the verification of the result.](image)
Now students separate each angle of this triangle which are m< G, m< H and m< I and place them at one point on a straight line as shown in Figure 6.8.

![Figure 6.8. Placement of angles of GHI on a straight line.](image)

In this way, all angles come in a sequence and it is also known that the measure of angle from one end of a line to another end is $180^\circ$. Therefore, it can be concluded that the measure of three angles of a triangle is $180^\circ$.

### 6.7 Deductive Method

This method is totally different from inductive method. In this method, we proceed from general to specific and rule to example. Already constructed formula, rule, method or principle is told to the students and they apply them to solve the problems (Sidhu, 1995). In this teaching approach, we can also prove a theorem with the help of undefined terms, defined terms, axioms and postulates. Then with the help of that theorem and different rules and principles we can derive other theorems as well (Singh, 2007).

#### 6.7.1 Procedure of Deductive Method

There are only two stages of deductive teaching method (Landmark, 2005):

- Presenting the concept and student practice.
a) **Presenting the concept:** First of all, teacher explains the rules or formulas related to one particular concept to the students. Then he or she solves some relevant examples before them.

b) **Student Practice:** After understanding the rules or formulas, students are given opportunity to apply them by themselves to solve different problems under the guidance of teacher unless they get master of that rules or formulas.

### 6.7.2 Merits and Demerits of Deductive Method

Deductive method has some merits and demerits as well (Sekhar, 2006).

**Merits:** This method has following merits.

- This method is very easy and short.
- To remember a formula or rule is not very difficult so this method is blessing for those students who can not remember complicated procedures (Brigham & Matins, 1999).
- Teachers can complete the syllabus easily by this method.
- This method helps to enhance the computational ability of the students.
- It is helpful in teaching those concepts in which derivation of rules or methods is not involved.
- With the help of this method, we can prove other theorems using already defined formulas or principles.

**Demerits:** This method has following demerits.

- It becomes very difficult for students when they have to remember so many rules and formulas.
- This method does not help to improve reasoning ability in the students (Landmark, 2005).
- It is not effective at lower level classes.
- This method is not constructivist so that if a student forgets a rule or principle then he or she can not reconstruct that easily (Sidhu, 1995).
- This method does not encourage discovery learning.
- It can not make the students creative.
- Students may be in doubt that why they are using one particular formula.

6.7.3 Application of Deductive Method in Secondary Mathematics

Deductive method is the highly used method in mathematics. It is used to solve those problems in which complicated procedures are not involved and they can be solved by applying different kinds of already established laws, methods, formulas and principles directly. Such kinds of problems can be found in all units of syllabus of mathematics at secondary level including sets, logarithms, algebra, matrices, variation, statistics, geometry and trigonometry.

6.7.4 Application of Deductive Method in Sets

**Problem:** Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

If $A = \{a,b,c\}$, $B = \{b,d,f\}$, $C = \{a,f,c\}$

**Solution:** First of all, teacher will explain the already defined terms and operations like intersection and union of sets to the students then students will apply them to solve the problem.

The given sets are $A = \{a,b,c\}$, $B = \{b,d,f\}$, $C = \{a,f,c\}$

$B \cup C = \{b,d,f\} \cup \{a,f,c\} = \{a,b,c,d,f\}$

$A \cap (B \cup C) = \{a,b,c\} \cap \{a,b,c,d,f\}$

$A \cap (B \cup C) = \{a,b,c\}$
A ∩ B = {a,b,c} ∩ {b,d,f} = {b}
A ∩ C = {a,b,c} ∩ {a,f,c} = {a,c}

(A ∩ B) U (A ∩ C) = {b} U {a,c}

Hence A ∩ (B U C) = (A ∩ B) U (A ∩ C)

6.7.5 Application of Deductive Method in Logarithms

Problem: Evaluate the following with the help of logarithms.
\[
\sqrt[3]{\frac{8}{3}} \frac{1}{\sqrt{22}}
\]

Solution: First of all, teacher will explain the methods of finding logarithm and anti-logarithm using tables. Then students will follow the same methods to evaluate the given expression.

Let \( x = \sqrt[3]{\frac{8}{3}} \frac{1}{\sqrt{22}} \)

\[ = \frac{\sqrt{25}}{\sqrt{3} \cdot \sqrt{22}} \]

\[ = \frac{\sqrt{25}}{\sqrt{3} \cdot \sqrt{22}} \]

\[ x = \frac{(25)^{\frac{1}{2}}}{(3)^{\frac{1}{2}} \cdot (22)^{\frac{1}{3}}} \]

Taking logarithm of both the sides

\[ \log x = \log \left( \frac{(25)^{\frac{1}{2}}}{(3)^{\frac{1}{2}} \cdot (22)^{\frac{1}{3}}} \right) \]
\[
\log \left(\frac{25}{3} \cdot \frac{22}{3}\right) = \frac{1}{2} \log (25) - \frac{1}{2} \log (3) - \frac{1}{3} \log (22)
\]
\[
= \frac{1}{2} (1.3980) - \frac{1}{2} (0.4771) - \frac{1}{3} (1.3424)
\]
\[
= 0.6990 - 0.2386 - 0.4475
\]

\[
\log x = 0.0129
\]
Taking anti-logarithm of both the sides
\[
antilog (\log x) = antilog (0.0129)
\]
\[
x = 1.030
\]

Thus \[
\sqrt{\frac{8}{3}} = 1.030
\]

Similarly, deductive method is used to solve all the problems of logarithm at secondary level. Moreover, laws of logarithm can also be proved with the help of this method.

6.7.6 Application of Deductive Method in Algebra

**Problem:** Find the solution set of the following equation by Quadratic formula.
\[
15x^2 - 13x + 2 = 0
\]

**Solution:** Here the teacher will explain the Quadratic formula first then students will apply that formula by themselves.
As \[
15x^2 - 13x + 2 = 0
\]
Here \[
a = 15, \quad b = -13 \quad \text{and} \quad c = 2
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
( Quadratic Formula )
Putting the value in the quadratic formula, we have

\[ x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(15)(2)}}{2(15)} \]

\[ = \frac{13 \pm \sqrt{169 - 120}}{30} \]

\[ = \frac{13 \pm \sqrt{49}}{30} \]

\[ = \frac{13 \pm 7}{30} \]

\[ x = \frac{13 + 7}{30}, \quad x = \frac{13 - 7}{30} \]

\[ x = \frac{20}{30}, \quad x = \frac{6}{30} \]

\[ x = \frac{2}{3}, \quad x = \frac{1}{5} \]

so the solution set of the equation is

\[ \text{Solution set} = \left\{ \frac{2}{3}, \frac{1}{5} \right\} \]

Similarly, students can apply other formulas and theorems like Invertendo, Alternendo, Componendo, Dividendo and K-Method in proportion related problems.

6.7.7 Application of Deductive Method in Matrices

**Problem:** Find the multiplicative inverse of the following matrix.

\[ \begin{pmatrix} 3 & -8 \\ 4 & 9 \end{pmatrix} \]

**Solution:** First of all, teacher will explain the whole procedure to find out the multiplicative inverse including formula then students will apply that formula to solve the problem.
Let \( P = \begin{pmatrix} 3 & -8 \\ 4 & 9 \end{pmatrix} \)

Multiplicative inverse of \( P = P^{-1} = \frac{\text{Adj } P}{|P|} \)

\[
\text{Adj } P = \begin{pmatrix} 9 & 8 \\ -4 & 3 \end{pmatrix}
\]

Determinant of \( P = |P| = \begin{vmatrix} 3 & -8 \\ 4 & 9 \end{vmatrix} \)

\[
|P| = 27 - (-32) = 27 + 32
\]

\[
|P| = 59
\]

\[
P^{-1} = \frac{\text{Adj } P}{|P|} = \begin{pmatrix} 9 & 8 \\ -4 & 3 \end{pmatrix} \]

\[
P^{-1} = \begin{pmatrix} 9/59 & 8/59 \\ -4/59 & 3/59 \end{pmatrix}
\]

Similarly, students can directly apply other rules of matrices according to the situation. For example, Cramer’s rule can be used for finding out the solution of matrices based on two simultaneous linear equations.

**6.7.8 Application of Deductive Method in Information Handling (Statistics)**

**Problem:** Find the variance and standard deviation of the data:

\[4 , 9 , 3 , 15 , 19\]

**Solution:** First of all, teacher will explain the formulas of Mean, Variance and Standard Deviation then students will apply them to solve the problem.

Here \( X = 4 , 9 , 3 , 15 , 19 \)

Number of observations = \( n = 5 \)
Arithmetic mean = $\bar{X} = \frac{\sum X}{n}$

$\sum X = 4 + 9 + 3 + 15 + 19 = 50$

so $\bar{X} = \frac{50}{5} = 10$

Now we calculate sum of squares of the deviations from $\bar{X}$ as shown in Table 6.3.

<table>
<thead>
<tr>
<th>X</th>
<th>$X - \bar{X}$</th>
<th>$(X - \bar{X})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$4 - 10 = -6$</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>$9 - 10 = -1$</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>$3 - 10 = -7$</td>
<td>49</td>
</tr>
<tr>
<td>15</td>
<td>$15 - 10 = 5$</td>
<td>25</td>
</tr>
<tr>
<td>19</td>
<td>$19 - 10 = 9$</td>
<td>81</td>
</tr>
<tr>
<td>Sum: 50</td>
<td></td>
<td>Sum: 192</td>
</tr>
</tbody>
</table>

Variance $= S^2 = \frac{\sum (X - \bar{X})^2}{n} = \frac{192}{5} = 38.40$

Standard Deviation $= S = \sqrt{\frac{\sum (X - \bar{X})^2}{n}}$

$= \sqrt{\frac{192}{5}} = \sqrt{38.40}$

Standard Deviation $= 6.20$

Similarly, students can use formulas to calculate Mean, Median, Mode and Range of data as well in the unit of statistics.
6.7.9 Application of Deductive Method in Demonstrative Geometry

**Problem:** Prove the theorem that if in any correspondence of two triangles, two angles and one side of a triangle are congruent to the corresponding two angles and one side of the other, the triangles are congruent.

**Solution:** First of all, teacher will explain the necessary terms, definitions and postulates related to this theorem to the students. Then students can use them to prove this theorem.

**Given:** Let suppose the given triangles are \( \triangle ABC \) and \( \triangle DEF \) (see Figure 6.9).

![Figure 6.9. Triangles \( \triangle ABC \) and \( \triangle DEF \).](image)

In \( \triangle ABC \leftrightarrow \triangle DEF \)

\( m<B = m<E \)

\( m<C = m<F \)

\( BC = EF \)

**To Prove:** \( \triangle ABC = \triangle DEF \)

**Construction:** Suppose \( AB \neq DE \) and there is a point \( G \) on \( DE \) such that \( AB = GE \). Join \( G \) to \( F \) as shown in Figure 6.10.

![Figure 6.10. Joining of point \( F \) with point \( G \) on \( DE \).](image)
Proof of theorem is given below as shown in Table 6.4.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>In □ ABC ↔ □ GEF</td>
<td></td>
</tr>
<tr>
<td>AB = GE</td>
<td>(i) Construction / Supposition</td>
</tr>
<tr>
<td>BC = EF</td>
<td>(ii) Given</td>
</tr>
<tr>
<td>m&lt;B = m&lt;E</td>
<td>(iii) Given</td>
</tr>
<tr>
<td>Therefore □ ABC ≅ □ GEF</td>
<td>S.A.S. Postulate</td>
</tr>
<tr>
<td>So, m&lt; C = m&lt; GFE</td>
<td>Corresponding angles of congruent triangles.</td>
</tr>
<tr>
<td>But m&lt; C = m&lt; DFE</td>
<td>Given</td>
</tr>
<tr>
<td>Therefore □ DFE ≅ □ GEF</td>
<td>Both congruent to m&lt; C</td>
</tr>
<tr>
<td>This is possible only if D and G are the same points.</td>
<td></td>
</tr>
<tr>
<td>AB = DE</td>
<td>(iv) Proved that D and G are the same points.</td>
</tr>
<tr>
<td>Thus from (ii), (iii) and (iv), we have</td>
<td></td>
</tr>
<tr>
<td>□ ABC ≅ □ DEF</td>
<td>S.A.S. postulate</td>
</tr>
</tbody>
</table>

**Explanation:** Through deductive method, different kinds of defined terms, axiom and postulates are used to prove different theorems in demonstrative geometry, so same procedure is adopted in proving this theorem. Triangle, angle, corresponding angle and congruent angle are defined terms which are used in this proof. Moreover, S.A.S. postulate has been used as well. All geometry theorems at secondary level can be proved through deductive method.

6.7.10 Application of Deductive Method in Trigonometry

**Problem:** Prove that \( \frac{\tan \theta + \cot \theta}{\cosec \theta} = \sec \theta \)
Solution: Here teacher will explain the trigonometric ratios to the students then they will apply them to prove this equation.

\[
\text{L.H.S.} = \frac{\tan \theta + \cot \theta}{\csc \theta}
\]

\[
= \frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta} \csc \theta
\]

\[
= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \csc \theta \quad \text{As} \quad \tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{and} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}
\]

\[
= \frac{1}{\cos \theta \sin \theta} \quad \text{(As} \quad \sin^2 \theta + \cos^2 \theta = 1)
\]

\[
= \frac{\sin \theta}{\cos \theta} \sin \theta \quad \text{(As} \quad \frac{1}{\csc \theta} = \sin \theta)
\]

\[
= \frac{1}{\cos \theta} = \sec \theta \quad \text{R.H.S.}
\]

If students remember all predefined trigonometric ratios then they can prove other trigonometric equations through deductive method as well in the unit of trigonometry.

6.8 Heuristic Method

The word heuristic was drawn from a Greek word “heurisko” which means “I find out”. Heuristic method is based on child’s psychology that he or she always wants to discover something by himself or herself. That is why, it is also known as discovery method (Bruner, 1960, 1962, 1966). Sometimes teacher only focuses on delivering lectures through speech in which students do not actively participate. In this way, students get bored most of the time. In the heuristic method, students are encouraged
to reach the solution by constructing the knowledge themselves. Teacher only facilitates them by raising relevant questions before them. That is why, it is also called inquiry method (Suchman, 1962). As the students discover the solution under the guidance of teacher so it is also known as guided discovery method or programmed instruction. So many researches (Ashton, 1962; Wills, 1967; Wilson, 1967) have proved that heuristic or discovery method is more effective in teaching mathematics than expository approach.

6.8.1 Procedure of Heuristic Method

There are five phases in heuristic or discovery method to reach the solution (Arends, 1994; Sekhar, 2006).

a) Explaining the relevant concepts: First of all, the teacher will explain the main objectives and the concepts related to a lesson.

b) Presenting the problem: Teacher will describe the problem which will be a little bit puzzling and the solution of that problem can only be found out through heuristic method.

c) Information gathering and mental experimentation by the students: Students will start thinking and gathering required information to solve the problem. Teacher will help in this process.

d) Inquiring students: Teacher will start asking such kind of questions through which he or she can help students in discovery process. Teacher can provide clues as well.

e) Solving the problem by the students: Students will go through the discovery process to find out the solution by finding out the answers to the questions asked by the teacher at different steps.
6.8.2 Merits and Demerits of Heuristic Method

This method has merits and demerits as well (Singh, 2007; Sidhu, 1995).

**Merits:** This method has following merits.

- It is a student centred approach.
- It gives confidence to the students because they discover the solution by themselves.
- It makes students creative.
- It develops reasoning and thinking abilities in students.
- It clears the concepts in better way.
- Continuously inquiring the students keeps them active, so they do not get bored.

**Demerits:** This method has following demerits.

- It is quite time consuming.
- It is essential for all teachers to be properly skilled with heuristic approach; otherwise it is very difficult for them to apply this in the classroom.
- If any student has less aptitude towards discovery then it becomes very difficult for him or her to learn something through this method.
- It is only applicable if the strength of the class is low which is usually impossible in Pakistani context.
- If teacher fails to give proper guidance to every student then they may get discouraged.
- This method is not suitable for teaching all kinds of mathematical problems.
- Sometimes teacher fails to ask proper questions that distracts students.
- With the help of this method, lengthy syllabus can not be finished in time.
6.8.3 Application of Heuristic Method in Secondary Mathematics

Heuristic method can be used to teach all branches of mathematics. It is used when students are not master to solve problems related to one particular concept and they need guidance. When students get master then deductive or problem solving methods can be used to solve the same problems.

6.8.4 Application of Heuristic Method in Sets

Problem: Show that \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)

If \( A = \{0\} \), \( B = \{0,1\} \), \( C = \{\} \)

Solution: Before presenting the problem, teacher will explain the concepts related to sets then students will start thinking on this. Teacher will also help them by asking relevant questions. Questions asked by the teacher and expected answer given by students are listed below in Table 6.5 for fining out the solution.

Table 6.5. Discovery process to show that \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \).

<table>
<thead>
<tr>
<th>Questions Asked by The Teacher</th>
<th>Expected Answers From Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) What is given?</td>
<td>1) We have three sets</td>
</tr>
<tr>
<td></td>
<td>( A = {0} ), ( B = {0,1} ), ( C = {} )</td>
</tr>
<tr>
<td>2) What have to be proved?</td>
<td>2) We have to prove</td>
</tr>
<tr>
<td></td>
<td>( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) )</td>
</tr>
<tr>
<td>3) What can you find with the help of given sets?</td>
<td>3) We can find ( (B \cap C) ), ( A \cup (B \cap C) ), ( (A \cup B) ), ( (A \cup C) ) and ( (A \cup B) \cap (A \cup C) )</td>
</tr>
<tr>
<td>4) Can you explain the union and intersection of sets?</td>
<td>4) Yes, the union of two sets is a set consisting of all the elements of both the sets but the common elements are taken once. The intersection of two sets is a set consisting of all the common elements of both the sets.</td>
</tr>
</tbody>
</table>
5) Can you find \((B \cap C)\) and \(A \cup (B \cap C)\)?

6) Can you find \((A \cup B), (A \cup C)\) and then \((A \cup B) \cap (A \cup C)\)?

7) What can you conclude from this?

6.8.5 Application of Heuristic Method in Ratio and Proportion (Variation)

**Problem:** Find the fourth proportional of 7, 21, 3.

**Solution:** First of all, teacher will explain the concepts of ratio and proportion then students will start thinking themselves to solve the problem. Along with this, teacher will start asking questions to help them in finding out the solution as shown in Table 6.6.

<table>
<thead>
<tr>
<th>Questions Asked by The Teacher</th>
<th>Expected Answers From Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) What is given?</td>
<td>1) We have three numbers where 7 is first, 21 is second and 3 is third proportional.</td>
</tr>
<tr>
<td>2) What have to be found?</td>
<td>2) We have to find out fourth proportional.</td>
</tr>
<tr>
<td>3) How will you find the fourth one?</td>
<td>3) We will suppose it as X.</td>
</tr>
<tr>
<td>4) Can you define the proportion?</td>
<td>4) Yes, the statement of equality of two ratios is called proportion.</td>
</tr>
<tr>
<td>5) How will you write the proportion?</td>
<td>5) We will write it as (7:21::3:X)</td>
</tr>
<tr>
<td>6) Can you write it in another form?</td>
<td>6) Yes, (\frac{7}{21} = \frac{3}{X}) or (\frac{1}{3} = \frac{3}{X})</td>
</tr>
<tr>
<td>7) Can you simplify it further for getting the value of X?</td>
<td>7) Yes, (X = 9) or Fourth Proportional = 9</td>
</tr>
</tbody>
</table>
6.8.6 Application of Heuristic Method in Matrices

Problem: If \( A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} \), \( B = \begin{pmatrix} -2 & 4 \\ 4 & -5 \end{pmatrix} \) and \( C = \begin{pmatrix} 2 & 1 \\ 7 & 5 \end{pmatrix} \)

Then show that \( A (B + C) = AB + AC \)

Solution: Before presenting this problem, teacher will explain the concepts relevant to matrices then students will start thinking on this. Along with this, teacher will keep asking questions for finding out the solution as shown in Table 6.7.

Table 6.7. Discovery process to conclude that \( A (B + C) = AB + AC \).

<table>
<thead>
<tr>
<th>Questions Asked by The Teacher</th>
<th>Expected Answers From Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) What is given?</td>
<td>1) We have three matrices ( A = \begin{pmatrix} 0 &amp; 1 \ 2 &amp; 1 \end{pmatrix} ), ( B = \begin{pmatrix} -2 &amp; 4 \ 4 &amp; -5 \end{pmatrix} ) and ( C = \begin{pmatrix} 2 &amp; 1 \ 7 &amp; 5 \end{pmatrix} )</td>
</tr>
<tr>
<td>2) What have to be found?</td>
<td>2) ( A (B + C) = AB + AC )</td>
</tr>
<tr>
<td>3) What can you find with the given matrices?</td>
<td>3) We can find ((B + C)), ( A (B + C)), ( AB ) and ( AC ).</td>
</tr>
<tr>
<td>4) Do you think that the addition a and multiplication of these matrices are possible?</td>
<td>4) Yes, because the order of all the matrices is same that is (2 \times 2). Multiplication is also possible because the number of rows of one matrix is equal to the number of columns of another matrix.</td>
</tr>
<tr>
<td>5) Can you find ((B + C)) and ( A (B + C))?</td>
<td>5) Yes, ( B + C = \begin{pmatrix} -2 &amp; 4 \ 4 &amp; -5 \end{pmatrix} + \begin{pmatrix} 2 &amp; 1 \ 7 &amp; 5 \end{pmatrix} = \begin{pmatrix} 0 &amp; 5 \ 11 &amp; 0 \end{pmatrix} )</td>
</tr>
</tbody>
</table>
and \( A (B + C) = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 5 \\ 11 & 0 \end{pmatrix} \)

\[
A (B + C) = \begin{pmatrix} 0+11 & 0+0 \\ 0+11 & 10+0 \end{pmatrix} = \begin{pmatrix} 11 & 0 \\ 11 & 10 \end{pmatrix}
\]

6) Now, can you find \( AB, AC \) and \( AB + AC \)?

6) Yes,

\[
AB = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 4 \\ 4 & -5 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 0 & 3 \end{pmatrix}
\]

and \( AC = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 7 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 11 & 7 \end{pmatrix} \)

\[
AB + AC = \begin{pmatrix} 4 & -5 \\ 0 & 3 \end{pmatrix} + \begin{pmatrix} 7 & 5 \\ 11 & 7 \end{pmatrix} = \begin{pmatrix} 11 & 0 \\ 11 & 10 \end{pmatrix}
\]

7) So what can you conclude from this result?

7) We conclude that \( A (B + C) = AB + AC \)

---

6.8.7 Application of Heuristic Method in Basic Geometry

**Problem:** Analyse the following shape and try to understand the characteristics of it.
Solution: Teacher will explain the basic concepts of geometry before raising this problem then he or she will start asking questions, step by step to help the students in the analysis of given shape as shown in Table 6.8.

Table 6.8. Discovery process to analyse the shape WXYZ.

<table>
<thead>
<tr>
<th>Questions Asked by The Teacher</th>
<th>Expected Answers From Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) What is the type of this figure?</td>
<td>1) It is quadrilateral.</td>
</tr>
<tr>
<td>2) What do you understand after measuring the sides?</td>
<td>2) Opposite sides of this quadrilateral are equal.</td>
</tr>
<tr>
<td>3) Now what you think after taking these measurements?</td>
<td>3) WXYZ is a parallelogram.</td>
</tr>
<tr>
<td>4) What its diagonals show?</td>
<td>4) They bisect each other.</td>
</tr>
<tr>
<td>5) What you conclude from all these observations?</td>
<td>5) The diagonals of each parallelogram bisect each other.</td>
</tr>
</tbody>
</table>

6.8.8 Application of Heuristic Method in Demonstrative Geometry

Problem: Prove the theorem that if a transversal intersects two coplanar lines such that the pair of alternate angles is congruent, then the lines are parallel.

Solution: First of all, teacher will explain the concepts of congruent angles then students will draw a Figure 6.12 according to the statement of the problem and take measurements to prove this theorem.
Figure 6.12. A transversal $LM$ intersecting two coplanar lines $AB$ and $CD$.

Teacher will also start asking questions to help them out as shown in Table 6.9.

<table>
<thead>
<tr>
<th>Questions Asked by The Teacher</th>
<th>Expected Answers From Students</th>
</tr>
</thead>
</table>
| 1) What do you understand after measuring the angles? | 1) It shows that $m< P \cong m< T$, $m< S \cong m< W$  
$m< Q \cong m< U$, $m< R \cong m< V$ |
| 2) What it shows?               | 2) It shows that corresponding angles are equal.                                             |
| 3) What measurements of the angles show further? | 3) It shows that $m< R \cong m< T$ and $m< S \cong m< U$  
It means that alternate angles are congruent. |
| 4) If alternate angles are congruent then what do you conclude from this? | 4) We conclude that if a transversal intersects two coplanar lines such that the pair of alternate angles is congruent, then the lines are parallel. |

6.9 Analytic Method

In this method, we analyse the problem first which means we break up the problem in small segments and then move towards solution. It is also called descriptive method. It leads us from unknown or required conclusion to known which is given in the
problem. This method focuses on why we are applying different kinds of operations and what is the relationship between the conclusion and other portions of the problem (Rani, 2007; Singh, 2007).

6.9.1 Procedure of Analytic Method

There are three steps to analyse and solve the problem through analytic method (Sekhar, 2006).

a) **What is given?** It should be cleared that what is known or given and what are the conditions to be considered in the problem.

b) **What is to be proved?** It should also be cleared that what is unknown or to be proved by backward chaining of arguments and mathematical operations.

c) **What mathematical operations are required and why?** Some mathematical operations are defiantly required to prove something but there should be a proper logic for each of them.

6.9.2 Merits and Demerits of Analytic Method

Analytic method has some merits and demerits as well (Sidhu, 1995; Sekhar, 2006).

**Merits:** This method has following merits.

- This is a pure logical method so there is always less chance of doubts.
- This is the method to discover something so it promotes logical thinking and reasoning ability in the student (Agarwal, 1992).
- Students always play active role in this method.
- Students do not need to memorise any set procedure to solve a problem.
- It encourages scientific attitude.
Demerits: This method has following demerits.

- This method is quite lengthy and time consuming (Agarwal, 1992).
- This is not suitable for all kinds of problems.
- Only skilled teachers can apply this method.
- This is not suitable if the syllabus is so lengthy.

6.9.3 Application of Analytic Method in Secondary Mathematics

Because of discovery approach, only such kind of problems can be taught with the help of this method in which we have to prove something. At secondary level, such problems can only be found in the units of algebra, ratio & proportion (variation) and geometry.

6.9.4 Application of Analytic Method in Algebra

Problem: If \( \frac{P}{Q} = \frac{R}{S} \)

Prove that \( \frac{PR - 4Q^2}{Q} = \frac{R^2 - 4QS}{S} \)

Analysis:

a) What is given?

\( \frac{P}{Q} = \frac{R}{S} \)

b) What is to be proved?

\( \frac{PR - 4Q^2}{Q} = \frac{R^2 - 4QS}{S} \)

c) How can we simplify the equation?

\( PRS - 4Q^2S = QR^2 - 4Q^2S \) (by cross multiplication)
c) How can we further simplify the equation?

\[ \text{PRS} = QR^2 \quad \text{(by cancelling \(-4Q^2S\) from both sides)} \]

\[ \text{PS} = QR \quad \text{(dividing by R on both sides)} \]

\[ \frac{P}{Q} = \frac{R}{S} \quad \text{(known and true as well)} \]

Therefore by backward chaining of arguments, we can prove that

\[ \frac{PR - 4Q^3}{Q} = \frac{R^2 - 4QS}{S} \]

6.9.5 Application of Analytic Method in Ratio & Proportion (Variation)

Problem: If \(a : b = c : d\), then prove that \(5a + 4b : 5a - 4b = 5c + 4d : 5c - 4d\)

Analysis:

a) What is given?

\(a : b = c : d\)

b) What is to be proved?

We have to prove that \(5a + 4b : 5a - 4b = 5c + 4d : 5c - 4d\)

We start from it which is unknown and then proceed to known which is given.

d) How will we prove the equation?

\[ \frac{5a + 4b}{5a - 4b} = \frac{5c + 4d}{5c - 4d} \]

First of all, we write the equation in other form

\[ \frac{5a + 4b}{5a - 4b} = \frac{5c + 4d}{5c - 4d} \]

\[ \frac{(5a + 4b) + (5a - 4b)}{(5a + 4b) - (5a - 4b)} = \frac{(5c + 4d) + (5c - 4d)}{(5c + 4d) - (5c - 4d)} \]

\[ \frac{5a + 4b + 5a - 4b}{5a + 4b - 5a - 4b} = \frac{5c + 4d + 5c - 4d}{5c + 4d - 5c - 4d} \]

by Componendo - Dividendo theorem
\[
\frac{10a}{8b} = \frac{10c}{8d}
\]

\[
\frac{a}{b} = \frac{c}{d} \quad \quad \text{Multiplying by } \frac{8}{10} \quad \text{on both the sides}
\]

or \[a : b = c : d\] (which is given and true)

Therefore by backward chaining of arguments, we can prove that

\[5a + 4b : 5a - 4b = 5c + 4d : 5c - 4d\]

6.9.6 Application of Analytic Method in Demonstrative Geometry

**Problem:** Prove the theorem that if a diameter of a circle bisects a chord, it will be perpendicular to the chord.

**Analysis:**

a) What is given?

Suppose \(\overline{RQ}\) is a diameter of the given circle with centre O that bisects the chord \(\overline{PT}\) at point S (see Figure 6.13) so that \(\overline{PS} = \overline{ST}\).

![Figure 6.13. \(\overline{RQ}\), the diameter of the circle bisecting the chord at point S.](image)

b) What is to be proved?

\(\overline{RQ}\) is perpendicular to \(\overline{PT}\) or \(m\angle OST = m\angle OSP = 90^\circ\)
c) **How can we prove that \( m< OST \) and \( m< OSP \) are equal?**

For proving the equality of these two angles, we have to prove the congruency of the triangles which contain these angles.

d) **Where are the triangles?**

These triangles do not exist so we join \( O \) with \( T \) and \( P \) so that we get two triangles \( \triangle OPS \) and \( \triangle OTS \) as shown in Figure 6.14.

![Figure 6.14](image)

Figure 6.14. Generating the triangles \( \triangle OPS \) and \( \triangle OTS \) by joining the centre \( O \) with points \( T \) and \( P \).

e) **How can we prove the congruency of these two triangles?**

We know that \( PS = ST \) \hspace{1cm} (given)
and \( OS = OS \) \hspace{1cm} (Common)
\( OP = OT \) \hspace{1cm} (both are radii of the circle)

Hence it is proved that the triangles \( \triangle OPS \) and \( \triangle OTS \) are congruent so that \( m< OST = m< OSP = 90^\circ \). Therefore \( OS \) is perpendicular to \( PT \) or we can say that diameter of a circle \( RQ \) is perpendicular to \( PT \).
6.10 Synthetic Method

This method is completely reverse of analytic method in which we proceed from given or known to the desired conclusion or unknown. In this approach, we synthesise or put together given separate elements or small portions to draw a series of conclusions until the unknown or desired result is reached (Sidhu, 1995). This method is quite simple and lead by analytic method. Analytic method involves thinking that is why it clears the basics of concepts. On the other hand, synthetic method is based on already learnt concepts. Therefore it is very necessary to go through the process of analysis to become master of something then synthetic method can be used to solve the problems more quickly. In this method, students are not bound to give reason for each and every step during solving a mathematical problem. That is why, it can not be preferred alone to derive mathematical proofs (Butler & Wren, 1965).

6.10.1 Procedure of Synthetic Method

There are three steps in this method to solve a problem.

a) **What is given?** It should be cleared that what is known or given from where students can proceed to unknown.

b) **What is to be proved?** It should also be cleared to the students that what is unknown or to be proved.

c) **What mathematical operations are required?** Some mathematical operations are required to prove something but it is not necessary to give logic for them at each step in the case of synthetic method.
6.10.2 Merits and Demerits of Synthetic Method

This method has some merits and demerits as well.

**Merits:** It has following merits (Agarwal, 1992; Sekhar, 2006)

- Synthetic method is short and brief.
- It is quick because of deductive reasoning.
- It sharpens the memory of students.
- Teachers can finish the lengthy course in time through it.
- It provides opportunity to the students to practise mathematical formulas or procedures.

**Demerits:** It has following demerits (Sidhu, 1995, Singh, 2007).

- It is not student centred.
- It does not develop reasoning and thinking abilities in students.
- It does not possess heuristic approach.
- Students have to remember so many steps without reasoning.
- If student forgets any mathematical proof then it is very difficult to recall it, step by step.
- It does not clarify the concepts completely.
- It is neither psychological nor scientific in nature.

6.10.3 Application of Synthetic Method in Secondary Mathematics

Just like analytic method, this method can also be used for such problems in which we have to prove something or to reach something which is unknown with the help of given condition. These problems are part of algebra, ratio & proportion (variation) and geometry units of syllabus.
6.10.4 Application of Synthetic Method in Algebra

Problem: If \( \frac{P}{Q} = \frac{R}{S} \)

Prove that \( \frac{PR - 4Q^2}{Q} = \frac{R^2 - 4QS}{S} \)

Synthetic Proof:

a) What is given?
\( \frac{P}{Q} = \frac{R}{S} \)

b) What is to be proved?
\( \frac{PR - 4Q^2}{Q} = \frac{R^2 - 4QS}{S} \)

c) What mathematical operations are required?

We start from which is given
\( \frac{P}{Q} = \frac{R}{S} \)

\( \frac{P}{Q} - \frac{4Q}{R} = \frac{R}{S} - \frac{4Q}{R} \)  \(\text{Subtract } \frac{4Q}{R} \text{ from both sides but there is no reason for this. It is only a guess work.}\)

\( \frac{PR - 4Q^2}{QR} = \frac{R^2 - 4QS}{SR} \)

\( \frac{PR - 4Q^2}{Q} = \frac{R^2 - 4QS}{S} \)  \(\text{by cancelling } \frac{1}{R} \text{ from both sides.}\)

Hence it is proved that
\( \frac{PR - 4Q^2}{Q} = \frac{R^2 - 4QS}{S} \)

if \( \frac{P}{Q} = \frac{R}{S} \)
6.10.5 Application of Synthetic Method in Ratio & Proportion (Variation)

Problem: If \( a : b = c : d \), then prove that

\[
\frac{sa + tb}{sa - tb} : \frac{sc + td}{sc - td}
\]

Synthetic Proof:

a) What is given?

\( a : b = c : d \)

b) What is to be proved?

\[
\frac{sa + tb}{sa - tb} : \frac{sc + td}{sc - td}
\]

c) What mathematical operations are required?

We start from what is given to us

\[
\frac{a}{b} = \frac{c}{d}
\]

or \[
\frac{a}{b} = \frac{c}{d}
\]

\[
\frac{sa}{tb} = \frac{sc}{td}
\]

by multiplying with \( \frac{s}{t} \) on both the sides but there is no reason for this.

\[
\frac{sa + tb}{sa - tb} = \frac{sc + td}{sc - td}
\]

(by componendo-dividendo theorem)

or \( \frac{sa + tb}{sa - tb} : \frac{sc + td}{sc - td} \)

therefore, it is proved that

\[
\frac{sa + tb}{sa - tb} : \frac{sc + td}{sc - td}
\]

if \( a : b = c : d \)

6.10.6 Application of Synthetic Method in Demonstrative Geometry

Problem: Prove that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides (Pythagoras theorem).
**Synthetic Proof:** First of all, we draw a right angled triangle according to the statement as shown in Figure 6.15.

![Figure 6.15. A right angled triangle \( \triangle PQR \).](image1)

**a) What is given?**

A right angled triangle \( \triangle PQR \) in which \( \angle Q = 90^\circ \)

**b) What is to be proved?**

\[
PR^2 = PQ^2 + QR^2
\]

**c) What mathematical operations are required?**

We draw \( QS \perp PR \) (see Figure 6.16). But why we draw it, there is no logic for that. It is a guess work only.

![Figure 6.16. A right angled triangle \( \triangle PQR \) with a line \( QS \) perpendicular to \( PR \).](image2)
In $\square$ PSQ and $\square$ PQR

$m< P = m< P$  

( Common angle )

Hence $\square$ PSQ $\cong \square$ PQR

So $\frac{PS}{PQ} = \frac{PQ}{PR}$  

( In similar triangles, corresponding sides are proportional )

$PQ^2 = PS \cdot PR$  

----------(1)

Similarly, $QR^2 = SR \cdot PR$  

----------(2)

Adding (1) and (2), we get

$PQ^2 + QR^2 = PS \cdot PR + SR \cdot PR$

$PQ^2 + QR^2 = PR (PS + SR)$

$PQ^2 + QR^2 = PR (PR)$

$PQ^2 + QR^2 = PR^2$

or $PR^2 = PQ^2 + QR^2$

Hence Pythagoras theorem is proved.

6.11 Problem Solving Method

Instructing the students should develop thinking ability in them. In this way, they become capable to find out the solutions of different kinds of difficult problems not only during the studies but in the daily matters of life as well. That is why, problem solving became the major teaching method of mathematics (Collier & Lerch, 1969). Bruner, Oliver, Greenfield (1966) and Gagné (1970), the top psychologists also gave the top priority to this method.
Every child has the curiosity to explore the things himself or herself so the psychology of the children can be utilised in a better way through problem solving method. In this method, students are given such problems which can not be solved easily or their solutions are not obvious. Students try to reach the goals or solutions through the set of events or procedures. Gagné (1970) calls these events or procedures as lower order capabilities in which formulas, rules and concepts are used from which a student is already familiar. According to him, what the student learns is called a higher order principle which is the result of lower order capabilities.

6.11.1 Procedure of Problem Solving Method

Pólya (1957) presented the procedure of problem solving method by integrating the work of great mathematicians, psychologists and educationists Dewey (1910), Parker (1923) and Maier (1931). According to him, there are four stages to solve the problem: Understanding the problem, devising a plan, carrying out the plan and looking back.

a) Understanding the problem: First of all, the nature of the problem should be clear. Then it should be made sure that what is to be found out and what data and conditions are given.

b) Devising a plan: Before devising a plan, student should think that whether he or she has seen such kind of problem before. If yes then solving the problem becomes easy otherwise student should take another look at unknown and make a plan that how it can be solved and which method, concept or formula can be used. It should also be assured that whether practical work would be involved or not. Then student can move towards solution.
c) **Carrying out the plan:** Before starting to solve the problem, it should be cleared that whether the plan is right and applicable. Can the student apply the necessary mathematical formulas or methods? After these assurances student should start to solve the problem according to the plan.

d) **Looking back:** After solving the problem, student should recheck the process and steps carefully then he or she should also try to verify the result through any other mathematical procedure.

### 6.11.2 Merits and Demerits of Problem Solving Method

This method has also some merits and demerits.

**Merits:** There are the following merits of problem solving method (Taplin, n.d.; Singh, 2007).

- This method is scientific in nature.
- This method is student centred.
- Reasoning ability is improved by this method.
- Students are provided opportunity to apply their previous knowledge by problem solving.
- Students learn how to face totally new situation by solving different kinds of questions.
- Teacher can assess the abilities of his or her students easily.
- This method improves logical thinking in the students which leads towards creativity.

**Demerits:** There are some demerits of this method as well (Sidhu, 1995; Singh, 2007).

- This method is quite time consuming.
• This is usually not recommended for lower classes.
• Textbooks do not provide enough help to follow this method because such books are usually written in a traditional way.
• Logical thinking is involved in this method therefore physical kinds of activities are totally neglected.

6.11.3 Application of Problem Solving Method in Secondary Mathematics

This method is used to solve those complicated problems which can not be solved with the help of single formula or method. Usually, word problems are solved with it. At secondary level, such kinds of problems can be found in the units of algebra, ration & proportion (variation) and trigonometry.

6.11.4 Application of Problem Solving Method in Algebra

Problem: Find the solution set of following simultaneous linear equations graphically and check when \( x, y \in \mathbb{R} \).

\[
x + 2y = 5, \quad 5x + y = 7
\]

Solution:

a) Understanding the problem: First of all, student will understand the problem in his or her own words that two linear equations are given to him or her to find out the solution set of them graphically. For this purpose he or she will have to draw the graphs of liner equations. The point where graphs as straight lines of both the equations intersect will give the solution set of the linear equations and if the graphs will not intersect then these linear equations will not have any solution set.
b) Devising a plan: After recognising the problem, student will make a plan to solve this problem that he or she will start from obtaining some ordered pairs for both the linear equations then draw the graphs with the help of those ordered pairs.

c) Carrying out the plan: As the linear equations are

\[ x + 2y = 5, \quad 5x + y = 7 \]

\[ x + 2y = 5 \quad \text{...........(1)} \]

\[ 2y = 5 - x \]

or \[ y = \frac{5 - x}{2} \]

Ordered pairs for the equation \( x + 2y = 5 \) are given below in the Table 6.10.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>5</th>
<th>-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>( (x, y) )</td>
<td>(1, 2)</td>
<td>(5, 0)</td>
<td>(-5, 5)</td>
</tr>
<tr>
<td>( P(x, y) )</td>
<td>( P_1(1, 2) )</td>
<td>( P_2(5, 0) )</td>
<td>( P_3(-5, 5) )</td>
</tr>
</tbody>
</table>

\[ 5x + y = 7 \quad \text{...........(2)} \]

or \[ y = 7 - 5x \]

Ordered pairs for the equation \( 5x + y = 7 \) are given below in Table 6.11.

<table>
<thead>
<tr>
<th>( x )</th>
<th>3</th>
<th>2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-8</td>
<td>-3</td>
<td>7</td>
</tr>
<tr>
<td>( (x, y) )</td>
<td>(3, -8)</td>
<td>(2, -3)</td>
<td>(0, 7)</td>
</tr>
<tr>
<td>( P(x, y) )</td>
<td>( P_4(3, -8) )</td>
<td>( P_5(2, -3) )</td>
<td>( P_6(0, 7) )</td>
</tr>
</tbody>
</table>

With the help of ordered pairs \( P_1, P_2, P_3, P_4, P_5, P_6 \), student will draw graphs for both the equations as shown in Figure 6.17.
Both the graphs intersect at P1(1, 2). Hence solution set is {1, 2}.

d) **Looking back:** Student will check all the steps including the construction of the graph then he or she will verify the solution set by putting the values of x and y in both the linear equations.

As x + 2y = 5  

1 + 2(2) = 5  (by putting x = 1, y = 2 in equation-1)

1 + 4 = 5

5 = 5

1 = 1

Now 5x + y = 7  

5(1) + 2 = 7  (by putting x = 1, y = 2 in equation 2)

5 + 2 = 7

7 = 7
As the result of both the equation is same after putting values in them, hence the solution set \{1 , 2\} is right.

6.11.5 Application of Problem Solving Method in Ratio and Proportion (Variation)

**Problem:** The ratio between two numbers is 5 : 6. If 4 is added to each of them, the new ratio becomes 7 : 8. Find the numbers.

**Solution:**

a) **Understanding the problem:** First of all, student will analyse or interpret the problem in his or her own words that he or she will have to find two numbers, suppose x and y. Ratio between them is given with two different conditions so that he or she will have two proportions.

b) **Devising a Plan:** Student can get the values of both numbers by applying the K-method on both proportions.

c) **Carrying out the plan:** Let the numbers be x and y. According to the given condition, proportion would be

\[
x : y = 5 : 6 \quad \text{...............(1)}
\]

\[
x : 5 = y : 6
\]

\[
\frac{x}{5} = \frac{y}{6} = k \quad \text{(because of K-Method)}
\]

or \(x = 5k\) , \(y = 6k\)

Now \(x + 4 : y + 4 = 7 : 8\) \hspace{1cm} (proportion according to second condition)

\[
\frac{x + 4}{y + 4} = \frac{7}{8} \quad \text{...............(2)}
\]

Putting the values of x and y in (2)
\[ \frac{5k + 4}{6k + 4} = \frac{7}{8} \]

\[ 8(5k + 4) = 7(6k + 4) \]

\[ 40k + 32 = 42k + 28 \]

\[ -2k = -4 \]

\[ k = 2 \]

The first number = \( x = 5k = 5(2) = 10 \)

The second number = \( y = 6k = 6(2) = 12 \)

Therefore, the required numbers are 10 and 12

d) **Looking back**: Student will check all the steps including application of K-Method then he or she will verify the result by putting the values of \( x \) and \( y \) in equation (1)

\[ x : y = 5 : 6 \]

\[ 10 : 12 = 5 : 6 \]

or \[ \frac{10}{12} = \frac{5}{6} \]

\[ \frac{5}{6} = \frac{5}{6} \]

\[ 1 = 1 \]

which shows that the values of required numbers are right.

**6.11.6 Application of Problem Solving Method in Trigonometry**

**Problem**: The angle of elevation of the top of a flag post from a point on the ground level 40m away from the flag post is 60\(^\circ\). Find the height of the post.
Solution:

a) **Understanding the problem:** First of all, student will analyse the problem in his or her own words that the angle of elevation of the top of a flag post from a point on ground is $60^\circ$. The distance of that point from the post is 40m and the height of that post is to be found.

b) **Devising a plan:** After analysing the problem, student can easily predict that the trigonometric ratios can be helpful for finding out the problem.

c) **Carrying out the plan:** Student will draw a triangle according to the given data as shown in Figure 6.18.

![Figure 6.18](image-url)

**Figure 6.18.** A triangle showing Point “A” on the ground level, 40m away from flag post.

Let $\overline{BC}$ be the flag

and $\overline{BC} = x = ?$

In $\triangle ABC$

$\overline{AB} = 40m$

$m<A = 60^\circ$

$\tan 60^\circ = \frac{x}{40}$
\[ \sqrt{3} = \frac{x}{40} \]

\[ x = 40 \left( \sqrt{3} \right) = 40 (1.732) \]

\[ x = 69.28 \]

so the height of the post = 69.28m

d) Looking back: Student will check all the steps carefully then he or she can take help of Pythagoras theorem so that the value of \( \overline{AC} \) will also be calculated.

Let \( \overline{AC} = y = ? \)

\[ \sin 90^\circ = \frac{40}{y} \]

\[ \frac{1}{2} = \frac{40}{y} \]

\[ y = 80m \]

or \( \overline{AC} = 80m \)

According to the Pythagoras theorem

\[ \overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 \]

\[ (80)^2 = (40)^2 + (69.28)^2 \]

\[ 6400 = 6400 \]

\[ 1 = 1 \]

It shows that 69.28m which is the height of the post or value of \( \overline{BC} \) is right.

6.12 Laboratory Method

Mathematics is not like those subjects in which only reading is involved. Practical work is also the major part of this subject. Laboratory method has the capacity to fulfil these requirements of mathematics. It is a method of “learning by doing”. That
is why, different kinds of tools and equipments are used in it to perform practical work in which drawing of different shapes, taking measurements of geometrical figures and making of charts and graphs are involved. Students go through from different experiments in laboratory or classroom and learn by observing and calculating themselves. During this process, they get opportunity to draw conclusions and generalise different laws and formulas. Therefore, this method can be said an extended form of inductive method (Sidhu, 1995).

The role of teacher in this method is to supervise the whole process and give proper instructions to the students at each step. He or she should keep some points in mind to make this method successful (Singh, 2007).

- Necessary equipments related to the laboratory work should be arranged in advance.
- Teacher should keep his or her eye on the practical work of every student and guide him or her accordingly.
- Teacher should also keep encourage every student throughout the practical work.
- All necessary concepts should be cleared before experimental work.
- If number of students is high and equipments are less then students can be divided in small groups.

6.12.1 Procedure of Laboratory Method

There are five steps in this method (Singh, 2007).

a) **Objective or what is to be proved:** It should be clear that which law or theorem is needed to be proved
b) **Apparatus needed for the problem:** Practical work requires necessary equipment or apparatus. Usually cardboard sheets, charts, markers, pencils, rubbers, scales and geometry related apparatus is needed at secondary level.

c) **Procedure:** It defines the basic practical steps to prove a law or theorem.

d) **Observations:** After following the procedures, students get some substantive results which should be observed carefully.

e) **Conclusion:** After observing the substantive results, students are asked to conclude. On the basis of that conclusion, a law or theorem can be proved.

### 6.12.2 Merits and Demerits of Laboratory Method

This method has some merits and demerits as well (Sekhar, 2006).

**Merits:** This method has following merits.

- It is student centred method.
- Students play active role so they do not get bored.
- It is based on discovery approach.
- Knowledge gained through practical work is long lasting.
- As students establish laws and formulas themselves so they gain confidence.
- Practical utilisation of mathematics is realised by the students.
- When students work in groups then their learning becomes fast because of sharing information and ideas.
- The teacher-student relationship gets strengthened.

**Demerits:** This method has following demerits.

- It is very lengthy process.
- It is restricted to those topics only in which practical work is involved.
In Pakistan, it is very difficult for so many schools to spend a lot of money on tools and equipments used in this method.

Teachers have to practise a lot before applying this method in the classroom or laboratory.

Students can not practise this method to establish laws or principles independently.

It is more effective in lower level classes as compare to secondary level.

6.12.3 Application of Laboratory Method in Secondary Mathematics

This method is mostly used for practical geometry. At the secondary level of mathematics, it can also be used to establish or verify the laws and theorems in sets and trigonometry. These laws and theorems are usually proved through inductive method but laboratory method can be used at alternative basis to create interest in students.

6.12.4 Application of Laboratory Method in Sets

Problem: Prove De Morgan’s laws through Venn diagram.

\[ (A \cup B)^c = A^c \cap B^c \]
\[ (A \cap B)^c = A^c \cup B^c \]

Solution:

a) Objective or what is to be proved: To prove De Morgan’s laws through Venn diagram.

b) Apparatus needed for this problem: Cardboard sheet, pencil, rubber and scale.

c) Procedure: Every student will be given cardboard sheet and necessary equipments then they will also be explained well that how to draw Venn diagram. Then students
will be explained that there might be three cases for which Venn diagrams will be drawn.

Case (i): The sets A and B are overlapping

Case (ii): The sets A and B are disjoint

Case (iii): One set is subset of the other

d) Observations: For first De Morgan’s law \((A \cup B)^c = A^c \cap B^c\), students will start drawing Venn diagrams according to the above mentioned cases one by one.

Case (i): The sets A and B are overlapping then \((A \cup B)^c\) can be represented through Venn diagram as shown in Figure 6.19.

![Figure 6.19](image)

Figure 6.19. Shaded regions represent \((A \cup B)\) and lined region represents \((A \cup B)^c\).

Similarly when the sets A and B are overlapping then \(A^c \cap B^c\) can be represented through Venn diagram as shown in Figure 6.20.

![Figure 6.20](image)

Figure 6.20. Double lined region represents \(A^c \cap B^c\). Horizontal lined and double lined regions represent \(A^c\) and vertical lined and doubled lined regions represent \(B^c\).

From both the figures for case (i), it can easily be observed that \((A \cup B)^c = A^c \cap B^c\).
Case (ii): The sets A and B are disjoint then \((A \cup B)^c\) can be represented through Venn diagram as shown in Figure 6.21.

\[ \text{Figure 6.21.} \quad \text{Lined regions jointly represent } A \cup B \text{ and shaded region represents } (A \cup B)^c. \]

Similarly when the sets A and B are disjoint then \(A^c \cap B^c\) can be represented through Venn diagram as shown in Figure 6.22.

\[ \text{Figure 6.22.} \quad \text{Double lined region represents } A^c \cap B^c. \text{ However horizontal and doubled lined regions jointly represent } A^c \text{ and vertical and double lined regions jointly represent } B^c. \]

From both the figures for case (ii), it can easily be observed that \((A \cup B)^c = A^c \cap B^c\).

Case (iii): One set is subset of the other then \((A \cup B)^c\) can be represented through Venn diagram as shown in Figure 6.23.

\[ \text{Figure 6.23.} \quad \text{Vertical lined region represents } (A \cup B) \text{ and horizontal lined region } (A \cup B)^c. \]
Similarly when one set is subset of the other then $A^c \cap B^c$ can be represented through Venn diagram as shown in Figure 6.24.

![Venn Diagram](image)

**Figure 6.24.** Double lined region represents $A^c \cap B^c$. However shaded and double lined regions jointly represent $A^c$ and vertical and double lined regions jointly represent $B^c$.

Double lined region represents $A^c$, vertical and double lined regions jointly represent $B^c$ and double lined region only represents $A^c \cap B^c$. From both the figures for case (iii), it can easily be observed that $(A \cup B)^c = A^c \cap B^c$.

e) **Conclusion:** As the result of each case is same then students can conclude that

$$(A \cup B)^c = A^c \cap B^c.$$  

Similarly, second De Morgan’s law can also be proved, that is

$$(A \cap B)^c = A^c \cup B^c.$$  

### 6.12.5 Application of Laboratory Method in Trigonometry

**Problem:** To prove Pythagoras theorem that is

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

**Solution:**

a) **Objective or what is to be proved:** To prove Pythagoras theorem.

b) **Apparatus needed for this problem:** Cardboard sheet, pencil, rubber and scale.

c) **Procedure:** First of all, all necessary equipments or apparatus will be provided to every student. Then teacher will explain the difference between the hypotenuse, perpendicular and base. After this he or she will ask the students to draw a triangle with some fictitious values, for example, hypotenuse is equal to 5cm, perpendicular is
equal to 3 cm and base is equal to 4 cm. Teacher will also ask them to draw the squares on three sides of this triangle which should be divided into small squares with the sides of 1 cm each. Students will make the diagram according to the given measures as shown in Figure 6.25.

![Figure 6.25. Triangle with the squares on its three sides including hypotenuse, perpendicular and base.](image)

d) Observations: Students will observe the squares drawn on the sides of triangle and which are already subdivided into mini squares. Students will count the mini squares of each big square.

Number of mini squares in LMQR on base = 16

Number of mini squares in MNST on perpendicular = 9

Number of mini squares in LNOP on hypotenuse = 25
e) Conclusions: After observing the mini squares, it can easily be concluded that the number of mini squares drawn on hypotenuse is equal to the total number of mini squares drawn on base and perpendicular. Hence it is proved that

\[(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2\]

6.13 Project Method

This method is also based on the philosophy of “learning by doing”. It was devised by famous educationist Prof. Dr. William H. Kilpatrick who defined this method as “whole-hearted purposeful activity” (Kilpatrick, 1918). In this method, students are engaged in such kind of projects in which they get opportunity to apply their theoretical knowledge and to learn practically. In these projects, students work in a natural environment within the boundary of school or outside. During this process they face different kinds of real life problems and then try to solve them with previously gained knowledge. Projects may be at individual level but usually they are divided in the small groups of students (Sidhu, 1995).

Project method provides cooperative learning in which not only students share the ideas and knowledge but they also get motivated to complete the tasks as soon as possible. Famous educationist John Dewey (1916) emphasised on social interaction of the learners for the first time. Then Herbert Thelen (1954, 1960) also gave importance to cooperative learning in small groups.

6.13.1 Procedure of Project Method

There are five steps to accomplish a project successfully (Singh, 2007).
a) **Presenting a situation:** First of all, teacher presents a situation before students in which they feel some problems that can only be solved through mathematical operations.

b) **Proposing a project:** According to the situation, teacher proposes a project in which students show interest.

c) **Planning:** Students prepare the plan with the help of teacher. At this step, everybody should be given the opportunity to participate properly. Planning includes identification of necessary equipments, estimating the budget and time duration to complete the project.

d) **Execution:** Every student should take part with full interest and devotion in the project. Teacher should assign the duties according to the interest of each student and he or she should also observe the performance of every student. During the project, students should also keep record of all activities which can be helpful in future.

e) **Evaluation:** At the end of the project, teacher evaluate the result that whether the objectives of the project has been met or not. Progress of student is also evaluated at individual basis.

**6.13.2 Merits and Demerits of Project Method**

This method has some merits and demerits as well (Sekhar, 2006).

**Merits:** This method has following merits.

- It is totally student centred method.

- It helps students to correlate the mathematical knowledge with real life problems.

- It is a social activity that helps to promote friendly environment among students.
• Students share the ideas and experiences with each other.
• It gives confidence to the students.
• Students learn so many other things in real life situations.
• Students remain active and enjoy throughout the project.

**Demerits:** This method has some demerits as well.

• It is quite time consuming.
• It is costly because so many equipments are involved in it.
• Because of excessive practical work, students can not give much attention to practise the mathematical operations.
• Usually textbooks are not designed according to this method.
• It is very difficult to complete the syllabus in time with the help of this method.

### 6.13.3 Application of Project Method in Secondary Mathematics

This method is not used to teach one particular concept of mathematics. When students get master of different kinds of concepts with the help of other methods then project method is used to provide opportunity to them to apply their already learnt knowledge related to algebra, geometry or trigonometry in real life scenarios.

### 6.13.4 Project Examples

**Project 1:** “Paint the school walls.”

**Detail:** If a teacher wants to teach the students that how to take measurements of their home or school then he or she can design a project to paint all the rooms of their school or a specific portion of that school. For this kind of activity, students will have to ask the rates of paints and necessary equipments. Along with this, they will
have to measure the length, width and height of all walls, doors, windows and the building. Then they will calculate the total area which would be painted. During this project, students will learn that how they can apply different arithmetic, geometry and trigonometry related concepts and formulas in their real life.

**Project 2:** “Gardening in the school.”

**Detail:** If a teacher wants to teach the geometry concepts and “time and work” related problems then he or she can design a project of gardening. In this project, it is necessary that there should be such piece of land in the school, where gardening is possible. Then teacher will divide the students in small groups and each group will be allocated a small portion. Students will prepare the map of the garden and take measurements of their portions with the help of scales. Then they will find out the volume of cultivated land. Moreover, they also calculate working hours of each group and ratio of their performance.

During the cultivation process, students will also compare the cost of gardening and the growth. In this whole process, students will go through many mathematical formulas and methods. Along with this, they will also learn so many other things associated with this whole activity which will help them in their future life.

**6.14 Summary**

In this chapter, pedagogy of mathematics at secondary level has been described which is also domain of online system PAHMS. Pedagogy comprises of application of teaching methods in mathematics. These methods include lecture method, inductive method, deductive method, heuristic method, analytic method, synthetic method, problem solving method, laboratory method and project method. In answer to
research Question 4, the pedagogy of mathematics at secondary level has been described through several examples from different branches of mathematics according to the new curriculum designed by Ministry of Education, Pakistan.

According to this pedagogy, lecture method can be used to explain basic concepts of all branches of mathematics. Inductive method can be used to establish laws and formulas related to algebra, matrices and geometry. Already established laws and formulas can be applied through deductive method to solve problems related to all branches of mathematics. If students have not proper command to solve problems then heuristic or discovery method can be applied in which inquiry approach is quite helpful to make students capable for this. It is quite time consuming but it makes students creative. The problems in which students have to prove something can be taught with the help of analytic method. Such kinds of problems can be found in the units of algebra, ratio & proportion (variation) and geometry. When students get master to analyse the problems then they are in the position to synthesise to reach the desired result by already learnt concepts more quickly. So the problems in which something has to be proved can also be taught through synthetic method. This method is short and brief as compared to analytic method.

There are also some lengthy problems which can not be solved directly by applying a single formula or a small procedure then problem solving method is adopted. Such kinds of lengthy word problems can be found in the units of algebra, ratio & proportion (variation) and trigonometry.

To prove laws and theorems related to sets and trigonometry involves practical work at secondary level. For this purpose, laboratory method can be used. Practical geometry is totally dependent on this method. There is another method called project method provides opportunity to the students to relate their theoretical knowledge
about mathematics with their real life scenarios. Students get involved in different
small projects with the help of project method and try to get solutions by applying
laws and formulas related to different branches of mathematics.

A teacher should be familiar with all of these methods because he or she can
get better results by applying appropriate method according to the nature of a
problem.
CHAPTER 7

DATA ANALYSIS AND INTERPRETATION

This chapter focuses on the data analysis collected through survey research and its interpretation to ascertain the effectiveness of PAHMS. Section 7.1 describes the criterion according to which the data have been interpreted. Section 7.2 describes the analysis and interpretation of results of survey research conducted from the teachers of mathematics at secondary level. Section 7.3 summarises the data analysis.

7.1 Criterion for Data Interpretation

To ascertain the effectiveness of PAHMS, survey research was employed. Data were collected from teachers of mathematics through questionnaire (see appendix B). It was consisted of 25 statements and each statement was related to one particular feature of the system. This questionnaire was designed on five point Likert scale. Teachers gave response about each statement against this scale. It was consisted of five values from 1 to 5 indicating strongly disagree, disagree, undecided, agree and strongly agree.

PAHMS is a pedagogical adaptive learning system so effectiveness of the system can be ascertained in three aspects: learning environment, adaptation features and pedagogy provided by PAHMS. In this regard, questionnaire was also divided in three clusters which are related with these aspects.
One-sample *t*-test was used to judge whether majority of the teachers are undecided, agreed or disagreed with the statements of questionnaire. With the help of *t*-test, mean response values for each statement, their clusters and all of them collectively were calculated. Mean response value for each statement may range from 1 to 5, so the test value would be 3. Thus if mean response value for any statement is less than 3 then it shows that the majority of the teachers disagree with it and if mean response value is higher than 3 then it indicates that majority of the teachers agree with the statement.

### 7.2 Effectiveness of PAHMS

To ascertain the effectiveness of PAHMS, the overall opinion of the teachers of mathematics at secondary level about PAHMS was taken. Average mean score for all the clusters of questionnaire was calculated for this purpose by applying *t*-test as shown in Tables 7.1 and 7.2.

#### Table 7.1. One-sample statistics for overall opinion of teachers about PAHMS.

<table>
<thead>
<tr>
<th>Overall Opinion</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>3.8324</td>
<td>.23978</td>
<td>.02398</td>
</tr>
</tbody>
</table>

#### Table 7.2. One-sample *t*-test for overall opinion of teachers about PAHMS.

<table>
<thead>
<tr>
<th>Overall Opinion</th>
<th>Test Value = 3</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>t</em></td>
<td>df</td>
</tr>
<tr>
<td></td>
<td>34.716</td>
<td>99</td>
</tr>
</tbody>
</table>

For overall opinion of the teachers about PAHMS, the value of *t*(100)=34.716, *p*=0.000 is significant at *α*=0.05. The mean value of 3.8324 higher than test value of 3, shows that the majority of the teachers agree with the overall qualities of PAHMS.
This shows that the teachers gave favourable opinion about PAHMS but it is also important to ascertain the main features of the system separately in which learning environment, adaptation features and pedagogy provided by the system are included.

### 7.2.1 Learning Environment of PAHMS

Learning environment covers communication and assessment facilities, supporting tools for learners and reference material provided by PAHMS. The cluster consisting of statements 1 to 10 of the questionnaire is related with the learning environment of the system. To ascertain the overall learning environment of the system, one-sample \( t \)-test was used to calculate mean response value for this cluster as shown in Tables 7.3 and 7.4.

#### Table 7.3. One-sample statistics for overall learning environment of PAHMS.

<table>
<thead>
<tr>
<th>Learning Environment</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>3.9610</td>
<td>.34316</td>
<td>.03432</td>
</tr>
</tbody>
</table>

#### Table 7.4. One-sample \( t \)-test for overall learning environment of PAHMS.

<table>
<thead>
<tr>
<th>Learning Environment</th>
<th>Test Value = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>28.005</td>
</tr>
</tbody>
</table>

For overall learning environment of PAHMS, the value of \( t(100)=28.005 \), \( p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 3.9610 higher than test value of 3, shows that the majority of the teachers agree with the effectiveness of overall learning environment of PAHMS. Now one-sample \( t \)-test is applied to calculate mean response values for all statements of this cluster separately which are related to the leaning environment of PAHMS as shown in Tables 7.5 and 7.6.
Table 7.5. One-sample statistics for each statement of cluster related to learning environment of PAHMS.

<table>
<thead>
<tr>
<th>Statements related to Learning Environment</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement 1</td>
<td>100</td>
<td>4.0600</td>
<td>.81427</td>
<td>.08143</td>
</tr>
<tr>
<td>Statement 2</td>
<td>100</td>
<td>4.2200</td>
<td>.69019</td>
<td>.06902</td>
</tr>
<tr>
<td>Statement 3</td>
<td>100</td>
<td>4.0500</td>
<td>.64157</td>
<td>.06416</td>
</tr>
<tr>
<td>Statement 4</td>
<td>100</td>
<td>4.0400</td>
<td>.92025</td>
<td>.09203</td>
</tr>
<tr>
<td>Statement 5</td>
<td>100</td>
<td>3.7500</td>
<td>.75712</td>
<td>.07571</td>
</tr>
<tr>
<td>Statement 6</td>
<td>100</td>
<td>3.6800</td>
<td>.76383</td>
<td>.07638</td>
</tr>
<tr>
<td>Statement 7</td>
<td>100</td>
<td>4.4100</td>
<td>.55222</td>
<td>.05522</td>
</tr>
<tr>
<td>Statement 8</td>
<td>100</td>
<td>3.6800</td>
<td>.73691</td>
<td>.07369</td>
</tr>
<tr>
<td>Statement 9</td>
<td>100</td>
<td>4.0700</td>
<td>.67052</td>
<td>.06705</td>
</tr>
<tr>
<td>Statement 10</td>
<td>100</td>
<td>3.6500</td>
<td>.90314</td>
<td>.09031</td>
</tr>
</tbody>
</table>

Table 7.6. One-sample t-test for each statement of cluster related to learning environment of PAHMS.

<table>
<thead>
<tr>
<th>Statements related to Learning Environment</th>
<th>Test Value = 3</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower</td>
</tr>
<tr>
<td>Statement 1</td>
<td>13.018</td>
<td>99</td>
<td>0.000</td>
<td>1.06000</td>
<td>.8984</td>
<td>1.2216</td>
</tr>
<tr>
<td>Statement 2</td>
<td>17.676</td>
<td>99</td>
<td>0.000</td>
<td>1.22000</td>
<td>1.0831</td>
<td>1.3569</td>
</tr>
<tr>
<td>Statement 3</td>
<td>16.366</td>
<td>99</td>
<td>0.000</td>
<td>1.05000</td>
<td>.9227</td>
<td>1.1773</td>
</tr>
<tr>
<td>Statement 4</td>
<td>11.301</td>
<td>99</td>
<td>0.000</td>
<td>1.04000</td>
<td>.8574</td>
<td>1.2226</td>
</tr>
<tr>
<td>Statement 5</td>
<td>9.906</td>
<td>99</td>
<td>0.000</td>
<td>.75000</td>
<td>.5998</td>
<td>.9002</td>
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<tr>
<td>Statement 6</td>
<td>8.903</td>
<td>99</td>
<td>0.000</td>
<td>.68000</td>
<td>.5284</td>
<td>.8316</td>
</tr>
<tr>
<td>Statement 7</td>
<td>25.533</td>
<td>99</td>
<td>0.000</td>
<td>1.41000</td>
<td>1.3004</td>
<td>1.5196</td>
</tr>
<tr>
<td>Statement 8</td>
<td>9.228</td>
<td>99</td>
<td>0.000</td>
<td>.68000</td>
<td>.5338</td>
<td>.8262</td>
</tr>
<tr>
<td>Statement 9</td>
<td>15.958</td>
<td>99</td>
<td>0.000</td>
<td>1.07000</td>
<td>.9370</td>
<td>1.2030</td>
</tr>
<tr>
<td>Statement 10</td>
<td>7.197</td>
<td>99</td>
<td>0.000</td>
<td>.65000</td>
<td>.4708</td>
<td>.8292</td>
</tr>
</tbody>
</table>
For statement 1, the value of $t(100)=13.018, p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.0600 higher than test value of 3, shows that the majority of the teachers agree with the statement 1 that is “PAHMS provides more attractive and self-paced learning environment as compared to conventional learning mode”.

For statement 2, the value of $t(100)=17.676, p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.2200 higher than test value of 3, shows that the majority of the teachers agree with the statement 2 that is “objectives and description of the system are clearly mentioned”.

For statement 3, the value of $t(100)=16.366, p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.0500 higher than test value of 3, shows that the majority of the teachers agree with the statement 3 that is “sources for reference material are available”.

For statement 4, the value of $t(100)=11.301, p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.0400 higher than test value of 3, shows that the majority of the teachers agree with the statement 4 that is “learning is assessed by different methods including tests, discussions and assignments”.

For statement 5, the value of $t(100)=9.906, p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.7500 higher than test value of 3, shows that the majority of the teachers agree with the statement 5 that is “learner is given opportunity of collaborative study”.

For statement 6, the value of $t(100)=8.903, p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.6800 higher than test value of 3, shows that the majority of the teachers agree with the statement 6 that is “discussion forums are used for discussing pedagogical issues”.
For statement 7, the value of $t(100)=25.533$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.4100 higher than test value of 3, shows that the majority of the teachers agree with the statement 7 that is “private messaging facility for learner to learner, learner to instructor and instructor to learner is available”.

For statement 8, the value of $t(100)=9.228$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.6800 higher than test value of 3, shows that the majority of the teachers agree with the statement 8 that is “live communication facilities are available including chatting, audio or video conferencing”.

For statement 9, the value of $t(100)=15.958$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.0700 higher than test value of 3, shows that the majority of the teachers agree with the statement 9 that is “learner can observe his or her educational performance”.

For statement 10, the value of $t(100)=7.197$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.6500 higher than test value of 3, shows that the majority of the teachers agree with the statement 10 that is “learner can manage his or her educational material online”.

Teachers gave favourable opinion for all statements of cluster related to learning environment of PAHMS. Hence it can be concluded that learning environment of PAHMS is quite effective.

7.2.2 Adaptive Environment of PAHMS

Adaptive environment covers adaptive presentation and adaptive navigation facilities provided by PAHMS. The cluster consisting of statements 11 to 20 of the questionnaire is related with the adaptation features of the system. To ascertain the
adaptive environment of the system as a whole, one-sample $t$-test was used to calculate mean response value for this cluster as shown in Tables 7.7 and 7.8.

**Table 7.7.** One-sample statistics for adaptive environment of PAHMS.

<table>
<thead>
<tr>
<th>Adaptive Environment of PAHMS</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>3.6570</td>
<td>.30260</td>
<td>.03026</td>
</tr>
</tbody>
</table>

**Table 7.8.** One-sample $t$-test for adaptive environment of PAHMS.

<table>
<thead>
<tr>
<th>Adaptive Environment of PAHMS</th>
<th>Test Value = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>21.712</td>
</tr>
</tbody>
</table>

For overall adaptive environment of PAHMS, the value of $t(100)=21.712$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.6570 higher than test value of 3, shows that the majority of the teachers agree with the effectiveness of adaptive environment of PAHMS. Now one-sample $t$-test is applied to calculate mean response values for all statements of the cluster separately which are related to the adaptive features of PAHMS as shown in Tables 7.9. and 7.10.
Table 7.9. One-sample statistics for each statement of cluster related to adaptive environment of PAHMS.

<table>
<thead>
<tr>
<th>Statements related to Adaptive Environment</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement 11</td>
<td>100</td>
<td>4.0800</td>
<td>.59764</td>
<td>.05976</td>
</tr>
<tr>
<td>Statement 12</td>
<td>100</td>
<td>4.2200</td>
<td>.85965</td>
<td>.08596</td>
</tr>
<tr>
<td>Statement 13</td>
<td>100</td>
<td>4.1800</td>
<td>.53899</td>
<td>.05390</td>
</tr>
<tr>
<td>Statement 14</td>
<td>100</td>
<td>3.8700</td>
<td>.81222</td>
<td>.08122</td>
</tr>
<tr>
<td>Statement 15</td>
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<td>.67353</td>
<td>.06735</td>
</tr>
<tr>
<td>Statement 16</td>
<td>100</td>
<td>2.4100</td>
<td>.87727</td>
<td>.08773</td>
</tr>
<tr>
<td>Statement 17</td>
<td>100</td>
<td>2.2100</td>
<td>.87957</td>
<td>.08796</td>
</tr>
<tr>
<td>Statement 18</td>
<td>100</td>
<td>3.8600</td>
<td>.97463</td>
<td>.09746</td>
</tr>
<tr>
<td>Statement 19</td>
<td>100</td>
<td>3.9800</td>
<td>.65103</td>
<td>.06510</td>
</tr>
<tr>
<td>Statement 20</td>
<td>100</td>
<td>3.7300</td>
<td>.72272</td>
<td>.07227</td>
</tr>
</tbody>
</table>

Table 7.10. One-sample $t$-test for each statement of cluster related to adaptive environment of PAHMS.

<table>
<thead>
<tr>
<th>Statements related to Adaptive Environment</th>
<th>t</th>
<th>Df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower</td>
</tr>
<tr>
<td>Statement 11</td>
<td>18.071</td>
<td>99</td>
<td>0.000</td>
<td>1.08000</td>
<td>.9614</td>
</tr>
<tr>
<td>Statement 12</td>
<td>14.192</td>
<td>99</td>
<td>0.000</td>
<td>1.22000</td>
<td>1.0494</td>
</tr>
<tr>
<td>Statement 13</td>
<td>21.893</td>
<td>99</td>
<td>0.000</td>
<td>1.18000</td>
<td>1.0731</td>
</tr>
<tr>
<td>Statement 14</td>
<td>10.711</td>
<td>99</td>
<td>0.000</td>
<td>.87000</td>
<td>.7088</td>
</tr>
<tr>
<td>Statement 15</td>
<td>15.293</td>
<td>99</td>
<td>0.000</td>
<td>1.03000</td>
<td>.8964</td>
</tr>
<tr>
<td>Statement 16</td>
<td>-6.725</td>
<td>99</td>
<td>0.000</td>
<td>-.59000</td>
<td>-.7641</td>
</tr>
<tr>
<td>Statement 17</td>
<td>-8.982</td>
<td>99</td>
<td>0.000</td>
<td>-.79000</td>
<td>-.9645</td>
</tr>
<tr>
<td>Statement 18</td>
<td>8.824</td>
<td>99</td>
<td>0.000</td>
<td>.86000</td>
<td>.6666</td>
</tr>
<tr>
<td>Statement 19</td>
<td>15.053</td>
<td>99</td>
<td>0.000</td>
<td>.98000</td>
<td>.8508</td>
</tr>
<tr>
<td>Statement 20</td>
<td>10.101</td>
<td>99</td>
<td>0.000</td>
<td>.73000</td>
<td>.5866</td>
</tr>
</tbody>
</table>
For statement 11, the value of $t(100)=18.071, p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.0800 higher than test value of 3, shows that the majority of the teachers agree with the statement 11 that is “global guidance is available by suggesting all possible links to follow”.

For statement 12, the value of $t(100)=14.192, p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.2200 higher than test value of 3, shows that the majority of the teachers agree with the statement 12 that is “local guidance is available by suggesting the next most appropriate link”.

For statement 13, the value of $t(100)=21.893, p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.1800 higher than test value of 3, shows that the majority of the teachers agree with the statement 13 that is “global orientation support is provided with the help of complete site map”.

For statement 14, the value of $t(100)=10.711, p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.8700 higher than test value of 3, shows that the majority of the teachers agree with the statement 14 that is “local orientation support is provided by suggesting most relevant links to the current web page”.

For statement 15, the value of $t(100)=15.293, p=0.000$ is significant at $\alpha=0.05$. The mean value of 4.0300 higher than test value of 3, shows that the majority of the teachers agree with the statement 15 that is “learner is given opportunity to change the layout of PAHMS”.

For statement 16, the value of $t(100)=-6.725, p=0.000$ is significant at $\alpha=0.05$. The mean value of 2.4100 less than test value of 3, shows that the majority of the teachers disagree with the statement 16 that is “pedagogical content is sorted according to the learner’s educational level with comparative explanation”.
For statement 17, the value of \( t(100) = -8.982, p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 2.2100 less than test value of 3, shows that the majority of the teachers disagree with the statement 17 that is “pedagogical content is presented in different shapes and mediums like text, animation, audio and video”.

For statement 18, the value of \( t(100) = 8.824, p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 3.8600 higher than test value of 3, shows that the majority of the teachers agree with the statement 18 that is “pedagogical content is presented according to the learner’s educational level”.

For statement 19, the value of \( t(100) = 15.053, p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 3.9800 higher than test value of 3, shows that the majority of the teachers agree with the statement 19 that is “prerequisite concepts are shown to the learner”.

For statement 20, the value of \( t(100) = 10.101, p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 3.7300 higher than test value of 3, shows that the majority of the teachers agree with the statement 20 that is “adaptive peers searching facility is available”.

Teachers gave favourable opinion for the statements of cluster related to adaptive environment of PAHMS except for the statements 16 and 17. Hence it can be concluded that adaptive environment of PAHMS is also quite effective and it supports both adaptive presentation and adaptive navigation but the system does not support three features related to adaptive presentation including adaptive sorting, comparative explanations and explanation variants.
7.2.3 Pedagogy Provided by PAHMS

Pedagogy includes application of teaching methods in different branches of mathematics at secondary level with the help of several examples from the curriculum of mathematics in Pakistan. The cluster consisting of statements 21 to 25 of the questionnaire is related with the pedagogy provided by the system. To ascertain the pedagogy of the system as a whole, one-sample \( t \)-test was used to calculate mean response value for this cluster as shown in Tables 7.11. and 7.12.

Table 7.11. One-sample statistics for pedagogy provided by PAHMS.

<table>
<thead>
<tr>
<th>Pedagogy</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>3.9260</td>
<td>.50983</td>
<td>.05098</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pedagogy</th>
<th>Test Value = 3</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td>df</td>
</tr>
<tr>
<td></td>
<td>18.163</td>
<td>99</td>
</tr>
</tbody>
</table>

For pedagogy provided by PAHMS, the value of \( t(100)= 18.163, p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 3.9260 higher than test value of 3, shows that the majority of the teachers agree with the pedagogy provided by the system. Now one-sample \( t \)-test is applied to calculate mean response values for all statements of the cluster separately which are related to the pedagogy as shown in Tables 7.13. and 7.14.
Table 7.13. One-sample statistics for each statement of cluster related to the pedagogy provided by PAHMS.

<table>
<thead>
<tr>
<th>Statements related to Pedagogy</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement 21</td>
<td>100</td>
<td>4.0700</td>
<td>.57305</td>
<td>.05730</td>
</tr>
<tr>
<td>Statement 22</td>
<td>100</td>
<td>4.3300</td>
<td>.84154</td>
<td>.08415</td>
</tr>
<tr>
<td>Statement 23</td>
<td>100</td>
<td>3.8300</td>
<td>.80472</td>
<td>.08047</td>
</tr>
<tr>
<td>Statement 24</td>
<td>100</td>
<td>3.7400</td>
<td>.73333</td>
<td>.07333</td>
</tr>
<tr>
<td>Statement 25</td>
<td>100</td>
<td>3.6600</td>
<td>.90140</td>
<td>.09014</td>
</tr>
</tbody>
</table>

Table 7.14. One-sample t-test for each statement of cluster related to the pedagogy provided by PAHMS.

<table>
<thead>
<tr>
<th>Statements related to Pedagogy</th>
<th>Test Value = 3</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td>df</td>
</tr>
<tr>
<td>Statement 21</td>
<td>18.672</td>
<td>99</td>
</tr>
<tr>
<td>Statement 22</td>
<td>15.804</td>
<td>99</td>
</tr>
<tr>
<td>Statement 23</td>
<td>10.314</td>
<td>99</td>
</tr>
<tr>
<td>Statement 24</td>
<td>10.091</td>
<td>99</td>
</tr>
<tr>
<td>Statement 25</td>
<td>7.322</td>
<td>99</td>
</tr>
</tbody>
</table>

For statement 21, the value of t(100)=18.672, p=0.000 is significant at α=0.05. The mean value of 4.0700 higher than test value of 3, shows that the majority of the teachers agree with the statement 21 that is “pedagogy provided by the system is according to the new curriculum of mathematics”.

For statement 22, the value of t(100)=15.804, p=0.000 is significant at α=0.05. The mean value of 4.3300 higher than test value of 3, shows that the majority of the teachers agree with the statement 22 that is “pedagogy covers all sections of curriculum”.
For statement 23, the value of $t(100)=10.314$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.8300 higher than test value of 3, shows that the majority of the teachers agree with the statement 23 that is “pedagogy includes application of all famous teaching methods in mathematics”.

For statement 24, the value of $t(100)=10.091$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.7400 higher than test value of 3, shows that the majority of the teachers agree with the statement 24 that is “pedagogy is explained with the help of appropriate examples from different branches of mathematics”.

For statement 25, the value of $t(100)=7.322$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.6600 higher than test value of 3, shows that the majority of the teachers agree with the statement 25 that is “the procedures of all teaching methods are easily understandable”.

Teachers gave favourable opinion for all statements of cluster related to the pedagogy provided by PAHMS. Hence it can be concluded that the pedagogy of mathematics at secondary level is quite effective.

7.2.4 Opinion about PAHMS at the Bases of Demographic Variables

Effectiveness of PAHMS was also ascertained at the basis of demographic variables. These variables include gender, age and teaching experience. To get the overall opinion of teachers about the effectiveness of PAHMS at the basis of teaching experience, average mean score for all the clusters of questionnaire was calculated by applying $t$-test as shown in Tables 7.15 and 7.16.
Table 7.15. One-sample statistics for overall opinion of both in-service and student teachers about PAHMS.

<table>
<thead>
<tr>
<th>Teachers</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-Service Teachers</td>
<td>50</td>
<td>3.8544</td>
<td>.16577</td>
<td>.02344</td>
</tr>
<tr>
<td>Student Teachers</td>
<td>50</td>
<td>3.8104</td>
<td>.29613</td>
<td>.04188</td>
</tr>
</tbody>
</table>

Table 7.16. One-sample t-test for overall opinion of both in-service and student teachers about PAHMS.

<table>
<thead>
<tr>
<th>Teachers</th>
<th>T</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower</td>
</tr>
<tr>
<td>In-Service Teachers</td>
<td>36.446</td>
<td>49</td>
<td>0.000</td>
<td>.85440</td>
<td>.8073</td>
</tr>
<tr>
<td>Student Teachers</td>
<td>19.351</td>
<td>49</td>
<td>0.000</td>
<td>.81040</td>
<td>.7262</td>
</tr>
</tbody>
</table>

For overall opinion of the in-service teachers about PAHMS, the value of $t(50)=36.446$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.8544 higher than test value of 3, shows that the majority of the in-service teachers agree with the effectiveness of PAHMS. Similarly, for overall opinion of the student teachers about PAHMS, the value of $t(50)=19.351$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.8104 higher than test value of 3, shows that the majority of the student teachers also agree with the effectiveness of PAHMS.

To get the overall opinion of teachers at the basis of gender, average mean score for all the clusters of questionnaire was calculated by applying $t$-test as shown in Tables 7.17 and 7.18.
Table 7.17. One-sample statistics for overall opinion of both male and female teachers about PAHMS.

<table>
<thead>
<tr>
<th>Gender</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>42</td>
<td>3.8762</td>
<td>.15650</td>
<td>.02415</td>
</tr>
<tr>
<td>Female</td>
<td>58</td>
<td>3.8007</td>
<td>.28249</td>
<td>.03709</td>
</tr>
</tbody>
</table>

Table 7.18. One-sample t-test for overall opinion of both male and female teachers about PAHMS.

<table>
<thead>
<tr>
<th>Gender</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower</td>
</tr>
<tr>
<td>Male</td>
<td>36.283</td>
<td>41</td>
<td>0.000</td>
<td>.87619</td>
<td>.8274</td>
</tr>
<tr>
<td>Female</td>
<td>21.586</td>
<td>57</td>
<td>0.000</td>
<td>.80069</td>
<td>.7264</td>
</tr>
</tbody>
</table>

For overall opinion of the male teachers about PAHMS, the value of \( t(42)=36.283, \ p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 3.8762 higher than test value of 3, shows that the majority of the male teachers agree with the effectiveness of PAHMS. Similarly, for overall opinion of the female teachers about PAHMS, the value of \( t(58)=21.586, \ p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 3.8007 higher than test value of 3, shows that the majority of the female teachers also agree with the effectiveness of PAHMS.

To get the overall opinion of teachers at the basis of age, average mean score for all the clusters of questionnaire was calculated by applying \( t \)-test as shown in Tables 7.19 and 7.20.

Table 7.19. One-sample statistics for overall opinion of teachers about PAHMS at the base of age.

<table>
<thead>
<tr>
<th>Age</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 40</td>
<td>61</td>
<td>3.7993</td>
<td>.27947</td>
<td>.03578</td>
</tr>
<tr>
<td>Above 40</td>
<td>39</td>
<td>3.8841</td>
<td>.14819</td>
<td>.02373</td>
</tr>
</tbody>
</table>
Table 7.20. One-sample $t$-test for overall opinion of teachers about PAHMS at the base of age.

<table>
<thead>
<tr>
<th>Age</th>
<th>T</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 40</td>
<td>22.339</td>
<td>60</td>
<td>0.000</td>
<td>.79934</td>
<td>Lower: .7278, Upper: .8709</td>
</tr>
<tr>
<td>Above 40</td>
<td>37.257</td>
<td>38</td>
<td>0.000</td>
<td>.88410</td>
<td>Lower: .8361, Upper: .9321</td>
</tr>
</tbody>
</table>

For overall opinion of the below 40 teachers about PAHMS, the value of $t(61)=22.339, p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.7993 higher than test value of 3, shows that the majority of the teachers who are below 40 agree with the effectiveness of PAHMS. Similarly, for overall opinion of the above 40 teachers about PAHMS, the value of $t(39)=37.257, p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.8841 higher than test value of 3, shows that the majority of the teachers who are above 40 also agree with the effectiveness of PAHMS.

7.3 Summary

To ascertain the effectiveness of PAHMS, survey was conducted from both in-service and student teachers through questionnaire designed at five point Likert scale. Data collected through survey were analysed to check the effectiveness of learning environment, adaptive features and pedagogy provided by the system. One-sample $t$-test was used for data analysis. Majority of the teachers agree with the statements of questionnaire depicting the features of PAHMS. Effectiveness was also ascertained at the basis of demographic variables including teaching experience, gender and age. There was no significant difference between the results. Everybody gave positive response about the system.
CHAPTER 8

SUMMARY, FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

This chapter summarises the results of this reach. Section 8.1 provides the brief summary of the thesis. Section 8.2 describes the findings of survey research about PAHMS. Section 8.3 describes the conclusions at the basis of findings. Section 8.4 gives some recommendations for further research and development.

8.1 Summary

The main purpose of this research was to develop pedagogical adaptive hypermedia based system for mathematics at secondary level (PAHMS). It provides teaching methodology for mathematics to both in-service and student teachers according to their needs and educational level. It supports adaptive presentation and adaptive navigation along with adaptive peers’ searching. PAHMS has been developed using an open source tool COFALE.

Pedagogy of mathematics provided by PAHMS comprises of application of famous teaching methods in different branches of mathematics. These methods include lecture method, inductive method, deductive method, analytic method, synthetic method, heuristic method, problem solving method, laboratory method and project method. Procedures, merits and demerits of all teaching methods have been explained in this pedagogy.
E-learning objects have also been developed including instructional, collaborative and assessment objects. Instructional objects include lessons, glossary and external web resources. Collaborative objects like discussion forums, inbox and chat are also part of PAHMS because of the framework of COFALE. Assessment objects include online tests designed for all teaching methods separately.

Effectiveness of PAHMS has also been ascertained by conducting survey from hundred teachers of mathematics teaching in different schools of Lahore city. They gave positive response about learning and adaptive environment of PAHMS and pedagogy provided by it. PAHMS is also compared with an existing adaptive e-learning system GRAPPLE Tutorial (see appendix E).

8.2 Findings

Online survey was conducted to ascertain the effectiveness of PAHMS. Both in-service and student teachers gave positive response about the system after using it. After getting their opinion about the system, the findings of the research can be summarised as follows:

- PAHMS provides self paced learning environment so that any teacher of mathematics can learn pedagogy at any time and any place.
- PAHMS is much better than classroom based teacher’s training.
- Teachers can easily understand the objectives of the system when they interact with it online.
- References material in the shape of external web links is available in the system.
- Assessment of learners can be done through online tests and discussion forums in PAHMS.
• Because of communication facilities, learners are given opportunity for collaborative study in PAHMS.

• Both synchronous and asynchronous ways of communication like inbox, chatting and announcements are available.

• Discussion forums are used for discussing different pedagogical issues.

• System also provides online tracking through which a learner can check his or her educational performance.

• Learners are given facility to manage their educational content online through document manager.

• During the learning process, a learner is given global guidance with the help of table of content and providing all related links to one particular concept.

• Local guidance is also provided by the system by indicating the next most relevant link.

• Global orientation is also provided in the shape of complete site map of the system.

• For the help of a leaner, most relevant links to the current web page are shown as a local orientation.

• Learners are given facility to change the layout of PAHMS according to his or her needs.

• PAHMS does not support adaptive sorting of content according to the educational background of a learner.

• Comparative explanations are also not provided by the system.

• PAHMS provides pedagogical content in the shape of text, figures and tables. It does not provide audio or video material.
• PAHMS provides pedagogy to the teachers according to their educational level.

• Prerequisites of a concept are shown to learner with the help of related links.

• Each learner can find most appropriate peers according to his or her learner model.

• PAHMS provides pedagogy according to the new curriculum of mathematics designed by Ministry of Education, Pakistan.

• Pedagogy covers all units of curriculum include sets, logarithm, algebra, variance, matrices, information handling, geometry and trigonometry.

• All teaching methods are explained with the help of appropriate examples from curriculum.

• The procedures to apply all the teaching methods in classroom are clearly explained by the system.

8.3 Conclusions

According to the findings of this research following conclusions can be drawn:

• Online teaching and learning is much better than classroom based instruction because it is fast, convenient and self paced.

• Adaptive presentation and adaptive navigation in PAHMS provide learners centred environment which makes the system more attractive for them.

• Reference material can be accessed more easily through PAHMS because there is no need to visit libraries as learners have to do it in conventional mode of instruction.

• Collaborative learning and adaptive peer searching in PAHMS make learning more effective and learner centred.
• Learner to learner, learner to instructor and instructor to instructor communication is the basic need of education. PAHMS fulfils this need through different communication tools.

• Learners’ assessment becomes more effective through the system because of online test and discussion forums as compared to conventional mode.

• Learners’ tools in PAHMS make learning process more convenient. For example, glossary provides definitions of all important terms related to pedagogy. Learners can also save notes, important websites and concept maps in PAHMS. They can observe their performance as well.

• Global and local guidance provided by PAHMS save time to search relevant links related to one particular concept.

• Global and local orientations prevent a learner being lost in hyperspace of the system.

• Learners can change the layout of the system which gives more personalised view to them.

• PAHMS deals with both in-service and student teachers. It provides pedagogy to them according to their teaching experience but it neither sorts the content according to their educational performance nor provides comparative explanations.

• Teachers of mathematics can access online pedagogy whenever they need to consult it.

• Teaching methods explained with the help of examples according to the new curriculum, so it is quite helpful for the teachers to access up-to-date
pedagogy. Moreover, pedagogy can be updated with the passage of time easily.

- Pedagogy can be enriched with the help of suggestions provided by different experts all over the country because it is available to everybody free of cost.

### 8.4 Recommendations

Both general and specific recommendations are given below:

- Adaptive presentation can be improved in PAHMS with the help of different methods like *comparative explanations*, *explanation variants* and *sorting* with the help of different techniques used for adaptive presentation.

- Adaptive navigation support in PAHMS can also be improved with the help of different techniques like *link sorting*, *link annotation*, *link hiding* and *link generation*.

- Mechanism of adaptive assessment can be incorporated in the system based on learner’s proficiency and previous educational level.

- Different images, tables and graphs as e-learning objects have been used in the system but audio and video content should also be added in it.

- Facility of audio and video chatting can also be provided through PAHMS.

- Research can be carried out to make this system for multiple languages.

- Adaptive problem solving support can be incorporated in PAHMS as well.

- PAHMS provides pedagogy to two types of users: student teachers and in-service teachers. There might be some learners within a specific type who vary with respect to their educational level or preferences. This issue can be resolved through user level ontology engineering.

- Adaptive curriculum sequencing can also be implemented in the system.
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APPENDIX A

ONLINE TESTS FOR PEDAGOGY OF MATHEMATICS IN PAHMS

Test 1: Pedagogy and Teaching Standards

Multiple Choice Questions

1. Appreciating different thinking and learning styles,
   a) gives confidence to the students.
   b) promotes creativity among students.
   c) helps in solving problems.

2. Teacher can guide students about ethical values in
   a) collaborative learning.
   b) students’ societies.
   c) workshops.

3. Instructional strategies should be adopted according to
   a) the learning environment.
   b) the subject.
   c) cultural background of students.

4. Engaging in educational research helps in
   a) students’ assessment.
   b) human growth and development.
c) continuous professional development of teacher.

5. Teacher’s collaboration and partnerships include
   a) relationships with students at individual level.
   b) working relationships with teachers and administration.
   c) relationships with professionals and parents of students.

**True / False Questions**

6. According to cognitivism, behaviours of learners can be changed or reshaped with the help of reinforcement.
   a) True  b) False

7. One of the aims of teaching mathematics is to enable the students to find out the similar patterns in one particular activity or phenomenon for generalising the results from them.
   a) True  b) False

8. Competitive environment can only be developed with the help of cooperative learning.
   a) True  b) False

9. Cooperative learning plays vital role in human growth and development.
   a) True  b) False

10. Informing parents about student’s performance leaves bad effects on student.
    a) True  b) False

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**Test 2: Lecture Method**

**Multiple Choice Questions**

1. Lecture method is more useful at
a) primary level.
b) secondary level.
c) higher secondary level.

2. Lecture method is helpful to explain
   a) basic concepts.
   b) complex mathematical methods.
   c) practical kind of problems.

3. Lecture method is helpful to
   a) develop thinking and reasoning ability in the students.
   b) raise the interest level of the students.
   c) make the environment interesting.

4. To keep the interest level high, teacher should
   a) ask questions continuously.
   b) teach them speedily.
   c) avoid giving multiple examples.

5. The best way of teaching in combination with other teaching methods is
   a) visual.
   b) verbal.
   c) manual.

**True / False Questions**

6. If strength of class is very high then lecture method does not remain effective.
   a) True  b) False

7. Lecture method is also called training model.
   a) True  b) False

8. Lecture method is student centred approach.
9. Teacher-student relationship can be developed in a better way with the help of
lecture method.
   a) True    b) False

10. Lecture method is applicable to teach all branches of mathematics.
    a) True    b) False

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**Test 3: Inductive Method**

**Multiple Choice Questions**

1. Inductive method is more suitable for
   a) secondary level classes.
   b) under-graduate level classes.
   c) Post-graduate level classes.

2. Inductive method develops
   a) problem solving ability in students.
   b) reasoning ability in students.
   c) creativity in students.

3. At secondary level, inductive method is mainly used for
   a) Statistics.
   b) Logarithms.
   c) Algebra.

4. Establishing laws, formulas or principals
   a) promotes collaboration among students.
b) helps in remembering them easily.

c) makes the teacher’s job easy.

5. Inductive method can easily be practised by

a) all the teachers.

b) experienced teachers only.

c) highly qualified teachers only.

**True / False Questions**

6. Inductive method helps to introduce a new concept in mathematics.

a) True  

b) False

7. Inductive method is logical in nature.

a) True  

b) False

8. Teaching becomes fast with the help of inductive method.

a) True  

b) False

9. It is always authentic to prove formulas, rules or laws by inductive method.

a) True  

b) False

10. Inductive method is excessively used for problem solving.

a) True  

b) False

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**Test 4: Deductive Method**

**Multiple Choice Questions**

1. Deductive method helps in

a) developing reasoning ability in students.

b) enhancing computational ability in students.
c) making students creative.

2. Deductive method is useful to
   a) establish new rules or formulas.
   b) complete syllabus easily.
   c) encourage cooperative learning.

3. Deductive method becomes blessing for those students
   a) who can remember formulas or rules easily.
   b) who have discovery approach in learning.
   c) who have strong analytical ability.

4. In deductive method, we proceed from
   a) known to unknown.
   b) specific to general.
   c) general to specific.

5. Deductive method is more suitable for
   a) primary level classes.
   b) middle level classes.
   c) secondary level classes.

True / False Questions

6. Deductive method is not suitable for those students who can not remember complicated mathematical procedures.
   a) True    b) False

7. Deductive method is quite suitable for derivation of formulas or rules.
   a) True    b) False

8. Deductive method is quite time consuming.
   a) True    b) False
9. We can prove theorems using deductive method with the help of already defined rules or principles.
   a) True  
   b) False

10. Deductive method is applicable in all fields of mathematics.
   a) True  
   b) False

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Test 5: Heuristic / Discovery Method

Multiple Choice Questions

1. Discovery nature of heuristic method
   a) develops creativity in students.
   b) makes learning process boring.
   c) is suitable to establish new rules, formulas or laws.

2. Teachers should adopt heuristic method when
   a) syllabus is quite lengthy.
   b) they are properly trained for this.
   c) strength of the class is high.

3. Continuously inquiring students in heuristic method
   a) becomes irritating for them.
   b) keeps them active.
   c) disturbs their thinking process.

4. Students get distracted in discovery solution
   a) if problem is quite lengthy.
   b) if there is a lack of coordination among students.
c) if teacher asks inappropriate questions.

5. Heuristic method clears the concepts in better way because
   a) teacher guides students at every step.
   b) students play active role in it.
   c) students construct their knowledge themselves.

**True / False Questions**

6. Heuristic method is purely teacher centred.
   a) True  b) False

7. Heuristic method is suitable to teach all kinds of mathematical problems.
   a) True  b) False

8. Children’s cognitive skill is the base of heuristic method.
   a) True  b) False

9. Heuristic method does not develop thinking and reasoning abilities in students.
   a) True  b) False

10. Heuristic method is not much time consuming.
    a) True  b) False

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**Test 6: Analytic Method**

**Multiple Choice Questions**

1. There is always less chance of doubts in analytic method because
   a) it is logical.
   b) it involves cooperative learning.
   c) it involves more teacher-student interaction.
2. Analytic method is not suitable if
   a) strength of the class is high.
   b) syllabus is lengthy.
   c) students are deprived of equipment for practical work.

3. Analytic method leads us from
   a) known to unknown.
   b) unknown to known.
   c) general to specific.

4. Student always play active role in analytic method because
   a) teacher interacts with them frequently.
   b) analyzing the mathematical problem is quite easy.
   c) they are involved in discovery process.

5. In analytic method, students do not need to
   b) follow any set procedure.
   c) memories specific formulas or rules.
   d) focus on mathematical operations and their proper reasons.

**True / False Questions**

6. Analytic method is suitable to solve all kinds of problems.
   a) True          b) False

7. Students can reach the solution in short time with the help of analytic method.
   a) True          b) False

8. Analytic method is not scientific in nature.
   a) True          b) False

9. Only skilled teachers can apply analytic method.
   a) True          b) False
10. Analytic method is very useful for solving problems related to information handling or statistics.

   a) True  b) False

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**Test 7: Synthetic Method**

**Multiple Choice Questions**

1. Synthetic method provides opportunity to the students to

   a) practise mathematical formulas.
   b) collaborate with each other.
   c) learn through practical work.

2. Synthetic method is fast because

   a) it involves deduction reasoning.
   b) there is less analytical process in it.
   c) teacher helps students a lot.

3. In synthetic method, student have to

   a) remember a set procedure.
   b) go through discovery process.
   c) give proper reasoning for each mathematical operation.

4. Synthetic method is useful

   a) to finish the lengthy course in short time.
   b) to clarify the concepts completely.
   c) because it is student centred.
5. Synthetic method is helpful to
   a) enhance cognitive skills of students.
   b) develop scientific approach in students.
   c) sharpen memory of students.

**True / False Questions**

6. Synthetic method is based on already learnt concepts.
   a) True   b) False

7. In synthetic method, we proceed from desired result to the given condition.
   a) True   b) False

8. Synthetic method is mainly used to derive mathematical proofs.
   a) True   b) False

9. Synthetic method helps to make the students creative.
   a) True   b) False

10. Better results can be achieved by combining analytic and synthetic methods.
    a) True   b) False

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**Test 8: Problem Solving Method**

**Multiple Choice Questions**

1. Problem solving method develops thinking ability in students because
   a) students have to devise a plan after analysing the whole problem.
   b) students go through discovery process.
   c) it involves learning by doing.
2. Problem solving method gives opportunity to the students to
   a) utilise their previous knowledge.
   b) learn in collaborative environment.
   c) practise formulas or rules.

3. Student reaches the goal in problem solving method through
   a) deductive reasoning.
   b) inductive reasoning.
   c) a set of procedures.

4. Problems solving method is more suitable to solve
   a) geometry related problems.
   b) word problems.
   c) problems related to sets.

5. Problem solving method is not recommended for
   a) primary level classes.
   b) secondary level classes.
   c) higher secondary level classes.

**True / False Questions**

6. Problem solving method is quite helpful to prove geometrical theorems.
   a) True  b) False

7. Problem solving method is used to solve those problems which can not be solved
   easily by applying single formula or method.
   a) True  b) False

8. Students learn that how to cope with a totally new situation though problem
   solving method.
   a) True  b) False
9. Logical approach of problem solving method presents students from practical work.
   a) True       b) False

10. Problem solving method is children’s psychology centred.
    a) True       b) False

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**Test 9: Laboratory Method**

**Multiple Choice Questions**

1. Laboratory method is called an extended form of
   a) inductive method.
   b) analytic method.
   c) synthetic method.

2. Role of teacher in laboratory method is to
   a) inquire students at each step.
   b) supervise students.
   c) provide facilitation for collaborative study.

3. The main focus of laboratory method is to
   a) give opportunity to the students to work in collaborative environment.
   b) establish laws and theorems through practical work.
   c) familiarise the students about apparatus related to mathematics.

4. Laboratory method is based on
   a) cooperative learning.
   b) discovery approach.
c) analytic reasoning.

5. Through laboratory method, students get opportunity to
   a) be familiarised with practical utilisation of mathematics.
   b) understand how to apply mathematical formulas or rules in practical work.
   c) apply mathematical knowledge on other fields of knowledge.

**True / False Questions**

6. Laboratory method is also called “learning by doing”.
   a) True  b) False

7. It is not necessary for teachers to clear all concepts before starting laboratory work.
   a) True  b) False

8. Laboratory method is the only method which can be applied to teach all branches of mathematics.
   a) True  b) False

9. Laboratory work is not a lengthy process.
   a) True  b) False

10. It is quite easy for students to practise laboratory work at their homes.
    a) True  b) False

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**Test 10: Project Method**

**Multiple Choice Questions**

1. Project method helps to
   a) develop reasoning ability in students.
   b) promote social interaction between students.
c) enhance creativity of students.

2. So many tasks involved in project method develop
   a) logical thinking in students.
   b) management skills in students.
   c) discovery approach.

3. In project method, students have to
   a) follow a set procedure.
   b) apply so many formulas of similar nature.
   c) follow a plan.

4. Project method mainly focuses on
   a) analysing different situations.
   b) learning by doing.
   c) teacher-student interaction.

5. It is difficult to apply project method because it is
   a) expensive.
   b) time consuming.
   c) both expensive and time consuming.

**True / False Questions**

6. Project method enables students to apply mathematical knowledge in real life situations.
   a) True  b) False

7. Students are provided opportunity to practise mathematical formulas and rules through project method.
   a) True  b) False
8. Different kinds of laws and theorems can be proved easily with the help of project method.
   a) True  b) False

9. The biggest hurdle in applying project method is that textbooks are not written according to project method.
   a) True  b) False

10. Project method is not considered as student centred because of teacher’s excessive involvement.
    a) True  b) False

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APPENDIX B

EFFECTIVENESS OF PEDAGOGICAL ADAPTIVE HYPERMEDIA BASED SYSTEM FOR MATHEMATICS AT SECONDARY LEVEL (PAHMS)

Questionnaire

Section 1: Particulars about the respondent.
Name: _____________________________
E-mail: _____________________________

Directions: Please check mark (✓) in the appropriate place against each item.

1. Gender:  ____ Male  ____ Female
2. Age:  ____ Below 40 years  ____ Above 40 years
3. Teaching Experience:  ____ In-Service Teacher
    ____ Student Teacher (Pre-service)
Section 2: Effectiveness Scale of Pedagogical Adaptive Hypermedia Based System for Mathematics at Secondary Level (PAHMS).

Directions: Some statements related to PAHMS are listed below. There is a five point scale against each statement: Strongly Agree (SA), Agree (A), Undecided (U), Disagree (D) and Strongly Disagree (SD). Please place a check mark (✓) at the appropriate point which represents your opinion.

<p>| 1. PAHMS provides more attractive and self paced learning environment as compared to conventional learning mode. | SA A U D SD |
| 2. Objectives and description of the system are clearly mentioned. | SA A U D SD |
| 3. Sources for reference material are available. | SA A U D SD |
| 4. Learning is assessed by different methods including tests, discussions and assignments. | SA A U D SD |
| 5. Learner is given opportunity of collaborative study. | SA A U D SD |
| 6. Discussion forums are used for discussing pedagogical issues. | SA A U D SD |
| 7. Private messaging facility for learner to learner, learner to instructor and instructor to learner is available. | SA A U D SD |
| 8. Live communication facilities are available including chatting, audio or video conferencing. | SA A U D SD |
| 9. Learner can observe his or her educational performance. | SA A U D SD |
| 10. Learner can manage his or her educational material online. | SA A U D SD |
| 11. Global guidance is available by suggesting all possible links to follow. | SA A U D SD |</p>
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<tr>
<td>12.</td>
<td>Local guidance is available by suggesting the next most appropriate link.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>13.</td>
<td>Global orientation support is provided with the help of complete site map.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>14.</td>
<td>Local orientation support is provided by suggesting most relevant links to the current web page.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>15.</td>
<td>Learner is given opportunity to change the layout of PAHMS.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>16.</td>
<td>Pedagogical content is sorted according to the learner’s educational level with comparative explanation.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>17.</td>
<td>Pedagogical content is presented in different shapes and mediums like text, animation, audio and video.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>18.</td>
<td>Pedagogical content is presented according to the learner’s educational level.</td>
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<td>U</td>
<td>D</td>
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<td>19.</td>
<td>Prerequisite concepts are shown to the learner.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
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<td>20.</td>
<td>Adaptive peers searching facility is available.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
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<td>21.</td>
<td>Pedagogy provided by the system is according to the new curriculum of mathematics.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>22.</td>
<td>Pedagogy covers all sections of curriculum.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>23.</td>
<td>Pedagogy includes application of all famous teaching methods in mathematics.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
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<td>24.</td>
<td>Pedagogy is explained with the help of appropriate examples from different branches of mathematics.</td>
<td>SA</td>
<td>A</td>
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<td>D</td>
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<td>25.</td>
<td>The procedures of all teaching methods are easily understandable.</td>
<td>SA</td>
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APPENDIX C

EXTERNAL WEB LINKS FOR PEDAGOGY OF MATHEMATICS IN PAHMS

Pedagogy

- Pedagogy Does Matter: http://fromnowon.org/sept03/pedagogy.html
- Teaching Methods:
  http://www.britannica.com/EBchecked/topic/448410/pedagogy
- Common Teaching Methods – Strengths, Limitations & Preparation:
  http://honolulu.hawaii.edu/intranet/committees/FacDevCom/guidebk/teachtip/comteach.htm
- Web Resources for Teaching Methods:
  http://www.mhhe.com/socscience/education/methods/resources.html
- Instructional Design Models:
  http://carbon.ucdenver.edu/~mryder/itc_data/idmodels.html
- Pedagogy of the Oppressed (Book) by Paulo Freire:
Professional Standards for Teachers

- Teaching to Standards:
  
  http://findarticles.com/p/articles/mi_m0HUL/is_3_31/ai_82092511/

- Standards and Focal Points for Math Teachers:
  

- Professional Development Resources:
  
  http://www.theteachersguide.com/Professionaldevelopment.htm

- Does Professional Development Change Teaching Practice? (Report):
  

Teaching of Mathematics

- A Guide for Teaching Mathematics:
  
  http://bearspace.baylor.edu/Alexander_Pruss/www/teachmath.html

- The Teaching of Mathematics (Online Journal):
  
  http://elib.mi.sanu.ac.rs/pages/browse_publication.php?db=tm

- The Teaching of Mathematics in the Elementary and the Secondary School (Book) by J. W. A. Young:
  
  http://openlibrary.org/b/OL7144679M/teaching_of_mathematics_in_the_elementary_and_the_secondary_school

- The Teaching of Mathematics:
  

- Teaching Methodology of Mathematics:
  
  hrcak.srce.hr/file/55086

- Teaching Mathematics (Book) by Michael Cornelius:
  
Lecture Method

- The Lecture Method of Class Time Organization:
  http://scidiv.bellevuecollege.edu/ls/Teaching/lecture.php


- Guidelines and Techniques for Lecture Method:

- Video Lectures on Mathematics: http://videolectures.net/Top/Mathematics/

- Free Video Lectures: http://freevideolectures.com/mathematics.html

Inductive Method


- The Inductive (Scientific) Method and its Comparison with Deductive Method:

- Inductive Teaching and Learning Methods:
  http://findarticles.com/p/articles/mi_qa3886/is_200604/ai_n17186573/

- The Many Faces of Inductive Teaching and Learning:
  http://mate.calpoly.edu/media/files/Prince_Felder.pdf

- Mathematical Induction and Induction in Mathematics:
  www.psych.northwestern.edu/~rips/documents/mathinduct3.pdf

Deductive Method

- Math Worksheets Using Deductive Reasoning:

• **Deductive vs. Inductive Teaching:** [http://knol.google.com/k/deductive-vs-inductive-teaching#](http://knol.google.com/k/deductive-vs-inductive-teaching#)

• **Deductive Geometry:**

**Heuristic / Discovery Method**

• **A Symposium of Heuristic Teaching:**

• **Introducing Math Teachers to Inquiry:**

• **Discovery Method:** [http://wik.ed.uiuc.edu/index.php/Discovery_method](http://wik.ed.uiuc.edu/index.php/Discovery_method)

• **Guided-Discovery Method of Instruction:**
  [http://www-users.math.umd.edu/~jnd/Discovery.htm](http://www-users.math.umd.edu/~jnd/Discovery.htm)

**Analytic Method**

• **Analytical Methods in Mathematics Education:**

• **Analytic Teaching and Philosophical Praxis (Online Journal):**

• **Application of Analytic Method in Mathematics:**
Synthetic Method

- **Analysis and Synthesis**: www.swemorph.com/pdf/anaeng-r.pdf
- **Synthetic geometry**: http://www.webcitation.org/5koAax8ct

Problem Solving Method

- **Problem Solving Literacy**: http://www.hawaii.edu/suremath/home1.html
- **Problem Solving in Mathematics**: http://library.thinkquest.org/25459/learning/problem/
- **Mathematical Problem Solving**: http://jwilson.coe.uga.edu/EMT725/PSsyn/PSsyn.html
- **Problem Solving Hints**: http://whyslopes.com/freeAccess/problems_solving_methods.html
- **Problem Solving Resources**: http://www.teachingideas.co.uk/maths/contents02problems.htm

Laboratory Method

- **Standards for the Use of the Laboratory Method**: http://www.eric.ed.gov/ERICWebPortal/contentdelivery/servlet/ERICServlet?accno=ED043844
• Mathematics Laboratory - an Alternate Method of Instruction:

Project Method

• The Project Method:
  http://www.planetvidyaschools.com/school/studyhallschool/home/Project.htm

• Definition and Principles of Project Method:
  http://59.163.61.3:8080/GRATEST/SHOWTEXFILE.do?page_id=user_image&user_image_id=1007

• Articles about Project Method:
  http://www.thefreelibrary.com/Project+method+(Education)-s192983

• The project method of teaching (Book) by John Alford:
  http://www.archive.org/details/projectmethodoft00steviala

• Project-Based Learning in the Middle and High School Mathematics Classroom:
  http://www.glencoe.com/sec/teachingtoday/subject/project_based.phtml

• Tips for Math Students in Project Based Instruction:
  http://www.ma.utexas.edu/users/mks/teachers/PBI.html
APPENDIX D

COFALE TECHNICAL REVIEW

Implementation of COFALE

PAHMS is powered by COFALE based on an open source online learning content management system (LCMS), ATutor. Complete technical details and implementation of COFALE is discussed by Chieu (2005) in his Ph.D. thesis titled “Constructivist learning: An operational approach for designing adaptive learning environments supporting cognitive flexibility”, Catholic University of Louvain, Belgium.

Twenty percent source code of ATutor was contributed by COFALE including five thousand lines of PHP code and fifteen hundred person-hours of programming work. General architecture of ATutor is shown in Figure D.1.

![Figure D.1. Architecture of ATutor (Chieu, 2005, p. 142).](image)

When a user logs into the system using a web browser then browser sends a request to the server where PHP was already installed. Depending on the request,
server will update MySQL database in which data related to the users, course content and forums are stored and indexed. Then server generates a HTML file and sends back to the browser. On the basis of this file, browser will generate a web page for the user.

Database structure in ATutor is simple. There are thirty tables organised in small groups. The source code of ATutor is organised into a tree-structured file system, which contains a variety of short PHP scripts. Figure D.2 shows three tables including “Courses”, “Content” and “Related_Content”.

![Figure D.2. Representation of courses and content objects in ATutor](Chieu, 2005, p. 231).

The attributes of table “Courses” consist of:

- **member_id:** The identity of the course designer.
- **cat_id:** Category of the course.
- **title:** Title of the course.
\begin{itemize}
  \item description: Description of the course.
  \item And so on

  The attributes of table "content" consist of:

  \begin{itemize}
    \item course_id: The identity of the course to which the content object belongs.
    \item content_parent_id: The identity of the parent content object of the current one.
    \item ordering: The order of the content object within the parent content object
    \item title: The title of the content object.
    \item text: The text describing the content object.
  \end{itemize}

  The relationships among the content objects are described by table "related_content". The data stored in this table is also helpful to construct "Related Topics" in COFALE. Table "g_click_data" contains navigation history of every user so it is used to create menu "Learning History". Similarly, to construct the option of students' summaries, a table "content_of_learners" is created in COFALE.

  Authoring tool Learner Model Manager in COFALE is based on table "mental_models" as shown in Figure D.3.

  \begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{mental_models.png}
  \caption{Representation of components of learner mental models (Chieu, 2005, p. 233).}
  \end{figure}

  The attributes of this table is consisted of:

  \begin{itemize}
    \item course_id: The identity of the course.
    \item member_id: The identity of the course designer.
  \end{itemize}
To display Learner Model Manager, a PHP file “tools/index.php” is modified. This tool is linked to another file “teachers/learner_model.php”. Another PHP file “teachers/edit_learner_model.php” is also created to enable the course designer to add new components or edit existing components.

There are three evaluation modes for a student in learner model: self-evaluation, the teacher’s evaluation, and the system’s evaluation. Each student may possess one or more components of learner models. So, table “learner_mental_model” is created. The system can automatically detect several components of learner models on the basis of students’ test results. Thus table “mental_model_test” to represent means for diagnosing certain components of learner models is also constructed.

A file “teachers/edit_learners_own_models.php” is created. When the course designer clicks on “Edit Learners’ Own Models” which is linked to this file, he or she is led to a set of tools for updating students’ learner model to the database. To let the system detect and update the learner model of a student, the PHP file “tools/tests/view_results.php” is modified.

To implement adaptive presentation of content in COFALE, a table “mental_model_content” is created through which association between learner models and content objects is defined as shown in Figure D.4.
The attributes of this table include:

- **course_id**: The identity of the course.
- **member_id**: The identity of the course designer.
- **mental_model_id**: The identity of the component.
- **content_id**: The identity of the content object defined by the course designer to be appropriate to students possessing the component.

File “teachers/edit_learner_model.php” is further modified for providing a tool to the course designer so that he or she can associate appropriate content objects with the learner models. All of these associations are stored in the table “mental_model_content”. A function “initcontent” defined in file “include/classes/ContentManager.class.php” is modified so that course content is adapted to the current learner model of the student. In this regard, table “mental_model_content” is used to retrieve the most appropriate course content for the learner according to his or her current learner model.

COFALE also supports adaptive communication support. For adaptive peers searching, a table “mental_model_help_relation” is created as shown in Figure D.5.
Figure D.5. Representation of help relations among components of learner models (Chieu, 2005, p. 236).

The attributes of this table consist of:

- course_id: The identity of the course.
- member_id: The identity of the course designer.
- Students possessing the components identified by model_left_id can help students possessing the component identified by model_right_id.

For producing a tool to define help relations among components of learner models, a file “teachers/edit_model_constraints.php” is created. Another file “search_peers.php” is also created through which a list of appropriate peers is shown to a learner when he or she makes a demand.

Reference

APPENDIX E

COMPARISON OF THE PROPOSED SYSTEM WITH AN EXISTING SYSTEM

We have compared our system PAHMS based on COFALE version 1.0 in terms of complexity, cohesion, coupling and usability with another adaptive tutorial for GRAPPLE (Generic Responsive Adaptive Personalized Learning Environment) designed using an open source adaptive learning environment GALE version 1.2.3 (De Bra et al., 2010). GALE (GRAPPLE Adaptive Learning Environment) is developed using Java servlet technology and MySQL.

![Figure E.1. Link adaptation view in GRAPPLE Tutorial.](image)

To evaluate PAHMS and GRAPPLE Tutorial on technical grounds, it is necessary to analyse the designs and source code of frameworks provided by
COFALE and GALE on which these systems are based. For this purpose, we have done reverse engineering with the help of commercially available tools including Understand version 3.0.631 (Scientific Toolworks, Inc., 2012) and Enterprise Architect version 8.0.860 (Sparks, 2010). The results generated by these tools can be verified.

**Complexity of the systems**

Complexity is computed to predict the reliability and maintainability of a system. Before calculating complexity of both systems we have gathered some information about source code of both systems. Few statistics related to classes, functions and lines within source code of COFALE on which PAHMS is based are given in the Table E.1.

<table>
<thead>
<tr>
<th>No. of Classes:</th>
<th>83</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Files:</td>
<td>509</td>
</tr>
<tr>
<td>Program Units (Functions):</td>
<td>1317</td>
</tr>
<tr>
<td>No. of Lines:</td>
<td>105770</td>
</tr>
<tr>
<td>Blank Lines:</td>
<td>11416</td>
</tr>
<tr>
<td>Lines of Code:</td>
<td>59524</td>
</tr>
<tr>
<td>Comment Lines of Code:</td>
<td>26136</td>
</tr>
<tr>
<td>Lines Inactive:</td>
<td>0</td>
</tr>
<tr>
<td>Executable Statements:</td>
<td>36442</td>
</tr>
<tr>
<td>Declarative Statements:</td>
<td>3405</td>
</tr>
</tbody>
</table>

We have computed cyclomatic complexity metrics (McCabe & Watson, 1994) for all files and functions to show the complexity of programme control flow within COFALE. Average cyclomatic complexity of the system is 5.589 as shown in Figure E.2. This shows that the overall complexity of COFALE is not high.
We have also computed the complexity of GALE. Statistics related to the source code of GALE are given in Table E.2.

**Table E.2.** Source code statistics of GALE.

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Classes:</td>
<td>843</td>
</tr>
<tr>
<td>No. of Files:</td>
<td>379</td>
</tr>
<tr>
<td>Program Units (Functions):</td>
<td>4006</td>
</tr>
<tr>
<td>No. of Lines:</td>
<td>108434</td>
</tr>
<tr>
<td>Blank Lines:</td>
<td>11341</td>
</tr>
<tr>
<td>Lines of Code:</td>
<td>77167</td>
</tr>
<tr>
<td>Comment Lines of Code:</td>
<td>20474</td>
</tr>
<tr>
<td>Lines Inactive:</td>
<td>0</td>
</tr>
<tr>
<td>Executable Statements:</td>
<td>21922</td>
</tr>
<tr>
<td>Declarative Statements:</td>
<td>15614</td>
</tr>
</tbody>
</table>

Cyclomatic complexity metrics for all files and methods within GALE are also computed and average cyclomatic complexity of the system is 2.442 as shown in Figure E.3. This shows that the overall complexity of GALE is also not high.
Figure E.3. Cyclomatic complexity of GALE.

**Cohesiveness of Classes within Systems**

High cohesion promotes information-hiding which means that any module or class encapsulates those functions and attributes that are related to each other and to that particular module or class only. The design of COFALE is object oriented but not modular or component based so we have computed lack of cohesion in methods (LCOM2) metrics (Henderson-Sellers, Constantine & Graham, 1996; Borkar & Khonde, 2012) to determine the overall cohesion of methods in classes of the system. Sum of values of computed LCOM2 metrics of 83 classes within COFALE is 30.59 so the average value of LCOM2 metrics for all classes is 0.368. It means there is 36.8 percent lack of cohesion in methods of classes. As value of 0.368 is less than maximum limit that is 1.0, it shows that methods in classes of COFALE are highly cohesive.
GALE is also object oriented. Sum of values of computed LCOM2 metrics of 843 classes of GALE is 201.43 so the average value of LCOM2 metrics for all classes is 0.238 so there is 23.8 percent lack of cohesion in methods of classes. As value of 0.238 is less than 1.0, it shows that methods in classes of GALE are also highly cohesive.

### Coupling between Classes within Systems

Coupling between components or classes is directly related to reusability. If coupling increases, not only reusability of components or classes decreases but it also complicates modifications, testing and maintenance afterwards. Because of object oriented design of COFALE, coupling between object classes (CBO) metrics (Chidamber & Kemerer, 1994) are computed for each class of the system. Sum of values of CBO metrics of 83 classes of the system is 160 so the average value of CBO metrics for all classes of COFALE is 1.927 which is less than maximum limit that is 4.0. It shows that classes within system are loosely coupled which is good.

Sum of values of CBO metrics of 843 classes of GALE is 1702 so average value of CBO metrics for all classes is 2.019 which is less than 4.0. It shows that classes of GALE are also loosely coupled.

### Usability of the Systems

Usability is a measure that how easily and efficiently a user interacts with a system. As both PAHMS and GRAPPLE Tutorial are adaptive and facility of adaptation or personalisation is directly related to usability (Benyon, 1993), so we also adopted the criteria to measure the usability of these systems on the basis of adaptive features provided by these systems. We have already ascertained the effectiveness of PAHMS
in terms of learning environment, adaptive support and pedagogy of mathematics provided by the system (see chapter 7).

Brusilovsky (1996) discussed the detailed taxonomy of adaptive hypermedia including different methods and techniques for adaptive presentation and adaptive navigation (see chapter 2). On the basis of this taxonomy, we evaluated the usability in terms of adaptive features provided by PAHMS and GRAPPLE Tutorial through survey which was conducted from thirty students of BS (Computer Science) who had interacted with the systems. These students had been studying at University of Management and Technology (UMT), Lahore, Pakistan. For this purpose, a questionnaire consisted of ten statements was developed. It was designed on five point Likert scale ranges from 1 to 5 indicating strongly disagree, disagree, undecided, agree and strongly agree. Statements of the questionnaire are as follows:

<table>
<thead>
<tr>
<th>Table E.3. Statements of the questionnaire designed for comparison of the systems.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement 1</td>
</tr>
<tr>
<td>Global guidance is available by suggesting all possible links to follow.</td>
</tr>
<tr>
<td>Statement 2</td>
</tr>
<tr>
<td>Local guidance is available by suggesting the next most appropriate link.</td>
</tr>
<tr>
<td>Statement 3</td>
</tr>
<tr>
<td>Global orientation support is provided with the help of complete site map.</td>
</tr>
<tr>
<td>Statement 4</td>
</tr>
<tr>
<td>Local orientation support is provided by suggesting most relevant links to the current web page.</td>
</tr>
<tr>
<td>Statement 5</td>
</tr>
<tr>
<td>User is given opportunity to change the layout of the system.</td>
</tr>
<tr>
<td>Statement 6</td>
</tr>
<tr>
<td>Content is sorted according to the user’s educational level with comparative explanation.</td>
</tr>
<tr>
<td>Statement 7</td>
</tr>
<tr>
<td>Content is presented in different mediums like text, animation, audio and video.</td>
</tr>
<tr>
<td>Statement 8</td>
</tr>
<tr>
<td>Content is presented according to the user’s educational background.</td>
</tr>
<tr>
<td>Statement 9</td>
</tr>
<tr>
<td>Prerequisite concepts are shown to the user.</td>
</tr>
<tr>
<td>Statement 10</td>
</tr>
<tr>
<td>Adaptive peers searching facility is available for collaborative learning.</td>
</tr>
</tbody>
</table>

One-sample $t$-test was used to judge the opinion of the users whether they are undecided, agreed or disagreed with the statements of questionnaire. Mean response values for all statement were calculated with the help of $t$-test. Mean response value
for each statement may range from 1 to 5, so the test value is 3. Thus if mean response value for any statement is higher than 3 then it means that majority of the users agree with the statement otherwise they disagree with it.

First of all, average mean score for all statements of questionnaire was calculated to judge the opinion of users about adaptive features provided by PAHMS as shown in Tables E.4 and E.5.

**Table E.4.** One-sample statistics for adaptive features provided by PAHMS.

<table>
<thead>
<tr>
<th>Overall Opinion of users</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>3.6767</td>
<td>0.28246</td>
<td>0.05157</td>
</tr>
</tbody>
</table>

**Table E.5.** One-sample t-test for adaptive features provided by PAHMS.

<table>
<thead>
<tr>
<th>Overall Opinion of users</th>
<th>Test Value = 3</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>13.122</td>
<td>29</td>
<td>0.000</td>
<td>0.67667</td>
<td>0.5712, 0.7821</td>
</tr>
</tbody>
</table>

For adaptive features provided by PAHMS, the value of t(30)=13.122, \( p=0.000 \) is significant at \( \alpha=0.05 \). The mean value of 3.676 higher than test value of 3, shows that the opinion of the users is in the favour of overall adaptive environment of PAHMS. One-sample t-test is also applied to calculate mean response values for all statements of questionnaire regarding PAHMS as shown in Table E.6.

**Table E.6.** Mean response values for all statements regarding PAHMS.

<table>
<thead>
<tr>
<th>Statements</th>
<th>N</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement 1</td>
<td>30</td>
<td>4.4667</td>
</tr>
<tr>
<td>Statement 2</td>
<td>30</td>
<td>4.4333</td>
</tr>
<tr>
<td>Statement 3</td>
<td>30</td>
<td>4.3333</td>
</tr>
<tr>
<td>Statement 4</td>
<td>30</td>
<td>3.8000</td>
</tr>
<tr>
<td>Statement 5</td>
<td>30</td>
<td>4.1000</td>
</tr>
<tr>
<td>Statement 6</td>
<td>30</td>
<td>1.5000</td>
</tr>
<tr>
<td>Statement 7</td>
<td>30</td>
<td>2.4333</td>
</tr>
<tr>
<td>Statement 8</td>
<td>30</td>
<td>3.7333</td>
</tr>
<tr>
<td>Statement 9</td>
<td>30</td>
<td>4.0000</td>
</tr>
<tr>
<td>Statement 10</td>
<td>30</td>
<td>3.9667</td>
</tr>
</tbody>
</table>
Table E.6 shows that the users are agreed with the statements of questionnaire regarding PAHMS except statements 6 and 7 because mean response values for these statements are less than test value that is 3.0. This shows that PAHMS does not provide content in different formats. Moreover, it does not support some adaptive features related adaptive presentation including *comparative explanations, explanation variants and sorting*.  

For GRAPPLE Tutorial, average mean score for all statements of questionnaire was also calculated to judge the opinion of users about adaptive features provided by this system as shown in Tables E.7 and E.8.

**Table E.7.** One-sample statistics for adaptive features provided by GRAPPLE Tutorial.

<table>
<thead>
<tr>
<th>Overall Opinion of users</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>3.10</td>
<td>0.21173</td>
<td>0.03866</td>
</tr>
</tbody>
</table>

**Table E.8.** One-sample $t$-test for adaptive features provided by GRAPPLE Tutorial.

<table>
<thead>
<tr>
<th>Overall Opinion of users</th>
<th>Test Value $= 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.587</td>
</tr>
</tbody>
</table>

For adaptive features provided by GRAPPLE Tutorial, the value of $t(30)=2.587$, $p=0.000$ is significant at $\alpha=0.05$. The mean value of 3.10 higher than test value of 3, shows that the opinion of the users is in the favour of overall adaptive environment of GRAPPLE Tutor but the result is not satisfactory as the average mean value is quite closer to the test value. One-sample $t$-test is also applied to calculate mean response values for all statements of questionnaire regarding GRAPPLE Tutor as shown in Table E.9.
Table E.9. Mean response values for all statements regarding GRAPPLE Tutorial.

<table>
<thead>
<tr>
<th>Statements</th>
<th>N</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement 1</td>
<td>30</td>
<td>4.5000</td>
</tr>
<tr>
<td>Statement 2</td>
<td>30</td>
<td>4.5333</td>
</tr>
<tr>
<td>Statement 3</td>
<td>30</td>
<td>4.3667</td>
</tr>
<tr>
<td>Statement 4</td>
<td>30</td>
<td>3.9667</td>
</tr>
<tr>
<td>Statement 5</td>
<td>30</td>
<td>1.2667</td>
</tr>
<tr>
<td>Statement 6</td>
<td>30</td>
<td>1.9000</td>
</tr>
<tr>
<td>Statement 7</td>
<td>30</td>
<td>2.8333</td>
</tr>
<tr>
<td>Statement 8</td>
<td>30</td>
<td>2.1333</td>
</tr>
<tr>
<td>Statement 9</td>
<td>30</td>
<td>4.3667</td>
</tr>
<tr>
<td>Statement 10</td>
<td>30</td>
<td>1.1333</td>
</tr>
</tbody>
</table>

Table E.9 shows that the users are agreed with only five statements of questionnaire regarding GRAPPLE Tutorial. Mean response values for the statements 5, 6, 7, 8 and 10 are less than test value that is 3.0. This shows that GRAPPLE Tutorial does not provide content in different formats. Moreover, it does not support some adaptive features related to adaptive presentation and adaptive navigation including comparative explanations, explanation variants, sorting and managing personalised view. Adaptive peer searching is also not provided by the system.

After analysing the environments of PAHMS and GRAPPLE Tutorial and considering the results of survey, we have compared adaptive features provided by these systems as shown in Table E.10.

Table E.10. Adaptive features provided by PAHMS and GRAPPLE Tutorial.

<table>
<thead>
<tr>
<th>Adaptive features</th>
<th>PAHMS based on COFALE</th>
<th>GRAPPLE Tutorial based on GALE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptive Presentation</td>
<td>Additional explanations</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Prerequisite explanations</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Comparative explanations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Explanation variants</td>
<td></td>
</tr>
<tr>
<td>Sorting</td>
<td>Global guidance</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Local guidance</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Global orientation support</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Local orientation support</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Managing personalised view</td>
<td>X</td>
</tr>
<tr>
<td>Adaptive Navigation</td>
<td>Adaptive Peer Searching for collaborative study</td>
<td>X</td>
</tr>
</tbody>
</table>
Conclusion

On the basis of “three Cs”: complexity, cohesion and coupling, we can ascertain the object oriented design of PAHMS based on COFALE and GRAPPLE Tutorial based on GALE. Complexity and coupling between classes of both the systems are not high. Similarly, classes of both the systems are cohesive as well. Although complexity and percentage of lack of cohesion of methods within object oriented design of COFALE are a little bit higher than that of GALE but that are quite normal. Result of coupling between object classes within framework of COFALE is slightly better than that of GALE but these systems are more or less same in this case.

As for as usability in terms of adaptation is concerned, PAHMS exhibits satisfactory results but GRAPPL Tutorial needs a lot of improvements. This tutor lacks some features related to adaptive presentation and adaptive navigation support. It does not provide facility of adaptive peer searching for collaborative study as well. As a whole, PAHMS based on COFALE seems better than GRAPPL Tutorial based on GALE.

References


