Nonlinear Dynamics of Large-Scale Vortical Motions in Electron-Positron Plasmas and Other Multiple Components Plasmas

By

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DEPARTMENT OF PHYSICS
GOVERNMENT COLLEGE UNIVERSITY
LAHORE, PAKISTAN
2011
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The thesis is submitted to GC University Lahore in partial fulfillment of the requirements for the award of degree of

DOCTOR OF PHILOSOPHY

IN

PHYSICS

By

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DEPARTMENT OF PHYSICS

GOVERNMENT COLLEGE UNIVERSITY

LAHORE, PAKISTAN

2011
DEDICATED

TO

MY BELOVED PARENTS, WIFE

AND DAUGHTER (NAJEEHA MAZHER)
DECLARATION

I, Mr. Mazher Shad, Reg. No. 89-GCU-PhD-Phy-2009, student of Prof. Dr. Tamaz David Kaladze in the subject of physics, hereby declare that the matter printed in the thesis titled “Nonlinear Dynamics of Large-Scale Vortical Motions in Electron-Positron Plasmas and Other Multiple Components Plasmas” is my own work and has not been printed, published and submitted as thesis or in any form in any University, Research Institution, etc. in Pakistan or abroad.

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It is certified that the research work contained in this thesis titled “Nonlinear Dynamics of Large-Scale Vortical Motions in Electron-Positron Plasmas and Other Multiple Components Plasmas” has been carried out and completed by Mr. Mazher Shad, Reg. No. 89-GCU-PhD-Phy-2009 under my supervision.

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ACKNOWLEDGEMENTS

With the grace of Almighty Allah, the most beneficent, and merciful, compassionate and the gracious, who is entire the source of all knowledge and wisdom endowed to mankind. It is glory to God for his mercy, who bestowed and bounteously blessed me so that I could complete this thesis inspite of numerous difficulties and acute frustrations.

I wish to acknowledge my indebtedness and great gratitude to my devoted and respected Foreign PhD Supervisor Prof. Dr. Tamaz David Kaladze for his support and invaluable supervision in the preparation of this thesis. He not only encouraged me, but also helped me in a time of turmoil during the course of my research work. His valuable suggestions, guidance, sympathetic attitude, analytical as well as critical scrutiny enabled me in broadening my knowledge and improving my research techniques and capability.

I am deeply indebted to L.V. Tsamalashvili from I. Vekua Institute of Applied Mathematics, Tbilisi State University, Georgia, O.A. Pokhotelov from Institute of Physics of the Earth, Russian Academy of Sciences, Moscow, and G.V. Jandieri from Georgian Technical University, Tbilisi, for very active collaboration in some of the work that I have undertaken in this thesis.

My cordial thanks to Prof. Dr. H.A. Shah, Chairman of Physics Department at Government College University (GCU), Lahore and Dr. G. Murtaza, Salam Professor of Salam Chair in Physics, at GCU for fruitful scientific collaboration. I am so grateful to Prof. Dr. N.L. Tsintsadze, foreign faculty at GCU for significant discussions on different aspects of plasma Physics.

I would like to thank Prof. Dr. Riaz Ahmed, (former chairman), now Director of the Centre for Advanced Studies and Physics for providing me an excellent working environment and facilities in the Department of Physics.

I am also thankful to all members of Theoretical Plasma Physics Group at Department of Physics GCU Lahore, specially Dr. Nouman Sarwar Qureshi, Laila Zafar Kahlon, Zubia Kiran, Dr. Atif Shabbaz and my friend Dr. Mushtaq Ahmed, from Pakistan Atomic Energy Commission (PAEC).
Finally, I take this opportunity to offer my gratitude to my beloved family, parents, brothers and sisters, who always pray to God for my success and inspired me through out the span of my PhD research work.

Mazher Shad
The possibility of zonal flows generation by low-frequency waves in magnetized space and laboratory plasmas is studied. Namely the zonal flows generation in the Earth’s ionospheric E- and F-layers by Rossby waves and in electron-positron-ion (EPI) plasmas by electrostatic drift waves is investigated. The modified parametric approach is used considering the arbitrary spectrum of primary modes. The driving forces of zonal flows are Reynolds stresses. An important nonlinear mechanism for the transfer of spectral energy from small-scale pumping waves to large-scale enhanced zonal flows (inverse cascade) is investigated.

The dynamics of Rossby waves in the electrically conducting ionospheric layers strongly depends on the interaction of inductive currents with the geomagnetic field. Such interaction in the ionospheric E-layer due to the prevalent effect of Hall conductivity gives rise to, so called magnetized Rossby (MR) waves to be propagating. But in the ionospheric F-layer, under such interaction dissipation arises due to Pedersen conductivity acting as the inductive (magnetic) inhibition. Modified by the interaction of inductive currents with the geomagnetic field Charney equation is used as the basic nonlinear equation. Considering comparatively short-scale perturbations only vector nonlinearity is responsible for the coupling between different modes in Charney equation. The nonlinear interaction of short-scale pump Rossby waves, two satellites of the pump waves (side-band waves) and a large-scale shear zonal flow is studied.

Propagating in the ionospheric E-layer MR waves do not significantly perturb the geomagnetic field. Zonal flow dispersion relation for an arbitrary spectrum of MR waves is obtained. Monochromatic and non-monochromatic wave packets of primary modes are discussed. In the case of monochromatic wave packet the instability is of the hydrodynamic type. It is found that the broadening of the wave packet spectrum of pump MR waves leads to a resonant interaction with a growth rate of the order of the monochromatic case. In the case when zonal flow generation by MR modes is prohibited by the Lighthill stability criterion, the so-called two-stream-like mechanism for the generation of sheared zonal flows by finite-amplitude MR waves in the ionospheric E-
layer is possible. The growth rates of zonal flow instabilities and the corresponding conditions are determined.

The possibility of zonal flow generation and appropriate distinctive properties are revealed when Rossby waves are propagating through the dissipative ionospheric F-layer.

To describe the nonlinear propagation of electrostatic drift waves the generalized Hasegawa-Mima (HM) equation containing one vector (Jacobian) and two scalar nonlinearities of different nature for the case of EPI plasma is obtained. The drift waves are supposed to have arbitrary wavelengths (as compared with the Larmor radius of plasma ions at the plasma electron temperature). Temperature inhomogeneity of electrons and positrons is taken into account, while ions are considered to be cold. The new space structure of drift waves is obtained. Spatial increase of the linear plasma-potential perturbations in the direction of density and temperature inhomogeneities is shown.

As long as under the zonal flow action different vortical structures can be maintained, possibility of the existence of drift vortical motions and the appropriate properties also are investigated in case of EPI plasma. It is shown that the vector nonlinearity is responsible for the existence of small-scale dipole-type solitary vortical structures. One of the scalar nonlinearities of KdV-type is responsible for the existence of the intermediate-scale vortical structures. The other scalar nonlinearity under the time derivative creates intermediate and large-scale monopole vortical structures and plays an essential role in different possibilities of zonal flows generation. It causes nonlinear interaction with vector and KdV-type nonlinearities and itself also. It is shown that the dynamics of low-frequency waves studied in usual electron-ion (EI) plasmas is generally modified in EPI plasmas. A new self-organization mechanism of formation of large-scale electrostatic drift vortical structures in EPI plasmas based on the competition between scalar and vector nonlinearities has been discussed.

Generation of large-scale zonal flows by relatively small-scale electrostatic drift waves of arbitrary wavelength in a nonuniform EPI plasma is studied. To describe the generation of zonal flow the generalized Hasegawa-Mima equation containing both vector and two scalar (of different nature) nonlinearities is used. The system of coupled equations describing the nonlinear interaction of drift waves and zonal flows is derived. Enriched possibilities of zonal flow generation with different growth rates are revealed.
Explicit expressions for the appropriate maximum growth rates are obtained. Obtained results may be useful to explain different observations on zonal flows and vortical motions in laboratory and astrophysical plasmas.
Obtained in the given thesis results are published in the following having Impact Factor International Journals and were represented at the International Conferences listed below:

**List of Publications**


**Presentations at Conferences**


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Chapter 1

Introduction

1.1 The Earth’s Ionosphere and its Structure

The Earth’s upper atmosphere is called the ionosphere which occupies the height from 60 to 1000 km from the Earth’s surface. This weakly ionized atmospheric gas is in the plasma state. The ionosphere is a transition layer between the neutral lower atmosphere and the fully ionized plasma of the magnetosphere. Therefore, the constitutional structure and properties of the ionosphere change rapidly with altitude. The dynamics of the ionospheric plasma is strongly influenced by the wave and corpuscular radiation of the sun, with processes in the magnetosphere and variation of the Earth’s magnetic field, with various activities in the upper atmosphere, and so on. That is why the ionosphere varies so rapidly in time and with geographic latitude.

The main source of the ionospheric ionization is the influence of solar wind and solar ultraviolet light. The solar wind’s streaming plasma flows are affecting the Earth’s magnetic field, ionosphere and magnetosphere. Free electrons are produced by energetic radiations from the sun on the Earth’s upper atmosphere. Starting from the height of 60 km the concentration of these free electrons sufficiently influences the propagation of electromagnetic waves. In addition passing through the atmosphere solar radiation is also absorbed. Owing to such absorption less radiation reaches the lower atmospheric levels and the ionization intensity is also reduced. But above 80 km altitudes collisions also become infrequent to result in rapid recombination and produced ionized particles are kept forming the ionosphere. High energy particles release from the sun and cosmic rays also contribute in the ionization process. But the rate of ionization in this case is generally much less in comparison with the electromagnetic radiation. However, at nights time with very little or no solar illumination, at the lowest heights that is out of range of electromagnetic radiations ionization by cosmic rays becomes essential. The ionosphere is under permanent observation by satellites and ground-based stations. Using of rockets also supply scientists with very useful data.
The ionosphere consists of several parts: the D, E, and F are main parts (see Fig. 1). The lower part (from 50 to 80 km) is called the D layer. Owing to its comparatively low position this region is relatively weakly ionized. Mainly this layer is ionized in day time. The higher region (from 80 to 150 km) is called E layer and that above 150 km the F layer. Ionospheric electron density is highest in this layer. Sometimes we make distinction between the F_1 layer, up to approximately 250 km, and the F_2 layer, above 250 km. At the ionospheric heights the ionosphere is weakly ionized and the ionization rate can be written as:

\[ \frac{N_e}{N_n} \ll 1, \]  

(1.1)

where \( N_e \) and \( N_n \) are the number densities of electrons and molecules, correspondingly. In case of the E region \( N_e/N_n \sim 10^{-10} - 10^{-7} \), and in case of the F region \( N_e/N_n \sim 10^{-5} - 10^{-3} \) (see table 1). Note that E region consists mostly of \( NO^+ \), \( N_2^+ \) and \( O_2^+ \) multiple ions, but F region consists of \( NO^+ \) and \( O^+ \) ions at the bottom and \( H^+ \) and \( He^+ \) ions at top. With the increase of heights (\( h \sim 1000 \text{ km} \)) the values \( N_e/N_n \sim 1 \) are achieved for outer ionosphere.

A detailed review of the Earth’s ionosphere may be found in Refs. [1-3].

1.1.1 Generalized Ohm’s Law for the Earth’s Ionosphere

Generally, the current density \( j \) in a conducting fluid depends not only on the electric field \( E \), but on various other fluid properties also. This dependence is described by the “generalized Ohm’s Law”. However, generalized Ohm’s Law is still under the investigation [4].
The generalized Ohm’s Law for weakly ionized ionospheric gas consisting of electrons, ions and neutral particles embedded in the geomagnetic field $\mathbf{B}_0$ may be presented in the following form

$$
\left( v_e + \frac{\omega_{Be}\omega_{Bi}}{v_{in}} \right) \mathbf{j} + \omega_{Be} \mathbf{j} \times \mathbf{b} - \frac{\omega_{Be}\omega_{Bi}}{v_{in}} \mathbf{b}(\mathbf{j} \cdot \mathbf{b}) = \frac{ne^2}{m_e} \mathbf{E}',
$$

(1.2)

where $\mathbf{E}' = \mathbf{E} + \mathbf{v}_n \times \mathbf{B} = \mathbf{E} + \mathbf{E}_d$, here $\mathbf{E}'$ and $\mathbf{E}$ represent the electric fields in the moving frame when the medium is a neutral wind and in the Earth’s rotational frame, respectively. The dynamo field $\mathbf{E}_d$ is given by the relation $\mathbf{E}_d = \mathbf{v}_n \times \mathbf{B}$. The occurrence of the field $\mathbf{E}_d$ has the general character and reflects the relation between electric fields in moving with the velocity $\mathbf{v}_n$ and immovable frames. Further designations in Eq. (1.2) are $\nu_e = \nu_{ei} + \nu_{en}$, $\nu_{\alpha\beta}$ is the elastic collision frequencies with $\beta$ representing the species with which the species $\alpha$ is colliding, $\omega_{Bi} = ZeB/m_i$ and $\omega_{Be} = eB/m_e$ are cyclotron frequencies of ions and electrons, respectively, $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the total magnetic field $\mathbf{B}$, and $n$ is the number density of charged particles, and quasineutrality condition $Zn_i = n_e = n$ is also imposed.

It is also convenient to represent Eq. (1.2) in the form

$$
\mathbf{j} = \sigma_{||}\mathbf{E}_d' + \sigma_{\perp}\mathbf{E}_d' + \sigma_H \mathbf{b} \times \mathbf{E}',
$$

(1.3)

where conductivities $\sigma_{||}$, $\sigma_{\perp}$ and $\sigma_H$ are named as parallel, perpendicular (or Pedersen) and Hall conductivities, respectively given as:

$$
\begin{align*}
\sigma_{||} &= \frac{ne^2}{m_e v_e}, \\
\sigma_{\perp} &= \frac{ne^2 v_{in}(\nu_e v_{in} + \omega_{Be}\omega_{Bi})}{m_e(\omega_{Be}^2 v_{in}^2 + \nu_e^2 v_{in}^2 + \omega_{Be}^2 \omega_{Bi}^2)}, \\
\sigma_H &= \frac{ne^2 v_{in}^2 \omega_{Be}}{m_e(\omega_{Be}^2 v_{in}^2 + \nu_e^2 v_{in}^2 + \omega_{Be}^2 \omega_{Bi}^2)}.
\end{align*}
$$

(1.4)

### 1.1.2 Earth’s Ionospheric Parameters

The following data for the different Earth’s ionospheric regions give average values of parameters. At equatorial latitudes the terrestrial dipole magnetic field in the lower ionosphere is
approximately $B_0 = 0.5 \times 10^{-4} T$, angular velocity of the Earth’s rotation $\Omega_0 = 7.27 \times 10^{-5} s^{-1}$ and radius of the Earth $R = 6371 \ km$.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>D Region</th>
<th>E Region</th>
<th>F Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude $(km)$</td>
<td>50 – 90</td>
<td>90 – 160</td>
<td>160 – 500</td>
</tr>
<tr>
<td>Neutral particles number density $(m^{-3})$</td>
<td>$10^{16} – 10^{22}$</td>
<td>$10^{18} – 10^{19}$</td>
<td>$10^{15} – 10^{16}$</td>
</tr>
<tr>
<td>Charged particles number density $(m^{-3})$</td>
<td>$10^8 – 10^{10}$</td>
<td>$10^9 – 10^{11}$</td>
<td>$10^{11} – 10^{12}$</td>
</tr>
<tr>
<td>Electron gyrofrequency $(s^{-1})$</td>
<td>$10^7$</td>
<td>$10^7$</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Ion gyrofrequency $(s^{-1})$</td>
<td>150 – 200</td>
<td>150 – 200</td>
<td>230 – 300</td>
</tr>
<tr>
<td>Electron-neutral collision frequency $(s^{-1})$</td>
<td>$10^6 – 10^7$</td>
<td>$10^3 – 10^5$</td>
<td>$10 – 100$</td>
</tr>
<tr>
<td>Electron-ion collision frequency $(s^{-1})$</td>
<td>$10 – 10^2$</td>
<td>$10^3$</td>
<td>$10^3$</td>
</tr>
<tr>
<td>Ion-neutral collision frequency $(s^{-1})$</td>
<td>$(1 – 7) \times 10^5$</td>
<td>$(30 – 7 \times 10^3)$</td>
<td>$0.05 – 0.5$</td>
</tr>
<tr>
<td>Neutral particles mass density $(kg m^{-3})$</td>
<td>$10^{-5}$</td>
<td>$10^{-8} – 10^{-6}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>Electron temperature $(^\circ K)$</td>
<td>$2 \times 10^2$</td>
<td>$(2 – 3) \times 10^2$</td>
<td>$10^3$</td>
</tr>
</tbody>
</table>

### 1.2 Electron-Positron-Ion Plasmas

Unlike the ordinary plasmas during the past several years a great deal of attention has been paid to investigate the electron-positron-ion (EPI) plasmas. Such plasma may consist of electron-ion plasma with small fraction of positrons. EPI plasmas are not only basic component of early universe [5-9], but also widely spread in nature. There are several evidences pointing out that our Universe was hot EPI plasma during the earliest time of its existence [10]. Furthermore, EPI plasma is a specific case of “amiplasma -a quasineutral space plasma containing electrons, positrons, protons, and antiprotons”. The term amiplasma was introduced in 1965 by Alfven [11]. In his book [12], he proposed several models describing the formation of ambistars (i.e., stars consisting of amiplasma) due to star-antistar collisions. EPI plasmas can occur, e.g., in the central zone of accretion disks surrounding the central black holes [13-15], in magnetospheres of neutron stars [15-19], in compact astrophysical objects (e.g., giant planetary interiors, white dwarfs, neutron stars/magnetars) [20, 21], in active galactic cores [22], and also
ubiquitous ingredient of solar flare [23, 24]. Recently, narrow-collimated extended relativistic jets of EPI plasma were observed in the vicinity of blazars and microquasars [22, 25-27]. Active Galactic Nuclei (AGN) [22, 28], the centre of our own galaxy [29], pulsars magnetospheres and supernova remnants [17, 30-34], fireballs producing $\gamma$-ray bursts [35, 36], different astrophysical sources of synchrotron radiation, the intergalactic jets and Van Allen radiation belts are believed to contain EPI plasma. It is observed by the Advanced Satellite for Cosmology and Astrophysics that have confirmed the existence of some concentration of ions in astrophysical two component electron-positron system [37].

In laboratory experiments, propagation of a short relativistically intense laser pulse in matter can produce EPI plasma because of pair production [38-43]. When electrons are accelerated relativistically, in the fields of high-power laser beams, it can lead to three component EPI plasma [44, 45]. The EPI plasma is also generated in the high intensity laser-matter interaction processes, and positron densities, as high as $10^{16} \text{ cm}^{-3}$, have been achieved [46]. EPI plasmas in laboratories have been produced [47] by introducing positrons in two component electron-ion systems. Various techniques have been devised to accumulate positrons in the laboratory, and work is in progress to realize neutral plasmas containing positron as one of the constituents [48]. One of the main drives behind these efforts, to create and study positron-rich plasmas in the laboratories, is to mimic the astrophysical environments so that the underlying physics of various astrophysical objects mentioned above could be comprehended.

Investigations carried out on plasmas in fusion device and other confinement systems are under observations. It is observed that in tokamak, like JET and JT-60U electron-positron pair production takes place and almost $10^{14}$ positrons may be produced in collisions [49]. Charged positron or quasineutral electron-positron beams can be injected into tokamak plasma in order to determine the rate of particle transport from the positron lifetime or the annihilation line in the $\gamma$ spectrum [50, 51]. In this case, a small region occupied by EPI plasma can form in the bulk plasma. Due to long life time the positrons have been injected to electron-ion tokamak plasmas for diagnostic purposes in some experiments [47, 51-53]. In fact, EPI plasma has also been investigated in laboratory experiments [50, 54-56] carried out with positrons as probe to study transport in tokamak.

The experimental work in this direction has increased the interest of the researches in EPI plasma. Properties of EPI plasmas are entirely different from those observed in the usual, plasmas with single electron species. Review of the theoretical study of EPI plasmas will be given in chapter 3.
1.3 Nonlinear Coherent Vortical Structures

In plasma physics a large number of phenomena may be reduced to a general problem of free (quasi) two-dimensional (2D) anisotropic turbulence. In terms of waves this sequence may be electromagnetic electron oscillations of inhomogeneous plasma, drift waves, ion waves trapped in a tokamak, etc. [57-59]. In geophysics these processes are large-scale Rossby waves and geostrophic turbulence, [60]. In astrophysics these are density waves, exist in galaxies [61]. Dimensionless “Charney and Hasegawa-Mima” equations are used to describe the latter and having the same structure as follows:

\[
\frac{\partial}{\partial t}(\Delta \psi - a^2 \psi) + \beta \frac{\partial \psi}{\partial x} + \epsilon J(\psi, \Delta \psi) = 0. \tag{1.5}
\]

here \( \psi = \psi(x, y) \), \( J(f, g) = \partial f / \partial x \partial g / \partial y - \partial f / \partial y \partial g / \partial x \) is the Jacobian, and parameters \( a, \epsilon, \beta \) depend on the system under observation. In the absence of dissipation the 2D model (1.5) is the Charney equation for the nonlinear Rossby waves, in the physics of the atmosphere [62] with the stream function \( \psi \) and the Hasegawa-Mima equation for drift wave turbulence, in plasma physics [57] with the electrostatic potential \( \psi \). The parameter intrinsic length, is peculiar property of these equations and it can be observed that there is no space scale invariance, in contrast to the Euler equation [63]. For the atmosphere this length represents the Rossby radius but for the magnetized plasma this length is the ion-sonic Larmor radius.

A great attention has been paid to investigate the Evolution of motions governed by Eq. (1.5) [57-61, 64-66]. A long series of observations, experiments, analytical and numerical studies have recognized that the fluids display an intrinsic trend to organization. This is very clear at relaxation from turbulent conditions and is expressed by function \( \psi \) (see, e.g. [63, 67-69]). A distinctive quality of 2D fluid as well as plasma turbulence is the appearance of long-lived coherent structures by self-organization mechanism [67, 68, 70, 71]. Spatially solitary monopolar and dipolar vortices appear from these structures (see Figs. 2, 3) and their role is consistent in the overall transport of particles as well as energy [72]. That influences the thermonuclear plasmas confined under a magnetic field [73]. The investigation of these solutions gives awareness about the behavior of vortices and their values in turbulent transport. The 2D simulations of “ion temperature gradient (ITG) mode” also disclose other features, like the occurrence of long-lived, large-scale structures when magnetic shear is considered and in a local coordinate system [74, 75]. Anomalous transport level can be affected by these structures in plasmas in a magnetic confinement system. Nycander et al. [76] have verified the monopolar
vortex in $\eta_i(=d\ln T_i/d\ln N)$ mode turbulence. Moreover the presence of quasi coherent structures in the ITG turbulence has also been demonstrated experimentally [77].

Initially vortices were investigated in fluid dynamics. Solution of 2D uniform fluid equations gives rise a monopole vortex, known as “Lamb-Batchelor vortex”, observed earlier [78, 79]. Larichev and Reznik [80] investigated the same solution for nonuniform fluids and showed that Charney equation (1.5) gives solutions in terms of dipole vortices. The theory related to vortices in plasmas was promoted after the papers presented by Hasegawa et al. [81, 82] and Petviashvili [83], where an essential analogy was observed. According to the analogy, Hasegawa-Mima equation which describes nonlinear perturbations in inhomogeneous magnetized plasmas is equivalent to the Charney equation. This analogy is due to the similarity between the Coriolis force in a rotating fluid and the magnetic force in a plasma. This analogy is useful to discover drift-electrostatic vortices of the Larichev-Reznik type in a plasma [84, 85]. Several 2D fluid models in geophysical fluid dynamics as well as plasma physics are studied to determine monopole vortex solutions using so called center-of-mass (charge) velocity method [86].

Fig. 2. Monopolar vortex structure.
Some distinctive properties and formation mechanisms of nonlinear solitary vortical structures should be noted here. Such formations have dualistic properties. On the one hand, they have properties of solitary waves because nonlinearity compensates their wave dispersion, this is why, they behave like long-living structures compared to the linear wave packets. On the other hand, they have vortex properties because they rotate with a velocity greater than their translational velocity. It means that, in the system of the structure streamlines, there exists a separatrix, enclosed trapped particles of a medium. Thus, the structure transforms into a “real vortex” carrying along the trapped particles. This drastically changes the properties of the structure namely; the characteristic size of the structure which is not associated with the amplitude (in contrast to the KdV-type soliton whose size is inversely proportional to the
Vortical solitary structures differ from ordinary solitons in terms of their mutual collisions which unlike collisions of the classical solitons, lead to merging, i.e. these collisions are inelastic. In addition, the existence condition for the solitary vortical structure requires that the velocity of propagation should be greater than the linear-wave velocities. Otherwise, due to the Cherenkov resonance with linear waves, the solitary structure will lose its energy, that is, it will not exist long enough to be called a solitary structure. Therefore, the velocity of the solitary structure must exceed the limiting linear-wave speed. In spite of all the beauty of the analogy mentioned above, one should bear in mind that there is the important restriction that the Hasegawa-Mima and Charney equations only describe small-scale structures. This corresponds to the quasi-geostrophic (QG) approximation in geophysical hydrodynamics, which observes purely 2D structures.

In several subsequent papers ([83], see, e.g. [87-91] and the literature cited therein), instead of small-scale structures only large-scale structures having dimensions larger than the “characteristic Larmor radius of plasma ions or the Rossby radius”, were observed. Ultimately, the generalized Charney as well as Hasegawa-Mima equations were obtained, which correspond to the “so-called intermediate-geostrophic (IG) approximation in geophysical hydrodynamics”. The free-surface perturbation plays crucial role under this approximation. So, the Charney and Hasegawa-Mima equations exhibit the symmetrical behavior regarding the transformation of a cyclone into an anticyclone or may be an anticyclone into cyclone. The mentioned equations predict that solitary structure exists in the shape of a dipolar vortex which consists of essentially two closed packed regions of opposite vorticity, that is a cyclonic-anticyclonic pair (see Fig. 3). The dipolar vortex possesses a net linear momentum and consequently, appears as a moving structure. When two poles of a dipole are equal (symmetric), then the vortex travels along a straight line. In the case when one of the poles is stronger than the other (or asymmetric dipoles), they travel along curved path. Solitary monopolar vortices cannot be described on the basis of Charney and Hasegawa-Mima equations.

The generalized Charney and Hasegawa-Mima equations presume the occurrence of solitary monopolar vortical structure (see Fig.2). Monopolar vortex is exponentially localized, and mainly circularly symmetric, it has a spatial structure with a swirling motion of only one orientation, and is characterized by a single set of closed concentric stream lines. In contrast to the dipole vortex monopole vortex possesses only angular momentum.

We will discuss the mechanism of self-organization of a medium into monopole and dipole vortical structures in detail in chapter 3.
1.4 Zonal Flow

Zonal flows are associated with azimuthally symmetric band-like shear flows and are ubiquitous phenomena in nature and the laboratory. Term zonal flow is usually used in meteorology which means large-scale atmospheric flow may be from west to east, or east to west, parallel to the lines of latitude. We can give the further qualification that zonal flows are restricted to bands (zones) of latitude. In general such flows are sheared and change both the flow velocity value and its direction several times from equator to pole. Examples associated with zonal flows are jet streams that exist in the Earth’s atmosphere and the Jovian belts. Their frequent existence has also been observed in the Venus atmosphere as well as in the transition region of the sun which is called tachocline, where their role is consistent in the process of solar dynamo. The striped atmospheres of giant planets (sometimes also known as jovian planets) have been investigated by zonal flows [92-95]. Voyager space-craft [96] has been used to point out zones of sheared flow on Saturn and Jupiter (see Fig. 4).

![Fig. 4. Zonal flows in Jupiter’s image.](image)

In the context of plasma experiments where it is confined toroidally, term zonal flow means a plasma flow within a magnetic surface primarily in the poloidal direction. So, zonal flows are explained by

a) actually, restricted in their radial extent perpendicular to the magnetic surfaces,

b) having small or no change in either poloidal or toroidal direction – their mode nature is \( m = n = 0 \) and \( k_\parallel = 0 \) (\( m \) and \( n \) represent the poloidal or toroidal mode numbers,
respectively, and $k_\parallel$ is the wave number parallel to the magnetic field), these modes have minimal Landau damping,
c) having almost zero real frequency when observed in the state of toroidal equilibrium.

Thus the zonal flow is a toroidally symmetric electric field perturbation with finite radial wave numbers in a toroidal plasma, which is constant on the magnetic surface but rapidly varies in the radial direction. Furthermore, they are elongated, asymmetric vortex modes, and thus have zero frequency.

It has been observed that zonal flow has the sheared property that has strongly beneficial effect of reducing radial transport by suppressing turbulence, thus improving the confinement of heat required to achieve fusion conditions. In gyrokinetic simulations [97] it was shown that the inclusion of zonal flows strongly decreases the ion thermal transport. Influences of zonal flows on ITG turbulence are explained in Fig. 5 where “Isodensity contours” are shown. Fluctuations when zonal flow is included (see Fig. 5(a)), have minor saturation, corresponding to the case where zonal flows are suppressed, (see Fig. 5(b)).

![Fig. 5. Radial size of turbulent eddies shown in coloured contour of ambient density fluctuation gets reduced due to random shearing by self-regulated $\vec{E} \times \vec{B}$ zonal flows from gyrokinetic particle simulation with zonal flow (a) and with zonal flow suppressed (b).](image)

The sheared property of zonal flows plays a crucial role in the laboratories. The sheared $\vec{E} \times \vec{B}$ zonal flows describe well “L-mode confinement”, “the L - H transition”, and internal transport barriers [98]. Experimental work regarding zonal flows is explained in detail [98-100]. The important component in the dynamics of zonal flow is the mechanism of shearing of turbulent eddies through larger-scale flows ($L_s > \Delta x_e$, where $L_s$, $\Delta x_e$ represent the Shear-length and eddy scale, respectively). In reality, the turbulence as well as transport is decreased by such shearing. The zonal flow pattern is oscillatory and complex, exhibiting structure on
scales of 10-20 $\rho_i$ (here $\rho_i$ is the ion Larmor radius). The mean sheared flow evolves on transport time scales, whereas the zonal flows can evolve on turbulent time scales. The scale of sheared mean flow is macroscopic, while the scale of zonal flow is mesoscopic scale. The difference of sheared mean flow and zonal flow is illustrated in Fig. 6.

![Fig. 6. (a) Sheared mean flow, (b) Zonal flow.](image)

As it was mentioned zonal flows are $n = 0$, $m \approx 0$ symmetric electric field fluctuations with finite radial wave number $q_r$. Thus, they cannot govern radial transport and free energy sources ($\nabla n, \nabla T, \eta$ etc.) are out of their range. The Modified Parametric method is used for the zonal flow generation. Turbulent stress is the process that plays role in generating zonal flow. They arise via a self-organization phenomenon driven by low-frequency drift-type modes, in which energy is transferred to longer wavelengths by modulational instability or turbulent inverse cascade. Three wave coupling is represented by nonlinear interaction, this interaction is observed between two drift waves and one small-scale (small wave number, $\tilde{q} = q_r \tilde{r}$) zonal flow. The problem drift wave-zonal flow is remarkable, since the wave numbers and frequencies of drift waves are high ($k_\perp \rho_i \sim 1$, $\omega_k \sim \omega_z$, and $\omega_z$ is the drift frequency) as compared to zonal flows ($q_r \rho_i \ll 1$, $\Omega \sim 0$ and $\Omega$ is the rate of the change or frequency of the zonal flow). Thus the method of multiscale expansions also can be used.

### 1.5 Layout of the Thesis

The thesis is arranged in the following fashion: The first chapter provides the justification to study the nonlinear phenomena in the Earth’s ionosphere and EPI plasmas. Namely, we give a brief description of the Earth’s ionosphere structure, electron-positron-ion plasmas, nonlinear coherent vortical structures and zonal flow.

Chapter 2 is devoted to the elucidation of the possibility of large-scale zonal flow
generation in different layers (E- and F) of the ionosphere by small-scale Rossby waves. The interaction of zonal flow with Rossby wave turbulence and its nonlinear dynamics is investigated on the basis of plain but illuminating Charney model. The modified parametric approach is used with an arbitrary spectrum of primary modes.

Chapter 3 deals with the dynamics of large-scale vortical structures in electron-positron-ion plasmas. We derive the generalized Hasegawa-Mima equation to describe the nonlinear propagation of electrostatic drift waves in EPI plasmas. The generalized Hasegawa-Mima equation contains one vector (Jacobian) and different nature two scalar nonlinearities. Unperturbed plasma densities and temperatures of electrons and positrons are assumed to be inhomogeneous. We discuss a new self-organization mechanism of formation of large-scale electrostatic drift vortical structures in EPI plasmas based on the competition between scalar and vector nonlinearities.

Chapter 4 is devoted to the investigation of the generation of large-scale zonal flows by comparatively small-scale electrostatic drift waves in EPI plasmas. The generation mechanism is based on the concept that we have used in the second chapter.

The final chapter 5 provides the summary of the new results obtained in the thesis and at the end the references are listed.
Chapter 2

Zonal Flow Generation by Rossby Waves in Ionospheric E- and F-layers

2.1 Introduction

Large-scale planetary processes exist in the Earth’s ionosphere. To study their dynamics, two factors, the turbulent state of the Earth’s ionosphere as well as non-uniform electromagnetic forces are essential.

Large-scale planetary waves having several days period and several hundred kilometers of wavelength are discussed because of their fascinating role to change the global atmospheric circulation [101, 102]. Sufficient observational data has now been accumulated [103-120] verifying that such ultra-low frequency (ULF) electromagnetic perturbations exist in the conductive layers of the Earth’s ionosphere. The majority of the ionospheric phenomena such as the ionospheric precursors of some extraordinary phenomena, e.g., earthquakes, volcano eruptions, etc. [121-123] and the ionospheric reaction on the anthropogenic action [124-127] are comparable to the frequency of planetary waves. Recently, it is observed that forced oscillations of that kind appear during magnetospheric storms [121].

The study of the generation and dynamics of planetary Rossby type waves has accordingly been a subject of a great deal of theoretical and experimental investigations in recent years and these waves are induced by the spatial inhomogeneity of both the Earth’s angular velocity and the geomagnetic field in the ionospheric plasma (see e.g. [128] and references therein). The presence of charged particles in the ionosphere may substantially enrich the class of possible ULF wave modes in the ionosphere. Dokuchaev [129] not only observed the interaction of induction electric current with the geomagnetic field, but also deviation of zonal flows from their geostrophic values by electromagnetic forces or latitudinal gradient of geomagnetic field. Tolstoy [130] used this effect for Rossby type waves in the E-layer and named them as “hydromagnetic gradient (HMG) waves”. He pointed out these waves could change the geomagnetic field strongly (from a few to several tenths of nT). In addition, it was
shown that HMG waves could couple with Rossby waves at the E-layer altitudes.

New type of waves was investigated by Kaladze et al. [131-135] in the ionospheric E-region, which was the generalization of tropospheric Rossby waves in the ionosphere. In contrast to HMG waves, these waves do not significantly perturb the geomagnetic field and are generated by the dynamo field \( E_d = \mathbf{v} \times \mathbf{B}_0 \). These waves were named as magnetized Rossby (MR) waves by Kaladze [133]. Some properties of these waves can be found in the experiments [103-107, 119].

At the same time both ground-based and satellites observations clearly demonstrate the permanent existence of zonal flows at different layers of the Earth’s atmosphere. Their velocities are inhomogeneous along the lines of latitude [136-138] and are driven by low-frequency planetary-scale perturbations [101]. Most important of them are Rossby type. These Rossby type planetary-scale perturbations propagate azimuthally around the Earth at a fixed latitude. It is shown that velocity shear causes the dispersion and nonlinear effects in wave perturbations [73]. The propagating waves and zonal flows are supported by the Earth’s atmosphere. These structures constitute a dynamic system that exhibits complex nonlinear interactions.

In this chapter we present a detailed calculation for the generation of zonal (sheared) flows in the Earth’s ionosphere from the parametric decay of finite amplitude MR waves.

On the basis of the kinetic equation for wave packets Smolyakov et al. [139] put forward the theory related to generation of zonal flows by tropospheric Rossby waves in the Earth’s ionosphere. Shukla and Stenflo [140] and Onishchenko et al. [141] promoted this theory but they showed the zonal flow generation by Rossby waves. They used the standard method of parametric instabilities, and showed that in a neutral atmosphere when it was rotating non-uniform the finite-amplitude Rossby waves could excite zonal flows. Accordingly, in these papers the interaction of pump waves (Rossby waves), a sheared flow and side-band waves called two satellites of the pump wave is investigated. Smolyakov et al. [139] used the same technique which was alternative to the standard weak turbulence approach. The Reynolds stresses is the driving mechanism of the zonal flow instability. Because of this nonlinear mechanism, inverse cascade, i.e. spectral energy transfers from small-scale Rossby waves to large-scale enhanced zonal flows in the Earth’s neutral atmosphere takes place. Further, classical nonlinear two-dimensional Charney equation was used to observe the zonal flow generation within Rossby wave turbulence model. The necessary condition for zonal flow generation similar to the Lighthill criterion for modulation instability in nonlinear optics [142] was found.
Now the question arises: are there other zonal flow generation mechanisms? To this end, Kaladze et al. [143, 144] added a Korteweg–de Vries type nonlinearity to the Charney equation. It was shown that in this case zonal flow generation by the Rossby waves always exists and needs no criterion for fulfillment. In Refs. [140, 141, 143, 144] generation of zonal flow by a monochromatic Rossby wave packet was presented. The research was carried out to explore the influence of non-monochromaticity of wave packet in general by Smolyakov et al. [139] and Malkov et al. [145]. Smolyakov et al. [139] showed that wave packet broadening generates the possibility of considering zonal flow generation in the ‘hydrodynamic’ and ‘kinetic’ regimes, similarity was found with beam plasma instabilities. In the kinetic regime resonant-type instability takes place, whereas the hydrodynamic regime refers to a coherent instability.

Mikhailovskii et al. [146] studied the zonal flow generation by primary drift Alfven modes with arbitrary spectrum broadening, where the modified parametric approach was suggested. In this technique, the driving forces (Reynolds stresses) of zonal flows are given by a summation or integration over the primary modes wave packet, which gives the possibility of revealing additional zonal flow generation mechanisms. The idea of using two Gaussian wave packets in the problem of zonal flow generation comes from Malkov et al. [145]. This was used by Mikhailovskii et al. [147] to show zonal flow generation in a shallow neutral fluid that is rotated; this type of instability is called two-stream-like instability. Unfortunately, Mikhailovskii et al. [147] did not obtain any expression for the growth rate concerning the excitation of zonal flow by ordinary Rossby waves and there does not appear to be any discussion of this question in that paper. In this chapter, our concentration will be on the Earth’s ionospheric plasma of the E- and F-layers (≈ 90–400 km range above the Earth’s surface) which are incompressible.

Unlike the neutral atmosphere, such a gas becomes conductive and the influence of electromagnetic forces should be taken into account, physical modeling of the ionospheric motion in the E- and F-layers is described in Sec. 2.2. In such an ionospheric E-layer gas, MR waves can propagate (see Sec. 2.3). As we have mentioned, the problem of zonal flow generation by MR waves in the E-region by a monochromatic wave packet was initiated by Kaladze et al. [143, 144]. It was shown that the zonal flow instability by these monochromatic waves is also prohibited when the Lighthill instability criterion is not fulfilled. However, no investigation has been carried out so far into the influence of wave packet i.e., (non-monochromatic) on zonal flow generation by MR waves or to identify any additional mechanisms of instability.

In Sec. 2.3 our goal will be to investigate the zonal flow generation by small-scale (λ < r_R, and r_R is changed by the inhomogeneous geomagnetic field Rossby radius) MR waves...
when the medium is ionospheric E-layer and the broadening of the wave packet spectrum is considered. To this end we examine the problem by considering primary MR waves having a sufficiently broad-spectrum wave packet and will show that a two-stream-like instability is also an effective mechanism for the excitation of zonal flow instability. Dispersion relation for zonal flow is derived in Sec. 2.3. To this end, by analogy with Kaladze et al. [143, 144], the problem is examined by considering a three-wave resonant interaction. Then, following Mikhailovskii et al. [146], the driving force of zonal flow is calculated, which is the Reynolds stress, and may be expressed as a summation over the spectrum of pump modes. The results of Kaladze et al. [143, 144] for a small-scale (\( \lambda < r_R \)) monochromatic spectrum of pump MR modes are obtained as a limiting case in Sec. 2.3. The growth rates for the monochromatic and non-monochromatic wave packets of pump modes are determined. The influence of non-monochromaticity on zonal flow generation under Lighthill’s instability criterion is investigated in Sec. 2.3. Gaussian wave packet in case of ‘single-humped’ is considered for zonal flow generation. Separately, small spectrum broadening and a strong broadening are discussed. Wave packet of primary MR waves in case of ‘Two-humped’ is also investigated in this section to show the existence of two-stream-like mechanism of zonal flow generation when the Lighthill instability condition is not fulfilled.

In Sec. 2.4, generation of zonal flow by Rossby waves in the Earth’s dissipative ionospheric F-layer is investigated. In this case dissipation arises due to the Pedersen conductivity acting as viscosity and gives rise to inductive (magnetic) inhibition. The zonal flow growth rate is also calculated which does not depend on small wave vector component of zonal flow mode, needs no instability condition and the spectral energy transferring process unconditionally takes place. The main results obtained in chapter 2 are discussed in Sec. 2.5.

### 2.2 Physical Modeling for Ionospheric Motion in the E- and F-Layers

Let us consider the partially ionized ionospheric gas; it is immersed in a dipole geomagnetic field \( \mathbf{B}_0 \), the gas is composed of electrons, ions and neutral particles. The behavior of such a weakly ionized gas is determined by its massive neutral component under the condition \( n/N \ll 1 \), ratio of the charged particles number density to the neutral density. The presence of charged particles causes the medium to be electrically conducting. We will use quasi-hydrodynamic equations having “friction force” which is due to collision [2, 136]. Neglecting the thermal flux contributions, we get the following set of equations [2, 136]
\[ \rho_n \frac{d\mathbf{v}_n}{dt} = -\nabla p_n + N\mathbf{F}_n + \eta_n \Delta \mathbf{v}_n - \rho_i \nu_i(n\mathbf{v}_n - \mathbf{v}_i) - \rho_e \nu_{en}(\mathbf{v}_n - \mathbf{v}_e), \]  
(2.1)

\[ \rho_i \frac{d\mathbf{v}_i}{dt} = -\nabla p_i + n\mathbf{F}_i + \eta_i \Delta \mathbf{v}_i - \rho_i \nu_i(n\mathbf{v}_i - \mathbf{v}_n) - \rho_e \nu_{ei}(\mathbf{v}_i - \mathbf{v}_e) + enE + e\mathbf{v}_i \times \mathbf{B}, \]  
(2.2)

\[ \rho_e \frac{d\mathbf{v}_e}{dt} = -\nabla p_e + n\mathbf{F}_e + \eta_e \Delta \mathbf{v}_e - \rho_e \nu_{en}(\mathbf{v}_e - \mathbf{v}_n) - \rho_e \nu_{ei}(\mathbf{v}_e - \mathbf{v}_i) - enE - e\mathbf{v}_e \times \mathbf{B}, \]  
(2.3)

with the appropriate incompressibility conditions

\[ \mathbf{\nabla} \cdot \mathbf{v}_n = 0, \quad \mathbf{\nabla} \cdot \mathbf{v}_i = 0, \quad \mathbf{\nabla} \cdot \mathbf{v}_e = 0. \]  
(2.4)

Here indices \( n, i \) and \( e \) denote molecules (neutral particles), ions and electrons, respectively; \( \frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \), \( \mathbf{v} \) is a hydrodynamic velocity of corresponding components; condition \( n_e = n_i = n \) is also used, so \( \rho_n = NM, \rho_i = nM, \rho_e = nm \) are mass densities, mass of ions (molecules) and electrons are denoted by \( M \) and \( m \) respectively; \( \nu_{ei}, \nu_{en} \), are collision frequencies of electrons with ions and molecules, \( \nu_{in} \) is the collision frequency of ions with molecules; \( \mathbf{E} \) represents the induced electric field strength; \( \mathbf{B} = \mathbf{B}_0 + \mathbf{h} \), is the total magnetic field, and perturbation of the geomagnetic field is denoted by \( \mathbf{h} \); \( \mathbf{F}_n, \mathbf{F}_i, \mathbf{F}_e \) and \( \eta_{\alpha}(\alpha = n, i, e) \) are the nonelectromagnetic (hydrodynamic) forces, pressures and dynamic viscosities acting on the corresponding gases, respectively. Taking into account the small concentration of charged particles we add equations (2.1)-(2.3) to obtain the following hydrodynamic equation [2, 136]

\[ \rho_n \frac{d\mathbf{v}_n}{dt} = -\nabla p_n + N\mathbf{F}_n + \eta_n \Delta \mathbf{v}_n + \mathbf{j} \times \mathbf{B}, \]  
(2.5)

where \( \mathbf{j} = en(\mathbf{v}_i - \mathbf{v}_e) \) is the density of induced electric current and for the ionospheric gas under consideration which is defined by the equations (1.3) and (1.4). For the problems set in this chapter, we rewrite equations (2.5) for the following neutral gas momentum equation

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla p}{\rho} + \frac{1}{\rho} \mathbf{j} \times \mathbf{B} + \mathbf{g} + 2\mathbf{v} \times \Omega + \nu \mathbf{\nabla}^2 \mathbf{v}, \]  
(2.6)

where \( \mathbf{v} \) is the velocity of neutral gas which is incompressible, \( \Omega \) is the angular velocity of the Earth’s rotation, on the right-hand side fourth term represents the Coriolis acceleration, pressure and the density of the medium are represented by \( p \) and \( \rho \), respectively; \( \mathbf{g} \) is the gravitational acceleration, \( \nu \) is the kinematic viscosity. Our goal is to calculate the Ampère force \( \mathbf{F}_A = \mathbf{j} \times \mathbf{B}/N \), and ionospheric motion is well examined by this force. Let us estimate the value of
vortical part of electric field $E$ in (1.2). If we introduce the typical space variation scale $L$, time scale $T$, and velocity scale $U$, then from the Maxwell’s equation $\nabla \times E = -\partial h / \partial t$, we get $E \sim Lh / t \sim U h$, and as $|h| \ll |B_0|$ this term is much smaller than the dynamo field $E_d = \mathbf{v} \times \mathbf{B}_0$. As to the electrostatic part $E = -\nabla \varphi$, it differs from zero only in the presence of free charges which cannot exist in the conducting ionosphere during the long time. Here non-inductive approximation is developed and $R_m = L U \mu_0 \sigma_e \ll 1$, is the magnetic Reynolds number, $\mu_0$ is free space permeability, and $\sigma_e$ is the effective conductivity of ionosphere. This condition in the ionosphere is well satisfied. Certainly, for the given perturbations $L \sim 10^3 km$, $U \sim 10^2 ms^{-1}$, $\sigma_e \approx 10^{-4} S/m$, and $\mu_0 = 4\pi \cdot 10^{-7} Hm^{-1}$, we get $R_m \sim 10^{-2}$. In the noninductive approximation, self-generated magnetic fields are ignored, therefore $\mathbf{B}$ will be equal to the external geomagnetic field $\mathbf{B}_0$.

Hartmann number in square form is given by the ratio of volumetric electromagnetic $j \times \mathbf{B}_0$ force to the viscous friction force, $Ha^2 = \sigma_e B_0^2 L^2 / \eta$, where $\eta = \nu \rho$ is the dynamic viscosity, and $L$ is the typical scale of wind velocity variation (in fact the height of the layer). For the ionospheric motions in the corresponding layers $Ha^2 \gg 1$. For example in the E-layer with $L = 40 km$, $B_0 \approx 0.5 \times 10^{-4} T$ and $\eta \approx 10^{-5} kg/s.m$, we obtain $Ha^2 \approx 40$. In F-layer, for $L = 100 km$, we get $Ha^2 \approx 300$. Thus we find that for large-scale motions the viscous force $v \nabla^2 v$, is very small as compared to electromagnetic force, therefore, last term in equation (2.6) can be neglected. Consequently for large-scale motions dissipative processes are connecting with the Pedersen conductivity [129]. In case of small-scale motions $Ha^2 \ll 1$, conversely, one can ignore Joule’s dissipation and keep the action of ordinary viscosity.

We introduce a local Cartesian system of coordinates $(x,y,z)$ and express in terms of latitude $\lambda$ and longitude $\varphi$, where $y = (\lambda - \lambda_0) R$ and $x = \varphi R \cos \lambda$ are latitudinal and longitudinal coordinates, respectively, and $R$ is the distance from the center of Earth. In the given system, the $x$ axis directed from west-east, $y$ axis from south-north and the $z$ axis outward normal direction. And the corresponding derivatives are given by $\partial / \partial \varphi = R \cos \lambda \partial / \partial x$ and $\partial / \partial \lambda = R \partial / \partial y$. Let us find the modulus of the geomagnetic field $B_0 = B_{eq}(1 + 3 \sin^2 \lambda)^{1/2}$, and the local components of the geomagnetic field vector are

$$B_0 = (0, B_{0y}, B_{0z}) = (0, B_{eq} \cos \lambda, -2 B_{eq} \sin \lambda). \quad (2.7)$$
where $B_{eq}$ is the equatorial value of the geomagnetic field. This value is taken at a distance $R$ from the Earth’s center. For the angular velocity of the Earth’s rotation $\boldsymbol{\Omega}$ in the local system of coordinates we have the following expression

$$\boldsymbol{\Omega} = (0, \Omega_0 y, \Omega_0 z) = (0, \Omega_0 \cos \lambda, \Omega_0 \sin \lambda).$$

(2.8)

According to the experimental data [129], the large-scale motions under the consideration in the ionosphere are two-dimensional taking place in $(x, y)$ plane and consequently, we can assume $v_z = \partial / \partial z = 0$. Taking into account equation (1.3) and all approximations discussed above we represent $x$ – and $y$ – components of the momentum equation (2.6) as follows

$$\frac{d v_x}{dt} - \left(2 \Omega_{o z} + \frac{\sigma H B_0 B_{o z}}{\rho}\right)v_y + \frac{1}{\rho} \frac{\sigma B^2}{\rho} v_x = - \frac{1}{\rho} \frac{\partial p}{\partial x},$$

(2.9)

$$\frac{d v_y}{dt} + \left(2 \Omega_{o z} + \frac{\sigma H B_0 B_{o z}}{\rho}\right)v_x + \frac{1}{\rho} \frac{\sigma B^2}{\rho} v_y = - \frac{1}{\rho} \frac{\partial p}{\partial y},$$

(2.10)

and the operator $d / dt$ is also spatially two-dimensional, i.e.

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y}.$$  

(2.11)

The incompressibility condition (2.4) enables us to introduce the stream function $\psi$ as

$$v_x = - \frac{\partial \psi}{\partial y}, \quad v_y = \frac{\partial \psi}{\partial x}.$$  

(2.12)

Differentiate equation (2.9) and (2.10) with respect to the corresponding coordinates, we eliminate the pressure $p$ from these equations in order to obtain the nonlinear equation [131].

$$\frac{\partial \Delta \psi}{\partial t} + J(\psi, \Delta \psi) + \frac{\partial \psi}{\partial x} \frac{\partial \bar{\Omega}}{\partial y} + \frac{\partial \psi}{\partial y} \frac{\partial \Lambda_{10}}{\partial y} + \Lambda_{1z} \frac{\partial^2 \psi}{\partial x^2} + \Lambda_{10} \frac{\partial^2 \psi}{\partial y^2} = 0.$$  

(2.13)

Here the operator $\Delta = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the two-dimensional Laplacian, $J(a,b) = \partial_x a \partial_y b - \partial_y a \partial_x b$ is the Poisson bracket operator (Jacobian) representing so called vector nonlinearity, and
\[ \tilde{\Omega} = 2 \Omega_0 + \Lambda_H, \quad \Lambda_H = \frac{\sigma_H B_0 B_{0z}}{\rho}, \]

\[ \Lambda_{10} = \frac{\sigma_1 B_0^2}{\rho}, \quad \Lambda_{1z} = \frac{\sigma_1 B_{0z}^2}{\rho}. \quad (2.14) \]

The nonlinear equation (2.13) is the modified Charney equation valid for the conductive ionospheric layers. It describes the nonlinear vortical formations propagating in the zonal direction.

Let us consider the two-dimensional $\beta$--plane approximation for geophysical and planetary fluid flow, and has been studied in depth since the pioneering work by Rhines [64]. In this approximation in the vicinity of the latitude $\lambda = \lambda_0$ we represent the Coriolis parameter as

\[ f = 2 \Omega_0 z = 2 \Omega_0 \sin \lambda = 2 \Omega_0 (\sin \lambda_0 + \Delta \lambda \cos \lambda_0) = f_0 + \beta y \quad (2.15) \]

with

\[ f_0 = 2 \Omega_0 \sin \lambda_0 > 0, \quad \beta = \frac{\partial f}{\partial y} = \frac{1}{R} \frac{\partial f}{\partial \lambda} = \frac{2 \Omega_0 \cos \lambda_0}{R} > 0. \quad (2.16) \]

Analogously, the $z$--component of geomagnetic field varies as

\[ \gamma = B_{0z} = -2 B_{eq} \sin \lambda = \gamma_0 + \beta_B y, \quad (2.17) \]

with

\[ \gamma_0 = -2 B_{eq} \sin \lambda_0 < 0, \quad \beta_B = \frac{\partial \gamma}{\partial y} = \frac{\partial B_{0z}}{\partial y} = -\frac{2 B_{eq} \cos \lambda_0}{R} < 0. \quad (2.18) \]

The quantities $\beta_B$, $\beta$, $f_0$, and $\gamma_0$ are related to the latitude $\lambda = \lambda_0$ where we took $y = 0$. Thus, in the $\beta$--plane approximation we use the plane wave method for dynamical equations.

We apply $\beta$--plane approximation to produce simple results, thus it is easy to study the motion on a rotating sphere. Notice that this $\beta$--plane approximation breaks down at auroral and polar latitudes and along the equatorial zones.

### 2.3 Nonlinear Interactions of Magnetized Rossby Waves and Zonal Flows in the E-Layer

Let us now consider the ionospheric E-layer, range $100 - 150 \text{ km}$ above the Earth’s surface. The plasma conditions in this region are ($v_e \approx v_en$, $v_in v_en \ll \omega_B e \omega_B$, $v_in \gg \omega_B$).
equation (1.4) for conductivities can be simplified under these conditions. We get for the “Hall conductivity” $\sigma_H \approx en/B_0$, which is much greater than the “Pedersen conductivity” i.e., $\sigma_P \approx \sigma_H \omega_B l / v_in \ll \sigma_H$, therefore, we can neglect the ion friction due to the Pedersen conductivity in this region. Thus the modified Charney equation (2.13) in the E-layer [131-133] has the form

$$\frac{\partial \Delta \psi}{\partial t} + J(\psi, \Delta \psi) + (\alpha + \beta) \frac{\partial \psi}{\partial x} = 0, \quad (2.19)$$

where $\alpha = en \beta_B / \rho$. The value of $(\alpha + \beta)$ represents the “generalized Rossby parameter”, where $\alpha$ appears from the “Ampère force”. equation (2.19) shows that under the influence of the geomagnetic field, we have following transformation

$$2\Omega_0 \rightarrow 2\Omega_0 + \frac{en}{\rho} B_0, \quad (2.20)$$

where $\Omega_0$ is the planetary angular rotation vector.

2.3.1 Magnetized Rossby Waves in the E-layer

From equation (2.19) the dispersion relation for the magnetized Rossby waves [133] is obtained in the following form

$$\omega_k = -\frac{k_x (\alpha + \beta)}{k_\perp^2}. \quad (2.21)$$

Here $\omega_k$ and $k$ represent the frequency and the wave vector of the wave, respectively, $k_\perp = (k_x^2 + k_y^2)^{1/2}$, and $x, y$ component of the wave vector is represented by $k_{x,y}$. The propagation of the wave depends not only on the generalized Rossby parameter $(\alpha + \beta)$ but also on the modified Rossby radius. We have for the E-layer $(\beta \approx -\alpha \approx 10^{-11} m^{-1} s^{-1})$, where $\alpha$ depends on the fraction $n/N$. It is observed that this ionization fraction contains different values for the night and day sides of the Earth and the parameter $(\alpha + \beta)$ in equation (2.21) may change its sign. Thus the propagation of MR waves in this region of the ionosphere may be westward or eastward parallel to the lines of latitude. They are weakly damped modes. The MR waves belong to the ULF range $(10^{-6} - 10^{-4}) s^{-1}$, with wavelength of the order 1000 km and longer, and the phase velocity $\sim (1 - 100) ms^{-1}$. Correspondingly, the wave period varies from 2 hours to 14 days. Geomagnetic field is not strongly perturbed by MR waves. They are induced by the inhomogeneity of both the geomagnetic field and the angular
velocity of the Earth’s rotation along the latitude given by $\alpha$ and $\beta$, respectively. The role of the ionospheric dynamo electric field is positive to excite the MR waves when the Hall effect is included.

2.3.2 Zonal Flow Dispersion Relation

The shallow rotating water model needs the existence of a free surface, which is difficult to justify in the case of the ionospheric E-layer. We will consider, therefore, the nonlinear interactions of small-scale MR wave turbulence and zonal flows. Our choice is the modified Charney equation (2.19) to investigate this nonlinear mechanism. In this mechanism, when $k_zr_R \gg 1$ where $r_R \equiv c_s/|\alpha + \beta|$ is the modified by the geomagnetic field Rossby radius, and $c_s$ is the equivalent sound speed in the ionospheric E-layer, the parametric instability is only due to the vector nonlinearity $J(\psi, \Delta \psi)$ [143, 144].

As the zonal flows vary on time scales slower than those of small-scale MR waves, so multiple-scale expansion can be utilized. Standard method to investigate the evolution of coupled system of MR waves and zonal flows is used in literature [141, 144, 146, 147]. Split the perturbation of the stream function in equation (2.19) into three components

$$\psi = \tilde{\psi} + \check{\psi} + \hat{\psi},$$

(2.22)

where function $\check{\psi}$ as a spectrum of pump modes ($\check{\psi}_-(k) = \check{\psi}_+(k)$, and * means the complex conjugative)

$$\check{\psi} = \sum_k [\check{\psi}_+(k)\exp(ik \cdot r - i\omega_k t) + \check{\psi}_-(k)\exp(-ik \cdot r + i\omega_k t)]$$

(2.23)

For side-band modes, function $\hat{\psi}$ as a spectrum

$$\hat{\psi} = \sum_k [\hat{\psi}_+(k)\exp(ik_+ \cdot r - i\omega_+ t) + \hat{\psi}_-(k)\exp(ik_- \cdot r - i\omega_- t) + c.c.]$$

(2.24)

and for zonal flow modes

$$\tilde{\psi} = \tilde{\psi}_0\exp(-i\Omega t + iq_0 y) + c.c.$$  

(2.25)

where c.c. means the complex conjugate and $\tilde{\psi}_0$ is the amplitude (constant) of zonal modes. These modes vary only along the meridians. Both energy and momentum conservations $\omega_\pm = \Omega \pm \omega_k$ and $k_\pm = q_0 e_y \pm k$ imposed on frequency and wave vector are satisfied, and the pairs $(\omega_k, k)$ and $(\Omega, q_0 e_y)$ are used for pump and zonal flow modes, respectively.
We use the standard quasi-linear procedure, and substitute (2.22)–(2.25) into equation (2.19). The role of small nonlinear terms is neglected for the high frequency primary modes, then we get

$$\frac{\partial \Delta \tilde{\psi}}{\partial t} + \frac{\partial \tilde{\psi}}{\partial x} (\alpha + \beta) = 0,$$  

(2.26)

from which one may obtain the dispersion relation given by (2.21) for the MR waves.

But for low frequency zonal modes, we keep such nonlinear terms. Substituting (2.22)–(2.25) into equation (2.19) and averaging out, we get the evolution equation of low-frequency zonal flow:

$$i\Omega \tilde{\psi}_0 = \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} \right) = \left( \frac{\partial \tilde{\psi}}{\partial x} \frac{\partial \tilde{\psi}}{\partial y} + \frac{\partial \tilde{\psi}}{\partial y} \frac{\partial \tilde{\psi}}{\partial x} \right)$$

$$= \sum_k k_x \left[ 2k_y (\tilde{\psi}_x \tilde{\psi}_- + \tilde{\psi}_- \tilde{\psi}_x) + q_y (\tilde{\psi}_x \tilde{\psi}_- - \tilde{\psi}_- \tilde{\psi}_x) \right],$$  

(2.27)

The averaging over fast oscillations is denoted by angular brackets. The driving force of zonal flow modes is Reynolds stress and it is on the right-hand side of equation (2.27).

In order to calculate the Reynolds stresses in equation (2.27), we need to calculate the side-band amplitudes $\tilde{\psi}_\pm$. Turning to equation (2.19), we find the equation

$$\frac{\partial \Delta \tilde{\psi}}{\partial t} + \frac{\partial \tilde{\psi}}{\partial x} (\alpha + \beta) + J(\tilde{\psi}, \Delta \tilde{\psi}) + J(\tilde{\psi}, \Delta \tilde{\psi}) = 0,$$  

(2.28)

and for side-band amplitudes we get, respectively,

$$\tilde{\psi}_\pm = \mp i \frac{k_x^2}{k_{1\pm}^2} D_\pm \tilde{\psi}_0 \tilde{\psi}_\pm.$$  

(2.29)

We consider $\Omega$ and $q_y$ to be small parameters and have neglected $q_y^2$ in comparison with $k_{1\pm}^2$ in equation (2.29). In equation (2.29)

$$D_\pm = \omega_\pm \pm (\alpha + \beta) \frac{k_x}{k_{1\pm}^2},$$  

(2.30)

and

$$k_{1\pm}^2 = k_\pm^2 + (q_y \pm k_y)^2.$$  

(2.31)
Substituting equation (2.29) into equation (2.27) and making all necessary calculations [141, 143, 144], we get the following zonal flow dispersion equation:

$$1 - \sum_{k} \frac{F(k)}{(\Omega - q_y V_g)^2} = 0,$$

(2.32)

where

$$F(k) = \frac{q_z^2 k_x^2 k_y^2 V'_g}{\omega_k} |\bar{\psi}_+|^2 = \frac{q_z^2 k_x^2 k_y^2 V'_g}{2\omega_k} I_k,$$

(2.33)

with

$$I_k = 2|\bar{\psi}_+|^2.$$

(2.34)

Here \( V'_g(k) \) is the latitudinal (y - component) pump magnetized Rossby group velocity defined by

$$V'_g(k) = \frac{\partial \omega_k}{\partial k_y} = 2 \frac{k_x k_y (\alpha + \beta)}{k_4^2} = -2\omega_k k_y \frac{k_x}{k_4^2},$$

(2.35)

and \( V'_g \equiv \partial V'_g / \partial k_y \) is its derivative, so that

$$V'_g = \frac{\partial^2 \omega_k}{\partial k_y^2} = 2k_x (\alpha + \beta) \frac{k_x^2 - 4k_y^2}{k_4^2} = -2\omega_k \frac{k_x^2 - 4k_y^2}{k_4^2}.$$

(2.36)

It is noted that \( V'_g(k) \) and \( V'_g \) can get change sign due to \( \omega_k \) (see (2.21)) or when \( k_x = \pm \sqrt{3} k_y \).

Equation (2.32) is the dispersion equation for zonal flow, and is the generalization when the arbitrary wave packet spectrum of the primary MR waves [146, 147]. Thus it is possible to investigate different types of zonal flow excitation mechanisms. It should be noted that dispersion equation (2.32) coincides in structure with equation (19) given by Mikhailovskii et al. [147], but they have mistaken the sign before summation over \( k \).

**2.3.3 Zonal Flow Driven by a Monochromatic Wave Packet**

Let us consider the monochromatic wave packet, for which \( F(k) \sim \delta(k - k_0) \) and equation (2.32) reduces to a hydrodynamic-type coherent instability [141, 143, 144].
\[(\Omega - q_y V_{g_0})^2 = -\Gamma^2, \quad (2.37)\]

where

\[\Gamma^2 = \frac{q_y^2 k_x^2 k_{x_0}^2 |V'_{g_0}|}{2|\omega_{k_0}|} \quad (2.38)\]

and the subscript ‘0’ means that appropriate values are taken at \(k_0\). It is assumed that the necessary instability condition

\[\frac{V'_{g_0}}{\omega_{k_0}} < 0 \quad (2.39)\]

is fulfilled for not too large \(k_{y_0}\) (see (2.36)),

\[k_{y_0}^2 < \frac{1}{3} k_{x_0}^2. \quad (2.40)\]

Note, that the condition in (2.39) is the same as the “Lighthill criterion” for modulation instability in nonlinear optics [142]. It was investigated by Kaladze et al. [143, 144], the growth rate will be the maximum at \(k_y = 0\), when (see (2.38))

\[\Gamma^2 = 2q_y^2 k_{x_0}^2 |\tilde{\psi}_+|^2. \quad (2.41)\]

The obtained result is the standard mechanism of zonal flow generation similar to plasma beam instability.

### 2.3.4 Zonal Flow Driven by a Single-Humped Wave Packet

Following the method developed by Mikhailovskii et al. [146], we observe now, the influence of non-monochromaticity of wave packets on the zonal flow generation by MR waves. It should be noted that we will investigate in this section the zonal flow instability mechanism, which is provided by the realization of the Lighthill instability criterion given by equation (2.39).

Consider a single non-monochromatic packet of MR waves taking the spectrum of \(I_k\) in terms of Gaussian
The wave vector \( k_{y0} \) is taken at the center of the wave packet, and the width of wave packet is denoted by \( \Delta k_y \) (\( \Delta k_y > 0 \)). Suppose component \( k_x \) is same for all modes, \( k_x = k_{x0} \). The summation over \( \mathbf{k} \) in equation (2.32) is now understood as the integrals over \( k_y \), and we consider frequency \( \omega = \omega_k \) and group velocity \( V_g \) are function of \( k_y \), i.e. \( \omega = \omega(k_y), V_g = V_g(k_y) \). Then for finite values of \( \Delta k_y/k_{y0} \) the denominator in equation (2.32) becomes a function of the variable \( k_y \) and the zonal flow dispersion relation (2.37) obtained in the case of a monochromatic wave packet is not valid.

Thus, instead of equation (2.32), we will use for finite \( \Delta k_y/k_{y0} \) the following generalized dispersion equation:

\[
1 - F(k_0) \left( \frac{1}{(\Omega - q_y V_g)^2} \right)_{k_y} = 0, \tag{2.43}
\]

and the ‘resonant denominator’ \( (\Omega - q_y V_g)^{-2} \) is modified by the non-monochromaticity of wave packets. Here

\[
\langle \ldots \rangle_{k_y} = \frac{1}{\pi^{1/2} \Delta k_y} \int \langle \ldots \rangle \exp \left( -\frac{(k_y - k_{y0})^2}{(\Delta k_y)^2} \right) \, dk_y. \tag{2.44}
\]

### 2.3.4.1 Small Wave Packet Broadening

Here we consider the case when the broadening of the wave packet is sufficiently small i.e. \( \Delta k_y/k_{y0} \ll 1 \). Then expanding the latitudinal group velocity \( V_g \) in series in the vicinity of \( k_{y0} \), we obtain [145]

\[
V_g = V_{g0} + V_{g0}' (k_y - k_{y0}). \tag{2.45}
\]

Here prime represents that derivative is with respect to component \( k_y \) of the wave vector. Integrating over \( k_y \), we find

\[
\left( \frac{1}{(\Omega - q_y V_g)^2} \right)_{k_y} = \frac{1}{\Omega^2} \left( 1 + \frac{3}{2} \frac{q_y^2 V_{g0}'^2}{\Omega^2} (\Delta k_y)^2 \right). \tag{2.46}
\]
where
\[ \tilde{\Omega} \equiv \Omega - q_y V'_{g0}. \quad (2.47) \]

Then, we get the following expression for zonal flow dispersion relation:
\[
\Omega^2 = -\Gamma^2 \left( 1 - \frac{3 q_y^2 V'_{g0}^2}{2 \Gamma^2} (\Delta k_y)^2 \right). \quad (2.48)
\]

The growth rate decreases by the second term in the parentheses of equation (2.48). Hence the spectrum broadening under the condition
\[
\frac{\Delta k_y}{k_{y0}} < \frac{\Gamma}{q_y V'_{g0}}. \quad (2.49)
\]
can be neglected

2.3.4.2 Arbitrary Wave Packet Broadening

Now let us consider the case when the broadening of the wave packet \( \Delta k_y \) is arbitrary and zonal flow instability has a resonant character. Then we have
\[
\left( \frac{1}{(\Omega - q_y V_g)^2} \right)_{k_y} = \frac{1}{q_y V'_{g0} |\Delta k_y|} \frac{\partial}{\partial \tilde{\Omega}} Z \left( \frac{\tilde{\Omega}}{q_y V'_{g0} |\Delta k_y|} \right), \quad (2.50)
\]
where
\[
Z(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} dt \exp(-t^2) \frac{t - z}{t} \quad (2.51)
\]
is the plasma dispersion function defined for \( \text{Im} z > 0 \). Then the dispersion relation instead of equation (2.43) is given by
\[
1 = -\frac{\Gamma^2}{|q_y V'_{g0} |\Delta k_y|} \frac{\partial}{\partial \tilde{\Omega}} Z \left( \frac{\tilde{\Omega}}{q_y V'_{g0} |\Delta k_y|} \right). \quad (2.52)
\]

For \( \tilde{\Omega} \ll |q_y V'_{g0} |\Delta k_y \) we obtain from equation (2.50)
\[
\left( \frac{1}{(\Omega - q_y V_g)^2} \right)_{k_y} = -\frac{2}{(q_y V'_{g0} |\Delta k_y|)^2} \left( 1 + i \sqrt{\pi} \frac{\tilde{\Omega}}{q_y V'_{g0} |\Delta k_y|} \right). \quad (2.53)
\]
Finally, we get the following zonal flow dispersion relation

\[
\Omega = i \frac{q_y V'_0 |\Delta k_y|}{\sqrt{\pi}} \left( 1 - \frac{(q_y V'_0 \Delta k_y)^2}{2\Gamma^2} \right). \tag{2.54}
\]

This equation describes a kinetic zonal flow instability. We get the instability condition

\[
\Gamma^2 > \frac{1}{2} \left( q_y V'_0 \Delta k_y \right)^2, \tag{2.55}
\]

where \( \Gamma \) is defined by equation (2.41). Qualitatively, this condition has the same meaning as equation (2.49).

For the maximum value of the growth rate from equation (2.54), the spectral broadening

\[
\Delta k_y = \left( \frac{2}{3} \right)^{1/2} \frac{|\Gamma|}{|q_y V'_0|}, \tag{2.56}
\]

and in order of magnitude is equal to

\[
\gamma \approx |\Gamma| |q_y k x_0 \bar{\psi}_s|, \tag{2.57}
\]

Note, that the role of this resonance interaction was not correctly estimated by Mikhailovskii et al. [146], giving the conclusion that strong broadening of the wave packet suppresses the generation of zonal flow.

### 2.3.5 Zonal Flow Driven by a Two-Humped Wave Packet

In this section, we assume that Lighthill’s instability criterion (2.39) is not fulfilled. It is then clear that the results obtained in subsections 2.3.3 and 2.3.4 are not representative and the system becomes stable. Meanwhile, if, instead of ‘the single beam’ case investigated in the previous subsections, we use the two-humped wave packet distribution

\[
F(k) = F(k_1) \delta_{kk1} + F(k_2) \delta_{kk2}, \tag{2.58}
\]

suggested by Malkov et al. [145], indeed we may get two-stream-like mechanism of zonal flow instability studied by Mikhailovskii et al. [147], for ordinary Rossby waves in the scope of a shallow water model. Indeed, in this case, instead of equation (2.32), one has the following dispersion relation
\[
1 - \frac{\Omega_1^2}{(\Omega - q_y V_{g1})^2} - \frac{\Omega_2^2}{(\Omega - q_y V_{g2})^2} = 0,
\] (2.59)

where \((\Omega_1^2, \Omega_2^2) = [F(k_1), F(k_2)], V_{g1} = V_g(k_i)\) and \(i = 1, 2\). It is clear that when \(V_{g1} = V_{g2}\) equation (2.59) has no complex roots. Thus, all growth rates should be proportional to the difference \(V_{g1} - V_{g2}\).

Unlike Mikhailovskii et al. [148], we will consider two ‘strong’ wave packets of non-equal intensity, which is similar to the system of two beams with non-equal densities. Consider \(\Omega \approx q_y V_{g1} \approx q_y V_{g2}\), we can neglect 1 in equation (2.59) and then the solution obtained is

\[
\Omega = \frac{q_y (\Omega_1^2 V_{g2} + \Omega_2^2 V_{g1}) \pm i |q_y \Omega_1 \Omega_2 (V_{g2} - V_{g1})|}{\Omega_1^2 + \Omega_2^2}.
\] (2.60)

The above root is valid for not too large \(q_y\), i.e. when

\[
q_y^2 < \frac{\Omega_{1,2}^2}{V_{g1,2}^2},
\] (2.61)

and the corresponding growth rate is given by

\[
Im \Omega = \frac{|q_y \Omega_1 \Omega_2 (V_{g2} - V_{g1})|}{\Omega_1^2 + \Omega_2^2}.
\] (2.62)

When \(V_{g2} = -V_{g1} \equiv V_g\) and \(\Omega_2^2 = \Omega_1^2\), one of the four roots of equation (2.59) is purely imaginary with

\[
Im \Omega = |q_y V_g|.
\] (2.63)

This solution is also valid when the inequality given by equation (2.61) is satisfied. The maximal growth rate is attained for \(q_y \sim \Omega_1/V_{g1}\), and is

\[
Im \Omega \sim \Omega_1 \sim \Omega_2.
\] (2.64)

The roots obtained here describe the two-stream-like generation of zonal flows by MR waves.
2.4 Nonlinear Interactions of Rossby Waves and Zonal Flows in the F-Layer

In this section the ionospheric F-layer, which is located at altitude 150 − 400 km above the Earth’s surface, is considered. In this layer the dominant effect is transverse (Pedersen) conductivity, which is responsible for Joule losses, acts as viscosity and gives rise to additional, so-called inductive (magnetic) inhibition [149]. In the F-region the following inequalities are valid \( \omega_{Be} \omega_{Bi} \gg v_e v_{in}, \omega_{Bi} \gg v_{in} \) and from the expression for the conductivities (1.4) we get \( \sigma_\parallel \approx n e^2 / m_e v_e, \sigma_p \approx v_e v_{in} \sigma_\parallel / \omega_{Bi} \omega_{Be}, \) and \( \sigma_{H} \approx v_{in} \sigma_p / \omega_{Bi} \ll \sigma_p \). Further to reveal explicitly the role of the inductive inhibition we will consider rather high latitudes to ignore \( B_{0y} \) geomagnetic field component compared to \( B_{0z} \) (see equation (2.7)) and assume the magnitudes (2.14) to be constant. Under such assumptions we can rewrite the modified Charney equation (2.13) for the ionospheric F-layer in the following form [131-133, 150]

\[
\frac{\partial \Delta \psi}{\partial t} + \beta \frac{\partial \psi}{\partial x} + \Lambda \Delta \psi + J(\psi, \Delta \psi) = 0, \tag{2.65}
\]

where \( \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) is two-dimensional Laplacian and

\[
\Lambda = \frac{\sigma_p B_0^2}{\rho} = v_{in} \frac{n}{N}. \tag{2.66}
\]

Here \( \sigma_p = \sigma_\perp \) is the Pedersen conductivity, \( B_0 \) is the geomagnetic field induction, \( \rho \) is the mass density of the ionosphere gas, \( n \) and \( N \) are number densities of charged and neutral particles, \( v_{in} \) is the ion-neutral collision frequency.

In comparison to the ionospheric E-layer the wind ceases to be geostrophic because of the appearance of the wind component in the direction of the pressure gradient. Considering the low-frequency range of interest \( (d/dt \ll \Omega_{0z}) \), from equations (2.9) and (2.10) at the given pressure gradient, we obtain

\[
v_x = -\frac{1}{\rho \Lambda} \frac{\partial \rho}{\partial x}, \quad v_y = -\frac{1}{\rho \Lambda} \frac{\partial \rho}{\partial y}. \tag{2.67}
\]

Consequently, in the F-layer, the winds ultimately become ageostrophic, i.e., they blow in the direction of the pressure gradient. In this region inductive (magnetic) damping acts as a viscosity.

Now, we analyze the consequences of the nonlinear equation (2.65). This equation...
differs from the Charney equation in the sense that it contains an additional term \( \Lambda \Delta \psi \). As was already mentioned, the quantity \( \Lambda \), which is related to the geomagnetic field induction, gives rise to inductive damping and plays the role of viscosity. Indeed, from describing vortical motions equation (2.64), we may obtain the following energy equation [150]

\[
\frac{\partial \mathcal{E}}{\partial t} = -\Lambda \int (\nabla \psi)^2 \, dx \, dy,
\]

where \( \mathcal{E} \) is the kinetic energy of the Rossby vortex motions, given as

\[
\mathcal{E} = \frac{1}{2} \int (\nabla \psi)^2 \, dx \, dy > 0.
\]

Clearly, equation (2.68) describes dissipative processes, namely decrease in the kinetic energy of the vortex due to inductive damping. From equations (2.68) and (2.69), we can estimate the characteristic relaxation time of a vortex in the F-region [150]

\[
\tau = \frac{1}{\Lambda}.
\]

We can see that the time over which a vortex is damped is independent of its size.

The nonlinear equation (2.65) has a periodic solution \( \sim \exp(ik \cdot r - i\omega_k t) \) with the dispersion relation [150]

\[
\omega_k = -\beta \frac{k_x}{k^2} - i\Lambda.
\]

This indicates that in the ionospheric F-layer, the waves under discussion are Rossby waves with the frequency

\[
\omega_k = -\beta \frac{k_x}{k^2},
\]

and the damping rate

\[
\gamma = -\Lambda,
\]

which coincides with estimate (2.70).

To describe the evolution of the coupled system (Rossby waves plus zonal flow) we will use the modified parametric approach used in Sec. 2.3 and split the perturbation of the stream
function in equation (2.64) into three components (2.22). Note that in contrast to the previous
Sec. 2.3, according to the dispersion relation (2.71), the frequency $\omega_k$ should be assumed
complex in equation (2.23). Then instead of equation (2.27) we get

$$(i\Omega - \Lambda)\tilde{\psi}_0 = \sum_k k_x [2k_y (\tilde{\psi}_+ \tilde{\psi}_- + \tilde{\psi}_- \tilde{\psi}_+) + q_y (\tilde{\psi}_+ \tilde{\psi}_- - \tilde{\psi}_- \tilde{\psi}_+)].$$

(2.74)

Analogously, for the side-band amplitudes we obtain equation (2.29) with the following
modification in the denominator

$$D_\pm = \omega_\pm \pm \beta \frac{k_x}{k_\perp^2} + i\Lambda.$$ 

(2.75)

Assuming $q_y/k_\perp \sim \Omega/\omega_k \ll 1$, and $|\Lambda| \sim |q_y^2|$, we can represent

$$D_\pm \approx D^{(1)} + D^{(2)}_\pm,$$

(2.76)

where

$$D^{(1)} = \Omega - q_y V_g(k),$$

(2.77)

$$D^{(2)}_\pm = \pm \frac{1}{2} q_y^2 V'_g \mp i\Lambda + i\Lambda.$$ 

(2.78)

Here

$$V_g(k) = \frac{\partial \omega}{\partial k_y} = \frac{2k_x k_y \beta}{k_\perp^4},$$

(2.79)

is the group velocity of the primary Rossby waves modes and

$$V'_g(k) = \frac{\partial^2 \omega}{\partial k_y^2} = 2k_x \beta \frac{k_x^2 - 4k_y^2}{k_\perp^6}.$$ 

(2.80)

If we carry out analogous calculations made in Sec. 2.3 we obtain the following zonal flow
dispersion relation

$$(\Omega + i\Lambda) = - \sum_k q_y k_x^2 \frac{(q_y V'_g k_\perp^4 \Omega)}{2} \frac{k_x \beta}{(\Omega - q_y V_g)^2}.$$ 

(2.81)
Compared with equation (2.32) now we have the cubic equation with respect to \( \Omega \).

In case of the monochromatic waves we get from equation (2.81):

\[
(\Omega + i\Lambda)(\Omega - q_y V_g) + \frac{1}{2} \Omega k_x q_y^2 k_y^4 \frac{1}{\beta} V_g' I_k + 2i k_x^2 k_y \Lambda q_y I_k = 0.
\]

(2.82)

Let us consider the case when \( \Omega - q_y V_g = i\Gamma \), and suppose \( q_y V_g \gg \Lambda, \Gamma \). Then from equation (2.82) we find

\[
\Gamma^2 = k_x I_k \left( \frac{1}{2} \frac{q_y^2 k_y^4}{\beta} V_g' + \frac{2i k_x k_y \Lambda}{V_g} \right).
\]

(2.83)

Note that when the dissipation \( \Lambda = 0 \) we get the previous growth rate (2.38). To distinguish the importance of magnetic inhibition \( \Lambda \) we will consider the case when \( V_g' \) is small (see equation (2.80)). Then we can easily find the zonal flow growth rate as

\[
\Gamma \approx \left( \frac{k_x \Lambda}{\beta} \right)^{1/2} k_x^2 |\tilde{\Psi}_+|.
\]

(2.84)

Two distinctive properties should be emphasized here: 1) In contrast to previous investigations (see equation (2.38)) the zonal flow growth rate does not depend on small parameter \( q_y \) and no instability condition is required; 2) Even at the small value of the pumping intensity \( I_k \) the process of spectral energy transferring from small-scale Rossby waves to large-scale enhanced zonal flows (inverse cascade) in the Earth’s ionospheric F-layer always takes place. Notice that the analogous problem of zonal flow generation in the dissipative medium was undertaken in [151, 152] but the role of dissipation was skipped.

### 2.5 Conclusions

In Sec. 2.3 we have investigated the influence of non-monochromaticity on low-frequency, large-scale zonal flow nonlinear generation by small-scale \((k r_R \gg 1)\) MR primary modes in the E-region of the Earth’s ionosphere. The spectral analysis was carried out by using the modified parametric approach. Correspondingly, the interaction between pump waves having two side-band modes and a sheared zonal flow is observed. The driving forces (the so called Reynolds stresses) in the evolution equation of zonal flows are expressed in terms of summation which is over the primary modes spectrum of (see (2.27)). We have made such a generalization and thereby arrive at the zonal flow dispersion equation (2.32) for an arbitrary spectrum of the MR waves.
It is shown in our investigations (see (2.32) and (2.33)) that the zonal flow generation condition by MR modes in the ionospheric E-layer is strongly attached with the sign of $\partial^2 \omega / \partial k_x^2$, where $\omega$ is the frequency of primary modes. Under the Lighthill instability condition given by (2.39), Gaussian wave packet was used to analyze the mechanism of zonal flow generation (see (2.42)). In these investigations, two cases i.e., sufficiently small spectrum broadening for zonal flow generation and arbitrary spectrum broadening were discussed separately. The value of maximum growth for both monochromatic as well non-monochromatic wave packets was obtained (see (2.57)). Analysis supports that the instability is of the hydrodynamic type in case of monochromatic wave packet (see (2.37)), which is similar to that studied by Lawrence and Jarvis [118] for drift monochromatic wave packet. It has been shown that wave packet broadening may be neglected under condition (see (2.49)). But small broadening can also play a role to decrease the growth rate (see (2.48)). It is also shown that any increase in the broadening can cause the transform of instability into the resonant type described by (2.54). The band width is given by (2.56), which in turn gives the maximum value of growth rate (see (2.57)).

In the case when Lighthill stability criterion prohibits the generation of zonal flow by MR modes (inverse inequality of (2.39)), the investigation should be continued by the elucidation of different types of zonal flow instabilities. To this end we examined the more complicated situation of a two-humped wave packet of MR waves, which in the simplest case can be realized as two pump waves. Considering the more general case of two wave packets of non-equal intensity (see (2.62), (2.64)) we have obtained a new class of instability to add to the two-stream-like instability in the Earth’s ionosphere.

In our investigations, we obtain the maximum growth rate of the zonal flow generation and it is of the order of (see (2.57))

$$\gamma \approx |\Gamma| \sim |q_y k_{x0} r_R^3 (\alpha + \beta) \hat{\psi}_+|.$$

(2.85)

Here the stream function $\hat{\psi}_+$ of pump modes is normalized by $v_R r_R$, where

$$v_R = - (\alpha + \beta) r_R^2$$

(2.86)

is the modified Rossby velocity and $r_R$ is the modified Rossby radius, respectively [134]. For the regime considered here $q_y r_R \approx 1, k_{x0} r_R \gg 1$ and for typical parameters of the ionosphere $(\alpha + \beta) \approx 10^{-11} \text{ m}^{-1} \text{ s}^{-1}, k_{x0} r_R \approx 10, \hat{\psi}_+ \approx 10^{-2}, r_R \approx 10^6 \text{ m}$ we obtain $\gamma \approx 10^{-6} \text{ s}^{-1}$. This estimate coincides with the existing observations and our investigation provides an essential nonlinear mechanism for the transfer of spectral energy from short-scale MR waves to
long-scale enhanced zonal flows in the Earth’s ionosphere. The distinctive property of this instability is that it grows because of MR waves exist in the cone bounded by the caustics when $V'_{g}/\omega_{k} = 0$. In this situation caustic shadow develops in the spectrum of the MR waves.

Rasmussen et al. [153] demonstrated an experiment to explain the generation of zonal flow in a simple rotating fluid. It seems reasonable to simulate two-stream-like generation of zonal flows to confirm the theory of Rossby wave excitation provided in Sec. 2.3.

In Sec. 2.4 nonlinear interaction mechanism of Rossby waves and zonal flows in the dissipative ionospheric F-layer is investigated. Dissipation arises due to Pedersen conductivity acting as an inductive (magnetic) inhibition and leads to Joule damping of Rossby waves (see equation (2.71)). For the ionospheric F-layer typical parameters $n/N = 10^{-5} - 10^{-3}$, and $\nu_{in} = 5 \cdot (10^{-2} - 10^{-1})$s$^{-1}$ (see Table 1) for the damping rate of Rossby waves (see equation (2.66) and (2.71)) we get the following range $\Lambda = 5 \cdot (10^{-7} - 10^{-4})$s$^{-1}$, which is plausible for the propagating of the Rossby waves having the oscillation frequency in the ULF range $(10^{-6} - 10^{-4})$s$^{-1}$. On the other hand, suppose Rossby waves arose till nonlinear level under certain conditions in the dissipative F-layer. In this case one may state the following problem: how the spectral energy of pumping Rossby waves will transfer to small-scale direction? In Sec. 2.4, for the first time we initiated to investigate the problem of generation of zonal flow by Rossby waves in the dissipative ionospheric F-layer. It is shown that in contrast to the investigation carried out for the ionospheric E-layer in Sec. 2.3 the zonal flow growth rate does not depend on small wave vector component of zonal flow mode (see Eq. (2.84)), needs no instability condition and the spectral energy transferring (inverse cascade) process unconditionally takes place. To estimate numerically the appropriate growth rate we recall the validity condition of equation (2.84), $\Lambda \sim q_{f}^{2}$, which may be achieved for sufficiently high latitudes when $\nu_{in} \rightarrow 0$ sharply.

Then under the ionospheric parameters mentioned above we get $\Gamma \sim 10^{-8}$s$^{-1}$. 
Chapter 3

Dynamics of Large-Scale Drift Vortical Structures in Electron-Positron-Ion Plasmas

3.1 Introduction

To grasp the elementary physics of EPI plasmas consisting of electrons, positrons and ions, a lot of theoretical research has been carried out [42, 154-232] considering “multifluid theory” which is applied to both astrophysical and laboratory plasmas. Investigations reveal that the nonlinear dynamics of waves in plasma composed by electrons and positive ions is quite different from those in a plasma containing positrons also [155].

Development of nonlinear theory of waves led to the emergence of new concepts- solitary wave, solitary vortex, soliton, filament, convective cell, jet, double layers, shock, and zonal flow, which are intensively discussed for the past years in EPI plasma. Study of such self-organized structural formations has disclosed the macroscopic feature of plasmas not only in the laboratory but also in space. Most papers were devoted to the investigation of ion-acoustic solitons in EPI plasma under different regimes [158, 161, 163, 170, 176, 180, 195, 201-203, 205, 210, 213-215, 221, 227, 229, 232]. Ion-acoustic nonlinear periodic waves, namely ion-acoustic cnoidal waves have been also studied [228]. A finite amplitude theory for ion-acoustic solitary waves and double layers is presented in Ref. 216. Mushtaq and Shah [188, 191] studied the nonlinear propagation of two-dimensional magnetosonic waves and found the magnetosonic solitons. Envelope solitons of electromagnetic waves in EPI plasma were observed by Rizzato [155] and Berezhiani et al. [157]. Kourakis et al. [199] showed that localized envelope solitons and holes occur in EPI plasma. Ion-acoustic drift solitons in EPI plasma were studied by Mushtaq [204]. Solitons on electrostatic acoustic-like lower mode and Langmuir-like optic-type upper one were found in Ref. [197]. In Ref. [193] low-frequency solitons are found in EPI plasma. Modulational interactions of the electromagnetic waves and electron-acoustic waves with possible formation of solitons are studied in Ref. [162]. Soliton solutions and double layers on electrostatic electron-acoustic waves in EPI plasma were obtained in Ref. [198]. Formation of
double layers, associated with kinetic Alfven waves in magnetized EPI plasma, has been investigated in Ref. [167]. Formation of light bullets and solitons was discussed in Ref. [179]. Berezhiani and Mahajan [42, 159] showed that EPI plasma can sustain stable large-amplitude relativistic solitons. Formation of relativistic electromagnetic envelop solitons was studied in Ref. [166]. Kinetic relativistic solitons are found in EPI plasma [187]. Appropriate solitons for perpendicular magnetosonic waves were found in Ref. [169]. Development of different structures like solitons and shocks in the inhomogeneous EPI plasma are discussed in Ref. [196]. Ion acoustic shock and solitary waves were considered in Refs. [206, 218, 220, 223, 226, 230]. Nonlinear propagation of low-frequency electromagnetic shocks and fast magnetoacoustic waves are studied by Masood et al. [224]. Two-stream instabilities in EPI plasma under different limits were investigated in Refs. [194, 219]. Solitary wave solutions of nonlinear Schrödinger equation, namely dark and bright envelope solitons are found in EPI plasma [225]. Solitary structures in EPI plasma as the solutions of Zakharov-Kuznetsov equation are also found [212, 217]. Nonlinear excitations (solitons and double layers) of kinetic Alfven waves in low but finite-\(\beta\) EPI plasma is presented in Ref. [222]. Low-frequency electrostatic ion waves in EPI plasmas were also focused. The nonlinear amplitude modulation of these waves in collisionless magnetized EPI plasma is investigated in Ref. [209]. The “Faraday rotation” in a magnetized EPI is studied in Ref. [190]. Acceleration of positrons by oblique magnetosonic shock waves in EPI plasma was investigated by Hasegawa and Ohsawa [175, 183]. The structure and particle acceleration characteristics of relativistic, transverse magnetosonic shocks were studied in Ref. [156] in case of EPI plasma.

Nonlinear solitary structures on shear Alfven wave in EPI plasma were studied in Ref. [177]. Gogoi and Goswami [172] studied the drift wave vortices in a plasma contains both positive and negative ions and additional component electrons. Shear flow driven solitary vortical structures in inhomogeneous EPI plasma were studied in Ref. [192]. Vortex of electrostatic as well as electromagnetic nature in ideal EPI plasma has been focused in Ref. [178]. Jammalamadaka et al. [160] revealed vortical formations for low-frequency electrostatic and electromagnetic disturbances in EPI plasma. In Ref. [165] it is disclosed that the “finite Larmor radius” effect of ions overheads for the nonlinear wave steepening. Quadrupolar vortex solutions of nonlinear equations for EPI plasma with shear flow were found in Ref. [171]. In Ref. [181] the nonlinear behavior not only of ion-acoustic but also electrostatic drift waves in the presence of ion sheared flow in EPI plasma when ions are considered to be cold was investigated and monopolar vortex was found. Localized solutions (density dips) on shear Alfven waves were found in Ref. [174]. In Ref. [184] it was shown that low-frequency
electromagnetic drift waves might give rise to dipolar vortices in EPI plasma. Linear and nonlinear drift-Alfven waves are discussed analytically in nonhomogeneous EPI plasmas in Ref. [231]. Phase-space holes are found in a relativistically hot EPI plasma in Ref. [189]. In Ref. [164], it was shown that localized structures (light bullets) can exist. Vortical structures on coupled electrostatical drift and ion-acoustic modes were found by Shukla et al. [182] in magnetized EPI plasma with sheared ion flow. Current gradient driven Alfven dipolar vortices in EPI plasma were studied by Haque and Saleem [185]. Ion-acoustic drift-wave instability due to ion sheared-flow with quadrupolar vortices in magneto nonuniform EPI plasma were considered by Mirza and Azeem [186]. Pokhotelov et al. [168] studied the nonlinear behavior of drift-Alfven modes in inhomogeneous electron-positron plasma with a little concentration of massive ions and two-dimensional dipolar vortices were found. Haque and Saleem [180] investigated ion-acoustic as well as drift waves in EPI plasma and obtained dipolar vortices. Honda and Honda showed a framework for transportation of astrophysical EPI jets [173].

In this chapter we consider the dynamics of nonlinear propagation of electrostatic drift waves leading to the formation of large-scale vortical structures in EPI plasmas.

The study of low-frequency drift waves has attracted a great attention because of their applications to several laboratory, space, and astrophysical environments. The anomalous transport of a plasma perpendicular to a magnetic field was described by these waves. Zonal flows, generated by drift modes, also play a major role in controlling the level of anomalous transport in magnetic confinement systems. Different aspects of electrostatic drift waves dynamics [178, 180, 181, 192, 196] are discussed in EPI plasma, where electron and positron temperatures have been supposed to be equal. As well, the modified Hasegawa-Mima (HM) equation obtained is not correct. In Refs. [178, 192] solitary vortical structures are also found as a solution of the corresponding nonlinear HM equation. In these papers it is assumed that the characteristic size of the considered nonlinear structures is less or of the order of the ion Larmor radius at the plasma electron temperature. Such structures may be described in the framework of classical HM equation containing only vectorial (Jacobian) nonlinearity. In other words classical HM equation only describes small-scale structures. Vortices in the electrostatic limit in EPI plasma under high magnetic field were investigated by Shukla et. al. [182] via the HM equation with a vector nonlinearity and vortex solutions of different kinds were described. Extension to the large-scale electrostatic drift nonlinear structures “(having dimensions larger than the characteristic Larmor radius of plasma ions)” was discussed by Nezlin and Chernikov[91]. It was shown that such structures are described in the framework of the generalized HM equation containing vectorial and “Korteweg-de Vries (KdV)” type scalar nonlinearity. When wave
dispersion is mutually compensated by both scalar as well as vector nonlinearities, then solitary structures are formed. As a result, in the general case, a solitary structure must appear as anisotropic and it is a superposition of monopolar and dipolar vortices.

In Sec. 3.2 generalized HM equation for the electrostatic drift waves valid for arbitrary sizes of vortical structures is obtained. In addition temperatures of electrons and positrons are assumed to be arbitrary. In Sec. 3.3 the nonlinear dynamics of large-scale solitary vortical structures in EPI plasma is analyzed. Discussion of the results obtained is given in Sec. 3.4.

### 3.2 Generalized Hasegawa-Mima Equation for Electrostatic Drift Waves in EPI Plasmas

We consider, in the electrostatic approximation, our system a quasineutral EPI plasma, its motion is assumed to be quasi-two-dimensional. Suppose \( \varphi(t, x, y) \) is a plasma potential. Let us consider that along the \( z \)-axis plasma is uniform, and external magnetic field \( B_0 \) is also taken parallel to \( z \)-axis.

Considering for both electrons and positrons the unperturbed plasma densities \( n_{e0}(x) \), \( n_{p0}(x) \) and corresponding temperatures \( T_{e0}(x) \), \( T_{p0}(x) \), are inhomogeneous and assumed to decrease tediously along the \( x \)-axis. The ions are supposed to be “cold” and the quasineutrality condition in equilibrium

\[
n_{e0}(x) = Zn_{i0}(x) + n_{p0}(x)
\]

is fulfilled, where \( Z \) is the charge number of positive ions.

Let us assume that in this system the plasma density perturbation arises (corresponding to the plasma-potential perturbation \( \varphi \)), which excites a drift wave. Plasma motion in the \((x, y)\) plane is supposed to be very slow, so that electrons and positrons are fast moving along the magnetic field, therefore Boltzmann equilibrium is followed. Then from the plasma quasineutrality condition (3.1), the ion density is defined by the relationship

\[
Zn_i = n_{e0}(x)e^{e\varphi/T_{e}(x)} - n_{p0}(x)e^{-e\varphi/T_{p}(x)}.
\]

We start from equation of motion for the ion in the plasma

\[
v_t + (v \cdot \nabla)v = v \times \omega_B - \frac{Ze}{M} \nabla \varphi,
\]
where $\omega_{\text{Bi}} = \frac{Ze B_0}{M}$ is the cyclotron frequency of ions, $M$ is the mass of ion, $Ze$ is the charge of ion, and the velocity vector $\mathbf{v}(v_x, v_y, 0)$, where $v_x$ and $v_y$ are components of velocity.

Due to quasi-two-dimensional motion, there will be no velocity perturbation along the $z$-direction. Using the continuity equation for ions with equation (3.3), we get the following equation of conservation of the so-called potential vorticity, which is defined as

$$\frac{d}{dt} \left( \frac{\zeta + \omega_{\text{Bi}}}{n} \right) = 0,$$

where $d/dt = \partial_t + v_x \partial_x + v_y \partial_y$ and the vorticity ($\zeta = \nabla \times \mathbf{v})_z = v_x - v_y$. Here the lower indices refer to differentiation with respect to $x$, $y$, and $t$, the cyclotron frequency $\omega_{\text{Bi}} = e_z \cdot \mathbf{\omega}_{\text{Bi}}$ and the unit-vector along the $z$ direction is $e_z$.

The Lorentz force in the Euler equation (3.4) is large as compared to the inertia terms, in this situation the drift wave regime occurs in plasma, such an approximation implies the existence of a small parameter,

$$\frac{\omega}{\omega_{\text{Bi}}} \ll 1,$$  \hspace{1cm} (3.5)

where $\omega$ is the characteristic frequency of the perturbation. We also take into account the polarization drift (which is a higher order term) and following equation (3.5) we get for the ion velocity:

$$\mathbf{v}_\perp = -\frac{Ze}{M\omega_{\text{Bi}}} \nabla \varphi \times e_z - \frac{Ze}{M\omega_{\text{Bi}}} \left( \frac{\partial}{\partial t} + \frac{Ze}{M\omega_{\text{Bi}}} e_z \times \nabla_{\perp} \varphi \cdot \nabla_\perp \right) \nabla_\perp \varphi.$$  \hspace{1cm} (3.6)

Here the subscript $\perp$ denotes the plane transverse to magnetic field.

Using equations (3.2) and (3.6) into equation (3.4) we obtain the following generalized (containing both vector and scalar nonlinearities) HM equation valid for arbitrary sizes of vortical structures in EPI plasma:
\[-r_L^2 \left( 1 - \frac{n_{p0}}{n_{e0}} \right) \frac{\partial \Delta_{\perp} \varphi}{\partial t} + \frac{1}{Z} \left( 1 + \frac{n_{p0}}{n_{e0}} \frac{T_e}{T_p} \right) \frac{\partial \varphi}{\partial t} - \frac{\partial^2 \varphi}{\partial t^2} - \frac{r_L^2 \omega_{Bi}}{n_{e0}} \left( \frac{n_{e0}'}{n_{e0}} - \frac{n_{p0}'}{n_{p0}} \right) \frac{\partial \varphi}{\partial y} - \frac{r_L^2}{n_{e0}} \frac{\partial^2 \varphi}{\partial t \partial x} \]

\[-r_L^2 \frac{e}{T_e} \left( 1 + \frac{n_{p0}}{n_{e0}} \frac{T_e}{T_p} \right) \frac{\partial \Delta_{\perp} \varphi}{\partial t} - \frac{r_L^4 \omega_{Bi}}{T_e} \left( 1 - \frac{n_{p0}}{n_{e0}} \right) J(\varphi, \Delta_{\perp} \varphi) + \frac{e}{2Z} \left( \frac{n_{p0}}{n_{e0}} \frac{T_e}{T_p^2} \right) \frac{\partial^2 \varphi}{\partial t^2} - \frac{r_L^2 \omega_{Bi}}{n_{e0}} \Delta_{\perp} \varphi \frac{\partial \varphi}{\partial y} - \frac{r_L^2}{2Z} \frac{e}{T_e} \left( 1 + \frac{n_{p0}}{n_{e0}} \frac{T_e}{T_p} \right) \frac{\partial (\nabla \cdot \varphi)^2}{\partial t} \]

\[-r_L^4 \frac{Z \omega_{Bi}}{T_e} \frac{n_{e0}'}{n_{e0}} \frac{\partial \varphi}{\partial x} - \frac{e}{2M \omega_{Bi}} \left( \frac{n_{e0}'}{n_{e0}} + \frac{n_{p0}'}{n_{e0}} \frac{T_e}{T_p} - \frac{T_e'}{n_{e0}'} \right) \frac{\partial \varphi}{\partial y} = 0. \quad (3.7)\]

In obtaining the above equation the inequality (3.5) and condition $e \varphi / T_e \ll 1$ were used to retain the dominant nonlinear terms. In Eq. (3.7) \( J(a, b) = \partial a / \partial x \partial b / \partial y - \partial a / \partial y \partial b / \partial x \) is the Jacobian (vector nonlinearity), the operator \( \Delta_{\perp} = \nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 \), \( n_{L} = (T_e / M \omega_{Bi})^{1/2} \) is the ion (sonic) Larmor radius at electron temperature \( T_e \), and the derivative with respect to the \( x \) variable is assigned by prime.

In the linear regime equation (3.7) reduces to

\[
\frac{Z^2}{M \omega_{Bi}} \frac{n_{i0}}{n_{e0}} \frac{\partial \varphi}{\partial t} + \frac{\omega_{Bi}}{T_e} \left( 1 + \frac{n_{p0}}{n_{e0}} \frac{T_e}{T_p} \right) \frac{\partial \varphi}{\partial t} + \frac{Z^2 n_{i0}'}{M n_{e0}} \frac{\partial \varphi}{\partial y} + \frac{Z^2 n_{i0}'}{M \omega_{Bi}} n_{e0} \frac{\partial^2 \varphi}{\partial t \partial x} = 0. \quad (3.8)\]

The last term (which is of the order of $\omega / \omega_{Bi}$) is kept in order to obtain the spatial structure of the electrostatic drift waves. Indeed, if we introduce the inhomogeneity length \( 1/L = \left| n_{i0}' / n_{i0} \right| = \left| n_{e0}' - n_{p0}' \right| / (n_{e0} - n_{p0}) \) and look for the propagation of drift plane waves of the form

\[
\varphi \sim e^{\mp x / 2L} e^{ikx + ik_y y - i \omega_k t}, \quad \text{(3.9)}
\]

we get for the electrostatic drift frequency

\[
\omega_k = \frac{\mp k_y v_*}{\beta + r_L^2 \left( k^2 + \frac{1}{4l^2} \right)}, \quad \text{(3.10)}
\]

Here \( v_* = T_e / M \omega_{Bi} L \) is the drift velocity.
\[ \beta = n_{e0} \left( 1 + \frac{n_{p0}}{n_{e0}} \frac{T_e}{T_p} \right) / Z (n_{e0} - n_{p0}), \] 

(3.11)

\[ k_{1x}^2 = k_{2x}^2 + k_{3x}^2 \]

and \( \mp \) sign in equations (3.9) and (3.10) corresponds to positive and negative signs of \( \frac{\dot{n}_{i0}}{n_{i0}} \). We emphasize that the latter ratio can change the sign depending on the difference between \( n'_{e0} \) and \( n'_{p0} \). Thus the drift wave potential in EPI plasma is spatially increasing or decreasing in the direction of the inhomogeneity. The drift wave structure given in equations (3.9) and (3.10) is similar to that of acoustic-gravity waves propagating in an inhomogeneous atmosphere embedded in a gravitational field [233].

It is seen from equation (3.7) that the classical HM equation (containing only vector nonlinearity) can be isolated, which has the form

\[ - \frac{Ze}{M \omega_{Bi}} \left( 1 - \frac{n_{p0}}{n_{e0}} \right) \frac{\partial \Delta_1 \varphi}{\partial t} + \frac{e \omega_{Bi}}{T_e} \left( 1 + \frac{n_{p0}}{n_{e0}} \frac{T_e}{T_p} \right) \frac{\partial \varphi}{\partial t} - \frac{Ze}{M} \frac{n'_{e0} - n'_{p0}}{n_{e0}} \frac{\partial \varphi}{\partial y} \]

\[ - \frac{Ze}{M \omega_{Bi}} \frac{n'_{e0} - n'_{p0}}{n_{e0}} \frac{\partial^2 \varphi}{\partial t \partial x} - \frac{Z^2 e^2}{M^2 \omega_{Bi}^2} \left( 1 - \frac{n_{p0}}{n_{e0}} \right) \frac{\partial f(\varphi, \Delta_1 \varphi)}{\partial \varphi} = 0. \]

(3.12)

The HM equation obtained above is valid if the following inequalities are fulfilled

\[ \frac{a^2}{\eta_L^2} \ll \frac{\omega_{Bi}}{\omega}, \frac{L}{a}; \quad \frac{a^4}{\eta_L^4} \ll \frac{\omega_{Bi}}{\omega}; \quad \frac{L}{a} \gg 1. \]

(3.13)

Here \( a \) is the perpendicular size of the structure and \( L \) is the characteristic scale of the inhomogeneity. Thus, the classical HM equation validity is confirmed only when the ratio of the characteristic structure size and the ion (sonic) Larmor radius satisfies the inequalities (3.13) and the smallness of this ratio as given in Ref. [91] need not to be fulfilled. Thus the classical HM equation, deals only with “small-scale” structures. The classical HM equation predicts the existence of a solitary structure in the form of a dipolar vortex that is a cyclonic-anticyclonic pair. Solitary monopolar vortices (cyclones, anticyclones) are absent in the framework of the classical HM equation.

### 3.3 Nonlinear Dynamics of Large-Scale Solitary Vortical Structures

We now look for the intermediate-scale vortical structures, when
In this limit equation (3.7) reduces to the following generalized (containing both vector and scalar nonlinearities) HM equation for the potential perturbation $\varphi$ in EPI plasma:

$$\left(\frac{n_{e0}}{n_{i0}} + \frac{n_{p0}}{n_{i0}} \frac{T_e}{T_p}\right) \frac{\partial \varphi}{\partial t} - Z^2 r_L^2 \frac{\partial \nabla_\perp \varphi}{\partial t} - Z^2 r_L^2 \omega_B I_\perp \frac{n_i'}{n_{i0}} \frac{\partial \varphi}{\partial y} - Z^2 r_L^2 \frac{n_i'}{n_{i0}} \frac{\partial^2 \varphi}{\partial t \partial x}$$

$$+ \frac{e}{2n_{i0}} \left(\frac{n_{e0}}{T_e} - \frac{n_{p0}}{T_p} \right) \frac{\partial \varphi^2}{\partial t} - \frac{Z^3 e \omega_B I_\perp^4}{T_e} f(\varphi, \Delta_\perp \varphi) - \frac{Ze \omega_B I_\perp^2}{2HT_e} \frac{\partial \varphi^2}{\partial y} = 0. \quad (3.15)$$

Here we have introduced the inhomogeneity length $H$ defined as

$$\frac{1}{H} = \frac{1}{n_{i0}} \left(\frac{n_{e0}'}{T_e} + \frac{n_{p0}}{T_p} \right) \frac{T_e'}{T_e}$$

$$= \frac{T_e}{n_{i0}} \left(\frac{n_{e0}}{T_e} + \frac{n_{p0}}{T_p} \right) \frac{T_e'}{T_p}. \quad (3.16)$$

Generalized HM equation (3.15) contains three nonlinearities: the first scalar one $\propto \partial \varphi^2/\partial t$ originates from $\partial n_i/\partial t$ in equation (3.4) when expanding over the powers of $e\varphi/T$ (see equation (3.2)). This nonlinearity as it seen from equation (3.19) (see further) creates large-scale monopole vortical structures and plays an essential role in the process of zonal flow generation (see subsection 4.2.6 in chapter 4). As to the last scalar nonlinearity in equation (3.15), $\propto \partial \varphi^2/\partial y$, it is analogous to the KdV nonlinearity. The KdV scalar and vector nonlinearities are of the same order when

$$\frac{a^2}{r_L^2} \sim \frac{|H|}{a} \gg 1. \quad (3.17)$$

Analogous to equation (3.15) a more simplified equation obtained in Ref. [178] with the expression for $1/H$ analogous to equation (3.16) for equal electron and positron temperatures is not correct. Thus in the process of self-organization of large-scale vortical solitary structures, nonlinearities, scalar and vector, taking place are important. Scalar nonlinearity affects the large-scale structures. It is necessary to explain the process in which solitary drift vortical structures appear because of the competition of dispersion and nonlinearity [91]. The issue is as follows: In case of KdV type solitons, wave packet dispersion is balanced by its nonlinear steepening. This equilibrium, displays a relationship: The larger the amplitude (stronger
nonlinearity), the smaller the width (stronger dispersion). But, the situation is more complicated
when it was discovered in the model experiments [234] that the characteristic size of the
structures was independent on the amplitude. When a nonlinear dynamic equation (e.g. obtained
here in Eq. (3.15)) involves two types of nonlinearities (KdV scalar and vector) the new class of
solitary structures was revealed [91]. Especially, if structure amplitude is very large, means the
scalar nonlinearity beats the dispersion. Then the dispersion is compensated by the vector
nonlinearity. In this situation structure does not undergo unlimited steepening. And the vector
nonlinearity opposes the scalar one. Now, considering the case when at a given structure size,
the structure amplitude becomes very small. Then, the wave dispersion exceeds the scalar
nonlinearity, both nonlinearities oppose the dispersion together. Thus, this process linked with
the compensation of wave dispersion by both nonlinearities. As a result, a solitary structure is in
general intrinsically anisotropic and contains a circular (monopolar) vortex superimposed on a
dipole perturbation. When the KdV scalar nonlinearity overcomes the vector nonlinearity (in
case of sufficiently large sizes, see Eq. (3.17)) only monopolar structures exist. Solitary
structures of this kind were first of all displayed in Ref. [234]. Numerical calculations of the
generalized HM equation containing both scalar and vector nonlinearities were performed in
Ref. [235]. We have shown here that a large-scale dipole vortex transforms into two monopoles
where a vortex of one polarity is long lived as compared to the vortex of the opposite polarity.

In the case of large-scale structures with

$$\frac{\omega}{\omega_{Bi}} \gg r_L^4, \quad \frac{r_L^2 a}{a^2 |H|} \ll 1,$$

(3.18)

we may be convinced that the first scalar nonlinear term $\propto \partial^2 \phi / \partial t$ in equation (3.15) is more
important than others and we get the following equation with only the scalar nonlinearity:

$$\left(\frac{n_{e0}}{n_{i0}} + \frac{n_{p0}}{n_{i0}} \frac{T_e}{T_p}\right) \frac{\partial \phi}{\partial t} - Z^2 r_L^2 \frac{\partial \nabla_L^2 \phi}{\partial t} - Z^2 r_L^2 \frac{n_{i0}}{n_{i0}} \frac{\partial^2 \phi}{\partial t \partial x} - Z^2 r_L^2 \omega_{Bi} \frac{n_{i0}}{n_{i0}} \frac{\partial \phi}{\partial y}$$

$$\cdot \left[\frac{e}{2n_{i0}} \left(\frac{n_{e0}}{T_e} - \frac{n_{e0}}{T_p}\right) \frac{\partial^2 \phi}{\partial t} \right] = 0.$$

(3.19)

According to the above mentioned discussion, this equation with only scalar nonlinearity
of $\sim \partial^2 \phi / \partial t$ describes large-scale solitary monopole vortical structures. Further in equation
(3.19) we ignore the term $-Z^2 r_L^2 \frac{n_{i0}}{n_{i0}} \frac{\partial \phi}{\partial t \partial x}$ which is small as compared with the next
term by $\omega / \omega_{Bi} \ll 1$ (note that the ignored term plays essential role in the case of zonal flow
generation problem investigated in chapter 4.). Suppose $v_D$ is the drifts velocity (constant) of
the structure, then considering the stationary case of propagation, one may obtain \( \partial \varphi / \partial t \rightarrow -v_D \partial \varphi / \partial y \). In this way we may easily obtain the following equation:

\[
\nabla_1^2 \varphi = \left[ \frac{1}{Z^2 r_L^2} \left( n_{e0} + \frac{n_{p0} T_e}{n_{i0} T_p} \right) + \omega_{Bi} \frac{n_{i0}^'}{n_{i0} v_D} \right] \varphi + \frac{e}{2n_{i0} Z^2 r_L^2} \left( \frac{n_{e0} - n_{p0} T_e}{T_e} - \frac{n_{p0} T_e}{T_p^2} \right) \varphi^2. \tag{3.20}
\]

Approximate analytical solution and numerical simulation of equation (3.20) confirm the solution in the form of monopole vortical structures as it has the same form as equation (30) of Ref. [172].

### 3.4 Conclusions

In the chapter 3 we have discussed a new self-organization mechanism of formation of large-scale electrostatic drift vortical structures in EPI plasmas based on the competition between scalar and vector nonlinearities. Solitary structure thus developed is intrinsically anisotropic and is the composition of monopole vortex and dipole perturbation. Temperature inhomogeneity of electrons and positrons is taken into account. A new class of corresponding differential equations (3.15) and (3.19) with appropriate validity conditions (3.14), (3.17) and (3.18) is obtained. The generalized HM (equation (3.7)) valid for arbitrary sizes of structures is obtained. We have shown that due to the existence of positrons in the plasma, the sign of the derivative \( Z n_{i0}^'(x) = n_{e0}^'(x) - n_{p0}^'(x) \) may change which in turn enriches the class of solutions of the generalized HM equation. The new spatial structure given by equation (3.9) for the drift waves with dispersion relation given by equation (3.10) is obtained. Finally we note that the range of validity of the classical HM equation is revised and is given by equation (3.13).

The results we have introduced may be valuable to probe the properties of EPI plasmas in laboratories [47, 49, 55]. These results also have a connection with the localized nonlinear electrostatic structures [7]. After the identification of the drift modes and corresponding large-scale structures described in the given chapter, their eventual observation should be used in the diagnostic of EPI plasma.
Chapter 4

Generation of Zonal Flows by Electrostatic Drift Waves in Electron-Positron-Ion Plasmas

4.1 Introduction

Introduction given in section 3.1 presented a comprehensive review of theoretical publications on the nonlinear solitary structures in EPI plasmas. One can see that another very important nonlinear process, viz., the formation of zonal flows has not been reported so far. In the present chapter for the first time we will investigate the generation of zonal flows by electrostatic drift waves in EPI plasmas (Note that on this analogous problem, only one publication [236] appeared after our investigations, in which some of our results are repeated).

Fujisawa et al. [237-239] described the experimental proof of zonal flows in a plasma that is toroidally confined. The tokamak experiments demonstrated on zonal flows are intensively studied [98-100]. It is admitted that zonal flows are basic component almost in all regimes of drift wave turbulence, now this problem is presented as “drift wave-zonal flow turbulence” [98, 99]. Remarkable experimental findings were indicated recently by Arakawa et al. [240] concerning dynamic interaction between the solitary drift wave structures and zonal flows. The role of zonal structures in the dynamics of small-scale turbulence is consistent especially, in saturation. Zonal structures with great shear in velocity are a source of the suppression of the anomalous transport in magnetic systems. Because of the mentioned reason, recently, zonal flows driven by drift-type turbulence have been well probed theoretically [139, 145, 146, 241-255].

Another reason for the investigations carried out in the given chapter is the results obtained by us in the previous chapter 3. There it was shown that in EPI plasma three different types of nonlinear solitary vortical structures on the electrostatic drift waves can appear: (1) Short-scale vortical structures of dipole type stipulated by the vector nonlinearity only (see equation (3.12)); (2) Intermediate-scale vortical structures of mixed type (consisting of dipole and monopole structures) stipulated by the competition between vector and two different scalar
nonlinearities. One of them is of “Korteweg-de Vries (KdV)” type and it is proportional to the derivative over space coordinate \( \partial \varphi^2 / \partial y \), and another scalar nonlinearity is proportional to the time derivative \( \partial \varphi^2 / \partial t \) (see equation (3.15)); (3) Large-scale solitary vortical structures of monopole type existing due to \( \partial \varphi^2 / \partial t \) scalar nonlinearity only (see equation (3.19)).

Thus there is a problem to be investigated: what is the corresponding growth rate of zonal flow generation by the mentioned different-scale vortical structures existing on the electrostatic drift waves in EPI plasmas. To carry out the corresponding investigation is the main goal of the given chapter. In Sec. 4.2 using the analytic method of parametric instabilities [146, 249, 252, 253] the nonlinear interaction of electrostatic drift waves and zonal flows is shown by system of equations. Excitation of zonal flows is clarified by this system. In the case of monochromatic pumping waves wide spectra of zonal flow instabilities are analyzed in Sec. 4.3. Discussion and conclusions are given in Sec. 4.4.

### 4.2 Nonlinear Interactions of Electrostatic Drift Waves and Zonal Flows in EPI Plasmas

#### 4.2.1 Reduction of the Initial Equations

Now, we are going to rewrite equation (3.12), (3.15) and (3.19) in the appropriate form ready for investigations on zonal flow generation problem.

(1) In case of short-scale size structures (3.13) we represent the perturbed potential as

\[
\varphi(x, y, t) = e^{\mp x/2L} \bar{\varphi}(x, y, t).
\]

Then from equation (3.12), under the conditions (3.13), we get the following form of HM equation:

\[
\beta \frac{\omega_{Bi}}{T_e} \frac{\partial \varphi}{\partial t} - \frac{1}{M \omega_{Bi}} \frac{\partial}{\partial t} \left( \Delta_{\perp} \varphi - \frac{1}{4L^2} \varphi \right) \mp \frac{1}{ML} \frac{\partial \varphi}{\partial y} - \frac{Ze}{M^2 \omega_{Bi}^2} J(\varphi, \Delta_{\perp} \varphi) = 0.
\]

Here, for the nonlinear terms, we have omitted the factor \( \exp(\mp x/2L) \approx 1 \) and the bar over \( \varphi \) is also dropped. The sign \( (\mp) \) corresponds to positive and negative signs of \( n_{e_0}^t - n_{p_0}^t \) and the parameter \( \beta \) is defined by equation (3.11).

(2) In case of intermediate-scale vortical structures (3.14), analogously to case (1), from equation (3.15) we get the following equation:
\[
\beta \frac{\omega_{Bi}}{T_e} \frac{\partial \varphi}{\partial t} - \frac{1}{M \omega_{Bi}} \frac{\partial}{\partial t} \left( \Delta_\perp \varphi - \frac{1}{4L^2} \varphi \right) \mp \frac{1}{ML} \frac{\partial \varphi}{\partial y} - \frac{Ze}{M^2 \omega_{Bi}^2} J(\varphi, \Delta_\perp \varphi)
\]
\[+ \frac{e \omega_{Bi}}{2Z} \frac{n_{e0}}{n_{e0} - n_{p0}} \frac{\partial \varphi^2}{\partial t} - \frac{e}{2M} \frac{1}{n_{e0} - n_{p0}} \frac{\partial \varphi^2}{\partial y} = 0. \quad (4.3)\]

(3) In case of large-scale vortical structures (3.18), analogously to case (1), from equation (3.19) we get the following equation:

\[
\beta \frac{\omega_{Bi}}{T_e} \frac{\partial \varphi}{\partial t} - \frac{1}{M \omega_{Bi}} \frac{\partial}{\partial t} \left( \Delta_\perp \varphi - \frac{1}{4L^2} \varphi \right) \mp \frac{1}{ML} \frac{\partial \varphi}{\partial y} - \frac{Ze}{M^2 \omega_{Bi}^2} J(\varphi, \Delta_\perp \varphi)
\]
\[+ \frac{e \omega_{Bi}}{2Z} \frac{n_{e0}}{n_{e0} - n_{p0}} \frac{\partial \varphi^2}{\partial t} - \frac{e}{2M} \frac{1}{n_{e0} - n_{p0}} \frac{\partial \varphi^2}{\partial y} = 0. \quad (4.4)\]

4.2.2 Separation of Variables

We consider a method of parametric instability (three-wave nonlinear interaction) in EPI plasma, this method describes the coupling between the pump (primary) drift waves and sideband modes drives low-frequency large-scale one-dimensional modes propagating along the x-axis, i.e., zonal flows.

We divide the perturbed potential \( \varphi \) in the following fashion,

\[
\varphi = \tilde{\varphi} + \hat{\varphi} + \bar{\varphi}. \quad (4.5)
\]

The primary small-scale modes are denoted by \( \tilde{\varphi} \). The function \( \hat{\varphi} \) represents the secondary small-scale modes. The function \( \bar{\varphi} \) is taken for large-scale zonal flow in the following form:

\[
\bar{\varphi} = \bar{\varphi}_0 \exp(-i\Omega t + i\Omega x) + c.c., \quad (4.6)
\]

where \( \Omega \) and \( q_x \) are the frequency and wave number of zonal flow, respectively. The amplitude \( \bar{\varphi}_0 \) of the zonal flow mode is assumed to be constant. Also c.c. stands for complex conjugative. The function \( \tilde{\varphi} \) is presented as the spectrum

\[
\tilde{\varphi} = \sum_k [\tilde{\varphi}_+(k) \exp(i k \cdot r - i \omega_k t) + \tilde{\varphi}_-(k) \exp(-i k \cdot r + i \omega_k t)], \quad (4.7)
\]

where \( \omega_k \) and \( k \) represent the primary mode frequencies and wave vectors, \( \tilde{\varphi}_-(k) = \tilde{\varphi}_+^*(k) \), and * means the complex conjugative. The totality of the primary modes is obtained by using the summation symbol. Similarly, the function \( \hat{\varphi} \) is presented as the spectrum
\[ \hat{\phi} = \sum_k \left[ \hat{\phi}_+ (k) \exp(i k_+ \cdot r - i \omega_{k+} t) + \hat{\phi}_- (k) \exp(i k_- \cdot r - i \omega_{k-} t) + c. c. \right] \]  (4.8)

where \( \hat{\phi}_\pm (k) \) are the sideband amplitudes.

The conservation of energy and momentum is satisfied for the frequencies \( \omega_\pm \) and wave vectors \( k_\pm \).

\[ \omega_\pm = \Omega \pm \omega \quad \text{and} \quad k_\pm = q_x e_x \pm k. \]  (4.9)

There exist small parameters

\[ \frac{|\Omega|}{|\omega_k|} \sim \frac{|q_x|}{|k_\pm|} \ll 1, \]  (4.10)

which are typical for the zonal flow generation problem.

It is evident that equation (4.3) is the general equation which can be written in terms of equations (4.2) and (4.4) as particular cases. Consequently we will consider equation (4.3) as the starting nonlinear equation to investigate zonal flow generation by drift waves in EPI plasma.

### 4.2.3 Primary Modes

For the spectral components of pump drift waves we substitute equations (4.6)-(4.8) into equation (4.3). We neglect small nonlinear terms from the high frequency relations. But in case of low frequency zonal flow modes we will keep such nonlinear terms. Solving the homogeneous equation for \( \hat{\phi}_\pm \) we get the pump wave frequency (3.10).

### 4.2.4 Evolutionary Equation for Zonal Flow

The turbulence and amplitude evolution of the zonal flow modes are obtained from equations when we use equations (4.5)-(4.8) into equation (4.3). The zonal flow evolution equation:

\[ \frac{\partial}{\partial t} \left[ \beta \frac{\omega_{B_i}}{T_e} - \frac{1}{M \omega_{B_i}} (\Delta_\perp - \frac{1}{4L^2}) \right] \hat{\phi} = - \frac{Ze}{M^2 \omega_{B_i}^2} \frac{\partial^2}{\partial x^2} \left( \hat{\phi} \frac{\partial \hat{\phi}}{\partial y} + \frac{\partial \hat{\phi}}{\partial x} \frac{\partial \hat{\phi}}{\partial y} \right) \]

\[ - e \omega_{B_i} \frac{n_{e0}}{\tau_{e}^2} - \frac{n_{p0}}{\tau_{B_i}^2} \frac{\partial}{\partial t} \langle \hat{\phi} \hat{\phi} \rangle. \]  (4.11)

Here \( \langle \cdot \rangle \) represents the average over fast oscillations. For Fourier components (4.6)-(4.8) we get
\[
\begin{align*}
&i\Omega \left[ \beta \frac{\omega_{Bi}}{T_e} + \frac{1}{M \omega_{Bi}} \left( q_x^2 + \frac{1}{4L^2} \right) \right] \phi_0 = - \frac{Ze q_x^2}{M^2 \omega_{Bi}^2} \sum_k k_y [2k_x (\phi_+ \phi_- + \phi_- \phi_+)] \\
&\quad + q_x (\phi_+ \phi_- - \phi_- \phi_+) \right] - i\Omega \frac{e \omega_{Bi}}{Z} \sum_k \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} (\phi_+ \phi_- + \phi_- \phi_+). \quad (4.12)
\end{align*}
\]

The driving force of zonal flows is called mean Reynolds stress and it is given on the right-hand side of equation (4.12).

### 4.2.5 Satellite Modes

Reynolds stress can be determined if we know the satellite (sideband) amplitudes \( \phi_\pm \).

From equation (4.3) we get the following turbulent part contribution equation:

\[
\frac{\partial}{\partial t} \left[ \beta \frac{\omega_{Bi}}{T_e} - \frac{1}{M \omega_{Bi}} \left( \Delta_\perp - \frac{1}{4L^2} \right) \right] \phi_\pm \mp \frac{1}{ML} \frac{\partial}{\partial y} \phi_\pm - \frac{Ze}{M^2 \omega_{Bi}^2} \left[ J(\phi_\pm, \nabla^2 \phi_0) + J(\phi_0, \nabla^2 \phi_\pm) \right] \\
+ \frac{e \omega_{Bi}}{Z} \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} \frac{\partial}{\partial t} \phi_0 \pm \frac{e}{M} \left( \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} \right)' \frac{\partial}{\partial y} \phi_\pm \phi_0 = 0. \quad (4.13)
\]

The solution of this equation gives for the sideband amplitudes

\[
\phi_\pm = \frac{\pm i \frac{Ze k_y q_x}{M^2 \omega_{Bi}^2} k_\perp - \frac{e \omega_{Bi}}{Z} \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} \omega_\pm \mp \frac{e}{M} \left( \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} \right)'}{BD_\pm} \phi_\pm \phi_0, \quad (4.14)
\]

where

\[
D_\pm = \omega_\pm \pm \left( \mp \right) \frac{k_y}{MLB'}, \quad k_\perp^2 = (q_x \pm k_x)^2 + k_y^2,
\]

\[
B = \beta \frac{\omega_{Bi}}{T_e} + \frac{1}{M \omega_{Bi}} \left( k_\perp^2 + \frac{1}{4L^2} \right). \quad (4.15)
\]

Here, in the expression for \( D_\pm \), the first combination of signs \( \pm \) should be used for positive sign of \( n_{e0}' - n_{p0}' \), but the second one (in brackets), only for negative sign of \( n_{e0}' - n_{p0}' \) is used. Now we can expand \( D_\pm \) over the small parameters (4.10). We get

\[
D_\pm = D^{(0)} + D^{(1)}, \quad (4.16)
\]
where

\[ D^{(0)} = \Omega - q_x V_g(k), \quad \text{and} \quad D^{(1)} = \mp \frac{q_x^2 V_g}{2}. \]  

Here the group velocity \( V_g \) of the primary modes and its derivative \( V_g' \) are defined differently for positive and negative signs of \( n_{e0} - n_{p0} \),

\[ V_g(k) = \frac{\partial \omega}{\partial k_x} = \pm \frac{2k_x k_y}{M^2 L \omega_B \Omega C^2} = - \frac{2k_x}{M \omega_B C} \omega_k, \]  

\[ V_g'(k) = \frac{\partial V_g}{\partial k_x} = \pm \frac{2k_y}{M^2 L \omega_B C^2} \pm \frac{8k_y k_x^2}{M^3 L \omega_B^2 C^3} = - \frac{2\omega_k}{M \omega_B C} \left( 1 - \frac{4k_x^2}{M \omega_B C} \right). \]

In equations (4.18) and (4.19) the upper and lower signs correspond to the positive and negative values of \( n_{e0} - n_{p0} \), respectively, and

\[ C = \frac{1}{M \omega_B} \left( \frac{\beta}{r_L^2} + k_x^2 + \frac{1}{4L^2} \right). \]

Now we substitute the equation (4.16)-(4.20) into equation (4.14), and get

\[ \hat{\phi}_\pm = \frac{\pm \frac{2\Omega^2}{M^2 \omega_B} k_y^2 - \frac{\epsilon \omega_B \Omega}{n_{e0} - n_{p0}} (\Omega \pm \omega) \mp \frac{\epsilon}{M} \left( \frac{n_{e0}}{\tau_e} \frac{n_{p0}}{\tau_p} \right)'}{D^{(0)} \left[ 1 + \frac{D^{(1)}}{D^{(0)}} \right] C \left[ 1 + \frac{q_x^2 + 2k_x q_x}{M \omega_B C} \right]} \hat{\phi}_0. \]  

Using the expansion over the small parameters (4.10), we represent the sideband amplitude as

\[ \hat{\phi}_\pm = \hat{\phi}^{(-1)}_\pm + \hat{\phi}^{(0)}_\pm + \hat{\phi}^{(1)}_\pm, \]

where \( \hat{\phi}_\pm^{(-1)} \), \( \hat{\phi}_\pm^{(0)} \), and \( \hat{\phi}_\pm^{(1)} \) are terms of \( q^{-1}_x, q^0_x \) and \( q_x \) order, respectively, and

\[ \hat{\phi}_\pm^{(-1)} = \frac{\hat{\phi}_\pm \hat{\phi}_0}{C D^{(0)}} \left[ \pm \frac{\epsilon \omega_B}{Z} \frac{n_{e0} - n_{p0}}{\tau_e \tau_p} \omega_k \mp \frac{\epsilon}{M} \left( \frac{n_{e0}}{\tau_e} + \frac{n_{p0}}{\tau_p} \right)' k_y \right], \]  

(4.23)
\[
\frac{\phi^{(0)}_{\pm}}{CD^{(0)}} = \frac{i}{Z} \frac{Ze k_y q_x k_{1z}^2}{M^2 \omega_{Bi}^3} \left[ 1 + e \frac{n_{e0} - n_{p0}}{\tau_e^2} \frac{\Omega}{n_{e0} - n_{p0}} \right] + e \frac{n_{e0} - n_{p0}}{\tau_e^2} \frac{\Omega}{n_{e0} - n_{p0}} \frac{2 k_x q_x}{MC} + \frac{e}{M} \frac{\left( \frac{n_{e0} + n_{p0}}{\tau_e + \tau_p} \right)'} k_y \frac{2 k_x q_x}{MC} + e \frac{\left( \frac{n_{e0} + n_{p0}}{\tau_e + \tau_p} \right)'} k_y \frac{D^{(1)}_{\pm}}{D^{(0)}} + e \frac{\left( \frac{n_{e0} + n_{p0}}{\tau_e + \tau_p} \right)'} k_y \frac{D^{(1)}_{\pm}}{D^{(0)}} \right] \]

\[
\phi^{(1)}_{\pm} = \frac{i}{Z} \frac{Ze k_y k_{1z}^2}{M^2 \omega_{Bi}^3} \left[ 4 k_x^2 - 1 \right] + e \frac{n_{e0} - n_{p0}}{\tau_e^2} \frac{\Omega}{n_{e0} - n_{p0}} + e \frac{n_{e0} - n_{p0}}{\tau_e^2} \frac{\Omega}{n_{e0} - n_{p0}} \frac{2 k_x q_x}{MC} + \frac{e}{M} \frac{\left( \frac{n_{e0} + n_{p0}}{\tau_e + \tau_p} \right)'} k_y \frac{2 k_x q_x}{MC} \frac{D^{(1)}_{\pm}}{D^{(0)}} \right] \]

\[
4.2.6 \text{ Dispersion Relation for Zonal Flow} \]

If we substitute the expressions (4.22)-(4.25) into equation (4.12) we get the zonal flow dispersion equation

\[
1 - \sum_{k} \frac{F(k)}{(\Omega - q_x V_g)^2} = 0, \tag{4.26}
\]

where

\[
F(k) = F_1(k) + F_2(k) + F_3(k) + F_4(k) + F_5(k) + F_6(k). \tag{4.27}
\]
Here,

\[
F_1(k) = \frac{Z^2 e^2 q_x^4 k_y^2 k_z^2 V'_x}{2AM^2 \omega_{Bi}^2 \omega_k I_k} \tag{4.28}
\]

represents the contribution from the vector nonlinearity;

\[
F_2(k) = i \frac{Ze^2 q_x^3 k_y V'_x}{2AM^2 \omega_{Bi} \omega_k} \frac{\left( \frac{n_{e0}}{T_e} + \frac{n_{p0}}{T_p} \right)}{n_{e0} - n_{p0}} I_k \tag{4.29}
\]

represents the contribution from the KdV scalar nonlinearity; and

\[
F_3(k) = i \frac{e^2 q_x^2 k_y \frac{n_{e0}}{T_e} - \frac{n_{p0}}{T_p}}{2AM \omega_k} [2V_g(\Omega - q_x V_g) + q_x V'_g \omega_k] I_k \tag{4.30}
\]

represent the contribution from the \(\partial \phi^2 / \partial t\) scalar nonlinearity. Further,

\[
F_4(k) = \frac{e^2 \omega_{Bi}^2}{2AZ^2 C} \frac{\left( \frac{n_{e0}}{T_e} - \frac{n_{p0}}{T_p} \right)}{(n_{e0} - n_{p0})^2} [2(\Omega^2 - q_x^2 V_g^2) + q_x^2 V'_g \omega_k] I_k \tag{4.31}
\]

represents the contribution from the interaction of the \(\partial \phi^2 / \partial t\) scalar nonlinearity with itself;

\[
F_5(k) = \frac{e^2 \omega_{Bi} q_x k_y \frac{n_{e0}}{T_e} - \frac{n_{p0}}{T_p}}{2AZC M \omega_k} \frac{\left( \frac{n_{e0}}{T_e} + \frac{n_{p0}}{T_p} \right)}{(n_{e0} - n_{p0})^2} [2V_g(\Omega - q_x V_g) + q_x V'_g \omega_k] I_k \tag{4.32}
\]

represents the contribution from the interaction of the \(\partial \phi^2 / \partial t\) scalar nonlinearity with KdV scalar nonlinearity; and

\[
F_6(k) = -i \frac{e^2 q_x^2 k_y k_z^2}{2AZC^2 \omega_{Bi} \omega_k} \frac{\left( \frac{n_{e0}}{T_e} - \frac{n_{p0}}{T_p} \right)}{n_{e0} - n_{p0}} [2V_g(\Omega - q_x V_g) + q_x V'_g \omega_k] I_k \tag{4.33}
\]

represents the contribution from the interaction of the \(\partial \phi^2 / \partial t\) scalar nonlinearity with the vector nonlinearity. In the given expressions \(I_k = 2\tilde{\phi}_+ \tilde{\phi}_- = 2|\tilde{\phi}_+|^2\) is the intensity of pumping waves,

\[
A = \beta \frac{\omega_{Bi}}{T_e} + \frac{1}{M \omega_{Bi}} \left( q_x^2 + \frac{1}{4L^2} \right), \tag{4.34}
\]

and we have defined \(C\) by equation (4.20).
A) Let us consider the case when the contribution from the terms proportional to $V'_g$ is much more than from the corresponding additional terms in equations (4.30)-(4.33). As such terms are proportional to small value of $q_x^2$, thus the dispersion equation (4.26) is applicable only when $(\Omega - q_x V_g)$ is also a small parameter. Then from equation (4.30)–(4.33) the functions can be calculated for $\Omega \approx q_x V_g$,

$$F_3(k) = \frac{i e^2 q_x^2 k_y}{2AM} \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} V'_g I_k,$$  

$$F_4(k) = \frac{e^2 \omega_{Bi}^2 q_x^2}{2AZ^2 C} \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} V'_g I_k,$$  

$$F_5(k) = \frac{e^2 \omega_{Bi} q_x^2 k_y}{2AZC} \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} \left( \frac{n_{e0} + n_{p0}}{T_e + T_p} \right) V'_g I_k,$$  

$$F_6(k) = -i \frac{e^2 q_x^2 k_y k_1^2}{2ACM^2} \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} V'_g I_k.$$  

B) In case when the derivative of group velocity $V'_g$ (see equation (4.19)) is so small that its contribution in the expressions (4.27)-(4.33) is not important, then the zonal flow dispersion equation (4.26) is modified by

$$1 - \sum_k \frac{F(k)}{\Omega - q_x V_g} = 0,$$  

and for the contents of equation (4.27) we have

$$F_1(k) = F_2(k) \approx 0,$$  

$$F_3(k) \approx \frac{i e^2 q_x^2 k_y}{AM \omega_k} \frac{n_{e0} - n_{p0}}{n_{e0} - n_{p0}} V_g I_k,$$  

$$F_4(k) \approx \frac{e^2 \omega_{Bi}^2}{Z^2 CA} (\Omega + q_x V_g) \left( \frac{n_{e0} - n_{p0}}{T_e + T_p} \right)^2 I_k.$$  

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\[
F_5(k) \approx \frac{e^2 \omega_B k_y q_x}{ZAC \ M \omega_k} \frac{n_{e0}}{T_e} \frac{n_{p0}}{T_p} \left( \frac{n_{e0}}{T_e} + \frac{n_{p0}}{T_p} \right)^2 V_B l_k, \quad (4.43)
\]

\[
F_6(k) \approx -i \frac{e^2 k_y k_z q_x}{ACM^2 \omega_B \omega_k} \frac{n_{e0}}{T_e} \frac{n_{p0}}{T_p} V_B l_k. \quad (4.44)
\]

### 4.3 Zonal Flow Instabilities in Case of Monochromatic Wave Packet

Here we will consider items A) and B) of subsec. 4.2.6 separately.

A) In case of the monochromatic wave packet \( F(k) \sim \delta(k - k_0) \) and equation (4.26) reduces to a hydrodynamic-type coherent instability,

\[
(\Omega - q_x V_B)^2 = F(k_0) = -\Gamma^2,
\]

where \( \Gamma^2 = \Gamma_1^2 + \Gamma_2^2 + \Gamma_3^2 + \Gamma_4^2 + \Gamma_5^2 + \Gamma_6^2 \) is the square of zonal flow growth rate and

\[
\Gamma_1^2 = -\frac{Z e^2 q_x^4 k_y^2 k_0^2}{2AM^2 \omega_B \omega_k} V_B'(k_0) l_k
\]

(4.46)

is the contribution from the vector nonlinearity;

\[
\Gamma_2^2 = -i \frac{Ze^2 q_x^3 k_y^2 V_B'(k_0)}{2AM^2 \omega_B} \left( \frac{n_{e0}}{T_e} + \frac{n_{p0}}{T_p} \right)^2 l_k
\]

(4.47)

is the contribution from KdV scalar nonlinearity; and

\[
\Gamma_3^2 = -i \frac{e^2 q_x^3 k_y_0}{2MA} \frac{n_{e0}}{n_{e0} - n_{p0}} V_B'(k_0) l_k
\]

(4.48)

is the contribution from the \( \partial \phi^2 / \partial t \) scalar nonlinearity. Further,

\[
\Gamma_4^2 = -\frac{e^2 \omega_B^2 q_x^2 V_B'(k_0)}{2Z^2 CA} \left( \frac{n_{e0}^2}{T_e} - \frac{n_{p0}}{T_p} \right)^2 \frac{1}{(n_{e0} - n_{p0})^2} l_k
\]

(4.49)

is the contribution from the interaction of the \( \partial \phi^2 / \partial t \) scalar nonlinearity with itself;
\[
\Gamma_5^2 = -\frac{e^2 \omega_{Bi} q_x^2 k_y y_0 n_{e0} - n_{p0}}{2ZC MA} \left( \frac{n_{e0}}{T_e} - \frac{n_{p0}}{T_p} \right) \left( \frac{n_{e0}}{T_e} + \frac{n_{p0}}{T_p} \right)' V'_g(k_0) I_{k0}
\]  

(4.50)

is the contribution from the interaction between two scalar nonlinearities; and

\[
\Gamma_6^2 = \frac{e^2 q_x^2 k_y y_0 k_{1,0} n_{e0} - n_{p0}}{2ACM^2 \omega_{Bi}} \left( \frac{n_{e0}}{T_e} - \frac{n_{p0}}{T_p} \right) V'_g I_{k0}
\]  

(4.51)

is the contribution from the interaction of two nonlinearities, one of them is time dependent scalar and the other is vector nonlinearity.

As to the zonal flow oscillation frequency it is given by the following expression (see equation (4.18)):

\[
Re\Omega \approx q_x V_g(k_0) = -2 \frac{q_x k_x}{M \omega_{Bi} \xi} \omega_k.
\]  

(4.52)

It is seen from equations (4.46) and (4.49) that for instability \( V'_g/\omega_k < 0 \). This condition is similar to the Lighthill criterion for modulation instability in nonlinear optics. According to equation (4.19) the instability condition becomes

\[
\frac{\beta}{r_e^2} + \frac{1}{4L^2} + k_y^2 - 3k_x^2 > 0.
\]  

(4.53)

As to the cases described by equations (4.47), (4.48), and (4.51) instabilities always exist in spite of the sign of \( V'_g/\omega_k \). But in the case of equation (4.50) the instability exists only under the condition

\[
k_y y_0 \left( \frac{n_{e0}}{T_e} - \frac{n_{p0}}{T_p} \right) \left( \frac{n_{e0}}{T_e} + \frac{n_{p0}}{T_p} \right)' V'_g(k_0) < 0.
\]  

(4.54)

Now we can discuss the main problems under consideration described by equations (4.2)-(4.4). Below, we normalize the potential \( \phi_+ \) of pump modes by the ratio \( T_e/Ze \).

**1)** In case of the small-scale size structures (3.13) described by the nonlinear equation (4.2), the existence of the vector nonlinearity only causes the generation of zonal flow with the squared growth rate (see equation (4.46)),

\[
\Gamma_1^2 = -\frac{q_x^4 k_y y_0^2\omega_{Bi}^2 r_L^6 V'_g(k_0)}{2\omega \xi k_0} \left[ \beta + r_L^2 \left(q_x^2 + \frac{1}{4L^2}\right) \right] I_{k0}.
\]  

(4.55)

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The instability condition is expressed by equation (4.53). Temperature inhomogeneity effects have no influence in small-scale turbulence. We have obtained here unstable branch in such a form that it has a zonal flow growth rate proportional to \( \frac{2}{k_{\perp 0}} (k_{y 0}^2 - 3k_{x 0}^2) < 0 \). In this case the instability condition becomes

\[
\frac{V'_g}{\omega_{k_0}} = - \frac{2}{k_{\perp 0}^4} (k_{y 0}^2 - 3k_{x 0}^2) < 0. \tag{4.56}
\]

Thus the instability condition is used for drift waves with \(-k_y/\sqrt{3} < k_x < k_y/\sqrt{3}\). We put \( k_x = 0 \) for maximum growth rate. Under this condition the group velocity \( V_g = 0 \) (see equation (4.18)) and we have the aperiodic (\( Re \Omega = 0 \)) generation of zonal flows with the growth rate

\[
\Gamma_1 = \frac{q_z^2 \omega_{Bi} |k_{y 0}| n_L^3}{\left[ \beta + n_L^2 \left( q_z^2 + \frac{1}{4\ell^2} \right) \right]^{1/2} k_{k 0}^1}. \tag{4.57}
\]

The initial stage of zonal flow growth is given by this equation. For \( n_L q_x \sim 1 \) the growth rate can estimated to

\[
\Gamma_1 \approx \omega_{Bi}|k_{y 0}| n_L^{1/2}. \tag{4.58}
\]

This is the maximum growth rate. This approach indicates that by increasing \( k \) in the small wavelength limit \( (k_{\perp} n_L \gg 1) \), \( \Gamma \) increases. Physically, this instability is the expression of an inverse cascade where the spectral energy of the small-scale drift wave turbulence is transferred into the large scales of the zonal flows, i.e., the drift wave energy is converted into the energy of slow zonal motions. If values are taken for specific tokamak \( \phi_a \sim 10^{-1}, |k_{\perp} n_L| \sim 10, \) and \( \omega_{Bi} \sim 10^8 \text{s}^{-1} \), from Eq. (4.58) we obtain the maximum growth rate \( \Gamma_1 \sim 10^8 \text{s}^{-1} \).

(2) In case of large-scale structures existing under the conditions (3.18) and described by equation (4.4) only the \( \partial \phi^2 / \partial t \) scalar nonlinearity drives the generation of zonal flow. Appropriate squared growth rate of zonal flow consists of two terms (see equations (4.48) and (4.49)),

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\[\Gamma_3^2 + \Gamma_4^2 = -i \frac{q_x^3 k_y \omega}{2Z^2 \beta} \frac{\xi}{\tau_e} \frac{n_{e0} - n_{p0}}{\tau_p} V_g'(k_0) I_{k0} = -\frac{q_x^2 M^4 \omega_b^8 r_L^8}{2Z^2 \beta^2} V_g'(k_0) \omega_{k0} \]

\[\times \left( \frac{n_{e0} - n_{p0}}{\tau_e} \right)^2 \left( \frac{n_{e0} - n_{p0}}{\tau_p} \right)^2 I_{k0}. \quad (4.59)\]

As it was mentioned above that the last term describes the interaction of the \( \partial \varphi^2 / \partial t \) scalar nonlinearity with itself and its contribution is proportional to \( q_b^2 \), while the first term is \( q_x \)-times less. In spite of this fact the instability due to the first term always exists, but for the contribution from the last term the realization of necessary instability condition \( \beta^2 / r_L^2 > 0 \) is needed (see equation (4.53)), which is also always fulfilled. The contribution from the last term achieves its maximum value at \( k_x = 0 \) (see equation (4.19)) and the appropriate growth rate is equal to

\[\Gamma_4 = \frac{r_L}{Z^2 \beta^{3/2}} q_x \omega_{k0} \frac{n_{e0} - n_{p0}}{\tau_e} \frac{\tau_p^2}{\tau_p^2} I_{k0}^{1/2}. \quad (4.60)\]

Such generation doesn’t depend on \( k \) and is also aperiodic with \( Re \Omega = 0 \). The numeric value of the growth rate \( \Gamma_4 \approx 10^4 \text{s}^{-1} \), if we assume here \( |q_x/k_{y0}| = 10^{-1}, |k_{y0} r_L| = 10^{-1}, \omega_{k0} = 10^7 \text{s}^{-1}, \omega_{Bi} = 10^8 \text{s}^{-1}, \phi_+ = 10^{-1}, \beta = 1 \). This contribution in the growth rate increases as \( k_{y0} \) (see equation (3.10)).

As to the first contribution in equation (4.59) its appropriate growth rate is equal,

\[\Gamma_3 = \frac{r_L^2 \omega_{Bi}^{1/2}}{2^{1/2} Z \beta} \frac{q_x^2 k_y \omega_{k0}}{n_{e0} - n_{p0}} \left( \frac{\tau_p^2}{\tau_p^2} \right)^{1/2} I_{k0}^{1/2}. \quad (4.61)\]

This contribution increases as \( k_{y0} \) and is almost aperiodic instability. Numerically the appropriate growth rate \( \Gamma_3 \approx 10^3 \text{s}^{-1} \).

(3) In case of the intermediate-scale size structures (3.14) described by the nonlinear equation (4.3), three different (one vector and two scalar) nonlinearities give the contribution in the generation of zonal flow,
This instability exists always with the growth rate 

$$\Gamma_1^2 + \Gamma_2^2 + \Gamma_3^2 + \Gamma_4^2 + \Gamma_5^2 + \Gamma_6^2$$

$$= -\frac{q_x^4 r_L^2}{2} \frac{q_x^2 \omega_B T_e k_{y0}^2}{2} \left( \frac{n_e - n_{p0}}{T_e} + \frac{n_{p0}}{T_p} \right)^2 V_g' (k_0) I_{k0}$$

$$= -\frac{q_x^3 r_L^2}{2Z^2 \beta^2} \frac{e - n_{p0}}{n_e - n_{p0}} V_g' (k_0) I_{k0}$$

$$- i \frac{q_x^2 \omega_B T_e k_{y0}^2}{2Z^3 \beta^2} \left( \frac{n_e - n_{p0}}{T_e} + \frac{n_{p0}}{T_p} \right)^2 \left( \frac{n_{e} - n_{p0}}{T_e} + \frac{n_{p0}}{T_p} \right)^2 V_g' (k_0) I_{k0}$$

$$+ i \frac{q_x^2 r_L^6 M^2 T_e \omega_B M^2 k_{y0}^2}{2Z^2 \beta^2} \frac{n_e - n_{p0}}{n_e - n_{p0}} V_g' (k_0) I_{k0}.$$  \(4.62\)

As it was mentioned above that the first term in equation (4.62) gives the contribution from the vector nonlinearity in case of the large-scale structures. The appropriate growth rate is (cf. with equations (4.57) and (4.58))

$$\Gamma_1 = \frac{q_x^2 r_L^4 \omega_B}{\beta} \left| k_{y0} k_{\perp 0} \right|^{1/2}.  \(4.63\)$$

The growth rate is proportional to \(k_{y0}\). The instability condition \(\beta / r_L^2 > 0\) (see equation (4.53)) is always fulfilled and the growth rate achieves its maximum value at \(k_{x0} = 0\). Under the following numerical values \(|q_x / k_{y0}| = 10^{-1}, |k_{y0} r_L| = 10^{-1}, \omega_B = 10^8 \text{s}^{-1}, \phi_+ = 10^{-1}\) and \(\beta = 1\), we get \(\Gamma_1 \approx 10 \text{s}^{-1}\). This contribution is aperiodic.

The second term in equation (4.62) gives the contribution from KdV scalar nonlinearity. This instability exists always with the growth rate

$$\Gamma_2 = \frac{r_L^3 \omega_B}{2^{1/2} Z \beta} \left| q_x^{3/2} k_{y0} \right|^{1/2} I_{k0}^{1/2}.  \(4.64\)$$

The growth rate is proportional to \(k_{y0}\). According to equation (3.17) we take into account the value \(|k_{\perp 0} H| \approx 10\) and under the previous numerical values we get \(\Gamma_2 \approx 10^2 \text{s}^{-1}\). The contribution is almost aperiodic.

The third term in equation (4.62) is the contribution from \(\partial \phi^2 / \partial t\) scalar nonlinearity. This instability exists always having the growth rate (4.61).
The fourth term in equation (4.62) is the contribution from the nonlinear interaction of \( \partial \psi^2 / \partial t \) scalar nonlinearity with itself. The appropriate growth rate is expressed by equation (4.60). The fifth term in equation (4.62) is the contribution from the interaction of the \( \partial \psi^2 / \partial t \) scalar nonlinearity with the KdV scalar nonlinearity and the expression for growth rate is

\[
\Gamma_5 = \frac{r_L^2 \omega_B^1}{2Z^2 \beta^3} \left[ \frac{q_x^2 k_{y_0} \omega k_0 n_{e_0} - n_{p_0} \tau_e^2}{H} \right]^{1/2} \frac{i}{k_0}. \tag{4.65}
\]

However, the condition for instability is given by equation (4.54). The maximum value of the growth rate can be obtained at \( k_{x_0} = 0 \) and the instability is aperiodic one. If we suppose that \(|k_{y_0}H| \approx 10 \) for the numeric estimation we get \( \Gamma_5 \approx 10^3 s^{-1} \).

The last term in equation (4.62) gives the contribution from the nonlinear interaction of the \( \partial \psi^2 / \partial t \) scalar nonlinearity with the vector nonlinearity. The corresponding growth rate is

\[
\Gamma_6 = \frac{r_L^3 \omega_B^1}{2Z^2 \beta^3} \left[ \frac{q_x^3 k_{y_0} k_{L_0}^2 \omega k_0 n_{e_0} - n_{p_0} \tau_e^2}{H} \right]^{1/2} \frac{i}{k_0}. \tag{4.66}
\]

This aperiodic contribution always exists and numerically \( \Gamma_6 \approx 10^2 s^{-1} \).

B) This case is valid for the low-frequency intermediate structures and in contrast to the previous case existence of \( k_{x_0} \) component is essential. In case of the monochromatic wave packet \( F(k) \sim \delta(k - k_0) \) dispersion equation (4.39) for unstable branches reduces to

\[
\Omega - q_x V_g = F(k_0) = \Gamma_3 + \Gamma_6,
\]

where

\[
\Gamma_3 = \frac{i q_x^2 k_{y_0} n_{e_0} - n_{p_0} \tau_e^2}{Z^2 A M \omega k_0} V_g(k_0) i k_0.
\]

is the contribution from the \( \partial \psi^2 / \partial t \) scalar nonlinearity and

\[
\Gamma_6 = -\frac{i q_x^2 k_{y_0} k_{L_0}^2 n_{e_0} - n_{p_0} \tau_e^2}{Z^2 A C M^2 \omega_B \omega k_0} V_g(k_0) i k_0.
\]

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is the contribution from the interaction of the $\frac{\partial \varphi^2}{\partial t}$ scalar nonlinearity with the vector nonlinearity.

As seen from equations (4.67) and (4.68) the generation of zonal flow under the action of only $\frac{\partial \varphi^2}{\partial t}$ scalar nonlinearity holds when

$$\frac{k_{\gamma 0} V_g}{\omega_k} \frac{n_{e0}}{n_{e0} - n_{p0}} \frac{\tau_e^2}{\tau_p^2} > 0.$$  \hspace{1cm} (4.70)

But in case of the simultaneous action of the $\frac{\partial \varphi^2}{\partial t}$ scalar and vector nonlinearities for the instability condition from equation (4.69), we get the expression (4.70) with the opposite sign of inequality. It means that instability always takes place under the action of $\frac{\partial \varphi^2}{\partial t}$ scalar nonlinearity. For the contribution (4.68) we can estimate the corresponding growth rate

$$\Gamma_3 = -2i \frac{q_x^2 k_{\gamma 0} k_{\omega B}}{Z^2 \beta^2} \frac{n_{e0}}{n_{e0} - n_{p0}} \frac{\tau_e^2}{\tau_p^2} I_{k0}. \hspace{1cm} (4.71)$$

Numerically $\Gamma_3 \approx 1 \text{s}^{-1}$. As to the other contribution (4.69) we get

$$\Gamma_6 = 2i \frac{q_x^2 k_{\gamma 0} k_{\omega B}}{Z^2 \beta^3} \frac{n_{e0}}{n_{e0} - n_{p0}} \frac{\tau_e^2}{\tau_p^2} I_{k0}. \hspace{1cm} (4.72)$$

Numerically $\Gamma_6 \approx 10^{-2} \text{s}^{-1}$. In contrast to the previous cases of zonal flow generation, now there exists the real part of zonal flow frequency (see equation (4.52)),

$$\text{Re } \Omega = q_x V_g = -2 \frac{q_x k_{\gamma 0} r_L^2}{\beta} \omega_{k0} \approx 10 \text{s}^{-1}. \hspace{1cm} (4.73)$$

Here the following numeric values were used: $q_x/k_x = 10^{-1}$, $k_x r_L = 10^{-1}$, $\beta = 1$, and $\omega_{k0} = 10^4 \text{s}^{-1}$.

4.4 Conclusions

In the given chapter the problem of zonal flow generation by electrostatic drift waves having arbitrary wavelengths is investigated in EPI plasma. Our investigation provides an essential nonlinear mechanism for the transfer of spectral energy from small-scale drift waves to large-scale enhanced zonal flows in EPI plasmas. To describe this process extended for EPI plasma, generalized HM equation is obtained containing one vector and two different kinds
scalar nonlinearities. The vector nonlinearity (Jacobian) is responsible for the existence of small-scale \( k_\perp r_\perp \gg 1 \) dipole-type solitary vortical structures (see equation (4.2)). One of the scalar nonlinearities (of KdV type) is proportional to \( \partial \psi / \partial y \) and is responsible for the existence of the intermediate-scale vortical structures (see equation (4.3)). Another scalar nonlinearity is proportional to \( \partial \psi / \partial t \) and originates from \( \delta n_i / \partial t \) in equation (3.4). This nonlinearity creates intermediate- and large-scale monopole vortical structures (see equations (4.3) and (4.4)) and plays an essential role in different possibilities of zonal flow generation. It causes nonlinear interaction with vector and KdV-type nonlinearities and itself also. Note that temperature inhomogeneity effects are involved only by the KdV-type scalar nonlinearity.

A system of coupled equations telling the nonlinear interaction of drift waves and zonal flows is derived. The driving force (Reynolds stresses) controlling the evolution of zonal flows (see equation (4.12)) is obtained. We have made such a generalization and thereby obtained the dispersion relation for zonal flow and was shown by equations (4.26) and (4.39) for an arbitrary spectrum of drift pumping waves. These dispersion equations describe hydrodynamic-type instabilities. Zonal flow generation described by equation (4.26) is stipulated by the derivative of group velocity \( V'_{\psi}(k) \) (see equation (4.19)). The corresponding growth rates are comparatively high \( (10 - 10^8 \text{ s}^{-1}) \) and caused by both small- and large-scale solitary vortical structures. These instabilities are almost aperiodic with \( Re \Omega \approx 0 \) and the mode growing rapidly has a vector that is transverse with drift pump wave \( (k_x = 0) \). As to the dispersion equation (4.39) it describes the low-frequency generation of zonal flow and is valid when \( V'_{\psi} \approx 0 \). The generation is stipulated by the group velocity \( V_g \) and the corresponding growth rates lie in the region \( (10^{-2} - 1 \text{ s}^{-1}) \).

We have considered only monochromatic wave packet excitation of zonal flow. It has been shown apparently, not only the maximum growth rate but also optimal spatial dimensions of the zonal flows. It is shown that in contrast to usual electron-ion plasma, the existence of positrons in the plasma causes modification of both the zonal flow growth rate and instability conditions.

Small-scale turbulence is represented by the assembly of dipole vortical structures and causes the aperiodic generation of zonal flow. This instability is stipulated by the vector nonlinearity. In addition, the temperature inhomogeneity of electrons and positrons has no influence on the generation mechanism of zonal flows in three component EPI plasmas. In this chapter we get unstable branch has the zonal flow growth rate proportional to \( q^2 \) which is quite small. The matching instability condition is given by equation (4.53). The maximum growth rate
\( \Gamma_1 \approx 10^8 \text{s}^{-1} \) is being achieved at \( k_x = 0 \) and \( q_x r_L \approx 1 \) (see Eq. (4.58)). We note that in this case the growth rate does not depend on \( q_x \). The growth rate is proportional to \( k_{y0} \) and supports the existence of inverse cascade, i.e., the transferring of the spectral energy of the small-scale drift wave turbulence into the large-scales of the zonal flows. In other words the drift wave energy is converted into the energy of slow zonal motions.

In case of the large-scale turbulence represented by the assembly of monopole vortical structures (see equation (4.4)) zonal flow generation is caused only by \( \partial \varphi^2 / \partial t \) scalar nonlinearity. In the process of zonal flow generation there is the contribution from both the mentioned scalar nonlinearity \( \Gamma_3 \) and interaction of this nonlinearity with itself \( \Gamma_4 \) (see equations (4.59)-(4.61)). The contribution from \( \Gamma_4 \approx 10^4 \text{s}^{-1} \) is the ten times more than the contribution \( \Gamma_3 \). In addition \( \Gamma_3 \) and \( \Gamma_4 \) increase as \( k_{y0} \). Maximum growth rates are being achieved at \( k_x = 0 \) and are aperiodic. Instability condition \( \beta / r_L^2 > 0 \) is always fulfilled. In this case of the large-scale turbulence temperature effects have essential influence.

In case of the intermediate-scale turbulence when there exist both monopole and dipole structures formed under the competition between two scalar nonlinearities \( \propto \partial \varphi^2 / \partial t \) and KdV scalar nonlinearities \( \propto \partial \varphi^2 / \partial y \) in the zonal flow generation, there are six contributions (see equation (4.62)-(4.66)). (1) From the vector nonlinearity having the maximum growth rate \( \Gamma_1 = 10 \text{s}^{-1} \). The contribution is proportional to \( k_{y0}^2 \) and does not depend on temperature inhomogeneity of electrons and positrons. The instability condition \( \beta / r_L^2 > 0 \) is always fulfilled. (2) From KdV nonlinearity with the maximum growth rate \( \Gamma_2 = 10^2 \text{s}^{-1} \). This contribution is proportional to \( k_{y0} \) and the corresponding instability always exists. This contribution depends on the temperatures inhomogeneity. (3) From \( \partial \varphi^2 / \partial t \) scalar nonlinearity with the maximum value \( \Gamma_3 = 10^3 \text{s}^{-1} \). This contribution is the same as in the case of large-scale turbulence discussed above. (4) From the nonlinear interaction of \( \partial \varphi^2 / \partial t \) scalar nonlinearity with itself with the maximum value \( \Gamma_4 = 10^4 \text{s}^{-1} \). This contribution is the same as in the case of large-scale turbulence discussed above. (5) From nonlinear interaction of \( \partial \varphi^2 / \partial t \) scalar nonlinearity with KdV scalar nonlinearity \( \Gamma_5 = 10^3 \text{s}^{-1} \). This contribution in the growth rate is proportional to \( k_{y0} \) and involves the temperature influence. Instability condition is given by the condition (4.54). (6) From nonlinear interaction of \( \partial \varphi^2 / \partial t \) scalar nonlinearity with vector one having the \( \Gamma_6 = 10^2 \text{s}^{-1} \) contribution in the growth rate. This contribution in the total growth rate exists always, and is aperiodic with \( k_x = 0 \). This contribution is proportional to \( k_{y0}^2 \).

As it was mentioned above there is the other possibility of hydrodynamic-type instability described by the dispersion equation (4.39). This is the case of low-frequency intermediate-scale
excitation of zonal flow when the contribution in zonal flow generation is stipulated by $V_g$ group velocity in contrast to the previous case when the $V'_g$ has the main contribution (see equation (4.67)-(4.73)). The generation now is not aperiodic ($k_x \neq 0$) any more. The instability exists always and the temperature effects have essential influence. There are two contributions in the total growth rate. (1) The contribution from the $\partial \varphi^2 / \partial t$ scalar nonlinearity with the growth rate $\Gamma_3 \approx 1 \, s^{-1}$. The growth rate is proportional to $k^2$. (2) The contribution from the interaction of $\partial \varphi^2 / \partial t$ scalar nonlinearity with the vector nonlinearity has the growth rate $\Gamma_6 \approx 10^{-2} \, s^{-1}$. This contribution is important because the growth rate is proportional to $k^4$. The corresponding oscillation frequency is of the $10 \, s^{-1}$ order. Results obtained in this chapter can be used to explain different observations on zonal flows in laboratory and astrophysical plasmas consisting of electrons, positrons and ions.
Chapter 5

Summary

In the present thesis we investigate the mechanism of zonal flow generation by low-frequency Rossby waves in the Earth’s ionosphere. It is also investigated for electrostatic drift waves in EPI plasmas. The driving waves of this kind were promoted, because of significant physical resemblance between the drift waves in a magnetized plasma and the Rossby waves in rotating quasi-two-dimensional hydrodynamical media. This medium consists of planetary atmospheres as well as oceans. Nonlinear drift waves in a plasma are investigated by Hasegawa-Mima equation. Similarly nonlinear dynamics of Rossby waves in geophysical hydrodynamics is governed by Charney equation. The most impressive display of the resemblance is that the both equations turns out to be equivalent. The only way to distinguish one equation from the other is that we use stream function for Charney equation and perturbed plasma potential for Hasegawa-Mima equation.

The generation mechanism is associated with either Charney or Hasegawa-Mima equations in the Rossby wave-zonal flow or drift wave-zonal flow turbulence. Parametric instability technique to explain zonal flow generation based on three waves resonant nonlinear interaction is applied. This instability defines the interaction of a small-scale pump (Rossby, drift) waves, two satellites of the pump waves (side-band waves) and a large-scale sheared zonal flow. Reynolds stress is the driving force in the evolution equation of zonal flows. It is noted that, usually, slow timescale variation is associated with zonal flows, but it is fast in case of finite-frequency waves. Thus the important nonlinear mechanism for the transfer of spectral energy from small-scale pumping waves to large-scale enhanced zonal flows (inverse cascade) is investigated.

In chapter 2 zonal flow generation by Rossby waves in the Earth’s ionosphere is studied. The ionospheric E- and F-regions are considered where the weakly ionized gas exists. The medium becomes electrically conducting due to presence of charged particles. Existence of charged particles will also increase the wave motions degree of freedom. Massive neutral particles are in majority in comparison with charged particles, therefore they are useful to understand the behavior of such a gas. But the horizontal forces acting on the neutral particles
are of the same ratio, less than the Lorentz force experienced by the ions. Thus the dynamics of Rossby waves strongly depends on the interaction of inductive currents with the geomagnetic field. Such interaction in the ionospheric E-region is due to the dominant effect of Hall conductivity which gives rise to magnetized Rossby waves. In the case of ionospheric F-region, Pedersen conductivity gives rise to dissipation and this conductivity behaves as a inductive (magnetic) inhibition. Modified by the interaction exists between the geomagnetic field and inductive currents Charney equation is used as the basic nonlinear equation. Considering comparatively short-scale perturbations only vector nonlinearity causes the coupling between different modes in Charney equation.

It is observed that unlike the usual two components electron-ion plasma the nonlinear waves in electron-positron-ion plasmas behave differently. Due to such actuality in chapters 3 and 4 low-frequency electrostatic drift waves in electron-positron-ion plasmas are investigated. In our case the nonlinear dynamics of these waves are explored in a three component collisionless nonuniform magnetized plasmas.

As long as under the zonal flow action different vortical structures can be maintained, in chapter 3 possibility of the existence of drift vortical motions and the appropriate properties are investigated. In chapter 4 zonal flows (large-wavelength) generation by electrostatic drift waves (small-wavelength)is investigated in a nonuniform EPI plasmas.

In the present thesis the following novelties are deduced:

1. Considering the wave packet of magnetized Rossby (pumping) waves we have observed the influence of non-monochromaticity on zonal flow generation in the Earth’s ionospheric E-layer. The appropriate zonal flow dispersion relation (2.32) is obtained. Two equations (2.48) and (2.54) are obtained for different spectrum broadening. Variation in spectrum broadening causes the change in zonal flow growth rate. The band width is given by equation (2.56), which in turn gives the maximum growth rate (2.57), which is of the order of the hydrodynamic one. In the case when the zonal flow generation by magnetized Rossby modes is prohibited by the Lighthill stability criterion (inverse inequality of (2.39)), the new class of instability, so-called two-stream-like mechanism for the generation of zonal flows (sheared) by magnetized Rossby waves (finite-amplitude) is obtained. Corresponding growth rates are defined (see equations (2.62) and (2.64)).

2. Allowance of the inverse cascade process in dissipative medium is shown. Namely, the possibility of zonal flow generation and appropriate distinctive properties are revealed when Rossby waves are propagating through the dissipative ionospheric F-layer. Zonal flow growth rate is obtained in general form (see equation (2.82)). It is shown that the dissipative
contribution to the total growth rate is independent of zonal flow wave number and instability criteria are not needed (see equation (2.83)). Thus inverse cascade phenomenon always occurs in the Earth’s dissipative F-layer even for small pumping intensity.

3. Generalized Hasegawa-Mima equation containing one vector and two different nature scalar nonlinearities is obtained (see equation (3.7)) which describes the nonlinear dynamics of electrostatic drift waves of arbitrary wavelength propagating through the collisionless nonuniform magnetized electron-positron-ion plasmas. Temperatures of electrons and positrons are inhomogeneous and ions are supposed to be cold.

4. The new space structure of drift waves is obtained (see equation (3.9)) and the spatial increase of the linear plasma-potential perturbations in the direction of density and temperature inhomogeneities is shown.

5. It is shown that the vector nonlinearity is responsible for the existence of small-scale dipole-type solitary vortical structures in electron-positron-ion plasmas (see equation (3.12)). The appropriate conditions for that are found (see inequalities (3.13)).

6. Modified Hasegawa-Mima equation valid for the intermediate-scale (3.14) vortical structures is obtained (see equation (3.15)) for the electron-positron-ion plasmas.

7. Modified Hasegawa-Mima equation valid for the large-scale (3.18) vortical structures is obtained (see equation (3.19)) for the electron-positron-ion plasmas.

8. It is shown that the scalar nonlinearity of Korteweg-de Vries type is responsible for the existence of the intermediate-scale vortical structures.

9. The other scalar nonlinearity under the time derivative creates intermediate and large-scale monopole vortical structures and plays an essential role in different possibilities of zonal flows generation. It causes nonlinear interaction with vector and KdV-type nonlinearities and itself also.

10. It is shown that the dynamics of low-frequency waves investigated in electron-positron-ion plasmas instead of electron-ion plasma.

11. A new self-organization mechanism of formation of large-scale electrostatic drift vortical structures based on the competition between scalar and vector nonlinearities in electron-positron-ion plasmas is discussed.

12. Generation of zonal flows by electrostatic drift waves in a collisionless nonuniform magnetized electron-positron-ion plasmas is studied. To describe this process the generalized Hasegawa-Mima equation is used.
13. Nonlinear interaction of drift waves and zonal flows is given by equations (4.12) and (4.13). In the evolution equation of zonal flows (see equation (4.12)) the driving force is the Reynolds stress.

14. Considering the arbitrary spectrum of pumping drift waves two zonal flow dispersion relations (4.26) and (4.39) of different nature are deduced. They contain the contributions from the vector nonlinearity (4.28), from the Korteweg-de Vries type scalar nonlinearity (4.29), from the $\partial \phi^2 / \partial t$ scalar nonlinearity (4.30), from the interaction of the $\partial \phi^2 / \partial t$ scalar nonlinearity with itself (4.31), from the interaction of the $\partial \phi^2 / \partial t$ scalar nonlinearity with Korteweg-de Vries type scalar nonlinearity (4.32) and from the interaction of the $\partial \phi^2 / \partial t$ scalar nonlinearity with the vector nonlinearity (4.33). Different growth rates with zonal flow generation are developed for electron-positron-ion plasmas.

15. Zonal flow instabilities are observed in detail for different regimes of short-, intermediate- and large-scale structural turbulence when the wave packet is monochromatic. Suitable results for the maximum growth rates, corresponding instability criteria, and for the optimal spatial dimensions belong to zonal flows are found.

16. It is shown that in contrast to usual electron-ion plasma, the existence of positrons in the plasma causes modification of both the zonal flow growth rate and instability conditions.

17. Numerical estimations of the obtained growth rates are given throughout.

18. Obtained in the given thesis results can be used to explain different laboratory and space experiments.
References


