HYBRID QAM - FSK (HQFM) OFDM TRANSCEIVER WITH LOW PAPR

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In the name of Almighty Allah, the Beneficent, the Merciful
“After all these years, I do not know what I may appear to the world; but to myself, I seem to have been only a boy playing on the sea-shore and diverting myself in, now and then, finding a smoother pebble or a prettier shell than ordinary whilst the great ocean of truth lay all undiscovered before me.”

Isaac Newton
Dedicated to My Grandfather

Whose childhood training of learning and consistent efforts
have enabled me to complete this daunting task
DECLARATION

I hereby declare that this dissertation is my own work and that, to the best
of my knowledge and belief, it has not been submitted by another person or
in any form for another degree or diploma at any university or other
institution of higher learning. Information derived from published or
unpublished work of others has been acknowledged in the text and a list of
references is given.

Asma Latif
12th January, 2009
STATEMENT OF ORIGINALITY

1. A new modulator is proposed which make use of hybrid frequency shift keying (FSK) and quadrature amplitude modulation (QAM) exhibiting the advantages of both FSK (power efficiency) and QAM (bandwidth efficiency).

2. 90% and 99% power bandwidth is computed by evaluating fractional out-of band power. It is proved that 99% power bandwidth of the proposed modulator is same to that of QAM.

3. When employed in OFDM systems, it is shown that PAPR is reduced as compared to QAM-OFDM systems. The statistical dependence of PAPR on number of subcarriers, number of FSK tones employed and modulation index is also shown.

4. SER and BER expressions are derived and evaluated for different formats of the proposed modulator both in classical AWGN and Rayleigh fading channel. It is proved that the proposed modulator improves the BER performance as compared to QAM based systems.

5. A phase acquisition algorithm to correct the phase offset for QAM constellation points is developed. It is due to the detection of wrong frequency part of the demodulator in past which due to inherited memory property of FSK affects the current symbol.
ACKNOWLEDGMENTS

In the name of Almighty Allah who gave me courage and support to complete the research work presented in this dissertation.

The research work presented in this dissertation was carried out under the supervision of Dr. Nasir D. Gohar, whose kind attention and advice helped me in completing this project. Special thanks to Higher Education Commission (HEC) for awarding scholarship to pursue my PhD studies and Pakistan Atomic Energy Commission (PAEC) for granting me study leave. I want to pay tribute to Dr. Mohinder Jankiraman (Technical Consultant, Radar & Wireless Communications, Dallas, TX, USA), Dr. Syed Ismail Shah (Iqra University, Islamabad) and Dr. Wasim Q. Malik (MIT, Cambridge, MA, USA) to provide technical assistance which boost up my research. Also I want to appreciate the friendly behavior of many GIKI friends like Shaista Baber (my hostel mate), Saima Saleem (Hostel Warden) and many good undergraduate fellows who always relaxed me whenever I felt tired and hopeless during this work and stay at GIKI. Special thanks to Sobia Baig who always guide me in preparing technical papers and presentations. I am thankful to my PINSTECH/ PIEAS colleagues whose chitchats over a cup of tea make me comfortable which helped me in preparing this dissertation. Above all, I salute my parents who are always cooperative in all my fruitful educational endeavors and pray for my success all the time. My husband’s love will never be forgettable at this moment that always helped me in all situations. Lastly, special regards to my one year child, Aroomah, who is staying at her Granny’s home and waiting for me to complete this dissertation.
HYBRID QAM - FSK (HQFM) OFDM TRANSCEIVER WITH LOW PAPR

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ABSTRACT OF THE DISSERTATION

Orthogonal Frequency Division Multiplexing (OFDM) is an attractive multicarrier technique for mitigating the effects of multipath delay spread of radio channel, and hence accepted for several wireless standards as well as number of mobile multimedia applications. Alongside its advantages such as robustness against multipath fading, spectral efficiency and simple receiver design, OFDM has two major limitations. One of these is its sensitivity to carrier frequency offsets (CFO) caused by frequency differences between the local oscillators in the transmitter and the receiver and the other is high peak to average power ratio (PAPR). This high PAPR is due to the summation of sinc-pulses and non-constant envelope. Therefore, RF power amplifiers (PA) have to be operated in a very large linear region. Otherwise, the signal peaks get distorted, leading to intermodulation distortion (IMD) among the subcarriers and out-of-band radiation. A simple way to avoid is to use PA of large dynamic range but this makes the transmitter

1 Dr. Nasir D. Gohar is currently Professor/HoD, Communication Systems Engineering Department at NUST School of Electrical Engineering and Computer Science, Islamabad.
costly. Thus, it is highly desirable to reduce the PAPR.

In order to reduce the PAPR, several techniques have been proposed such as clipping, coding, peak windowing, Tone Reservation (TR), Tone Injection (TI), Selected Mapping (SLM) and Partial Transmit Sequence (PTS). After studying these schemes, it was found that most of these methods are unable to achieve simultaneously a large reduction in PAPR with low complexity, low coding overhead and without performance degradation and transmitter/receiver symbol handshake.

In this study, an OFDM transceiver is proposed which makes use of hybrid modulation scheme instead of conventional modulator like QAM or PSK. In addition to improved BER performance both in AWGN and frequency selective fading channel, it exhibits low PAPR. The modified OFDM transceiver makes use of multilevel QAM constellations, where the level of QAM is decided by specific number of bits chosen arbitrarily from a group of bits to be encoded in the QAM symbol. The simulated results show that PAPR is considerably reduced, though at the cost of a slight increase in detection complexity. Like PTS or SLM, it works with arbitrary number of subcarriers but needs no side information to be transmitted. It is also shown that PAPR reduction capability of the proposed system is comparable to PTS. However, to further reduce the PAPR, one has to alter this hybrid MQAM/LFSK (HQFM) signal sets like in PTS, but there is no need of transmitting any additional side information. At the receiver, these deformations can be removed in one or two iterations, thus, original data retrieved but with a little increase in the receiver complexity.
LIST OF PUBLICATIONS


Despite of publications cited above, the following list of publications is also the part of PhD research presented in this dissertation.


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**ABBREVIATIONS**

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<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>CCDF</td>
<td>Complementary Cumulative Distribution Function</td>
</tr>
<tr>
<td>CFO</td>
<td>Carrier Frequency Offset</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transforms</td>
</tr>
<tr>
<td>FOBP</td>
<td>Fractional Out-of-Band Power</td>
</tr>
<tr>
<td>FSK</td>
<td>Frequency Shift Keying</td>
</tr>
<tr>
<td>HPA</td>
<td>High Power Amplifier</td>
</tr>
<tr>
<td>HQFM</td>
<td>Hybrid QAM / FSK Modulation</td>
</tr>
<tr>
<td>ICI</td>
<td>Intercarrier Interference</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transforms</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>Identically Independently Distributed</td>
</tr>
<tr>
<td>ISI</td>
<td>Intersymbol Interference</td>
</tr>
<tr>
<td>PAPR</td>
<td>Peak to Average Power Ratio</td>
</tr>
<tr>
<td>p.d.f.</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>PTS</td>
<td>Partial Transmit Sequence</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>r.m.s.</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RS Codes</td>
<td>Reed Solomon Codes</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>WSSUS</td>
<td>Wide Sense Stationary Uncorrelated Scattering</td>
</tr>
</tbody>
</table>

1 The most common symbols and abbreviations are listed here for convenience, however, they are defined throughout the thesis.
SYMBOLS AND MATHEMATICAL CONVENTIONS

\( \in \) = belongs to / is a member of
\( \otimes \) = Convolution
\( \lfloor \ldots \rfloor \) = Floor function, \( \lfloor x \rfloor = \max \{ n \in \mathbb{Z} | n \leq x \} \)

\( \Psi \) = Set of Constellation Points
\( \xi \) = PAPR

\( n \) = Number of bits per HQFM symbol
\( Q = ML = 2^n \) = HQFM Signal size
\( k = \log_2(M) \) = Number of QAM bits
\( M = 2^k \) = QAM size
\( L = 2^{n-k} \) = Number of FSK Frequencies
\( f_\Delta \) = FSK Tone Separation (Hz)
\( h = f_\Delta T_s \) = Modulation Index

\( T_b \) = Bit duration (secs)
\( R_b = 1/T_b \) = Data Rate in bits per sec (bps)
\( T_s = T_b \log_2(Q) = T_b \log_2(ML) \) = HQFM Symbol Period (secs)
\( N \) = IFFT/FFT Length i.e. Number of subcarrier
\( N_{CP} \) = Number of subcarriers in Cyclic Prefix
\( T_{CP} \) = Cyclic Prefix Period (secs)
\( T = N T_s \) = OFDM Symbol Period (secs)
\( T_t = T + T_{CP} \) = Total OFDM Symbol Period (secs)
\( \Delta f = 1/T = 1/NT_s \) = OFDM Subcarrier Spacing (Hz)

\( \Phi(f) \) = Power Spectral Density (PSD)
\( B \) = Bandwidth
\( \eta_B = R_b/B = 1/B T_b \) = Null-to-Null Bandwidth Efficiency
\(\mu_B\) = Fractional Out-of-Band power (FOBP)

\(B_{90\%}\) = 90\% Power Bandwidth

\(B_{99\%}\) = 99\% Power Bandwidth

\(\mathcal{E}_b/N_0 = \gamma_b\) = Bit Energy to Noise Ratio

\(\mathcal{E}_s/N_0 = \gamma_s\) = Symbol Energy to Noise Ratio

\(P_b\) = Probability of Bit Error (BER)

\(P_e\) = Probability of Symbol Error (SER)

\(\binom{n}{r}\) = Binomial Coefficient, \(\binom{n}{r} = \frac{n!}{r!(n-r)!}\)

\((.)^*\) = Complex Conjugate

\((.)^H\) = Complex Conjugate Transpose

\(E\{.\}\) = Statistical expectation

\(Re(.)\) = Real part of complex number

\(j = \text{Imaginary unit}, j^2 = -1\)

\(p(u)\) = Probability Density Function (p.d.f.),

For Rayleigh \(p(u) = u/\sigma^2 \cdot e^{-u^2/2\sigma^2}; \quad u > 0\)

For Rice \(p(u) = u/\sigma^2 \cdot I_0(uA/\sigma^2) \cdot e^{-(u^2+A^2)/2\sigma^2}; \quad u > 0\)

\(Pr\{.\}\) = Outage Probability of Occurrence

\(Q(.)\) = Gaussian Q-Function, \(Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} \, dt\)

\(I_0(.)\) = \(0^\text{th}\) order modified Bessel function of \(1^\text{st}\) kind, \(I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{\cos\theta} \, d\theta\)

\(\mathbb{Z}^+\) = Set of Positive Integers

\(f_c\) = Centre Frequency

\(f_D\) = Doppler Shift

\(\tau_{\text{rms}}\) = Root Mean Square (r.m.s.) Delay spread

\(\tau_{\text{max}}\) = Maximum Delay Spread

\(\mathbb{F}\{.\}\) = Fast Fourier Transforms (FFT), \(X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, dt; \quad \forall \{f, t\} \in \mathbb{R}\)

\(\mathbb{F}^{-1}\{.\}\) = Inverse Fast Fourier Transform (IFFT), \(x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} \, df\)
Wireless communications is an emerging field, which has seen enormous growth in the last several years. The unprecedented and ubiquitous use of mobile phone technology, rapid expansion in wireless local area networks (WLAN) and the exponential growth of the Internet have resulted in an increased demand for new methods of establishing high capacity wireless networks.

As the wireless standards evolved, the access techniques used also exhibited increase in efficiency, capacity and scalability. The first generation wireless standards used Frequency Division Multiple Access (FDMA) or Time Division Multiple Access (TDMA). In wireless channels, FDMA consumed more bandwidth for guard to avoid intercarrier interference (ICI) and TDMA proved to be less efficient in handling high data rate channels as it requires large guard periods to alleviate the multipath impact. So, in 2G (2nd Generation) systems, one set of standard like Global System for Mobile communications (GSM) [Garg, Wilkes: 99] used combined TDMA and FDMA¹ and the

¹ FDMA in the sense that GSM uses two frequency band around 900MHz or 1800MHz. One for forward link and one for reverse link.
other set like Interim Standard 95 (IS-95) [Garg; 99] introduced a new access scheme called Code Division Multiple Access (CDMA). 2G, 2.5G and even 2.75G mobile networks, made possible to completely converge mobile phone technology with many multimedia applications such as high quality audio/video, computing and high-speed Internet access.

Usage of CDMA increased the system capacity and placed a soft limit on it rather than the hard limit. Data rate is also increased; hence, allowing greater cellular spectral efficiency, as this access scheme is efficient enough to handle the multipath channel. This enabled the 3G (3rd Generation) systems to use CDMA as the access scheme. 3G systems such as the Universal Mobile Telecommunications System (UMTS) [Richard: 00] [Berruto, Colombo: 97] [Dahlman, Gudmundson: 98] [Adachi, Sawahasi: 98] and International Mobile Telecommunications 2000 (IMT2000) support a wide range of services including wireless appliances, notebooks with built in mobile phones, remote logging, wireless web cameras and car navigation systems. These systems provide higher data rates (64 kbps - 2 Mbps [Richard: 00]), using either wide-band CDMA (WCDMA) [Dehghan, Lister: 00] or cdma2000 [Knisely, Kumar: 98] as carrier modulation scheme.

To go beyond 3G, 4G (4th Generation) mobile networks are evolving to provide a comprehensive IP-based integrated solution at an affordable price where voice, data and streamed multimedia can be given to users on an anytime, anywhere basis, and at higher data rates than previous generations. This will be achieved after the convergence of all types of wired and wireless technologies and will be capable of providing data rates between 100 Mbps and 1 Gbps (both indoors and outdoors), with premium quality and high security.

High data rate calls upon an improved spectral efficiency. Also, the demand for radio spectrum is becoming high, with terrestrial mobile phone systems being just one of many applications vying for more bandwidth. These applications require the system to operate reliably in non-line-of-sight environments with a propagation distance of 0.5 - 30 km, and at speeds up to 100 km/hr or higher. This limits the maximum RF frequency to 5 GHz, making the value of the radio spectrum extremely high, because operating above this frequency results in excessive channel path loss and Doppler spread at high velocity.

The only issue with the CDMA is that it suffers from poor spectrum flexibility
and scalability. Orthogonal Frequency Division Multiplexing (OFDM), an alternative wireless modulation technology, has the potential to surpass the capacity of CDMA systems. Hence, it is proved to be a potential candidate for the physical layer of next 4G mobile systems.

1.1 Orthogonal Frequency Division Multiplexing

OFDM is a modulation scheme that allows digital data to be efficiently and reliably transmitted over a radio channel and performs well even in multipath environments with reduced receiver complexity. Using OFDM, it is possible to exploit the time domain, the space domain, the frequency domain and even the code domain to optimize radio channel usage.

OFDM transmits data by using a large number of narrow-band subcarriers. These subcarriers are regularly spaced in frequency, forming a block of spectrum. The frequency spacing and time synchronization of the subcarriers is chosen in such a way that the subcarriers are orthogonal, meaning that they do not cause interference to one another. This is despite the subcarriers overlapping each other in the frequency domain. The name ‘OFDM’ is derived from the fact that the digital data is sent using many subcarriers, each of a different frequency (Frequency Division Multiplexing), which are orthogonal to each other, hence Orthogonal Frequency Division Multiplexing.

Figure 1.1(a) shows the construction of such an OFDM signal (real part only) with 5 subcarriers. Note that each subcarrier has an integer number of cycles per symbol, making them cyclic. Adding a copy of the symbol to the end would result in a smooth join between symbols (figure 1.1c). These subcarriers have sinc (sinx/x) response in frequency domain (figure 1.1b). This is a result of the symbol time corresponding to the inverse of the subcarrier spacing, \( \Delta f = 1/T \) where \( T \) is OFDM symbol period. The sinc shape has a narrow main lobe, with many side-lobes that decay slowly with the magnitude of the frequency difference away from the centre. Each subcarrier has a peak at the centre frequency and nulls evenly spaced with a frequency gap equal to the subcarrier spacing. The orthogonal nature of the transmission is a result of the peak of each subcarrier corresponding to the nulls of all other subcarriers. Therefore, there is no
intercarrier interference (ICI). Figure 1.1(d) shows the overall combined frequency response of OFDM. Since, the entire channel bandwidth is divided into many closely spaced sub-bands (or subcarriers); the frequency response over each subcarrier becomes relatively flat, making equalization potentially simpler than single-carrier system.

![Construction of OFDM with 5 subcarriers](image)

**Figure 1.1:** Construction of OFDM with 5 subcarriers (a) Time Domain Representation (b) Frequency Domain representation (c) Overall sum of subcarriers (Time Domain) (d) Overall combined frequency response of subcarriers

OFDM can be easily implemented using Fast Fourier Transforms (FFT), an efficient digital signal processing (DSP) realization of DFT. The reliance on DSP prevented the wide spread use of OFDM during its early development. It wasn’t until the late 1980’s that work began on the development of OFDM for commercial use, with the introduction of the digital audio broadcasting (DAB) system. Thanks to recent advances in integrated circuit technology that have made the implementation of OFDM cost effective.

OFDM also provides a frequency diversity gain, improving the physical layer performance. It can also be employed as a multiple access technology [Rohling, Gruneid: 97]. In Orthogonal frequency division multiple access (OFDMA), each OFDM symbol can transmit information to/from several users using a different set of subcarriers. This
not only provides additional flexibility for resource allocation (increasing the capacity), but also enables cross-layer optimization of radio link usage [Sari, Karam: 98] [Pietrzyk: 06]. In multcarrier CDMA (MC-CDMA), OFDM is combined with CDMA, for coding separation of users [Hara, Prasad: 97]. OFDM is also compatible with other enhancement technologies, such as smart antennas [Nasr, Costen: 04][Hu, Guo: 05] and multiple input multiple output (MIMO) systems [Jankiraman: 04] [Zelst:04].

1.2 Pros and Cons of OFDM

The key advantages of OFDM transmission systems are:

- OFDM is an efficient way to deal with multipath; for a given delay spread, the implementation complexity is significantly lower than that of single-carrier system with equalizer.
- In relatively slow time varying channels, it is possible to enhance capacity significantly by adapting the data rate per subcarriers according to SNR of that particular subcarrier.
- OFDM is robust against narrowband interference because such interference affects only a small percentage of the subcarriers.
- OFDM makes single frequency networks (transmitter macrodiversity) possible, which is especially attractive for broadcasting applications.

On the other hand, it has two main drawbacks:

- OFDM is more sensitive to carrier frequency offset (CFO) and phase noise.
- OFDM has relatively large peak to average power ratio (PAPR), which tends to reduce the power efficiency of RF power amplifier.
1.3 Peak-to-Average Power Ratio

Typically, the PAPR is not an issue with constant amplitude signals. With non-constant amplitude signals, however, it is important to deal with the PAPR of those signals. In fact, the PAPR problem also arises in many cases other than OFDM transmission. For example, a DS-CDMA signal suffers from the PAPR problem especially in the downlink because it is the sum of the signals for many users [Han, Lee: 05]. In this thesis, however, we limit our attention to the PAPR problem in OFDM transmission only.

The most severe disadvantage of using several subcarriers in parallel using IFFT is the highly non-constant envelope of the transmit signal, making OFDM very sensitive to nonlinear components in the transmission path. A key component is the high power amplifier (HPA). Due to cost, design and most importantly power efficiency considerations, the HPA cannot resolve the dynamics of the transmit signal and inevitably cuts off the signal at some point causing additional in-band distortions and adjacent channel interference. The power efficiency penalty is certainly the major obstacle to implement OFDM in low-cost applications. Moreover, in power-limited regimes determined by regulatory bodies, the average power is reduced compared to single-carrier systems reducing in turn the range of transmission. The power control problem motivates further research since it touches on many of the advantages that originally made OFDM transmission popular, i.e. spectral efficiency and implementation issues.

1.4 Motivation

OFDM is generated by mapping the bits to some modulation scheme like quadrature amplitude modulation (QAM) or phase shift keying (PSK) and converted to time domain signal through IFFT operations. [Greenstein, Fitzgerald: 81], [Boyd: 86], [Bos: 87] and several authors reported that PAPR in OFDM is due to the summation of phase angles and amplitudes of different subcarriers. Therefore, efforts have been made to modify the PSK (phase information) or QAM (both phase and amplitude information) symbols so that the PAPR of the OFDM is reduced.
In 2002, [Tassaduq, Rao: 02 j-a & -b] used continuous phase modulation (CPM) instead of PSK or QAM to reduce PAPR in OFDM systems. They exploit the constant envelope and correlated phase states property of the CPM. This motivates the use of orthogonal frequency shift keying (FSK), a member of general class CPM, to be used in OFDM systems instead of QAM or PSK as mapping of information bits and study the PAPR properties of OFDM. But $M$-ary FSK with modulation index $h = 1$, is not bandwidth efficient and we cannot use it in conjunction with OFDM for $M > 4$. On the other side due to orthogonality, its power efficiency increases with increasing $M$.

Linear modulation, like PSK or QAM, in contrast to non-linear (FSK), is bandwidth efficient but power inefficient scheme. The idea to get the bandwidth efficiency of linear modulation and power efficiency of non-linear modulation leads to the birth of hybrid modulation like hybrid QAM-FSK modulation (HQFM) which is main theme of this thesis. In this thesis, the peak power problem in OFDM using HQFM is analyzed. Several other properties like bandwidth occupancy, fractional out-of-band power and power efficiency are also discussed in this dissertation.

We concentrate on a single-user point-to-point communication link and investigate the impact of the non-constant signal envelope on system performance. We would like to emphasize that there are several other disadvantages such as synchronization effects, frequency offsets and channel estimation etc. that we do not comment on here. Also the multi-user or broadcast case and multiple antennas will not be considered in this dissertation.

1.5 Dissertation Outline and Contributions to Field

The dissertation is divided in 7 chapters with classification of three main parts: i) OFDM ii) HQFM signaling and its PAPR properties and iii) Transceiver’s performance in classical AWGN and Fading Channels. The organization of dissertation is described below:

**Fundamentals of OFDM:** Chapter 2 describes the fundamental concept of OFDM. A detail mathematical model of continuous time OFDM signal is described in this chapter. After describing the research challenges, the reason and influence of high
PAPR in OFDM is mentioned. This chapter also discusses several methods to alleviate this problem. One such method, known as Partial Transmit sequences (PTS) is discussed in detail as its PAPR reduction capability is compared with the proposed scheme in next chapters.

**Hybrid MQAM-LFSK (HQFM) Signaling:** In Chapter 3, this hybrid modulation scheme is introduced using the signal space concept. The power spectral density (PSD) and fractional out-of-band power (FOBP) expressions for HQFM has been evaluated. It has been shown in this chapter that the bandwidth efficiency of HQFM in terms of 99% of power containment is almost equal to MQAM systems.

Chapter 4 describes the PAPR properties of HQFM when employed in OFDM systems. Several factors like modulation index \( h \), QAM size \( M \), number of FSK frequencies \( L \) and number of subcarriers \( N \) are varied to show their dependence on PAPR’s statistics.

**BER Performance:** Chapter 5 describes the bit error rate (BER) performance of HQFM in classical AWGN channel. Expression for symbol error rate (SER) was evaluated using maximum a posteriori probability (MAP) criterion [Proakis: 89] by choosing the largest posterior probability. Then bit error probabilities are expressed in terms of symbol error rate expression. These expressions are then confirmed through Monte Carlo simulations.

Chapter 6 describes the BER performance of HQFM and expressions are evaluated for slow Rayleigh fading channel by averaging the probability of symbol error in AWGN (evaluated earlier in chapter 5) over the possible strength in fading channels. The dependence of QAM size \( M \) or number of frequencies \( L \) on BER performance is observed. The results are also obtained by passing the systems through several standard channels.

Chapter 7 concludes the dissertation and directions of research in future are mentioned.


1.6 Terms and Notations

Although $M$ is used for number of signals in a particular modulation scheme, e.g. MPSK, MQAM or MFSK, but since HQFM is hybrid of two modulation schemes so $L$ is used for number of FSK frequencies and $M$ is used as QAM size in a particular $L/M$ HQFM signal set.

Also, to distinguish two different types of the frequency separation encountered in HQFM-OFDM, $f_\Delta$ is used for FSK tone separation within the HQFM symbol and $\Delta f$ is frequency spacing among OFDM subcarriers. To further avoid the confusion, the term modulation index, $h = f_\Delta T_s$, is used, where $T_s$ is the symbol period of HQFM signal.

If $Q = ML$ are total number of HQFM signals and $n$ is the total number of bits used to represent a single HQFM signals then $n = \log_2 ML$ or $ML = Q = 2^n$. If $T_s$ is the symbol period and $T_b$ is the bit duration then $T_s = T_b \log_2 ML$. If $N$ is the number of OFDM subcarriers then the OFDM symbol duration is defined as $T = NT_s$ and frequency separation then becomes $\Delta f = 1/T = 1/NT_s$. The total OFDM symbol period is the sum of OFDM symbol period $T$ and the duration of cyclic extension $T_{CP}$ i.e. $T_t = T + T_{CP}$.

REFERENCES


Multimedia is effectively an infrastructure technology with widely different origins in computing, telecommunications, entertainment and publishing. New applications are emerging, not just in the wired environment, but also in the mobile one. At present, only low bit-rate data services are available to the mobile users. However, demands of the wireless multimedia broadband system are anticipated within both public and private sector.

In multimedia communication, a demand for high-speed, high-quality digital mobile portable reception and transmission is emerging all around the globe. A receiver has to cope with a signal that is often weaker than desirable and that contains many echoes. Simple digital systems do not work well in the multipath environment. Multimedia communication has rather large demands upon bandwidth and quality of service (QoS) compared to what is available today to the mobile user. Bit-rates for multimedia span from few kbps (for voice) to about 20 Mbps (for HDTV) or even more in the peak hours. When solving this problem, the question is how to put this large bit
stream on air with sufficient QoS guarantees, i.e. which modulation can compromise all contradicting requirements in the best manner.

The radio environment is harsh, due to many reflected waves and other effects. Using adaptive equalization techniques at the receiver could be the solution, but there are practical difficulties in operating this equalization in real-time at several Mbps with compact, low-cost hardware. A promising candidate that eliminates the need for the complex equalizers is the Orthogonal Frequency Division Multiplexing (OFDM) [Bingham: 90], a multicarrier modulation technique.

2.1 Historical Background of OFDM

It is reported that OFDM based systems were present during World War II (1939-1945). US military systems used OFDM in several high frequency applications e.g. KINEPLEX, ANDEFT and KATHRYN [Parasad: 04] [Pennington: 89]. In KATHRYN, 34 low rate channels, with 82 Hz channel spacing, using PSK modulation were generated by orthogonal frequency multiplexing [Zimmerman, Kirsch: 67].

For the first time in literature, [Chang: 66] introduced the concept of using parallel data transmission by means of frequency division multiplexing (FDM) with overlapping subcarriers. This was an efficient way to avoid the use of high-speed equalization and to combat impulsive noise, and multipath distortion as well as to fully use the available bandwidth. Soon after Chang’s paper, [Saltzberg: 67] made a performance analysis of OFDM transmission system. A theoretical performance analysis of OFDM subject to a number of degrading factors normally encountered by a practical operating system was also studied in 1968 [Chang, Gibby: 68]. The factors considered jointly were sampling time error, carrier phase offset, and imperfect phase characteristics of transmitting and receiving filters. A U.S. patent was filed in November 1966 and issued in January 1970 [Chang: 70].

For a large number of subcarriers, the arrays of sinusoidal generators and coherent demodulators required in a parallel system become unreasonably expensive and complex. The receiver needs precise phasing of the demodulating subcarriers and sampling times in order to keep crosstalk between subcarriers acceptable. [Weinstein,
Ebert: 71] gave an idea of replacing large numbers of coherent demodulators by making IDFT and DFT as modulation and demodulation respectively. Now, after evolutionary development in DSP and VLSI technologies, high speed chips can be built around special-purpose hardware performing the large size Fast Fourier Transform (FFT) (efficient algorithm for DFT) at affordable price [Despain: 79] [Bidet, Castelain: 95].

Another important contribution was due to [Peled, Ruiz: 80], who introduced the cyclic prefix (CP), solving the problem of maintaining the orthogonality among subcarriers. This was an era, when OFDM was studied for high-speed modems [Trailblazer: 85] [Hirosaki: 85], digital mobile communications [Cimini: 85] and high-density recording. One of the systems used a pilot tone for stabilizing subcarrier and clock frequency control and trellis coding was implemented [Höher: 91].

Since 1990s, OFDM has been explored for wideband data communications over mobile radio FM channels [Casas, Leung: 91-92], high-bit-rate digital subscriber lines (HDSL; 1.6 Mbps) [Chow, Dhahir: 93], asymmetric digital subscriber lines (ADSL; 1.536 Gbps) [Chow, Tu: 91 j-a], very high-speed digital subscriber lines (VHDSL; 100 Mbps) [Chow, Tu: 91 j-b], high definition television (HDTV) terrestrial broadcasting [Toutier, Monnier: 93] and digital terrestrial television broadcasting (DTTB) [Sari, Karam: 95] [Zou, Wu: 95]. Digital Audio Broadcasting (DAB) was the first standard to use OFDM [ETSI: 97]. It intended to replace analog technologies such as AM and FM and is designed to be a single frequency network (SFN), in which the user receives same signals from several different transmitters. Later on, in Europe and USA, OFDM was used as Digital Video Broadcasting (DVB) [ETSI: 98] standard. DVB is intended for broadcasting digital television over satellites, cables and thorough terrestrial (wireless) transmission.

A first prototype of OFDM-based wireless ATM network demonstrator (WAND) was implemented by Magic©. It largely impacted standard activities in high-rate wireless communications around 5 GHz band and formed the basis for (high performance local area networks version 2) hiperLAN/2 [Aldis, Althoff: 96].

Many standards have been proposed for wireless local area networks (WLANs) for 900 MHz, 2.4 GHz and 5 GHz ISM band based on spread-spectrum technology. In June 1997, first OFDM-based WLAN standard, IEEE 802.11, was released supporting 1
Mbps and optionally 2 Mbps using 2.4 GHz frequency band. Later on, in July 1999, IEEE approved 802.11a for packet-based radio transmission of 5 GHz at a rate up to 54 Mbps using OFDM. Meanwhile, ETSI BRAN standardized another standard hiperLAN/2, which adopted OFDM for their PHY standards [ETSI : 99]. In early 2001, the FCC announced new rules allowing additional modulations in the 2.4 GHz range, allowing IEEE to extend 802.11b (11 Mbps using DSSS\(^1\) technique) to support 54 Mbps, resulting in an OFDM based 802.11g standard. [Parasad: 04].

### 2.2 Qualitative Description of OFDM

In OFDM the available spectrum is divided into many closely spaced parallel subcarriers, each one being modulated by a low rate data stream [Bingham: 90]. By reducing the bit-rate per subcarrier (not the total bit-rate), the influence of ISI is significantly reduced. As the frequency response over each subcarrier is relatively flat, equalization is potentially simpler than in a serial data system.

Spectral efficiency is achieved by making all the OFDM subcarriers orthogonal, i.e. it is possible to arrange OFDM subcarriers such that the signals can still be received without adjacent carrier interference. In order to do this, all subcarriers must be mathematically orthogonal i.e. if given a set of signals \( \Psi \), each with symbol period \( T \), the signals are orthogonal if

\[
\int_0^T \psi_p(t)\psi_{p'}(t)dt = \begin{cases} 
\delta[p-p'] & p = p' \\
0 & p \neq p'
\end{cases} \tag{2.1}
\]

where the \( \delta[.] \) and \( (.)^* \) indicates the Kronecker delta function and complex conjugate respectively. In order to preserve orthogonality among OFDM subcarriers, following must be true:

1. The receiver and the transmitter must be perfectly synchronized. This means they both must assume exactly the same modulation frequency and the same time-scale for transmission (which usually is not the case).

---

\(^1\) DSSS stands for Direct Sequence Spread Spectrum
2. The analog components in transmitter and receiver must be of very high quality.

3. There should be no multipath channel, which is always present when considering the wireless environment. An easy solution for this problem is to add the cyclic prefix (CP) to each OFDM symbol. Of course this is not for free, since by preceding the useful part of the symbol, some parts of the signal is lost that cannot be used for transmitting information.

2.3 OFDM Generation

OFDM is generated by firstly choosing the spectrum required, based on the input data, and modulation scheme \( \{\Psi\} \) used (typically PSK [Zimmermann, Kirsch: 67], DPSK [Pennington: 89], QAM [Hirosaki: 81], CPM [Tassaduq, Rao: 02], ASK [Xiong: 03] or FSK [Wetz, Periša: 07]). Each subcarrier to be produced is assigned some data to transmit. The spectrum is then converted to its time domain representation using IFFT. The IFFT provides a simple way of ensuring that the subcarriers produced are orthogonal. If the number of complex data points, \( \{N_{\text{used}}\} \) (taken from \( \Psi \)) is less than the required length, \( \{N\} \) (must be an integral power of 2 i.e. \( \log_2 N \in \mathbb{Z}^+ \)) then zero padding is done.

One of the most important properties of OFDM transmissions is the robustness against multipath delay spread. This is achieved by having a long symbol period i.e. by addition of guard periods between transmitted symbols to minimize ISI. This guard period allows time for multipath signals from the previous symbol to die away before the information from the current symbol is gathered. The most effective guard period to use is a cyclic extension of the symbol [Peled, Ruiz: 1980] [Henkel, Tuböck: 2002]. If a mirror in time, the end of the symbol waveform is put at the start of the symbol as the guard period (see figure 2.1); this effectively extends the length of the symbol, while maintaining the orthogonality of the waveform.

Using this cyclic prefixed symbol, the samples required for performing the FFT (to decode the symbol), can be taken anywhere over the length of the symbol. This
provides multipath immunity as well as symbol time synchronization tolerance. The ratio of the cyclic prefix \( T_{CP} \) to useful symbol duration \( T \) is application-dependent. Since CP insertion will reduce data throughput, therefore \( T_{CP} \leq T/4 \) is usually considered. Another reason to use cyclic prefix is that the cyclic convolution can still be applied between the OFDM signal and the channel response to model the transmission system.

![Cyclic Prefixed OFDM](image)

**Figure 2.1:** Cyclic Prefixed OFDM

After appending CP, the parallel discrete data symbols are again made serial, converted to analog and low-pass filtered for RF up conversion. The receiver performs the inverse process of the transmitter. A typical discrete time model of OFDM systems is given in figure 2.2.

Following assumptions are employed while running the simulations:

- A cyclic prefix is used.
- The impulse response of the channel is shorter than the cyclic prefix.
- Transmitter and receiver are perfectly synchronized.
- Channel noise is complex, additive and white Gaussian.
- The fading is slow enough for the channel to be considered constant during one OFDM symbol interval.
- Input data is considered as statistically independent and identically distributed random variables.
2.4 Mathematical Description of OFDM

A continuous time OFDM model presented below is considered an ideal OFDM system, which in practice can be digitally synthesized [Edfors, Sandell: 96].

Assume an OFDM system with $N$ subcarriers, a bandwidth of $1/T_s$ Hz and a symbol length of $T_s = T + T_{CP}$ seconds, the transmitter uses a following waveform

$$
\phi_p(t) = \begin{cases}
  \frac{1}{\sqrt{T_s - T_{CP}}} e^{j2\pi \frac{1}{NT_s} p(t - T_{CP})} & t \in [0,T_s] \\
  0 & \text{otherwise}
\end{cases}
$$

(2.2)

where $T = NT_s$ and $T_{CP}$ is the length of cyclic prefix in seconds. The waveforms $\phi_p(t)$, used in modulation and transmitted baseband signal for OFDM symbol number $q$, are

$$
s_q(t) = \sum_{p=0}^{N-1} x_{p,q} \phi_p(t - qT)
$$

(2.3)

where $x_{p,q} = [x_{0,q}, x_{1,q}, \ldots, x_{N-1,q}]$ are complex numbers from a set of signal constellation points, $\{\Psi\}$. 

\textbf{FIGURE 2.2:} Basic OFDM Transmitter and Receiver
Combining equations (2.2) and (2.3), the actual expression for the OFDM symbol $q$, $\{s_q(t)\}$, can be rewritten as

\[
s_q(t) = \frac{1}{\sqrt{T_s - T_{CP}}} \sum_{p=0}^{N-1} x_{p,q} e^{j2\pi \frac{p}{N} (t-qT-T_{CP})}
\]  (2.4)

Ignoring the length of the cyclic prefix, $\{T_{CP}\}$, equation (2.4) reduces to the formal expression for IFFT i.e.

\[
s_{r,q} = \frac{1}{\sqrt{N}} \sum_{p=0}^{N-1} x_{p,q} e^{j2\pi \frac{p}{N} r} = \mathbb{Z}^{-1} \{x_{p,q}\} \quad 0 \leq r < N \quad 0 \leq p < N
\]  (2.5)

where $s_{r,q} = [s_{0,q}, s_{1,q}, \ldots, s_{N-1,q}]$ are carrier amplitudes associated with OFDM symbol. An equivalent OFDM modulator can be depicted as shown in figure 2.3(a). When an infinite sequence of OFDM symbols is transmitted, the output is

\[
s(t) = \sum_{q=-\infty}^{\infty} s_q(t) = \sum_{q=-\infty}^{\infty} \sum_{p=0}^{N-1} x_{p,q} \phi_p(t-qT)
\]  (2.6)

If the impulse response of the channel is within $T_{CP}$, then, the OFDM signal received is

\[
r_q(t) = (h_q \otimes s_q)(t) + n_q(t) = \int_{-\infty}^{T_{CP}} h_q(\tau) s_q(t-\tau) d\tau + n_q(t)
\]  (2.7)
where \( n_q(t) \) is additive, white and complex Gaussian noise.

The OFDM receiver (figure 2.3b) can be considered to be consisted of a filter bank, matched to the last part \([T_{CP}, T_i]\) of the transmitted waveform \( \phi_p(t) \), i.e.

\[
\psi_p(t) = \begin{cases} 
\phi_p^*(T_i - t) & t \in [0, T_i - T_{CP}] \\
0 & \text{otherwise}
\end{cases}
\]  

Effectively this means that cyclic prefix is removed. Since cyclic prefix contains all the ISI from the previous symbols, the sampled output from the receiver’s filter bank contains no ISI. The sampled output of the \( p' \)-th matched filter in the \( q \)-th interval can be derived using equations (2.6) (2.7) and (2.8)

\[
y_{p,q} = (r_q \otimes \psi_{p'})(t) \big|_{t=T_i} = \int_{-\infty}^{\infty} r_q(t) \psi_{p'}(T_i - t) \, dt
\]

\[
= \int_{T_{CP}}^{T_i} \left( \int_{0}^{T_{CP}} h_q(\tau) \left[ \sum_{p=0}^{N-1} x_{p,q} \phi_p(t - qT - \tau) \right] d\tau \right) \phi_{p'}^*(t) \, dt
\]

\[
+ \int_{T_{CP}}^{T_i} n_q(T_i - t) \phi_{p'}^*(t) \, dt
\]

\[
= \sum_{p=0}^{N-1} x_{p,q} \int_{T_{CP}}^{T_i} \left( \int_{0}^{T_{CP}} h_q(\tau) \phi_p(t - qT - \tau) d\tau \right) \phi_{p'}^*(t) \, dt + n_{p',q}
\]  

where \( n_{p',q} \) is AWGN. The integration intervals \( T_{CP} < t < T_i \) and \( 0 < \tau < T_{CP} \) implies \( 0 < t - \tau < T_{CP} \). Consider the inner integral

\[
\int_{0}^{T_{CP}} h_q(\tau) \phi_p(t - qT - \tau) \, d\tau = \frac{1}{\sqrt{T_i - T_{CP}}} \int_{0}^{T_{CP}} h_q(\tau) e^{-j2\pi p(t - qT - \tau - T_{CP})/NT_i} \, d\tau
\]

\[
= \frac{e^{j2\pi p(t - T_{CP})/NT_i}}{\sqrt{T_i - T_{CP}}} \int_{0}^{T_{CP}} h_q(\tau) e^{-j2\pi p\tau NT_i} e^{-j2\pi pqT/NT_i} \, d\tau
\]

\[
= \frac{e^{j2\pi p(t - T_{CP})/NT_i}}{\sqrt{T_i - T_{CP}}} \Im \left( h_q(\tau) \right)
\]

\[
= \frac{e^{j2\pi p(t - T_{CP})/NT_i}}{\sqrt{T_i - T_{CP}}} h_{p,q}
\]  

From equation(2.2), equation(2.10) becomes

\[
\int_{0}^{T_{CP}} h_q(\tau) \phi_p(t - qT - \tau) \, d\tau = \phi_{p'}(t) h_{p,q}
\]  

20
FIGURE 2.4: OFDM System can be interpreted as parallel Gaussian channels

Now equation (2.9) can be rewritten as

\[
y_{p,q} = \sum_{p=0}^{N-1} x_{p,q} h_{p,q} \int_{t_{CP}}^{T} \phi_p(t)\phi^*_p(t)dt + n_{p,q}
\]  
\[
(2.12)
\]

Since \(\phi_p\) and \(\phi^*_p(t)\) are orthogonal to each other, the received OFDM symbol simplifies to

\[
y_{p,q} = \sum_{p=0}^{N-1} x_{p,q} h_{p,q} \delta(p-p') + n_{p',q}
\]
\[
= h_{p,q}x_{p,q} + n_{p,q}
\]  
\[
(2.13)
\]

Thus OFDM can be viewed as set of parallel Gaussian channel (figure 2.4) [Edfors, Sandell: 96].

The transmitted energy per subcarrier is

\[
\int \left| \phi_p(t) \right|^2 dt = \int \left| \frac{1}{\sqrt{T_t-T_{CP}}} e^{j2\pi \frac{1}{T_t} t(t-T_{CP})} \right|^2 dt
\]
\[
= \frac{1}{T_t-T_{CP}} \int dt = \frac{T_t}{T_t-T_{CP}}
\]  
\[
(2.14)
\]

Here an interesting point is to be noted. The transmitted energy increases as the length of the cyclic prefix increases but the expression for received signal \(y_{p,q}\) (equation (2.13)) remains the same.
2.5 Research Challenges

OFDM has several advantages such as robustness to multipath distortions, high spectral efficiency as compared to FDM or TDM and provides better system throughput. But, it has certain associated problems as well. These problems can limit its utility, so we, have to overcome these limitations.

- **Sensitivity to Carrier Frequency Offset (CFO):** CFO causes number of impairments including phase noise introduced by nonlinear channels, attenuation and rotation of each of the subcarriers and intercarrier interference (ICI). ICI, disturbing the orthogonality among subcarriers, is caused due to the relative movement between transmitter and receiver resulting in doppler frequency shifts. Another destructive effect caused by CFO in OFDM systems, is the reduction of signal amplitude. The sinc functions are shifted and no longer sampled at the peak. A number of methods have been developed to reduce this sensitivity to frequency offset [Beek, Sandell: 97] [Landström, Arenas: 97].

- **Symbol Synchronization:** The objective here is to know when the symbol starts. The receiver has to estimate the symbol boundaries and the optimal timing instants that minimize the effects of ICI and ISI. Due to the centre frequency difference of the transmitter and receiver, each signal sample at time $t$ contains an unknown phase factor $e^{j2\pi f_c t}$. This unknown phase factor must be estimated and compensated for each sample before FFT at the receiver; otherwise the orthogonality is lost. See Ref: [Muller, Huber: 98] [Landström, Petersson: 01]

- **Phase Noise:** Another associated problem with OFDM systems is the effect of phase noise [Armada: 01]. Phase noise is present in all practical oscillators and it manifests itself in the form of random phase modulation of the carrier. The effect of
both phase noise and frequency offsets is worse in OFDM than single-carrier systems. The use of efficient frequency and phase estimation schemes can help reduce these effects.

- **Peak to Average Power Ratio (PAPR):** OFDM signal exhibits a very high PAPR, which is due to the summation of sinc-waves and non-constant envelope. Therefore, RF power amplifiers have to be operated in a very large linear region. Otherwise, the signal peaks get into non-linear region causing signal distortion. This signal distortion introduces intermodulation among the subcarriers and out-of-band radiation. A simple way to avoid is to use an RF amplifier of large dynamic range but this makes the transmitter costly. Thus, it is highly desirable to reduce the PAPR. Various techniques have been investigated with a view to reducing the problems caused by PAPR. Theoretically, the difference of PAPR between any multicarrier and single-carrier is a function of number of subcarriers \( N \) i.e. \( \Delta(dB) = 10\log N \). When \( N = 1000 \), the difference can be as large as 30dB. However, this theoretical value rarely occurs. Well scrambled input data lowers the chances of reaching this maximum limit, especially, when constellation size is large [Merchan, Armada: 98].

### 2.6 Impact of HPA on OFDM

The basic function of any kind of high power amplifier (HPA) is always the same: to boost a low-power signal to a higher power level, to be delivered to the amplifier load. It fails, however, when the demands placed on an amplifier are extreme. It is difficult to satisfy the requirement for maximum capability of two or more conflicting parameters, such as the demand for broad bandwidth and high power in the same package.

The most common RF amplifiers used in digital communication system are
traveling wave tube amplifier (TWTA) and solid state power amplifier (SSPA). Satisfying the need for broadband capability, high output power and particularly high DC-to-RF conversion efficiency, TWTA finds it application in many satellite transponders. While, low power-efficient, SSPA (GaAs FET Amplifier) are suitable in non-constant envelope modulation system. It exhibits a more linear behavior as compared to TWTA [Aghvami, Robertson: 93].

Several mathematical models are available which describes the non-linear behavior of HPA. Some of them can be derived from physical principle of amplifier’s technology and usually leads to some power series or complex Voltera series. If $v_{in}$ is the input to HPA, then the output of HPA can be written as

$$v_{out} = A(v_{in})e^{j\Theta(v_{in})}$$

where $A(.)$ and $\Theta(.)$ denotes the amplitude and phase distortions introduced at the output of an HPA respectively. For TWTA, as proposed by [Saleh: 81], the amplitude and phase distortions are

$$A(v_{in}) = \frac{\alpha_{\alpha}v_{in}}{1 + \beta_{\alpha}v_{in}^2}; \quad \Theta(v_{in}) = \frac{\alpha_{\phi}v_{in}^2}{1 + \beta_{\phi}v_{in}^2}$$

where $\alpha_{\alpha}$, $\beta_{\alpha}$, $\alpha_{\phi}$ and $\beta_{\phi}$ are positive real constants.

One of models proposed by [Rapp: 91], particularly suitable for SSPA, defines the amplitude distortion as

$$A(v_{in}) = \nu \frac{v_{in}}{1 + \left(\frac{v_{in}}{A_0}\right)^2}$$

with $p > 0$; $A_0 \geq 0$ is the limiting output amplitude and $\nu$ is a small signal gain. The phase distortion for SSPA is small so it can be neglected for simulating the HPA.

Because of high PAPR in OFDM, RF power is always a critical issue. The high PAPR of OFDM system makes it susceptible to non-linear or clipping distortions, as the signal peaks may occasionally thrust into or near the saturation region of the power
amplifier. This distorts the signal, therefore, resulting in BER degradation [Merchan, Armada: 98] and adjacent channel interference. The conflicting requirements of high power and signal distortions need to be balanced carefully. [Bogenfeld, Valentin: 93] [Chini, Wu: 98] [Banelli, Baruffa: 01] [Costa, Pupolin: 02] discusses the performance of OFDM system in the presence of a non linear amplifier. Also, [Santella, Mazzenga: 98] [Dardari, Tralli: 00, 02] evaluated analytically the degradation in OFDM.

There are two ways to deal with the problem of distortions introduced by HPA in OFDM systems. One is to develop methods to enhance the linearity of HPA e.g. predistortion [Brajal, Chouly: 94], cartesian feedback and feedforward [Faulkner: 98], linear amplification with non-linear components (LINC) [Elaal, Ghannouchi: 06] and many more. The second way, which covers the scope of thesis, is to avoid the non linear amplification of OFDM i.e. to develop methods to reduce PAPR before HPA.

2.7 Peak-to-Average Power Ratio (PAPR)

2.7.1 PAPR Defined

The crest factor\(^1\) of any signal, \(C_{ri}\) is defined as ratio of the peak magnitude value and the square root of the average of that given signal. PAPR, widely used in literature, is simply the square of this crest factor. Mathematically it is defined as

\[
\xi = \max_{q,r\in[0,N-1]} \frac{|s_{r,q}(t)|^2}{E\{|s_{r,q}(t)|^2\}}
\]

(2.18)

where \(E\{\cdot\}\) denotes the statistical expectation.

If all active subcarriers \(N_{used}\) are drawn from the same signal constellation \(\{\Psi\}\), then they have same variance \(\sigma_s^2\). The multiplication of the individual components with the complex factor \(e^{j2\pi pr/N}\) does not affect the variance. According to Parseval’s theorem [Lathi: 98], the variance of OFDM symbol is then \(\sigma_s^2 = E\{|s_{r,q}|^2\} = E\{|x_{pq}|^2\}\), thus

\(^1\)Crest factor of baseband signal. It is approximately 3dB lower than the crest factor of bandpass signal, provided RF frequency is considered considerably larger than the transmission bandwidth (\(\Delta f\)).
\[ \sigma_s^2 \cong E\left(\left|s_{r,q}\right|^2\right) = E\left(\left|x_{r,p}\right|^2\right) = \frac{N_{\text{used}}}{N} \sigma_s^2 \quad (2.19) \]

Recall that if \(N_{\text{used}}\) is less than the required IFFT length \(N\) which is the general case, then zero-padding is done. By definition of IFFT, the maximum instantaneous power, which has to be encountered in an OFDM signal, is

\[ \max_{\forall q,0 \leq r < N} \left|s_{r,q}\right|^2 = \max_{\forall q,0 \leq p < N} \left|\frac{1}{\sqrt{N}} \sum_{p=0}^{N-1} x_{p,q} e^{j2\pi r/Np}\right|^2 \leq \frac{N_{\text{used}}^2}{N} \max_{x_{r,q} \in \Psi} \left|x_{r,q}\right| \quad (2.20) \]

From equations (2.19) and (2.20), the PAPR finally becomes

\[ \xi \leq N_{\text{used}} \frac{\max_{x_{r,q} \in \Psi} \left|x\right|^2}{\sigma_s^2} \quad (2.21) \]

Also, the equality in equation (2.21) is achieved when all the subcarriers have the same phase i.e. \(\arg\{x_{0,q}\} = \arg\{x_{r,p}\} \forall p = 0, 1, \ldots, N-1\).

\[ \xi_{\text{max}} = N_{\text{used}} \xi\Psi \quad (2.22) \]

where \(\xi\Psi\) is the PAPR of input signal constellation \(\Psi\). It means that PAPR grows linearly with the number of active subcarriers, \(N_{\text{used}}\), and is proportional to PAPR of the constellation \(\Psi\). If, for instance, \(\Psi\) are MQAM alphabets, then

\[ \max_{x_{r,q} \in \Psi} \left|x\right| = \begin{cases} \sqrt{2} \left(\sqrt{M} - 1\right) & n \text{ is even} \\ \sqrt{M} + 2 & n \text{ is odd} \end{cases} \quad (2.23) \]

where \(M = 2^n\), \(n\) being the number of bits per subcarrier. The variance of MQAM is [Praokis: 89] is \(\sigma_s^2 = 2(M-1)/3\). Thus PAPR of MQAM-OFDM becomes

\[ \xi_{\text{max}} = \begin{cases} 3N_{\text{used}} \frac{\sqrt{M} - 1}{\sqrt{M} + 1} & n \text{ is even} \\ 1.5N_{\text{used}} \frac{M + 2}{M - 1} & n \text{ is odd} \end{cases} \quad (2.24) \]

Hence maximum PAPR depends on the number of active subcarrier \(\{N_{\text{used}}\}\) and PAPR of input signal constellation \(\{\xi\Psi\}\). For example, for 16QAM-OFDM with \(N_{\text{used}} = N = 512\) (no zero padding applied), then \(\xi_{\text{max}} = 29.65\) dB whereas for 256QAM-OFDM with
same number of subcarriers, $\xi_{\text{max}} = 31.32$ dB. Doubling the number of active subcarriers increases this PAPR by 3 dB. But equation (2.24) yields upper bound only and the occurrence of OFDM with this theoretical maximum PAPR is rare. It can be shown that for $M = 4$, there are utmost 16 patterns that yield the theoretical maximum PAPR namely $N_{\text{used}}$ and the probability of occurrence is $16/4^{N_{\text{used}}}$ [Ochiai, Imai: 01]. Even for small number of $N_{\text{used}}$ say 25, the probability is $1.42 \times 10^{-14}$, which is negligibly small. Thus, 4QAM-OFDM symbol, with $N = 32$, supporting a data rate of 100 Mbps, the theoretical maximum PAPR is observed statistically once 14281 years!

Therefore, the upper bound may not be meaningful for characterizing the PAPR of OFDM signals, and the statistical distribution, complementary cumulative distribution function (CCDF), is a commonly used performance measure for OFDM’s PAPR. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold. [Müller, Bäuml: 97] derives a simple mathematical approximation of CCDF of PAPR in OFDM systems with Nyquist rate sampling.

### 2.7.2 Statistical Distribution of PAPR

According to central limit theorem (CLT) [Spiegel, Schiller: 00], for large number of subcarriers $N$, typically $\geq 64$, and uncorrelated input data, the transmitted signal becomes nearly complex Gaussian distributed. With zero mean and variance $\sigma_s^2$, the amplitude of the OFDM signal, therefore, has a Rayleigh distribution. The probability that the magnitude of one single signal sample that does not exceed a certain amplitude threshold $s_0 > 0$ then becomes

$$
\Pr\{ |s_{r,d}| < s_0 \} = \int_0^{s_0} p_{\text{used}} (u) du
$$

$$
= \int_0^{s_0} \frac{2u}{\sigma_s^2} \exp \left( -\frac{u^2}{\sigma_s^2} \right) du = 1 - \exp \left( -\frac{s_0^2}{\sigma_s^2} \right) \quad (2.25)
$$

where $\Pr\{}$ denotes outage probability.

Assuming the samples to be statistically independent, the probability that at least one magnitude of the entire OFDM symbol that exceeds a certain threshold can be approximated as
\[
\Pr \left\{ \text{max}_{q \in \{0,N-1\}} |s_{r,q}| \geq s_0 \right\} = 1 - \prod_{q=0}^{N-1} \Pr \left\{ |s_{r,q}| < s_0 \right\} \\
= 1 - \left( 1 - \exp \left( - \frac{s_0^2}{\sigma_s^2} \right) \right)^N
\]  
(2.26)

Finally the theoretical expression of the probability that the crest factor of one OFDM symbol at any time instant \( q \) exceeds a certain crest factor, \( C_{\xi_0} = s_0/\sigma_s \), follows

\[
\Pr \{ C \geq C_{\xi_0} \} = 1 - \left( 1 - e^{-\xi_0^2} \right)^N
\]

(2.27)

Simply changing the variable as \( \xi = C_{\xi_0}^2 \) yields the CCDF of PAPR

\[
\Pr \{ \xi \geq \xi_0 \} = 1 - \left( 1 - e^{-\xi_0} \right)^N
\]

(2.28)

Hence, the probability of occurrence of an OFDM symbols having PAPR higher than a given threshold, which is rather seldom, neither depend on \( N_{\text{used}} \) (number of active subcarriers) nor on \( \xi_0 \). It is merely the function of IFFT length \( N \) [Müller, Bäuml: 97].

Figure 2.5 shows the probability of PAPR of OFDM symbols, \( \xi \), that exceeds a given threshold, \( \xi_0 \), by evaluating equation(2.28) (solid line). This figure also plots the simulated data for different number of subcarriers (dotted line). The results shows that equation (2.28) is valid for sufficiently large value of \( N \) (i.e. \( N \geq 64 \)) at values of practical interest, i.e. \( \Pr \{ \xi_0 \} \geq 10^{-5} \). The theoretical calculations are tight for \( N \geq 64 \) only. It is interesting to note that the number \( N_{\text{used}} \) does not influence the probability, \( \Pr \{ \xi_0 \} \), as long as the assumption of statistical independence of OFDM symbols is justified, i.e. \( N_{\text{used}} \) must not be significantly smaller than \( N \). Although PAPR increases by 3 dB if \( N_{\text{used}} \) is just doubled, the probability of occurrence for different number of subcarriers does not vary to that extent. Interestingly, the values for PAPR occurring with \( \Pr \{ \xi_0 \} \leq 10^{-4} \) are within a range of 1 dB for a wide range of subcarriers number \( N \). Therefore, if OFDM is already applied as modulation scheme and its disadvantage of high PAPR is accepted then one should at least use large number of subcarriers. Of course increasing \( N \) causes other problems like increase in sensitivity to frequency offsets or increase in time delay. In practice there should be a trade off in the design of OFDM systems.
For larger values of $N$, the graph decreases steeply. It is to be noted that probability that single sample per unit time that exceeds the threshold $\xi_0$ is $e^{-\xi_0}$ (equation (2.28)). Hence, this probability is independent of $N$. Unfortunately the probabilities $\Pr\{\xi_0\}$ decrease quite slowly with increased threshold $\xi_0$. Thus, power amplifiers have to be operated with a high back-off to count for rather unlikely peak values. It is highly desirable to improve the statistical characteristics by modifying the transmitter to obtain more abrupt decrease of $\Pr\{\xi_0\}$ and thus a lower PAPR of OFDM.

Equation (2.28) is not accurate for a small number of subcarriers since a Gaussian assumption does not hold in this case. Therefore, there have been many attempts to derive more accurate distribution of PAPR. [Ochiai, Imai: 01] derived an expression using theory of level crossing rate analysis of the envelope process

$$\Pr\{\xi \geq \xi_0\} = 1 - e^{\frac{\xi_0}{\sqrt{3}e^{-\xi_0}}}$$  

(2.29)

Later on [Wei, Goeckel: 02] comes to a simple and well justified expression by applying the theory of extreme values of Chi-squared random processes

$$\Pr\{\xi \geq \xi_0\} = 1 - e^{N\frac{\xi_0}{\sqrt{\log N}e^{-\xi_0}}}$$  

(2.30)
2.7.3 Continuous Time PAPR

Equation (2.28) assumes that the $N$ time domain signal samples are mutually independent and uncorrelated. This is not true, however, when oversampling is applied because the PAPR of the continuous-time signal cannot be obtained precisely by the use of Nyquist rate sampling ($J = 1$). It is more than the discrete time transmit signal.

[Tellambura: 01] proved that four time oversampling can provide sufficiently accurate PAPR results i.e. for $J \geq 4$, difference between the PAPR of discrete- and continuous-time OFDM is negligible as shown in figure 2.6. Also, one can interpret from the figure, that for $J \geq 8$, one cannot feel much difference than for $J = 4$.

In [Sharif, Alkhansari: 03], authors derives an upper bound on the distribution of PAPR when oversampling $J > 2$ is applied

$$\Pr\{\xi \geq \xi_0\} < J N e^{-\frac{\xi_0^2}{2J}}$$

(2.31)
2.8 Different PAPR Reduction Schemes

Over the years, different solutions have been proposed to combat this problem. The first solution, in the history of OFDM, was proposed by [Greenstein, Fitzgerald: 81], about ten years after the discovery of OFDM [Weinstein, Ebert: 71]. Although addressing the same basic issue, these solutions differ greatly in the specific approach taken. Furthermore, different researchers do not entirely agree on the impact of the high signal peaks on the system performance. As a consequence, no general overview or consistent treatment of this problem is available in literature to the best of our knowledge. Categorically, depending on the demand of user and system, the PAPR mitigation techniques can be classified as:

- Non-Linear Transformation (Clipping and Windowing)
- Coding Techniques
- Multiple Signal Representations (MSR)

2.8.1 Non-Linear Transformation

Large peaks occur with a very low probability, therefore clipping the large PAPR symbols is an effective technique for PAPR reduction [Li, Cimini 98] [Bahai, Singh: 02] but this has the disadvantage that some useful information carried by these symbols is destroyed. However, clipping is a nonlinear process and may degrade the BER performance and increase the out-of-band radiation significantly. Therefore, the spectral efficiency is reduced. Filtering after clipping can reduce the spectral splatter but may also cause some peak regrowth. If digital signals are clipped directly, the resulting clipping noise will all fall in-band and cannot be reduced by filtering. One solution is to oversample each OFDM block by factor $J$ and taking $JN$-IFFT instead of $N$-IFFT. To improve the BER, forward error-correction coding (FEC) is used before bandpass filtering and clipping.

A different approach is to multiply large signal peak with a Gaussian shaped window [Parasad: 04] or any other window with good spectral properties i.e. it should be as narrow as possible in frequency domain. On the other hand, it should not be too long
in the time domain, in order to avoid large BER. PAPR reduction could be achieved independent from number of subcarriers, at the cost of a slight increase in BER and out-of-band radiation.

### 2.8.2 Coding Schemes

A block-coding scheme [Jones, Wilkinson: 94] for reduction of PAPR is to find codewords with minimum PAPR from a given set and to map the input data blocks of these selected codewords. Thus, it avoids transmitting the code-words which generates high peak envelop power. But, this reduction of PAPR is achieved at the expense of a decrease in coding rate. It reduces PAPR by 2.48 dB with $\frac{3}{4}$ rate block code for four subcarriers. For large number of subcarriers, necessary code sets exist but encoding and decoding is also difficult task. It is not suitable for higher order bit rates or large number of subcarriers.

The achievable PAPR is only between 5dB to 7.3 dB by using $m$-sequences for $m$ between 3 and 10 [Tellumbura: 97]. The problem with this approach is the extremely low rate for large values of $m$.

Another solution is to transmit information by mapping each data word with complementary Golay sequences [Davis, Jedwab: 99], prior to OFDM modulation. The achievable PAPR is typically 3 - 6 dB, however, the coding rate is poor, typically $\frac{1}{2}$, resulting in a large bandwidth increase. The disadvantage associated with Golay sequences is that they are only valid for MPSK based OFDM. Also, a lookup table with all Golay sequences is needed which becomes impractical, especially when Golay-code length and number of base Golay sequences used are large.

### 2.8.3 Multiple Signal Representations

 Tone Reservation (TR) [Tellado, Cioffi: 98], initially suggested for DMT applications, modifies the noise spectrum in such a way so that the noise is concentrated at high frequencies where the SNR is low. Since, bit allocation algorithms allocate bits only to those subcarriers (tones) with sufficient SNR; typically some of them will carry
no data at all. The clipping noise is then present on those unused tones. When unused tones are all clustered together, the PAPR reduction is not as good as with randomly selected tones. But, random selection reduces the data rate; therefore there is a tradeoff between PAPR reduction and data rate loss. An iterative algorithm is employed to recover data, therefore, increasing the receiver's complexity.

Another method, known as Tone Injection (TI) [Tellado, Cioffi: 98], transform the input constellations to choose OFDM symbol which carry identical information, but with low PAPR. Several identical representation of MQAM symbols are obtained in a complex plane by shifting the original location of these symbol by some constant value \( D \), either for the real part, imaginary part or both. The possible value for the shift range \( D \) should not reduce the minimum euclidean distance \( d \), in order to keep the same symbol error rate at the receiver. Although, this method does not require side information at all and only the value \( D \) is known to receiver, there is large complexity at the transmitter side. Due to this complexity, practical implementation is almost impossible. The transmit power also increases, as a large constellation is required to represent the same information. This power increase depends primarily on the symbols that are shifted.

In Selected Mapping (SLM) [Bäuml, Fischer: 96] [Müller, Bäuml: 97], \( U \) statistically independent alternations of OFDM symbol, \( s_q^{(u)} \), representing the same information, are generated. One possible method to generate \( s_q^{(u)} \) is: After mapping the information to carrier amplitudes, each \( x_{p,q} \), is point-wise multiplied with \( U \) vectors resulting in \( x_{p,q}^{(u)} = x_{p,q} e^{j\phi_{p}^{(u)}} \), \( \phi_{p}^{(u)} \in [0, 2\pi) \); \( 0 \leq p < N \); \( 1 \leq u < U \). For simplification, \( e^{j\phi_{p}^{(u)}} \in \{\pm 1, \pm j\} \) is chosen as it can be implemented without any multiplications; simply by interchanging, adding and subtracting the real and imaginary parts. Then, all \( U \) frames are transformed into the time domain to get \( x^{(u)} = \mathcal{F}^{-1}\{x^{(u)}\} \) and the one with the lowest PAPR is selected for transmission. Hence

\[
\tilde{u} = \arg \min_{0 \leq u < U} \left( \max_{0 \leq r < N} |x_{r,q}^{(u)}| \right) = \arg \min_{0 \leq u < U} \|s^{(u)}\|_\infty \quad (2.32)
\]

needs to be determined, usually by exhaustive search. Clearly, \( \|x\|_\infty \) denotes the \( \infty \)-norm (Chebyshev norm) of the vector in the argument. To recover data, the receiver has to know which vector \( e^{j\phi_{p}^{(u)}} \) has actually been used in the transmitter. The straightforward
method is to transmit the number $\hat{u}$ as side information.

A variant of SLM, known as Partial Transmit Sequence (PTS), was proposed [Müller, Hüber: 97] [Latif, Gohar: 02, 03] where the subcarrier vector $x_p$ is partitioned into $V$ pair-wise disjoint sub-blocks $x_q^{(v)}$, $v = 1, 2...V$. All subcarrier positions in $x_q^{(v)}$, which are already represented in another sub-block, are set to zero (Figure 2.7) so that $x_q = \sum_{v=1}^{V} x_q^{(v)}$. Now a complex valued rotational factors $b_q^{(v)} = e^{i\phi_q^{(v)}}; \phi_q^{(v)} \in [0, 2\pi)$ is introduced. Then, modified subcarrier vector

$$\tilde{x}_q = \sum_{v=1}^{V} b_q^{(v)} x_q^{(v)}$$

(2.33)

represents the same information as $x_p$, if the set \{${b_p^{(v)}; v = 1, 2...V}$\} is known (side information) for each $p$. To calculate $\tilde{s}_p = \mathcal{F}^{-1}\{\tilde{x}_p\}$, the linearity of IFFT is exploited i.e.

$$\tilde{s}_q = \mathcal{F}^{-1}\{\tilde{x}_q\} = \mathcal{F}^{-1}\left\{\sum_{v=1}^{V} b_q^{(v)} x_q^{(v)}\right\} = \sum_{v=1}^{V} b_q^{(v)} \mathcal{F}^{-1}\{x_q^{(v)}\} = \sum_{v=1}^{V} b_q^{(v)} s_q^{(v)}$$

(2.34)

**Figure 2.7:** PTS Sub-block Partitioning for $V = 4$. The sub-blocks are either contiguously (left) or randomly (right) placed with either equal (upper) or unequal (lower) number of subcarriers.
Thus, subblocks may be transformed by $V$ separate and parallel IFFTs. These PTSs are jointly orthogonal and based upon these PTSs, peak value optimization is done by choosing a free parameters $b_q^{(v)}$ such that PAPR is minimized for $\tilde{b}_q^{(v)}$.

The optimum parameters for OFDM symbol $q$ are given by

$$\left\{\tilde{b}_q^{(1)}, \tilde{b}_q^{(2)}, \ldots, \tilde{b}_q^{(V)}\right\} = \arg\min_{\left\{\bar{b}_q^{(1)}, \bar{b}_q^{(2)}, \ldots, \bar{b}_q^{(V)}\right\}} \left( \max_{v=1}^V \left| \sum_{r=1}^V b_q^{(r)} s_q^{(r)} \right| \right)$$

(2.35)

where $\arg\min(.)$ yields the argument for which the given expression achieves the global minimum. Then, the optimum transmit sequence is $\tilde{s}_q = \sum b_q^{(v)} s_q^{(v)}$. The OFDM transmitter employing this scheme is shown in Figure 2.8. This scheme requires that the receiver has knowledge about the generation of the transmitted OFDM in symbol period $q$. Thus, the set with all rotation factors $\tilde{b}_q^{(v)}$ has to be transmitted to the receiver so that it can rotate back the subcarriers appropriately. This side information is the redundancy introduced by PTS.

![Figure 2.8: OFDM Transmitter Employing PTS scheme](image)

2.9 Hybrid FSK-QAM Modulation (HQFM): A Novel Technique with Low PAPR

It was shown in [Tasadduq, Rao: 02], that employing CPM instead of QAM or PSK for drawing complex numbers from a constellation set $\Psi$, the PAPR of OFDM is reduced as compared to conventional QAM/PSK OFDM signal. Efficient new or existing algorithms like PTS can be used to further reduce PAPR in these systems.
Continuous phase FSK (CP-FSK or merely FSK) is a special case of full response CPM with modulation index \( h = 1 \). When the signal size \( Q \) increases beyond 4, it becomes bandwidth inefficient. So we cannot use such system with only 2 bits/subcarriers in OFDM which demands for more bits/subcarriers for modern applications. For instance, OFDM is employed in IEEE 802.11a/g and HIPERLAN/2 in their PHY layer which supports 6 bits/carriers (i.e. 64QAM). QAM on the other hand, a highly bandwidth efficient, does not show good power efficiency i.e. the relationship of \( E_b/N_0 \) with the signal size \( Q \). This puts a limitation on the symbol size \( Q \) and bandwidth efficiency. In this thesis, a novel modulation scheme is introduced which is simultaneously bandwidth/power efficient. Thus now the complex number drawn from the signal constellation \( \{\Psi\} \) is modified which shows a better \( E_b/N_0 \) and PAPR reduction capability with no scarification of bandwidth efficiency. It is also shown that this modulation can be used in OFDM.

REFERENCES


[ETSI: 97]: RADIO BROADCASTING SYSTEMS; DIGITAL AUDIO Broadcasting (DAB) TO MOBILE, PORTABLE AND FIXED RECEIVERS, ETS 300 401, 2nd Edition, May 97

[ETSI: 98]: DIGITAL VIDEO BROADCAST (DVB); FRAMING STRUCTURE, CHANNEL CODING AND MODULATION FOR DIGITAL TERRESTRIAL TELEVISION, ETSI EN 300 744 v1.1.2 (1997 - 08)


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CHAPTER 2  FUNDAMENTALS OF OFDM

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Quadrature multiplexing techniques have been used for a long time both in analog and digital communications. One digital modulation scheme of this type is the quadrature amplitude modulation (QAM), where both in phase \( I \) and quadrature \( Q \) components, are modulated by an independent series of \( M_1 \)-ary and \( M_2 \)-ary rectangular pulses respectively. The sum of the both \( I \) and \( Q \) components yields \( M = M_1 M_2 \) rectangular signal constellation. As \( M \) increases, the bandwidth efficiency is increased at the expense of power efficiency (measure of \( E_b/N_0 \)). In contrast, with multi-dimensional, \( L \)-ary orthogonal frequency shift keying (FSK) signal sets, power efficiency increases with \( L \) at the expense of the bandwidth efficiency. The hybrid of both of the modulation schemes, referred to as hybrid MQAM/ LFSK modulation (HQFM), can offer the advantages of both techniques. The results obtained are quite favorable. This hybrid technique applies the principle of quadrature-carrier multiplexing to \( L \)-dimensional orthogonal constellations in order to enhance the bandwidth efficiency and at the same time retain the power efficiency of orthogonal signaling.
It is appropriate to compare the proposed HQFM scheme with QAM. Like QAM, HQFM yields a signal set of size $Q$ ($= ML$) with a non-constant envelope and energy. HQFM is simultaneously power and bandwidth efficient modulation scheme where its error performance is always better than that of unfiltered QAM. Interestingly enough, however, under certain conditions (e.g., in terms of 99%-power containment bandwidth) the bandwidth efficiency of HQFM is approximately the same as that of $Q$-QAM.

In this chapter, only the signaling properties of the proposed HQFM are discussed. Its error rate performance will be discussed in chapter 5 and 6. Before discussing the signaling properties, brief review of general digital modulation scheme (MPSK, MQAM and MFSK) in terms of signal waveforms, energy characteristics, correlation or Euclidean distance properties is given. A literature review of hybrid modulation based on general modulation schemes is also given. Power spectral density, null-to-null bandwidth occupancy and bandwidth efficiency in terms of 90% and 99% power containment of HQFM is compared with conventional $L$-FSK, $M$-, $Q$- ($Q = ML$) QAMs in this chapter.

### 3.1 Brief Review of MPSK, MQAM and MFSK

In general, any $M$-ary signaling system, the waveform used to transmit the information will be encoded as $\{s_m(t); \ 1 \leq m \leq M; \ 0 \leq t \leq T_s\}$ where $M = 2^n$, $n$ being number of bits per encoded symbol [Proakis: 89]. There bandpass generic representation, with initial phase $\{\theta_0\}$ and centre frequency $\{f_c\}$ is

$$x_m(t) = \Re \{u_m(t)e^{(j2\pi f_c t + j\theta_m)}\} \quad (3.1)$$

The equivalent lowpass representation of equation (3.1) is

$$u_m(t) = A_m(t)e^{(jf_m(t) + j\theta_m)}g(t) \quad (3.2)$$

where $I_m(t) = A_m(t)e^{(j\omega(t) + j\theta_m(t))}$ and $g(t)$ are information bearing signal and pulse shaping filter respectively. For the sake of convenience, $\theta_0 = 0$ and $g(t)$ is assumed to be rectangular pulse shaping filter with period $T_s = T_b\log_2 M$ where $T_b$ being bit duration in
CHAPTER 3  HYBRID MQAM-LFSK (HQFM) SIGNALING

seconds i.e.

\[ g(t) = \frac{\sqrt{2E_s}}{T_s} \quad (3.3) \]

Equation (3.2) shows that \( I_m(t) \) in the \( p^{th} \) transmission interval, \( (p-1)T_s \leq t \leq pT_s \), may differ either in amplitude \( \{A_m(t)\} \), or phase \( \{\theta_m(t)\} \) or frequency \( \{f_m(t)\} \) or combination of all or some of these parameters and is chosen according to the designer’s need and requirement. These \( M \) signals, are also characterized by their energy

\[ E_m = \int_0^{T_s} s_m^*(t)g(t)dt = \frac{1}{2}|u_m(t)|^2 \quad (3.4) \]

and mutually by their complex cross correlation coefficients

\[ \rho_{ij} = \frac{1}{\sqrt{E_iE_j}} \int_0^{T_s} s_i(t)s_j^*(t)dt = \frac{u_i^*u_j^*}{|u_i||u_j|} \quad (3.5) \]

where \( u_m = [u_{m1}, u_{m2}, \ldots, u_{MN}] \) is equivalent complex valued lowpass \( N \)-dimensional signal vector. Another parameter measuring the similarity or dissimilarity of the set of \( M \) signal waveforms is Euclidean distance, which is defined as

\[ d_{ij}^2 = |\mathbf{s}_i - \mathbf{s}_j|^2 = E_i + E_j - 2\sqrt{E_iE_j}\text{Re}\{\rho_{ij}\} \quad (3.6) \]

Figures 3.1: Signal Space diagram of different MPSK

If \( u_m(t) = e^{(i\theta_m)}g(t) = (\cos \theta_m + j\sin \theta_m)g(t) \); \( 1 \leq m \leq M \), then \( s_m(t) \) corresponds to phase shift keying (PSK). \( \theta_m(t) = \theta_m = 2\pi(m-1)/M \) are the possible \( M \) phases of the carrier that convey the transmitted information. These signal waveforms, having equal energies i.e. \( E = \frac{1}{2}E_s \) and minimum Euclidean distance of \( \sqrt{2E_g(1 - \cos 2\pi / M)} \),

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produces a circular signal space diagram as shown in figure 3.1. One drawback of MPSK is that increasing $M$ brings the signal points closer to each other so the probability of detecting correct MPSK signals decreases with increasing $M$, therefore are less power efficient [Proakis: 89].

![Signal Space Diagnostics](image)

**FIGURE 3.2:** Signal Space diagram of different MQAM

If $u_m(t) = (A_{ml} + jA_{mq})g(t)$ then like MPSK a two dimensional signal space is produced where $A_{ml}$ and $A_{mq}$ defines the I- and Q-axis of the signal space diagram. This type of signaling is called quadrature amplitude modulation (QAM). An alternative way of representing the amplitudes is $|A_m|e^{j\theta_m}$, where $A_m^2 = A_{ml}^2 + A_{mq}^2$ and $\theta_m = \tan^{-1}\left(\frac{A_{ml}}{A_{mq}}\right)$ [Proakis: 89]. From this, it is apparent that QAM signals can be viewed as combined amplitude $\{|A_m|\}$ and phase $\{\theta_m\}$ modulation. In special case where the signal amplitudes take the set of discrete values $(2m_1-1-M_1) + j(2m_2-1-M_2)$, \{1$\leq m_1 \leq M_1; 1 \leq m_2 \leq M_2; M = M_1M_2\}$, the signal space diagram is rectangular as shown in figure 3.2. In this case each $n$ bit symbol is divided into $(n_1, n_2)$ bits and each $n_1 = \log_2(M_1)$ and $n_2 = \log_2(M_2)$ bits are ASK-encoded separately and combined. These signal waveforms have energies $\mathcal{E}_m = \frac{1}{2}A_m^4\mathcal{E}_g$ with minimum Euclidean distance of $2\sqrt{\mathcal{E}_g}$ between adjacent points. The bandwidth efficiency of MQAM is same as that of MPSK but are more power efficient than MPSK i.e. the probability of detecting correct MQAM symbol is $10\log[3M^2/(2(M-1)\pi^2)]$ dB more than MPSK for same number of signaling waveforms, $M$ [Proakis: 89]. For example, 16PSK requires at least 4.14dB more power than 16QAM, which justifies the use of 16QAM over 16PSK. Thus, QAM can be easily utilized in high data rate applications giving more power efficiency for the same bandwidth efficiency as compared to MPSK.
Now consider \( u_m(t) = e^{j\phi_m(t)}g(t) \), then \( M = 2^n \) signals are generated by shifting the carrier by an amount \( f_m(t) = \pi(2m-1-M)f_\Delta t; \ 1 \leq m \leq M \), where \( f_\Delta \) is the minimum frequency spacing between two adjacent signals or tones. This type of signaling is referred as frequency shift keying (FSK). In contrast to MQAM or MPSK, MFSK signaling is a function of time, so it is termed as non-linear modulation [Lathi: 98].

To corporate smooth transition from one frequency to another, \( f_m(t) \) takes the form

\[
f_m(t) = \varphi(t; 1) = 2\pi h \sum_{i=-\infty}^{p} I_i \int_{0}^{t-pT_s} g(\tau)d\tau \quad pT_s \leq t \leq (p+1)T_s
\]

where \( I_i \) is the sequence of \( M \)-ary information symbols selected from the set \( \{2m-1-M, 1 \leq m \leq M\} \) and \( h = f_\Delta T_s \) is known as the modulation index. This type of FSK is called continuous phase FSK (CP-FSK) [Proakis: 89]. Note that in interval \( pT_s \leq t \leq (p+1)T_s \), these signals are represented in terms of symbol denoting the accumulation of all symbols until time \( (p-1)T_s \). Being orthogonal to one another, these waveforms are no more 2-D, rather are \( M \)-dimensional [Proakis: 89], therefore, cannot be represented as discrete points in a signal space diagram, in contrast to MQAM or MPSK. To meet the condition of orthogonality, the minimum frequency separation, \( \{f_\Delta\} \), must satisfy

\[
h = f_\Delta T_s = \begin{cases} 
0.5 & \text{coherent} \\
1 & \text{non-coherent}
\end{cases}
\]

Schemes discussed so far are not bandwidth and power efficient simultaneously. For the case of orthogonal MFSK signals and increase in signal dimensions, there is no over crowding in the signal space. Hence, MFSK is power efficient i.e. the probability of detecting correct symbols increases with increasing \( M \) and approaches Shannon’s channel capacity as \( M \to \infty \). On the other hand, the channel bandwidth required for each MFSK signal is \( f_\Delta \), hence, total transmission bandwidth required is \( Mf_\Delta \), which decreases the bandwidth efficiency. While MQAM or MPSK, are highly bandwidth efficient i.e. more symbols can be accommodated in a given bandwidth as number of \( M \) signals increases. But, due to overcrowding of discrete points in a signal space diagram, the probability of detecting correct symbol decreases with increasing \( M \).
3.2 Hybrid Modulation: Literature Review

Many hybrid modulation techniques and their applications are discussed in literature. Generally all these schemes make use of hybridization of orthogonal FSK and MPSK. Neither of these schemes considers the combination of orthogonal FSK with MQAM. This section briefly reviews these available hybrid modulation techniques.

The concept of making simultaneous use of phase and frequency was first introduced by Reed and Scholtz in 1966 as N-Orthogonal Phase modulated codes [Ref: Ghareeb, Yongaçoğlu: 94]. Quadrature-Quadrature Phase Shift Keying (Q^2PSK) [Saha, Birdsall: 89] uses two parallel MSK signals (similar to binary CP-FSK with modulation index \( h = 0.5 \)), one is cosinusoidal with frequency either of \( f_c \pm 1/4 T \), the other is sinusoidal with frequency either of \( f_c \pm 1/4 T \).

A study of coded modulation format, referred as Quadrature Frequency Phase Modulation (QFPM), based on quadrature multiplexing of two \( L \)-dimensional biorthogonal set with modulation index \( h = 1 \) was carried out by [Periyalwar, Fleisher: 92] [Fleisher, Qu: 95]. These signals were of type LFSK/2PSK and have focused on the performance of the coherent systems only. The constant envelope QFPM (CEQFPM), which is in fact \( 4L \)-ary LFSK/QPSK, are of particular interest for satellite communication channels. These are derived from quadrature-sum of two biorthogonal \( L \)-dimensional NFSK/2PSK signal sets [Ref: Periyalwar, Fleisher: 92]. Symbol-by symbol differential detection of LFSK/ MDPSK and differential Q^2PSK for various satellite mobile channels were studied in [Wei, Korn: 95]. Although these studies have demonstrated the possibility of differential detection, these systems cannot be readily generalized to differential detection of differential QFPM for a general case of \( L \).

[Chung: 99] proposed and analyzed the maximum-likelihood differential-detection (ML-DD) algorithm for pilot symbol assisted differentially-encoded QFPM (PSA-DQFPM) signaling format in AWGN channel for a general case of \( N \).

[Ghareeb: 95] generalized the concept of [Fleisher, Qu: 95] and extendend the idea to LFSK/MPSK modulation schemes and named it as Joint Frequency Phase Modulation (JFPM). In his classical paper, he proposed a non-coherent detection of LFSK frequencies (\( h = 1 \)) and differential detection of MPSK (DPSK).
BER and spectral performance characteristics were explored for the orthogonally multiplexed orthogonal amplitude modulation (OMOAM) [Chung: 02] and orthogonally multiplexed orthogonal phase modulation (OMOPM) [Chung: 03] signals constructed from various time-limited rectangular and sinusoidal pulsed basis sets and bandlimited root-raised-cosine pulsed basis sets. It is to be noted that the OMOAM format, taking discrete amplitude levels, explicitly unify several hybrid modulations discussed above. Later on, two new modulation families were introduced by substituting the on–off-keyed signaling for the orthogonal group signaling adopted in OMOAM and OMOPM [Chung, Liaw.: 05].

The hybrid schemes discussed so far, employs both MFSK and MDPSK simultaneously. Since MQAM is spectrally as efficient as MDPSK but shows better error rate performance for \( M > 8 \), so replacing QAM with PSK is a better replacement to all the above mentioned hybrid modulation schemes, which is the main idea presented in this dissertation.

Recently a variable-rate, variable-power scheme is proposed which employs both non-coherent MFSK and MQAM [Digham, Alouini: 03a & b]. The authors presented an idea of using adaptive MFSK when the MQAM system declares an outage, so that a better channel utilization and a reliable communication link is provided. Depending on the channel conditions the system employs either MFSK \( \{M = 2, 4, 8 \text{ and } 16\} \) or MQAM \( \{M = 2, 4, 8, 16, 32, 64\} \)\(^1\). A power loading function in terms of total channel power gain is defined which is used to determine when to switch from FSK to QAM\(^2\) [Digham, Alouini: 06].

### 3.3 HQFM Signals

In \( L/M \) HQFM [Latif, Gohar: 06 c-a & -b], each \( n \) bit symbol is divided into \( n-k = \log_2 L \) and \( k = \log_2 M \) bits. The arbitrarily chosen \( n-k \) bits are used to select the frequency \( f_l \) from non-coherent LFSK according to \( f_l = (2l-1-L)f_\Delta; \quad 1 \leq l \leq L \). The minimum frequency separation, \( f_\Delta \), satisfies the condition of orthogonality stated in

\(^1\) The whole bits in a data \( (\log_2 M = n) \) is either FSK symbol or QAM alphabet.

\(^2\) See figure in Ref [Digham, Alouini: 06] showing a switching mechanism between the two modulation schemes.

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equation (3.8). During the same symbol period, $T_s$, the remaining $k = \log_2 M$ bits are then mapped using ordinary MQAM. Unlike the adaptive hybrid modulator described in [Digham, Alouini: 03a & b, 06], L/M HQFM employs simultaneously FSK and MQAM. It is a hard-cored hybrid system defining no switching threshold.

The complex form of HQFM signal in the $q^{th}$ signaling interval can then be expressed as

$$u_{ml,q} = x_{ml,q}(t)$$

(3.9)

where $g(t)$ is rectangular pulse shaping filter defined in equation (3.3) and

$$x_{ml,q} = A_{ml,q} e^{j(\mathcal{G}_{ml,q} + \phi_{q})}$$

(3.10)

$$A_{ml,q} = \sqrt{A_{ml,q}^2 + A_{mQ,q}^2}$$

(3.11)

$$\mathcal{G}_{m,q} = \tan^{-1}(A_{ml,q} / A_{mQ,q})$$

(3.12)

$$\phi_{q,l} = \pi h l_{q,l} + \pi h \sum_{j=0}^{q-1} l_{s,j}$$

(3.13)

where $A_{ml}$'s and $A_{mQ}$'s can take up discrete values $(2^{m_1-1}-M_1)$ and $(2^{m_2-1}-M_2)$; \{1≤$m_1$≤$M_1$; 1≤$m_2$≤$M_2$; $M = M_1 M_2$\} respectively, defining the $I$- and $Q$-axis of the signal space diagram. For square MQAM, $M_1 = M_2$. Also $I_l$'s can take up discrete values $\lambda_l = 2l-1-L \{1 ≤ l ≤ L\}$. In equation (3.13), $h = f_sT_s$ is the modulation index of the CP-FSK employed. Also, it is noted that at any time index $t$, the value of $\phi_q$ depends not only on the current data, but also on the past data. Therefore, equation (3.10) can also be rewritten as

$$x_{ml,q}(t) = A_{ml,q} e^{j(\mathcal{G}_{ml,q} + \pi h l_{q,l}/T_s)}$$

(3.14)

These waveforms have energies $E_{ml} = \frac{1}{2} A_{ml,q}^2 E_g$ and cross correlation coefficient

$$\rho_{l_{j,k},l_{j,k}} = \frac{1}{2 \sqrt{E_{l_{j,k}} E_{l_{j,k}}}} \int_{0}^{T_s} u_{l_{j,k}}(t) u_{l_{j,k}}^*(t) dt$$

$$= 1/T_s \int_{0}^{T_s} x_{l_{j,k}}(t) x_{l_{j,k}}^*(t) dt$$

$$= A_{l_{j,k}} A_{l_{j,k}}^* e^{j(\theta_{l_{j,k}} - \theta_{l_{j,k}})} \sin(\pi |\lambda_{l_{j,k}} - \lambda_{l_{j,k}}| f_s T_s) e^{j\pi |\lambda_{l_{j,k}} - \lambda_{l_{j,k}}| f_s T_s}$$

(3.15)
To meet the condition of orthogonality, the cross correlation coefficient in equation (3.15) must be zero. For non-coherent $L/M$ HQFM signal set, the phase reference for FSK is absent i.e. $|\rho_{il,k}| = 0$. In other words,

$$\sin(\pi |\lambda_l - \lambda_k| f_{\Delta} T_s) = \sin(2\pi |l - k| f_{\Delta} T_s) = 0 \quad 1 \leq l,k \leq L$$

Since adjacent frequency slot $|l-k| = 1$, therefore, it implies that $f_{\Delta} = 1/T_s$ for $l \neq k$ is the required minimum frequency separation to satisfy the condition of orthogonality. Or in other words, $f_{\Delta}$ must be the integral multiple of $1/T_s$. This is the same as required by non-coherent CP-FSK (or merely FSK) defined in equation (3.8).

From equation (3.15), it can be observed that $L/M$ HQFM signal set constitute of $L$ subsets, each with MQAM modulated symbols, where each member in one subset is orthogonal to every member in other subsets. The points lying in the HQFM signal space can be viewed as points lying in a QAM with less order on $L$ different orthogonal planes, where each plane is distinguished by its corresponding FSK frequency (figure 3.3). In other words, we can say that equivalent QAM\(^1\) signal set of size $Q$ can be split into small $Q/2$, $Q/4$, $Q/8$... QAMs with frequencies taken from 2, 4, 8... FSK respectively.

\(^1\) Equivalency in the sense of same number of bits per symbol when HQFM is compared with pure QAM.
During each symbol time $T_s$, \( n = (n-k) + k \) bits of information are transmitted. Only \((n-k)\) bits are transmitted, however, during each symbol time $T'_s$, by LFSK having the same number of tones $L = 2^{n-k}$. The symbol time of FSK must then be reduced to $T'_s = T_s \frac{(n-k)}{n}$, in order to maintain the same information bit rate. The minimum frequency separation between two adjacent tones in this equivalent FSK system is $1/T'_s$, and is $\frac{(n-k)}{n}$ wider than the HQFM tone separation of $1/T'_s$.

It is worth noting that QAM uses 2D constellations while HQFM uses $2^{L+1}$ dimensional signaling. Also, for ordinary MQAM, we have $L = 1$. For $L = 2$ and $M = 4$, HQFM reduces to a special modulation format known as $Q^2$PSK [Saha, Birdsall: 89] which is a member of a general class of modulation formats known as Joint Phase Frequency Modulation (JPFM) [Ghareeb: 95]. A generic modulator representing the generation of HQFM signals is shown in figure 3.4 where LFSK and MQAM signals are produced conventionally described in [Proakis:89] [Couch: 02].

![Figure 3.4: HQFM (Hybrid Quadrature Frequency Modulation)]

### 3.4 Power Spectral Density (PSD)

The available channel bandwidth is limited in most of the digital communication systems, so one has to consider the spectral contents of the digital modulation under consideration. Power spectral density (PSD) is one of the measures of channel bandwidth required to transmit an information bearing data signals.

Consider again equation (3.14). The random process representing the equivalent low-pass waveform of the HQFM has the form
where \( x_q = A_{m,q}e^{j\theta_q} \) is a random variable which can take up the value 
\((2m_1-1-M_1) + j(2m_2-1-M_2)\), \(1 \leq m_1 \leq M_1 \); \(1 \leq m_2 \leq M_2\); \(M = M_1M_2\) and \(\lambda_{i,q} = \lambda_i = 2l-1-L\).

Since the random process \( u(t) \) is non-stationary process so its Fourier transform 
\( U(f) = \Im \{ u(t) \} \) exists [Lathi: 98]. Taking the Fourier transform of equation(3.16)

\[
U(f) = \Im \left\{ \sum_{q=-\infty}^{\infty} x_q g(t - qT_s; \lambda_q) \right\} \\
= \sum_{q=-\infty}^{\infty} x_q \Im \{ g(t - qT_s; \lambda_q) \} = \sum_{q=-\infty}^{\infty} x_q G(f; \lambda_q) e^{-j2\pi f_q T_s} 
\]

where \( G(f; \lambda_q) \) is the Fourier transform of \( g(t; \lambda_q) \).

The PSD of \( u(t) \), \( \Phi(f) \) can be obtained by finding the line masses of the function \( \Upsilon(f_1, f_2) \) defined by

\[
\Upsilon(f_1, f_2) = E \left\{ U(f_1)U^*(f_2) \right\} 
\]

which is located on the bisector of the plane \((f_1, f_2)\). The operator \( E\{.\} \) and \((.)^*\) in equation(3.18) denotes statistical expectation and complex conjugation respectively.

\[
\Upsilon(f_1, f_2) = E \left\{ \sum_{q=-\infty}^{\infty} x_q G(f_1; \lambda_q) e^{-j2\pi f_q T_s} \left( \sum_{p=-\infty}^{\infty} x_p G(f_2; \lambda_q) e^{-j2\pi f_q T_s} \right)^* \right\} 
\]

Assuming \( \{x_q\} \) and \( \{\lambda_q\} \), both as stationary and statistically independent random variables and letting \( p = q + r \), equation(3.19) can be written in the form

\[
\Upsilon(f_1, f_2) = \sum_{q=-\infty}^{\infty} \sum_{r=-\infty}^{\infty} E\left\{ x_q^* x_r \right\} E\left\{ G(f_1; \lambda_q) G^*(f_2; \lambda_r) \right\} e^{-j2\pi (f_1 - f_2) q T_s} e^{j2\pi f_1 r T_s} \\
= \sum_{q=-\infty}^{\infty} \sum_{r=-\infty}^{\infty} \phi(r) E\left\{ G(f_1; \lambda_q) G^*(f_2; \lambda_r) \right\} e^{-j2\pi (f_1 - f_2) q T_s} e^{j2\pi f_1 r T_s} \\
= E\left\{ G(f_1; \lambda_q) G^*(f_2; \lambda_r) \right\} \sum_{q=-\infty}^{\infty} e^{-j2\pi (f_1 - f_2) q T_s} \sum_{r=-\infty}^{\infty} \phi(r) e^{j2\pi f_1 r T_s} 
\]
where $\phi(r) = E\{x_q x_{q+r}\}$ is the autocorrelation function of QAM symbols drawn from $\{x_q\}$. Assuming $\{x_q\}$ as identically distributed uncorrelated random variables with mean $\mu$ and variance $\sigma^2$, then [Praokis: 89]

$$
\phi(r) = \begin{cases} 
\sigma^2 + \mu^2 & r = 0 \\
\mu^2 & r \neq 0 
\end{cases} \tag{3.21}
$$

Substituting $\phi(r)$ from equation (3.21) and using identity [Lathi: 98]

$$
\sum_{r=-\infty}^{\infty} e^{j2\pi fr} = \frac{1}{T_s} \sum_{r=-\infty}^{\infty} \delta\left(f - \frac{r}{T_s}\right) \tag{3.22}
$$

equation (3.20) can be rewritten as

$$
Y(f_1, f_2) = E\{G(f_1; \lambda_\ell)G^*(f_2; \lambda_\ell)\} \sum_{q=-\infty}^{\infty} e^{-j2\pi(f_1-f_2)qT_s} \left(\sigma^2 + \frac{\mu^2}{T_s} \sum_{r=-\infty}^{\infty} \delta\left(f_2 - \frac{r}{T_s}\right)\right) \tag{3.23}
$$

Equation (3.23) shows that the function $Y(f_1, f_2)$ contains both continuous and discrete spectrum. Since the elements of the sequence $\{x_q\}$ are equally likely and symmetrically positioned on complex plane, therefore mean $\mu = 0$. Equation (3.23) then becomes

$$
Y(f_1, f_2) = \frac{\sigma^2}{T_s} E\{G(f_1; \lambda_\ell)G^*(f_2; \lambda_\ell)\} \sum_{r=-\infty}^{\infty} \delta\left(f_1 - f_2 - \frac{r}{T_s}\right) \tag{3.24}
$$

Equation (3.24) is obtained by using identity (3.22) again.

The PSD $\Phi(f)$, comprised of the line masses of the function $Y(f_1, f_2)$ located on the bisector of the plane $(f_1, f_2)$, is given by

$$
\Phi(f) = \frac{\sigma^2}{T_s} E\{|G(f; \lambda_\ell)|^2\} = \frac{\sigma^2}{T_s} \sum_{i=1}^{L} P_i |G(f; \lambda_i)|^2 \tag{3.25}
$$

where $P_i$ is the probability of occurrence of $G(f; \lambda_i) = G(f - \frac{1}{2} f_s \lambda_i)$. For equal probable symbols $P_i = 1/L$ for all $q$, equation (3.25) becomes

$$
\Phi(f) = \frac{\sigma^2}{LT_s} \sum_{i=1}^{L} |G(f; \lambda_i)|^2 \tag{3.26}
$$
Thus, the spectrum of HQFM does not contain a discrete spectral component but consists of a continuous spectrum whose shape depends only on the spectral characteristic of the signal pulse. The above analysis can be concluded by observing two factors which affect the shape of the signal spectrum independently. The first factor is the shape of the complex envelope waveform $s(t)$. The second factor is the correlation of the sequence $\{\varepsilon_q\}$. When no spectral shaping is employed, and for frequencies far from the center frequency, the spectra of HQFM signals fall off at a rate proportional to the inverse of the frequency difference squared.

Now consider the $G(f; \lambda_t)$ as the Fourier transform of a rectangular pulse $g(t; \lambda_t)$ with unit amplitude $A = 1$ and width $T_s$ [Couch: 02] i.e.

$$G(f; \lambda_t) = G\left(f - \frac{1}{2} f_s \lambda_t\right) = T_s \frac{\sin \pi \left(f - \frac{1}{2} f_s \lambda_t\right) T_s}{\pi \left(f - \frac{1}{2} f_s \lambda_t\right) T_s} e^{-j\pi \left(f - \frac{1}{2} f_s \lambda_t\right) T_s}$$

Putting $f_s T_s = h$

$$G(f; \lambda_t) = T_s \frac{\sin \pi \left(f T_s - \lambda_t h / 2\right)}{\pi \left(f T_s - \lambda_t h / 2\right)} e^{-j\pi \left(f T_s - \lambda_t h / 2\right)}$$

Hence equation (3.26) becomes

$$\Phi(f) = \frac{\sigma^2 T_s}{L} \sum_{l=1}^{L} \left( \frac{\sin \pi \left(f T_s - \frac{1}{2} \lambda_t h\right)}{\pi \left(f T_s - \frac{1}{2} \lambda_t h\right)} \right)^2$$

For non-coherent HQFM system, $h = f_s T_s = 1$ and by substituting the value of $\lambda_t = 2l-1-L$ into (3.29), we get [Latif, Gohar: 06 c-a & -b, 07]

$$\Phi(f) = \frac{\sigma^2 T_s}{L} \sum_{l=1}^{L} \left( \frac{\sin \pi \left(f T_s - [2l-1-L] / 2\right)}{\pi \left(f T_s - [2l-1-L] / 2\right)} \right)^2$$

Our observation on (3.30) is that, when $L = 1$, the power spectral density of a HQFM signals is reduced to the power spectrum density of an MQAM [Couch: 02] i.e.

$$\Phi(f) = \sigma^2 T_s \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2$$
For MPSK, \( \sigma^2 = 1 \). Equation (3.31) for square QAM with different values of \( M \) is plotted in figure 3.5 for comparison only. The origin refers to centre frequency, \( \{f_c\} \). Only the right half is shown because the power spectral density is symmetrical around \( f_c \) or origin. The figure clearly shows that the nulls for \( M = 4 \) occurs at integral multiple of \( fT_b = 0.5 \). Similarly, in case of \( M = 16, 64 \) and 256, nulls occurs at integral multiple of \( fT_b = 0.25, 0.1667, 0.125 \) respectfully. Hence the null-to-null bandwidth for MQAM is \( 1/T_b \log_2 M \). The bandwidth efficiency \( \eta_B = 1/BT_b = \log_2 M \), therefore increases by increasing \( M = 2^n \) i.e. the number of bits per signal.

For orthogonal LFSK, the PSD obtained is

\[
\Phi(f) = \frac{1}{T_s L} \left\{ \sum_{i=1}^{L} (L+2B_i)|G_i|^2 + 4\sum_{i=1}^{L} \sum_{j=1}^{L} B_y |G_i||G_j| \right\}
\] (3.32)

where \( U_i = U(f; \lambda_i) \) is the pulse shape used and

\[
B_y(f) = \frac{\cos \pi \left(2fT_s -(i+j-1-L)h\right) - \psi \cos \pi \left(i + j - 1 - L\right)h}{1 + \psi^2 - 2\psi \cos 2\pi fT_s}
\] (3.33)
Here \( \psi \equiv \psi(j\theta) \) is the characteristic function of the random information sequence. For rectangular pulse shape it is defined by equation (3.28). It can be shown that sidelobes in the PSD of LFSK falls with the rate of fourth power of frequency difference from centre frequency (see appendix A for proof).

In the following paragraphs different power spectral densities for different HQFM formats will be discussed. Different plots employing equation (3.30) are shown. The resulting PSDs are also compared with \( ML\)-QAM. Also, only the use of non-coherent FSK is considered i.e. \( h = f_T/s = 1 \).

Figure 3.6(a) shows the power spectral density of 4/4 HQFM \( \{L = 4; M = 4\} \) compared by 16QAM \( \{ML = Q = 16\} \). The PSDs of individual 4QAM and 4FSK using equation (3.31) and (3.32) respectively, is also shown for comparison. The figure clearly illustrates that the main lobe of 4/4 HQFM occurs at \( f_{Tb} = 0.625 \) which is greater than both \( M = 4 \) \( \{f_{Tb} = 0.5\} \) and \( ML = 16\)QAMs \( \{f_{Tb} = 0.25\} \), therefore occupying more bandwidth. On the other hand the main lobe of 4FSK is approximately 50% wider than 4/4 HQFM. Thus, the bandwidth occupancy of HQFM is less than the bandwidth occupancy of FSK.

Other possible HQFM format for \( \{\log_2 ML = 4\} \) bits symbol is 2/8 \( \{L = 2; M = 8\} \). The possibility of 8/2 HQFM \( \{L = 8; M = 2\} \) is not considered due to the non availability of MQAM with \( M < 4 \). Figure 3.6(b) compares the PSD of 2/8 HQFM with 16QAM. Comparing figure 3.6 (a) & (b), it is shown that the main lobe of the PSD widens by increasing the number of keying frequencies, \( \{L\} \), therefore increasing the bandwidth occupancy.

Figure 3.7 compare PSD plots when number of bits per HQFM symbol increases. Keeping \( \{L\} \) constant i.e. for 2/\( M \) HQFM \( \{L = 2; M = 8, 32, 128\} \) in figure 3.7(a) and 4/\( L \) HQFM \( \{L = 4; M = 16, 64, 256\} \) in figure 3.7(b) clearly shows a spectral narrowing of the main lobe with increasing \( M \). Therefore for fixed \( L \), the bandwidth occupancy is the inverse function of \( M \). This in turn, increases the bandwidth efficiency for fixed number of keying frequencies \( \{L\} \). This is also true for ordinary QAM for which \( L = 1 \). Thus, by increasing the number of bits per symbol and QAM size \( \{M\} \), the spectral occupancy of HQFM decreases.
FIGURE 3.6: Normalized Power Spectral Densities, $\Phi(f)/T_b$ for 4 bit HQFM formats (a) $4/4$ HQFM compared with $M = 4$, 16 QAM and 4FSK (b) Behavior of PSD with increasing $L = 2, 4$
FIGURE 3.7: Normalized Power Spectral Densities, $\Phi(f)/T_b$ for HQFM formats with fixed number of frequencies, $L$ (a) $2/M$ HQFM $\{M = 8, 32, 128\}$ (b) $4/M$ HQFM $\{M = 16, 64, 256\}$
By comparing PSDs in figure 3.8, it is evident that for fixed \{M\}, the main spectral lobe of \(L/M\) HQFM becomes wider with increasing \{L\}, although a reduction in the out-of-band spectral tails is observed. In all cases (figure 3.6, 3.7 and 3.8) the rate of fall of sidelobes decreases at the same rate as QAM (figure 3.5) i.e. squared frequency offset from centre frequency \(f_c\) (in our case origin).

### 3.5 Bandwidth Efficiency

In \(L/M\) HQFM, the required transmission bandwidth will almost always be determined by its frequency components. For orthogonal HQFM with modulation index \(h = 1\), this can be approximated by \(B = L/T_s\). The bit rate, \(R_b = 1/T_b\), is defined as \(R_b = n/T_s = \log_2ML/T_s\). Hence, for non-coherent HQFM, the bandwidth efficiency is [Latif, Gohar: 06 c-a & -b, 07]

\[
\eta_B = \frac{R_b}{B} = \frac{1}{BT_b} = \frac{\log_2 L + \log_2 M}{L}
\]  

(3.34)
Equation (3.34) shows that $\eta_B$ is directly proportional to QAM level $\{M\}$ and is an inverse function of number of FSK tones $\{L\}$. Also, the bandwidth efficiency of HQFM lies between the bandwidth efficiency of FSK and QAM. It is also evident that this equation reduces to the bandwidth efficiencies of pure QAM for $L = 1$ or pure FSK when $M = 1$.

Table 3.1 relates the bandwidth efficiencies of different HQFM formats. It can be observed that similar bandwidth efficiencies can be obtained for higher data rates e.g. 2/8, 4/64 HQFM has same efficiency as 4QAM. Similarly 2/128 has same bandwidth efficiency as 16 QAM but it supports higher data rate (double) i.e. 8 bits per symbol (2/128 HQFM) as compared to 4bits per symbol (16QAM).

**Table 3.1: Bandwidth Efficiencies ($\eta_B$) for Different HQFM Formats**

<table>
<thead>
<tr>
<th>$L$</th>
<th>FSK</th>
<th>$L/M$ HQFM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$L/4$</td>
</tr>
<tr>
<td>1</td>
<td>---</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.375</td>
<td>0.625</td>
</tr>
<tr>
<td>16</td>
<td>0.25</td>
<td>0.375</td>
</tr>
<tr>
<td>32</td>
<td>0.156</td>
<td>0.2188</td>
</tr>
<tr>
<td>64</td>
<td>0.094</td>
<td>0.125</td>
</tr>
</tbody>
</table>

### 3.6 Fractional Out-of-Band Power (FOBP)

There are several definitions of bandwidth that are popular in literature. The first null-to-null bandwidth is defined as the width of main lobe of the signal’s PSD in frequency domain. Although most of the signal’s energy lies in its main lobe, this measure does not fully describe where the majority of signal’s power is contained. Therefore, null-to-null bandwidth efficiency, defined in equation (3.34), cannot be used. In fact, it depends entirely upon the pulse shaping filter employed. From a practical

---

1 $L = 1$ corresponds to the bandwidth efficiency of pure MQAM
point of view, the bandwidth efficiency of a modulated signal is calculated in terms of fractional out-of-band power (FOBP) containment and is fixed to some value typically 0.9 or 0.99 [Couch: 02]. In general, the main lobe contains more than 90% of the total energy. Also, it is stated in US FCC regulations, the power in a certain band should be 99% of the total power.

Let the bandwidth $B$ be defined as the percent fractional power bandwidth, and let the total power be $P_t$. The fractional out-of-band power is defined as

$$\mu_B = 1 - \frac{P_B}{P_t}$$

(3.35)

where $P_t = \int_{-\infty}^{\infty} \Phi(f)df$ is the total power confined and $P_B = \int_{-B/2}^{B/2} \Phi(f)df$ is the power confined by the modulator over a bandwidth $B$.

To calculate the total power $P_t$, consider equation (3.29) again

$$P_t = \int_{-\infty}^{\infty} \Phi(f)df$$

$$= \frac{\sigma^2 T_s}{L} \sum_{l=1}^{L} \int_{-\infty}^{\infty} \left( \frac{\sin \pi (f T_s - \frac{1}{2} \lambda_l h)}{\pi (f T_s - \frac{1}{2} \lambda_l h)} \right)^2 df$$

$$= \frac{\sigma^2}{\pi L} \sum_{l=1}^{L} \int_{-\infty}^{\infty} \sin^2 \frac{z}{z^2} dz$$

(3.36)

where $z = \pi \left( f T_s - \frac{1}{2} \lambda_l h \right)$. In general,

$$\int_{-\infty}^{\infty} \sin^2 \frac{z}{z^2} dz = 2 \int_{0}^{\infty} \sin^2 \frac{z}{z^2} dz = \int_{0}^{\infty} 1 - \cos 2z \frac{dz}{z^2}$$

(3.37)

As we know that $\int_{0}^{\infty} 1 - \cos \frac{pz}{z^2} dz = \frac{p \pi}{2}$ [Spiegel: 68]. The total power defined by equation (3.36) becomes

$$P_t = \int_{-\infty}^{\infty} \Phi(f)df = \sigma^2$$

(3.38)
Power confined by the modulator over a bandwidth $B$ is defined as

$$P_B = \int_{-B/2}^{B/2} \Phi(f) df$$

$$= \sigma^2 T_s \sum_{l=1}^{L} \int_{-B/2}^{B/2} \left( \frac{\sin \pi(fT_s - \frac{1}{2} \lambda_l h) \pi(fT_s - \frac{1}{2} \lambda_l h)}{L} \right)^2 df = \frac{P_t}{L} \sum_{l=1}^{L} P_{b_l}$$  \hspace{1cm} (3.39)$$

Therefore, the FOBP of $L/M$ HQFM defined in equation (3.35) reduces to

$$\mu_b = 1 - \frac{1}{L} \sum_{l=1}^{L} P_{b_l}$$  \hspace{1cm} (3.40)$$

where

$$P_{b_l} = T_s \int_{-B/2}^{B/2} \left( \frac{\sin \pi(fT_s - \frac{1}{2} \lambda_l h) \pi(fT_s - \frac{1}{2} \lambda_l h)}{L} \right)^2 df$$  \hspace{1cm} (3.41)$$

Again, change $z = \pi(fT_s - \frac{1}{2} \lambda_l h)$, the limits in $P_{b_l}$ becomes from $f = \{-\frac{1}{2} B, \frac{1}{2} B\}$ to $z = \{\pi \left( BT_s + \lambda_l h \right), \pi \left( BT_s - \lambda_l h \right)\}$. The integrand (3.41) becomes

$$P_{b_l} = \frac{1}{\pi} \int_{-\pi \left( BT_s + \lambda_l h \right)}^{\pi \left( BT_s - \lambda_l h \right)} \frac{\sin^2 z}{z^2} dz = \frac{1}{2\pi} \int_{-\pi \left( BT_s + \lambda_l h \right)}^{\pi \left( BT_s - \lambda_l h \right)} \frac{1 - \cos 2z}{z^2} dz$$

$$= \frac{1}{2\pi} \left( \int_{-\pi \left( BT_s + \lambda_l h \right)}^{\pi \left( BT_s + \lambda_l h \right)} \frac{dz}{z^2} - \int_{-\pi \left( BT_s + \lambda_l h \right)}^{\pi \left( BT_s - \lambda_l h \right)} \frac{\cos 2z}{z^2} dz \right)$$

$$= \frac{1}{2\pi} \left( \frac{4BT_s}{\pi^2 BT_s^2 - \pi^2 \lambda_l h^2} + \int_{-\pi \left( BT_s + \lambda_l h \right)}^{\pi \left( BT_s + \lambda_l h \right)} \frac{\cos 2z}{z^2} dz \right)$$  \hspace{1cm} (3.42)$$

Since $\int_{0}^{\pi} \cos 2z/z^2 dz = 4(Si(2a) - Si(2b)) + (b \cos 2a - a \cos 2b) / ab$ where $Si(x)$ is known as sine integral and is defined as $Si(x) = \int_{0}^{x} \sin t/tdt$ [Spiegel: 68].

Equation (3.42) can now be rewritten as

$$P_{b_l} = \frac{2}{\pi} \left( Si(\pi BT_s - \lambda_l h) + Si(\pi BT_s + \lambda_l h) \right)$$

$$+ \frac{(1 + \pi \cos \pi BT_s \cos \pi \lambda_l h)BT_s + \pi \lambda_l h \sin \pi BT_s \sin \pi \lambda_l h}{(\pi BT_s)^2 - (\pi \lambda_l h)^2}$$  \hspace{1cm} (3.43)$$
Now FOBP of $L/M$ HQFM defined by (3.35) becomes

$$\mu_h = 1 - \frac{4}{\pi L} \sum_{l=1}^{L} \left\{ \text{Si} \left( \pi(BT_s - \lambda_l) \right) + \frac{(1 + \pi \cos \pi BT_s \cos \lambda_l h)BT_s + \pi \lambda_l h \sin \pi BT_s \sin \pi \lambda_l h}{\left( \pi BT_s \right)^2 - \left( \pi \lambda_l h \right)^2} \right\} \quad (3.44)$$

If FOBP defined in (3.44) is plotted against the bandwidth normalized by the bit rate ($BT_b$), bandwidth efficiency can be obtained directly for a given fractional power bandwidth.

**Special Cases**

$h = 0.5$

$$\mu_h = 1 - \frac{4}{\pi L} \sum_{l=1}^{L} \left\{ \text{Si} \left( \pi(BT_s - \frac{1}{2} \lambda_l) \right) + 2 \left( \frac{BT_s + (-1)^{(\lambda_l - 1)/2} \pi \lambda_l \sin \pi BT_s}{(2 \pi BT_s)^2 - (\pi \lambda_l)^2} \right) \right\} \quad (3.45)$$

$h = 1$

$$\mu_h = 1 - \frac{4}{\pi L} \sum_{l=1}^{L} \left\{ \text{Si} \left( \pi(BT_s - \lambda_l) \right) - \frac{(1 + \pi \cos \pi BT_s)BT_s}{(\pi BT_s)^2 - (\pi \lambda_l)^2} \right\} \quad (3.46)$$

When $L = 1$, equation (3.46) reduces to

$$\mu_h = 1 - \frac{4}{\pi} \left\{ \text{Si}(\pi BT_s) - \frac{(1 + \pi \cos \pi BT_s)}{\pi^2 BT_s} \right\} \quad (3.47)$$

This is the fractional out-of-band power expression for ordinary MQAM with symbol period $T_s = T_b \log_2 M$. This expression can also be obtained by replacing $\lambda_l = 0$ in equation (3.43) and substituting the resulting expression in equation (3.40). The FOBP containments of MQAM obtained by equation (3.47) are plotted versus the normalized bandwidth $BT_b$ in figure 3.9. This figure also plots the FOBP of LFSK by numerically solving equation (3.35) using equation (3.32). This figure clearly shows that LFSK is spectrally less efficient as compared to MQAM.
CHAPTER 3  HYBRID MQAM-LFSK (HQFM) SIGNALING

**FIGURE 3.9:** Comparison of FOBP as a function of Normalized Bandwidth $BT_b$ for square QAM, $M = \{4, 16, 64, \text{ and } 256\}$ and LFSK, $L = \{2, 4, \text{ and } 8\}$

**FIGURE 3.10:** Fractional Out-of-Band Power for $L/M$ HQFM with $L = \{2, 4\}$ and $M = \{4, 16, \text{ and } 64\}$
The FOBP of non-coherent \( L/M \) HQFM obtained by equation (3.46) are plotted versus the normalized bandwidth \( BT_b \) in figure 3.10 for \( L/M \) HQFM with \( L = \{2, 4\} \) and \( M = \{4, 16, 64\} \) respectively. From these plots, we conclude that, for any fixed \( \{L\} \), as order of QAM \( \{M\} \) increases, the bandwidth efficiency of HQFM is increased and the out-of-band spectral tails are reduced. Also, we see that MQAM signals have higher out-of-band spectral tails, whereas HQFM tends to reduce the out-of-band spectral tails, which make HQFM with fixed \( M \), more bandwidth efficient than MQAM.

FOBP for different formats of \( L/M \) HQFM using equation (3.46) and MQAM (case when \( L=1 \)) using equation (3.47) are given in Table 3.II and 3.III for 90% (\( \mu_B = 0.1 \)) and 99% (\( \mu_B = 0.01 \)) power containment, respectively. 90% and 99% power bandwidth of LFSK using numerical techniques is also given in these tables for comparison. Our observation of the data in Tables 3.II and 3.III is that, by using \( L/M \) HQFM, there are as many possibilities as could be desired for choosing the bandwidth efficiency (value of \( 1/BT_b \) at which \( \mu_B = -10 \) dB or -20dB respectively) of the desired system (as allowed in the design).

**Table 3.II: Comparison of 90% (-10dB) Power Bandwidth \((BT_b)\) for MQAM, LFSK and Different \( L/M \) HQFM**

<table>
<thead>
<tr>
<th>( L )</th>
<th>FSK</th>
<th>( L/M ) HQFM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( L/4 )</td>
</tr>
<tr>
<td>1</td>
<td>--</td>
<td>0.848</td>
</tr>
<tr>
<td>2</td>
<td>0.800</td>
<td>0.765</td>
</tr>
<tr>
<td>4</td>
<td>0.850</td>
<td>0.988</td>
</tr>
<tr>
<td>8</td>
<td>1.180</td>
<td>1.490</td>
</tr>
<tr>
<td>16</td>
<td>1.800</td>
<td>2.456</td>
</tr>
<tr>
<td>32</td>
<td>2.890</td>
<td>4.158</td>
</tr>
<tr>
<td>64</td>
<td>4.80</td>
<td>8.272</td>
</tr>
</tbody>
</table>

In terms of 90% power bandwidth efficiency, one can observe that, taking any combination of \( L/M \) HQFM, the bandwidth efficiency of HQFM is greater than that of the CPFSK when the size \( M \) of the signal set is greater than 4. It can also be observed that, for any fixed value of \( Q = ML \), the bandwidth efficiency of HQFM is smaller than...
that of $Q$-ary QAM. Comparing table 3.I and 3.II, the null-to-null bandwidth efficiency of HQFM is always greater than 90% power bandwidth.

**Table 3. III:** Comparison of 99% (-20dB) Power Bandwidth ($BT_b$) for MQAM, LFSK and different $L/M$ HQFM

<table>
<thead>
<tr>
<th>$L$</th>
<th>FSK</th>
<th>$L$/$M$ HQFM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$L/4$</td>
</tr>
<tr>
<td>1</td>
<td>--</td>
<td>10.30</td>
</tr>
<tr>
<td>2</td>
<td>1.200</td>
<td>6.720</td>
</tr>
<tr>
<td>4</td>
<td>1.266</td>
<td>5.070</td>
</tr>
<tr>
<td>8</td>
<td>1.380</td>
<td>4.320</td>
</tr>
<tr>
<td>16</td>
<td>1.950</td>
<td>4.020</td>
</tr>
<tr>
<td>64</td>
<td>5.300</td>
<td>8.005</td>
</tr>
</tbody>
</table>

In terms of 99% power bandwidth efficiency, we observe that there are strong analogies between $L/M$ HQFM and $Q$-ary QAM. Firstly the bandwidth efficiency of both increases as the size $Q = ML$ of the signal set increase and secondly, in both systems the bandwidth efficiencies are very close [Latif, Gohar: 07]. Compared to CPFSK, we see that the bandwidth efficiency of CPFSK decreases whereas in the case of HQFM, it increases as the signal size increases.

### 3.7 Spectral Properties of HQFM-OFDM

Lastly, we have to say that for OFDM, PSD of each orthogonal subcarrier, modulated by rectangular pulse is of the form of $\sin(x)/x$ pulses, thus the overall PSD for the complex envelope can be evaluated as [Couch: 02]

$$\Phi(f) = \sum_{q=0}^{N-1} \left( \frac{\sin \pi(f-f_q)T}{\pi(f-f_q)T} \right)^2$$

(3.48)

where $T = NT_s$ is OFDM symbol duration and $f_q = (2q-1-N)/NT_s$; $T_s = nT_b$; $n$ being number of bits per subcarrier. Equation (3.48) clearly states that PSD of OFDM
depends on frequency separation of each subcarrier ($\Delta f = 1/T = 1/NT_s$), irrespective of the modulation format used, so bandwidth occupancy of HQFM-OFDM, in terms of PSD/FOBP remains unchanged [Latif, Gohar: 07].

It is observed that by increasing the number of subcarriers, $N$, the side lobes of the OFDM power spectrum falls faster with increasing frequency. As $N$ goes to infinity, the PSD of OFDM signals converges to a rectangular function with duration $1/T_s$.

Observing FOBP of OFDM signals for different number of subcarriers, it can be shown that for large $N$, the bandwidth of OFDM signals can be approximated as [Ahlin, Zander: 06]:

$$B \approx \frac{N+1}{NT_s} \approx \frac{1}{T_s}$$  \hspace{1cm} (3.49)

REFERENCES


[LATIF, A.; GOHAR, N. D.: 06 c-a]: “A Hybrid MQAM-LFSK OFDM Transceiver with Low PAPR”, 2nd IEEE Int. Conf. on Wireless Communications, Networking and Mobile Computing (WiCom06), Sept. 22-24, 2006 China


In this chapter, an OFDM transceiver is proposed [Latif, Gohar: 06, 07, 08] which make use of hybrid modulation scheme instead of conventional modulator like QAM or PSK. It not only shows an improvement in BER performance, but also exhibits reduction in PAPR as compared to conventional MQAM-OFDM. The modified OFDM transceiver makes use of multilevel QAM constellations, where the level of QAM is decided by specific number of bits chosen from a group of bits to be encoded in the QAM symbol. The simulated results show that PAPR is considerably reduced but at the cost of a slight increase in detection complexity. Like PTS or SLM [Müller, Bäuml: 97][Müller, Hüber: 97 j & c] [Latif, Gohar: 02, 03], it works with arbitrary number of subcarriers but needs no side information to be transmitted. It is also shown that PAPR reduction capability of the proposed system is comparable to PTS. To further reduce the PAPR, one can alter the hybrid MQAM/LFSK (HQFM) signal sets like in PTS. At the receiver, these deformations can be recovered (needs not to be transmitted) in one or two iterations, thus increasing the detection complexity.
4.1 Hybrid MQAM-LFSK (HQFM) OFDM

As mentioned earlier, in a typical OFDM system, the bit rate per subcarrier (not the total bit rate) is reduced by converting binary serial bit stream into \( N_{\text{used}} \) parallel streams, with \( n \) bits in each stream. Then, a suitable modulation technique, MQAM/ MPSK \((M = 2^n)\), is applied to map these bits to \( N_{\text{used}} \) active subcarriers.

Here a novel modulator is proposed, which replaces QAM signals with hybrid LFSK modulated MQAM (HQFM) signals. Recall that in HQFM, instead of modulating all \( n = \log_2 Q = \log_2 ML \) information bits, arbitrarily chosen \( n-k = \log_2 L \) bits are used to select the tone \( f_c' \) from a LFSK according to \( f_c' = \lambda f_s/2, \lambda = 2l-1-L; 1 \leq l \leq L \). If \( T_s \) is the symbol period of a single HQFM signal, then the minimum tone separation \( f_A \) for HQFM to meet the condition for orthogonality is

\[
f_A = 1/T_s \quad (4.1)
\]

Hence, the modulation index \( h = f_A T_s = 1 \). The remaining \( k = \log_2 M \) bits are modulated using ordinary MQAM. Here \( Q = 2^n \) is the size of alphabet set e.g. for 6 bit symbol, there are \( 2^6 = 64 \) signals in an HQFM set.

For transformation of these HQFM signals to an OFDM symbol, usually \( N-N_{\text{used}} \) inactive subcarriers (set to zero) are added appropriately, and then \( N \)-point IFFT is applied. The zero padded signals are used to shape the power spectral density of the transmitted signal. In order to avoid ISI and ICI, the transmitted signal is made periodic by cyclically appending CP \((N_{CP} \leq 25\%)\) of the OFDM symbol. The signal is then D/A converted to produce the analog bandpass signal, up-converted to RF and then
transmitted (figure 4.1). A discrete-time OFDM symbol in the $p^{th}$ interval is expressed as

$$s_p(t) = \frac{1}{\sqrt{T}} \sum_{q=0}^{N-1} x_{p,q} e^{j2\pi f_{p,q} t}, \quad 0 \leq t \leq T$$  \hspace{1cm} (4.2)

where $N$ is number of OFDM subcarriers, $T = N T_s$ is the OFDM symbol period and $x_{p,q} = [x_{p,0}, x_{p,1}, \ldots, x_{p,N-1}]$ is a set of alphabet taken from HQFM alphabet set with the $q^{th}$ HQFM signal defined in equations (3.10) - (3.14) and $\Delta f$ is the OFDM subcarrier spacing, defined as

$$\Delta f = 1/T = 1/NT_s$$  \hspace{1cm} (4.3)

Figure 4.2 shows a portion of an arbitrary 512-carrier OFDM symbol, when 256QAM is employed and is compared with 16/16 HQFM OFDM symbol. The figure clearly shows that the peak of the OFDM symbol is drastically reduced when the hybrid signals are injected into the IFFT resulting in low PAPR.

![Figure 4.2: Amplitude and Mean of a Single 512-OFDM symbol $\xi_{256QAM} = 11.706$dB, $\xi_{16/16\text{ HQFM}} = 8.784$dB](image_url)
4.2 PAPR as a function of \( N \) (Number of Subcarriers)

Figure 4.3 shows the performance of 16/16 HQFM OFDM with different number of subcarriers and is compared with 256QAM OFDM. The conclusion drawn by viewing this graph is that using HQFM with \( 4N \) subcarriers allows transmission with PAPR significantly below the original OFDM system with \( N \) subcarriers.

![Figure 4.3: CCDFs of PAPR of 256QAM-OFDM compared with 16/16 HQFM using \( N = \{256, 512, 1024\} \)](image)

4.3 PAPR as a function of \( L \) (Number of FSK Tones)

Equation (4.3) using equation (4.1) can be rewritten as

\[
\Delta f = f_\Delta / N \iff f_\Delta = N \Delta f
\]

or

\[
N = f_\Delta / \Delta f
\]

As stated in chapter 2, increase in PAPR is linear function of \( N \). According to equation(4.5), PAPR is, therefore, linear function of \( f_\Delta / \Delta f \). Hence, for fixed \( N \) (in other words \( \Delta f \)), PAPR can be reduced by decreasing \( f_\Delta \). Bringing FSK tones closer to each
other while maintaining their orthogonality, means that more frequencies can be adjusted in a given frequency band. Therefore PAPR decreases by decreasing $f_A$ or increasing $L$.

![Figure 4.4: CCDF of PAPR of different formats of HQFM OFDM compared to (a) 64QAM OFDM (each with 6 bits/subcarrier) (b) 256QAM-OFDM (each with 8 bits/subcarrier)](image)

This is justified for HQFM-OFDM, for which PAPR decreases by increasing the number of FSK tones as compared to $2^n$QAM-OFDM ($L = 1$) as shown in figure 4.4(a) $n = 6$ (b) $n = 8$. The outermost line, in this figure, shows the $\Pr(\xi \geq \xi_0)$ against a specified threshold $\xi_0$ for conventional OFDM ((a) 64QAM-OFDM (b) 256QAM-
OFDM) with $N = 512$. From this figure, it is obvious that HQFM makes the probabilities to decay faster, yielding a more desirable statistical behavior.

Also, it shows that for fixed number of bits per subcarrier $\{n\}$, PAPR decays more fast if the number of FSK frequencies $\{L\}$ increases. For instance, PAPR for HQFM does not exceed approximately 13 dB (figure 4.4a) and 13.8 dB (figure 4.4b) at $\Pr(\xi_0) = 10^{-6}$ when $L = 16$. Although, by increasing the number of FSK frequencies, one can achieve more PAPR reduction, but, all this is achieved at the cost of reduced bandwidth efficiency. There is no drastic improvement in PAPR statistics if $L \geq 32$. Therefore, 16FSK is enough to reduce the OFDM’s PAPR.

\[ \text{Figure 4.5: CCDF of PAPR of different 4/M HQFM \{L = 4, M = 4, 16, and 64\} compared with } Q = 4M = \{16, 64, 256\} \text{ QAM OFDM} \]

Figure 4.5 compares the PAPR reduction capability of HQFM for $N = 512$ when the number of FSK frequencies $\{L\}$ is kept constant. In this case we compare our results with $L = 4$ for different values of $M = \{4, 16, 64\}$ and $Q = ML = 4M = \{16, 64, 256\}$ QAM OFDM.

The same behavior is shown in figure 4.6 when the QAM symbol size $\{M\}$ in HQFM is kept constant i.e. $M = 4$ and $L = \{2, 4, 8\}$ compared with $Q = 4L = \{8, 16, 32\}$ QAM OFDM accordingly. Both figures shows that $\Pr(\xi \geq \xi_0)$ does not depend on
the HQFM symbol size \( \{Q\} \). It is a function of number of subcarriers \( \{N\} \) only.

### Figure 4.6: CCDF of PAPR of different \( L/4 \) HQFM \( \{L = 2, 4, 8; M = 4\} \) compared with \( Q = 4L = \{8, 16, 32\} \) QAM OFDM

#### 4.4 PAPR as function of \( h \) (Modulation Index)

Previously, we conclude that PAPR reduces if the number of FSK frequencies \( \{L\} \) increases while keeping the number of bits per subcarrier \( \{n = \log_2 Q = \log_2 ML\} \) constant. Or we can say that PAPR decreases by reducing the QAM symbol size \( \{M\} \).

This section describes the PAPR reduction capability of HQFM when the frequency separation \( f_\Delta \) between FSK tones is altered while keeping both \( \{M\} \) and \( \{L\} \) i.e \( \{Q\} \) constant. To avoid the confusion between \( f_\Delta \) and \( \Delta f \) (frequency separation among OFDM subcarriers), the term modulation index, \( h = f_\Delta T_s \) will be used for LFSK.

Figure 4.7 compares the CCDF of PAPR of \( 4/16 \) \( \{L = 4; M = 16\} \) HQFM with \( Q = ML = 64\) QAM OFDM with \( N = 512 \) when \( h < 1 \). It is obvious that the PAPR reduction capability of HQFM with \( h = 0.5 \) (coherent FSK used) and \( h = 1 \) (non-coherent FSK used) shows the best results. Both are the cases for orthogonal FSK.
CHAPTER 4

PAPR ISSUES IN HQFM-OFDM

But for $h = 2$, which is also the case of non-coherent orthogonal FSK, there is no improvement in CCDF of HQFM based OFDM’s PAPR as shown in figure 4.8. This figure shows that $h = 1.5$ brings the CCDF of HQFM OFDM’s PAPR to the least value.
Similar figures can be drawn for even multiples of $h = 1$. Thus, keeping $h = \{2, 4, 6, 8 \ldots\}$ does not improve the statistics although these are the cases for non-coherent orthogonal FSK. The PAPR statistics only improves for odd multiples of either $h = 0.5$ (coherent orthogonal FSK) or $h = 1$ (non-coherent orthogonal FSK) as shown in figure 4.9.

**Figure 4.9**: CCDF of PAPR of 64QAM OFDM compared with 4/16 HQFM using odd multiples of (a) $h = f_\Delta T_\pi = 0.5$ (coherent FSK) (b) $h = f_\Delta T_\pi = 1$ (non-coherent FSK)
4.5 Modified HQFM-OFDM

The PAPR reduction capabilities of HQFM-OFDM are not as good as reduction algorithms applied to conventional $2^n$ QAM-OFDM. One such algorithm available in literature is PTS-OFDM [Latif, Gohar: 02, 03] which can be applied to QAM OFDM. Therefore, a modification is proposed, termed as HQFM-I.

It is proved that HQFM-OFDM shows a strong dependence of decrease in PAPR on number of keying frequencies \( L \) while keeping the symbol size \( Q = ML \) constant [Latif, Gohar: 06, 07]. Thus, one method to reduce PAPR of the HQFM-OFDM modulator is to use multi-FSK frequencies and choose that symbol for transmission that exhibits low PAPR.

**FIGURE 4.10:** HQFM-I Modulator

In HQFM-I [Latif, Gohar: 07], a multi-stage modulator is designed which uses variable FSK modulator to generate frequencies. In first stage, \( n-k_1 = \log_2 L_1 \) bits are used to generate \( L_1 \) frequencies and remaining \( k_1 = \log_2 M_1 \) bits are used for QAM modulation. Other stages generate \( L_2 = 2L_1, L_3 = 4L_1 \ldots L_K = 2^{K-1}L_1 \) frequencies which are used for \( M_2 = M_1/2, M_3 = M_1/4 \ldots M_K = M_1/2^{K-1} \) QAM Modulation respectively, so that the overall number of bits, \( n = \log_2 Q \), for HQFM signal in all \( K \) stages remains constant. Therefore, for all stages, \( Q = ML_1 = M_2L_2 = \ldots = M_KL_K \). Generally three stage modulator employing 4, 8 and 16- LFSK is used. The HQFM-I transmitter is shown in figure 4.10.
FIGURE 4.11: HQFM-I Demodulation

It is also assumed that there is a possibility of transmitting at least one HQFM signal in a set of $N$ signals, have maximum amplitude $A_{max,K}^2$ represented by

$$A_{max,K}^2 = \max_{i=0,1,\ldots,N-1} |x_{p,q}|^2 = \max_{i=0,1,\ldots,N-1} |A_{p,q}|^2$$

$$= (M_{1,K} - 1)^2 + (M_{2,K} - 1)^2$$

such that $M_{1,K}M_{2,K} = M_K$ are the number of QAM signals in $L_K/M_K$ HQFM signal set in stage $K$. These $K$ different values of $A_{max,K}^2$ is always known to receiver. After applying IFFT, only that HQFM-OFDM signal out of $K$ different representations is selected for transmission which exhibits the least PAPR.

The receiver, as shown in figure 4.11, first demodulates the OFDM symbols using $N$ point FFT, then after removal of zeros, determines the number of bits used by MQAM by observing the maximum amplitude in an entire symbol of $N_{used} L/M$ HQFM signals. It is usually done by calculating

$$(J_{min}, idx) = \min |\tilde{A}_{max} - A_{max,K}|^2; idx = 1,2,\ldots,K$$

Thus, $idx$ determines the maximum amplitude

$$A_{max, idx}^2 = (M_{1, idx} - 1)^2 + (M_{2, idx} - 1)^2$$
The number of bits used to encode QAM symbols in transmitted HQFM can readily be found using equation (4.8) as

\[ k_{idx} = \left\lfloor \log_2 \left( \frac{M_{1, idx}^2 + (M_{2, idx} - 1)^2}{n} \right) \right\rfloor \]  

(4.9)

where \( \lfloor . \rfloor \) denotes the floor function. Since \( n = \log_2 Q \) is constant for every transmitted HQFM signal, therefore it is always known by the receiver. Hence \( (n-k_{idx}) \) bits can be found after knowing \( k_{idx} \) using equation (4.9). Now a simple two-stage demodulation process is carried out to detect the HQFM. In first stage the correct number of frequencies \( L_{idx} = \log_2(n-k_{idx}) \) is detected using a bank of \( L_{idx} \) correlators. After a correct decision of frequencies, simple minimum distance criterion is used to detect the QAM symbols.

Monte Carlo simulations show that PAPR reduction capability is comparable to PTS-OFDM. Figure 4.12 compares the resulting OFDM transceiver (HQFM-I) with conventional OFDM and OFDM employing PTS. The results are comparable with PTS but, PTS utilizes side-information to be transmitted while, HQFM does not.

To further reduce the PAPR, HQFM-II [Latif, Gohar: 07] is proposed which is a combination of normal HQFM and PTS algorithm (HQFM-II). The whole HQFM
signal set containing \( N \) subcarriers, is divided into \( V \) adjacent subblocks, \( V \) being integral power of 2 (i.e. \( \log_2 V \in \mathbb{Z}^+ \)) with equal number of subcarriers \( N_v \geq 64 \) in each subblock.

Results show that phase vector of \( \{1, -1\} \) is sufficient to obtain desirable results, which can be detected, in one or two iterations, without transmitting it. From figure 4.13 it is obvious that at probability \( \leq 10^{-7} \), PAPR of 256QAM-OFDM with \( N = 512 \) is 15.8dB, which is 3dB higher than PAPR of PTS-OFDM symbol and 4.4dB higher than 16/16 HQFM-II.

\[
\Pr\{ \xi \geq \xi_0 \}
\]

\[
10^{-6} \quad 10^{-4} \quad 10^{-2} \quad 10^{0}
\]

6 8 10 12 14 16

**Figure 4.13:** Comparison of PTS-OFDM and HQFM-II-OFDM with SI \( \in \{1, -1\} \)

### 4.6 System’s Complexity

It is possible to derive a closed-form expression for the computational complexity of HQFM and compare it with OFDM and PTS-OFDM systems. For the analysis presented here, the coding overhead will be neglected, and attention will be focused solely on the FFT-based modulator. The systems complexity of different HQFM-OFDM can be compared with QAM-OFDM and PTS in terms of IFFT/FFT operations plus complexity of baseband modulation/demodulations. Table 4.1 compares number of IFFT/FFT operations and complexity of baseband modulation/demodulation processes employed in different forms of HQFM with OFDM and PTS.
From the baseband representation of QAM, PSK [Proakis: 89] or HQFM (equations (3.10)) systems, it is obvious that they do not require complex additions. The addition complexity exclusively depends upon the number of complex additions required for IFFT operations which is $N \log_2 N$ additions per IFFT operation. From table 4.1, it is obvious that OFDM and HQFM requires $N \log_2 N$ additions while PTS, HQFM-I and HQFM-II requires $4N \log_2 N$, $3N \log_2 N$ and $2N \log_2 N$ additions respectively. Thus any form of HQFM achieves less addition complexity as compared to PTS systems.

It can be shown that radix-2 $N$ point IFFT performs $(N/2) \log_2 N$ complex multiplications[Proakis, Manolakis: 95]. For OFDM and PTS, only IFFT operation plays the significant role. Therefore the total number of complex multiplications required for OFDM is $(N/2) \log_2 N$, while it is $v$ times greater for PTS. If $v = 4$, then total number of complex multiplications is $2N \log_2 N$. For HQFM, extra $N_{used}$ complex multiplications are required in modulation process. For statistical independence in OFDM symbols, $N_{used}$ is not assumed significantly less than $N$. Typically $N_{used}$ is around $0.75N$. Thus, the total

<table>
<thead>
<tr>
<th></th>
<th>Transmitter</th>
<th>Receiver</th>
<th>Extra Overhead (SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IFFT Ops</td>
<td>Mod.</td>
<td>FFT Ops</td>
</tr>
<tr>
<td>OFDM</td>
<td>One</td>
<td>Simple $2^n$ QAM</td>
<td>One</td>
</tr>
<tr>
<td>OFDM PTS</td>
<td>4(^1)</td>
<td>Simple $2^n$ QAM</td>
<td>One</td>
</tr>
<tr>
<td>HQFM</td>
<td>One</td>
<td>$2^k QAM/2^n FSK$</td>
<td>One</td>
</tr>
<tr>
<td>HQFM-I</td>
<td>3(^2)</td>
<td>Multi FSK</td>
<td>One</td>
</tr>
<tr>
<td>HQFM-II</td>
<td>$2^l$</td>
<td>$2^k QAM/2^n FSK$</td>
<td>One</td>
</tr>
</tbody>
</table>

\(^1\) refers as length of phase rotation vector (4 for PTS and 2 is for HQFM-II)  
\(^2\) depends on number of FSK stages.
number of complex multiplications required is \((N/2)\log_2 N + 0.75N = N/4\log_2(8N^2)\). For HQFM-I with atmost three FSK stages, the complexity is 3 times greater than HQFM, but still less than PTS for \(N > 32\). For HQFM-II, \(v = 2\) and needs \(N_{\text{used}} = 0.75N\) extra multiplications for modulation, thus the total number of complex multiplications required is \(M\log_2N + 0.75N = N/4\log_2(8N^4)\). Also its complexity is less than PTS system. This discussion is summarized in Table 4.II for different number of subcarriers.

**Table 4.II: Number of Complex Multiplication Required for QAM, PTS and HQFM in OFDM with Different Number of Subcarriers**

<table>
<thead>
<tr>
<th>(N)</th>
<th>OFDM</th>
<th>PTS-OFDM</th>
<th>HQFM-OFDM</th>
<th>HQFM-I</th>
<th>HQFM-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>32</td>
<td>128</td>
<td>44</td>
<td>132</td>
<td>76</td>
</tr>
<tr>
<td>32</td>
<td>80</td>
<td>320</td>
<td>104</td>
<td>312</td>
<td>184</td>
</tr>
<tr>
<td>64</td>
<td>192</td>
<td>768</td>
<td>240</td>
<td>720</td>
<td>432</td>
</tr>
<tr>
<td>128</td>
<td>448</td>
<td>1792</td>
<td>544</td>
<td>1632</td>
<td>992</td>
</tr>
<tr>
<td>256</td>
<td>1024</td>
<td>4096</td>
<td>1216</td>
<td>3648</td>
<td>2240</td>
</tr>
<tr>
<td>512</td>
<td>2304</td>
<td>9216</td>
<td>2688</td>
<td>8064</td>
<td>4992</td>
</tr>
<tr>
<td>1024</td>
<td>5120</td>
<td>20480</td>
<td>5888</td>
<td>17664</td>
<td>11008</td>
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<tr>
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<td>11264</td>
<td>45056</td>
<td>12800</td>
<td>38400</td>
<td>24064</td>
</tr>
</tbody>
</table>

**REFERENCES**


[Latif, A.; Gohar, N. D.: 06]: “A Hybrid MQAM-LFSK OFDM Transceiver


In this chapter the basic performance properties of HQFM and HQFM-OFDM are studied. It is assumed throughout this dissertation that the transmitted HQFM signals employ FSK signals with modulation index $h = 1$. Consider such complex-valued HQFM signals (eq. (3.10) - (3.13)) which are passed through an ideal channel (with channel attenuation $\alpha = 1$) and only complex additive white Gaussian noise (AWGN) is added. The received signal then becomes

$$r(t) = x(t) + n(t) = A_m e^{j(\phi_m + \phi(t)) + 2\pi f_j t) + n(t)}$$

(5.1)

where $A_m e^{j\phi_m}$ is a set of transmitted QAM constellation points and takes up discrete values $(2m_1 - 1 - M_1) + j(2m_2 - 1 - M_2); \{1 \leq m_1 \leq M_1; 1 \leq m_2 \leq M_2; M = M_1 M_2\}$ and

$$f_j(t) = 2\pi \lambda_f t$$

(5.2)

with $\lambda_f = 2L - 1 - L; 1 \leq l \leq L$ and $n(t)$ is zero mean complex AWGN with autocorrelation function
Since the noise components are uncorrelated and because they are Gaussian, they are statistically independent. In this chapter, we shall be concerned with the problem of receiver structure and error rate performances of uncoded signals when the modulated signal is transmitted through an AWGN channel.

## 5.1 HQFM Demodulation

As discussed in chapter 3, HQFM signal set consists of $2^{n-k} \{L\}$ sets of $2^k \{M\}$-QAM signals where $n$-bit signals in each set are orthogonal to signals in other sets. Therefore, a two stage demodulator is suggested, which estimates the correct frequency first and then MQAM signals are estimated using conventional method. Before the estimation of correct QAM signals, phase acquisition algorithm is applied to remove any phase discrepancy caused during the frequency estimate stage. This algorithm is discussed in section 5.3 in detail.

Figure 5.1 shows such a demodulation process performed in $p^{th}$ signaling interval. This receiver structure requires that the LFSK detector determine the part of symbols used to generate the frequency $f_i$. In each branch of the frequency detector, an MQAM detector is used to estimate the other part of the symbols i.e. amplitudes $\{A_m\}$ and phase shift $\{\vartheta_m\}$. Finally the outputs of the two parts of the HQFM receiver are combined to obtain an estimate of the transmitted source symbol.

In stage I, the correct frequency estimate $f_i^{(p)}$ is made using bank of $L$ matched filters followed by an envelope detector. If $f_i^{(p)}$ is output of $i^{th}$ envelope detector in $p^{th}$ signaling interval then, the correct frequency estimate is given as

$$\hat{f}_i = \max_{i=1,2,\ldots,L} |f_i^{(p)}|$$

(5.4)

where $\hat{f}_i$ is the possible transmitted signal. The decision variables $f_i^{(p)}$, $i=1, 2, \ldots L$ are assumed to be mutually statistically independent.
The random variables $f_i$ for $i \neq l$ are described by Rayleigh probability density function (p.d.f.)

$$p(f_i) = \frac{f_i}{2 \sigma^2 N_0} e^{-f_i^2/4\sigma^2 N_0}$$

(5.5)

While for $i = l$, the random variable $f_i$ is described by Rice p.d.f.

$$p(f_i) = \frac{f_i}{2 \sigma^2 N_0} e^{\left(\frac{f_i^2 + 4\sigma^2}{4\sigma^2 N_0}\right)} I_0 \left(\frac{f_i}{N_0}\right)$$

(5.6)

where $I_0(.)$ is the zeroth order modified Bessel function of the first kind. The probability of correct decision is simply the probability that the output of $l^{th}$ envelope detector exceeds the output of other $i \{i = 1, 2, \ldots, L; i \neq l\}$ detectors. Since $f_i; i \neq l$ are statistically independent and identically distributed, the probability of correct decision is
CHAPTER 5 PERFORMANCE IN AWGN

\[
P_{e,\text{FSK}} = \int_{0}^{f_I} \int_{0}^{f_R} p(f_I) p(f_R) df_I df_R
\]

\[
= \int_{0}^{\pi} \left[ 1 - e^{-\gamma f_I^2/\ln N_0} \right]^{L-1} p(f_I) df_I
\]  

Solving the integral in equation (5.7) after substituting \( p(f_I) \) from equation (5.6), the probability of correct frequency estimate can be written as

\[
P_{e,\text{FSK}} = \frac{\sum_{q=0}^{L-1} (-1)^q \left( \frac{L-1}{q+1} \right) e^{-\gamma q/(q+1)}}{\sum_{q=0}^{L-1} (-1)^q \left( \frac{L}{q+1} \right) e^{-\gamma q/(q+1)}}
\]  

(5.8)

where \( \gamma = \mathcal{E}_s/N_0 \) is SNR per \( n \)-bit symbol, \( \mathcal{E}_s \) is average symbol energy and \( N_0 \) is single sided noise power spectral density.

In stage II, when correct frequency estimate is made, QAM symbols are detected by computing the distance between the received symbol \( \hat{A}_m = \hat{A}_m + j\hat{A}_mQ \) and \( M \) possible transmitted symbols. Decision is made in favor of the point closest to \( \hat{A}_m \) i.e.

\[
\hat{A}_m = \min_{1 \leq j \leq M} | A_m - \hat{A}_m^{(p)} |^2
\]  

(5.9)

where \( \hat{A}_m^{(p)} \) is \( m \)th transmitted signal in \( p \)th signaling interval.

Now consider a simple case of QAM modulation yielding a square constellation i.e. \( M = M_1 M_2 = M_1^2 \) for \( M_1 = M_2 \). This can be viewed as two \( M_1 \)-ASK (or \( \sqrt{M} \)-ASK) signals entrenched on quadrature/ in-phase carriers. Thus, probability of correct decision for MQAM given the correct frequency estimate made is

\[
P_{e,\text{QAM/FSK}} = (1 - P_{e,\sqrt{M} \text{-ASK}})^2
\]

\[
= 1 - 2P_{e,\sqrt{M} \text{-ASK}} + P_{e,\sqrt{M} \text{-ASK}}^2
\]  

(5.10)

where \( P_{e,\sqrt{M} \text{-ASK}} \) is the probability of error for \( \sqrt{M} \)-ASK and is given as [Lathi: 98]

\[
P_{e,\sqrt{M} \text{-ASK}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left( \frac{3\gamma}{\sqrt{M} - 1} \right)
\]  

(5.11)

where \( Q(\cdot) \) defines the Gaussian Q function. Hence equation (5.10) becomes
\[ P_{e,QAM/FSK} = 1 - 4 \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \sqrt{\frac{3\gamma}{M - 1}} \right) + 4 \left( 1 - \frac{1}{\sqrt{M}} \right)^2 Q^2 \left( \sqrt{\frac{3\gamma}{M - 1}} \right) \]  

(5.12)

If the transmitted QAM symbols belong to cross constellation i.e. number of bits, \( k \) is odd, there is no equivalent \( \sqrt{M} \)-ASK which can be used to equate the performance. One way is to extend each side of \( 2^{k-1} \)QAM square constellation by adding \( 2^{k-3} \) symbols and ignoring the corners [Cioffi:07]. Now assigning square decision regions around each point, the symbol error probability SER can be approximated as

\[ P_{e,QAM/FSK} \approx 1 - 4 \left( 1 - \frac{1}{\sqrt{2M}} \right) Q \left( \sqrt{\frac{3\gamma}{M - 1}} \right) + 4 \left( 1 - \frac{2}{\sqrt{M}} \right) Q^2 \left( \sqrt{\frac{3\gamma}{M - 1}} \right) \]

\[ \approx 1 - 4 \left( 1 - \frac{1}{\sqrt{2M}} \right) Q \left( \sqrt{\frac{3\gamma}{M - 1}} \right) + 4 \left( 1 - \frac{2}{\sqrt{M}} \right) Q^2 \left( \sqrt{\frac{3\gamma}{M - 1}} \right) \]  

(5.13)

Combining equations (5.12) and (5.13), probability of correct decision for MQAM given the correct frequency estimate made becomes

\[ P_{e,FSK} = P_{e,FSK} P_{e,QAM|FSK} \]

\[ P_{e,FSK} = 1 - 4 \left( 1 - \frac{2}{\sqrt{M}} \right) Q \left( \sqrt{2g\gamma} \right) + 4 \left( 1 - \frac{2}{\sqrt{M}} \right) Q^2 \left( \sqrt{2g\gamma} \right) \]

(5.14)

Here \( g = 1.5/(M-1) \) and \( C \) is a constant that depends on the nature of QAM constellation, i.e.

\[ C = \begin{cases} 
1 & M_1 = M_2 \\
0.5 & M_1 \neq M_2 
\end{cases} \]  

(5.15)

The total probability of correct decision is \( P_e = P_{e,FSK} P_{e,QAM|FSK} \). Using equation (5.8) and (5.14), the probability of symbol error then becomes

\[ P_e = 1 - P_{e,FSK} P_{e,QAM|FSK} \]

\[ = 1 - \frac{1}{L} \left[ 1 - 4 \left( 1 - \frac{C}{M} \right) Q \left( \sqrt{2g\gamma} \right) + 4 \left( 1 - \frac{1}{CM} \right)^2 Q^2 \left( \sqrt{2g\gamma} \right) \sum_{q=0}^{L-1} (-1)^q \left( \frac{L}{q+1} \right) e^{-q\eta/(q+1)} \right] \]  

(5.16)

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The SER for $L/M$ HQFM for $n = 4$ and $n = 6$ are plotted against $E_s/N_0$, using expression (5.16) and shown in figure 5.2 and 5.3 respectively. Figure 5.2 illustrates that SER of 4/4 HQFM is nearly equal to that of 4QAM at $P_e = 10^{-3}$. Also, it compares the
SER performance of 16QAM and 4FSK. There is a loss of about 3dB against FSK and a gain of 7dB as compared to 16QAM at $P_e = 10^{-5}$.

Keeping the same set of frequencies, $L = 4$, increasing the QAM size from $M = 4$ to $M = 16$, the size of corresponding HQFM signal set also increases, therefore, an increase in SER is expected as shown in figure 5.3. This figure clearly shows that the required SNR increases from 17 dB to 24 dB (SNR of pure 16QAM at $P_e = 10^{-5}$), therefore the same SNR is required for reliable transmission at $P_e = 10^{-5}$ by just increasing two bits per HQFM symbol. If the number of keying frequencies is increased and QAM size is reduced while keeping the number of bits per HQFM symbol constant, the required SNR decreases e.g. a gain of 8 dB can be achieved if 4 times more frequencies are used to decode a 6 bit HQFM symbol.

5.2 BER and SER Relationship

The relationship of BER $\{P_b\}$ and SER $\{P_e\}$ for simple QAM and FSK signals are given as [Proakis: 89] [Rappaport: 01]

$$P_{b,\text{MQAM}} = \frac{1}{\log_2 M} P_{e,\text{MQAM}}$$

(5.17)

$$P_{b,\text{LFSK}} = \frac{L}{2(L-1)} P_{e,\text{LFSK}}$$

(5.18)

But the relationship between $P_e$ and $P_b$ for HQFM signals is not straightforward as mentioned in equations (5.17) and(5.18), because of two types of errors occurring at two different stages [Latif, Gohar: 06]. If $P_{b,1}$ and $P_{b,2}$ are bit error probability occurring at stage 1 and II respectively then total bit error probability is

$$P_b = P_{b,1} + P_{b,2}$$

(5.19)

A simple way to calculate errors occurring at different stages are to consider the transmission of all-zero or all-one bit pattern and calculate the probability of occurrence of either 1’s or 0’s accordingly.

Suppose all-zero bit pattern is transmitted and probability of occurrence of 1’s is
calculated. We know that

Number of 0’s or 1’s in all possible combinations of $n \ (\log_2 Q)$ bits = $n^{2^n - 1} = nQ/2$

where $Q = ML$ are the total number of HQFM signals.

In stage I, when decision is taken on account of correct frequency, error occurs when at least one bit out of $n \ (\log_2 Q)$ bits is wrong i.e. there are $Q-M = (L-1)M$ equally likely incorrect-frequency $n$-bit patterns. Therefore

Total number of incorrect frequency patterns = $n(L-1)M$

In the remaining $M$ correct frequencies patterns,

Number of 1’s = $kM/2$

Therefore,

Total number of 1’s in incorrect pattern = $nQ/2-kM/2 = \frac{1}{2} M(nL-k)$

The probability of bit error due to stage I can be found by taking the ratio of total 1’s to the number of incorrect-frequency patterns i.e.

\[
P_{b,1} = \frac{\text{Number of 1’s in incorrect pattern}}{\text{Number of incorrect frequency patterns}} (1-P_{c,FSK}) = \frac{1}{2} \left(\frac{nQ}{2} - kM\right) \left(1-P_{c,FSK}\right)
= \frac{nL-k}{2n(L-1)} \left(1-P_{c,FSK}\right)
\]

(5.20)

When a symbol error is due to an error in the stage II, $(n - k)$ bits out of $n$-bit pattern are correct, and the decision affects the $k$ bits only. This causes at least one bit in error out of $k \ (\log_2 M)$ bits per $n \ (\log_2 Q)$ bit symbol

\[
P_{b,2} = \frac{1}{n} \left( P_e - (1-P_{c,FSK}) \right) \]

(5.21)

Putting the value of $P_e$ from equation (5.16) in equation(5.21), we get

\[
P_{b,2} = \frac{1}{n} \left[ 1 - P_{e,\text{QAM}/FSK} \right] P_{c,FSK}
\]

(5.22)
Adding equations (5.20) and (5.21), equation (5.19) can be rewritten as

\[
P_b = P_{b,1} + P_{b,2}
\]
\[
= \frac{nL-k}{2n(L-1)}(1-P_{c,FSK}) + \frac{1}{n}\left\{1-P_{c,QAM|FSK}\right\}P_{c,FSK}
\]
\[
= \frac{1}{n}\left\{\frac{nL-k}{2(L-1)} + \left\{1-\frac{nL-k}{2(L-1)}-P_{c,QAM|FSK}\right\}P_{c,FSK}\right\}
\]

(5.23)

Rewriting equation (5.23) in terms of equations (5.8) and (5.14)

\[
P_b = \frac{1}{n}\left\{\frac{nL-k}{2(L-1)} + \frac{1}{L}\left\{4\left(1-\sqrt{\frac{C}{M}}\right)Q\left(\sqrt{2g\gamma}\right)\right\}
\]
\[
-4\left(1-\sqrt{\frac{1}{CM}}\right)^{2C}Q^2\left(\sqrt{2g\gamma}\right) - \frac{nL-k}{2(L-1)}\sum_{q=0}^{L-1}(-1)^q\left\{\frac{L}{q+1}e^{-\gamma q/(q+1)}\right\}\right\}
\]

(5.24)

Here \(g = 1.5/(M-1)\), \(n = \log_2 ML\), \(k = \log_2 M\) and \(\gamma = n\gamma_b\); \(\gamma_b\) is average SNR per bit.

For \(L = 1\), \(\log_2 L = 0\) which implies \(n = k\), equation (5.24) reduces to

\[
P_b = \frac{1}{k}\left\{4\left(1-\sqrt{\frac{C}{M}}\right)Q\left(\sqrt{2g\gamma}\right) - 4\left(1-\sqrt{\frac{1}{CM}}\right)^{2C}Q^2\left(\sqrt{2g\gamma}\right)\right\}
\]
\[
= \frac{1}{k}\left\{1-P_{c,MQAM}\right\}
\]

(5.25)

which is actually equation (5.17). Similarly, for \(M = 1\), \(k = \log_2 M = 0\), hence equation (5.24) reduces to

\[
P_b = \frac{L}{2(L-1)}\left\{1-\sum_{q=0}^{L-1}(-1)^q\left\{\frac{L}{q+1}e^{-\gamma q/(q+1)}\right\}\right\}
\]
\[
= \frac{L}{2(L-1)}\left\{1-P_{c,FSK}\right\}
\]

(5.26)

which is actually equation (5.18). Hence the whole HQFM system reduces to pure QAM when \(L = 1\) and pure FSK when \(M = 1\).
Comparing figure 5.4 with figure 5.2, the relationship between BER \( P_b \) and SER \( P_e \) is not related to each other in a simple way as in the case of LFSK and MQAM. However, bounds on the relationship are

\[
\frac{ML}{2(ML-1)} P_e \leq P_b \leq \frac{1}{n} P_e
\]  

When viewed in term of SER, (figure 5.2) at \( P_e = 10^{-5} \), the required \( E_b/N_0 \) for 4FSK is less than that of either 4QAM or 4/4HQFM. On the other hand, figure 5.4 shows that the required \( E_b/N_0 \) for FSK lies between both i.e \( E_b/N_0 = 9.27 \) dB (4/4HQFM) < \( E_b/N_0 = 10.6 \) dB (4FSK) < \( E_b/N_0 = 12.6 \) dB (4QAM).

BER performance for different HQFM formats except \( M = 4 \), shows that the system performs better than both MQAM and ML-QAM but the performance is degraded as compared to non-coherent orthogonal LFSK. The power efficiency (required \( E_b/N_0 \)) increases by increasing the number of keying frequencies \( \{L\} \) or decreasing the size of QAM constellation while keeping the number of bits \( \{n\} \) per HQFM symbol constant.
Figure 5.5: BER of 4/16 and 16/4 HQFM compared with 64QAM.

Figure 5.6: BER comparison of 4/64 and 16/16 HQFM with 256QAM.

This fact is illustrated in figure 5.5 and 5.6 for \( n = 6 \) and \( n = 8 \) respectively. These curves also compare the BER performance of 64- and 256-QAMs. Also,
increasing the constellation size \( \{M\} \), the power efficiency of the \( L/M \) HQFM system decreases for fixed number of frequencies \( \{L\} \). But in chapter 3, table 3.1 states that null to null bandwidth efficiency \( \{\eta_B\} \) of \( L/M \) HQFM can be improved by increasing the size of QAM constellations \( \{M\} \) for fixed number of frequencies \( \{L\} \). Thus for \( L/M \) HQFM, there is a tradeoff between the power efficiency of LFSK and bandwidth efficiency of MQAM employed.

### 5.3 Phase Acquisition in QAM Constellations

An HQFM signal contains information about the combined amplitude-phase \( A_m e^{j\theta_m} \) and frequency \( \phi(t) \). The value of \( \phi(t) \) depends not only on the current data, but also on the past data i.e.

\[
\phi_{l,q} = \pi h_{l,q} + \pi h \sum_{s=0}^{q-1} I_{r,s}
\]

(5.28)

where \( h = f_s T_s \) is the modulation index of the signal.

Therefore, the current HQFM signal contains the information about the current data as well the accumulation of all the previous data. When such signal is passed through a channel which corrupts the data, an incorrect decision made by the receiver on frequency part of the HQFM signal in past will effectively affect the current QAM
symbol part even if the frequency estimate is correct in the current symbol. Thus, the QAM constellation received (shown as hollow dots in figure 5.7) can be observed as a rotated constellation by an uncorrected frequency (phase) offset $\delta\theta$. Such situation may arise in presence of a phase offset of say $\delta\theta = \pi/5$ radians and an AWGN channel with SNR = 10 dB.

For QAM, usually square regions are assigned around each data symbol in the constellation so even with small $\delta\theta$, the data symbol may move in a different decision region and hence increases the bit error rate, $P_b$, in stage II of the demodulation process. For example say following data is sent to the receiver using 16QAM modulation using gray coding

\[
T_x = \{0, 12, 14, 9, 8, 13, 5, 6, 7, 8, 11\}
\]
\[
T_x = \{0000, 1100, 1110, 1001, 1000, 1101, 0101, 0110, 0111, 1000, 1011\}
\]

and suppose, the received data is:

\[
R_x = \{0, 14, 11, 12, 8, 12, 1, 4, 6, 8, 3\}
\]
\[
R_x = \{0000, 1110, 1011, 1100, 1000, 1100, 0001, 0100, 0101, 1000, 0011\}
\]

resulting in an SER of 8/11 i.e. 73\% of the received data is incorrect. The resulting BER is $10/44 = 0.227$.

The problem becomes even worse, as larger constellation e.g. $M = 32, 64, 128, 256 \ldots$ are used to improve the bandwidth efficiency. For large constellations, the problem of phase offset correction become more critical, imposing strict constraints on the quality of acquisition algorithm used. This fact implies that the algorithm must be devised which performs well in harsh channel condition (high SNR) and at the same time should be simple to implement.

In literature, the most popular method for phase offset correction for QAM constellation of any size is $P^b$ Power Law Estimator [Moenclaey, Jonghe: 94]. This estimator produces a phase estimate according to

\[
\hat{\delta}\theta = \frac{1}{P} \arg \left[ \mathbb{E}\left\{ A_p e^{-j\beta_k} \right\} \sum_{q=1}^{N} r_q^p \right]
\]

(5.29)
where \( r_q \) is the received symbol in the \( q^{th} \) signaling interval, \( W \) is the length of observation window (length of sequence of consecutive symbols) and \( \mathbb{E}\{\cdot\} \) denotes the expectation. Equation (5.29) is the general expression for rotationally symmetric constellations like MQAM/ MPSK. The value of \( P \) is 4 for QAM and \( M \) for MPSK. The \( P^{th} \) power estimate method assumes that the carrier phase is constant over an observation window, and returns an estimate of the carrier phase for the sequence. The length of observation window must be large enough to collect the information about the correct phase estimate.

**Figure 5.8:** Pictorial View of Phase Offset Acquisition Algorithm

In our case, this method fails because the phase offset does not remain constant over an observation window even on low SNR or small window size. Therefore, an algorithm is proposed which estimate the continuously changing phase offset \( \hat{\delta} \). The algorithm is summarized as follows:

1. Initialize \( \hat{\delta} = 0 \).
2. Before starting transmission, send a pilot signal (known data) whose position in the constellation is fixed during transmission. On reception calculate the phase offset:
   \[
   \hat{\delta} = \arg(R_{x,\text{pilot}}) - \arg(R_x)
   \]
3. Two points ([A, B] in figure 5.8) are computed in the
constellation which are the possible neighbors of the next received point \( R_{x_1} \) having same average power i.e. these points have same amplitudes \( A_m(0) \). Now the decision regions are circular surrounding points having equal energies (figure 5.8).

4. Calculate \( [A',B'] = e^{i\theta}[A,B] \) and select amongst them the closest point to the received symbol.

5. Step 3 is repeated till \( R_{x_2} \) is received for which only one possible neighbor exists i.e. for this received signal \([A = B]\). Update the phase offset parameter

\[
\delta \hat{\theta} = \arg(R_{x_2}) - \arg(A)
\]

6. Receive next symbol, \( R_{x_3} \) and compute the neighboring points \([C, D]\) according to step 3.

7. Go to step 3.

The proposed algorithm is a slight variation in the algorithms explained in [Georghiades: 97] [Yamanaka, Takeuchi: 97]. In the absence of this algorithm, a high BER is achieved which was not fully agreed with the equation (5.24). As described above, the phase ambiguity is achieved on symbol by symbol basis. This algorithm needs a single pilot tone thus do not affect the transmitter’s throughput.

### 5.4 Monte Carlo Simulations

Monte Carlo (MC) simulations are based on game of chance. This is of course the reason for the name “Monte Carlo’, the Mediterranean city famous for casino gambling. In this section we will validate the BER expression (5.24) evaluated for \( L/M \) HQFM using MC techniques.

Using equation (5.24), different power efficiencies at \( P_b = 10^{-5} \) for different HQFM formats are calculated and tabulated in Table 5.I. These values are also compared to pure QAM \((L = 1)\) and FSK \((M = 1)\). In the table, it is evident that BER is not a monotonic function of \( M \), i.e. there is a minimum at \( M = 4 \).
TABLE 5.1: THEORETICAL PROBABILITY OF BER (dB) AT $P_b = 10^{-5}$ FOR DIFFERENT HQFM FORMAT

<table>
<thead>
<tr>
<th>$L$</th>
<th>FSK</th>
<th>$L/M$ HQFM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$L/4$</td>
</tr>
<tr>
<td>1</td>
<td>---</td>
<td>12.6</td>
</tr>
<tr>
<td>2</td>
<td>13.35</td>
<td>10.77</td>
</tr>
<tr>
<td>8</td>
<td>9.09</td>
<td>8.31</td>
</tr>
<tr>
<td>16</td>
<td>8.08</td>
<td>7.32</td>
</tr>
<tr>
<td>32</td>
<td>7.33</td>
<td>6.71</td>
</tr>
<tr>
<td>64</td>
<td>6.56</td>
<td>6.01</td>
</tr>
</tbody>
</table>

To explain this let $M = 4$. Putting $k = \log_2 M = 2$, equation (5.24) becomes

$$P_b = \frac{nL-2}{2n(L-1)} + \frac{1}{nL} \left\{ 2Q\left(\sqrt{\gamma}\right) - Q^2\left(\sqrt{\gamma}\right) - \frac{nL-2}{2(L-1)} \sum_{l=0}^{L-1} (-1)^l \left( L \atop l+1 \right) e^{-\gamma(l+1)} \right\}$$

(5.30)

Practically, for reliable transmission, it is assumed that SNR $\gamma \gg 1$ (generally $> 10$ dB). Thus, Q function can be approximated as [Lathi: 98]

$$Q\left(\sqrt{\gamma}\right) = \sqrt{\frac{1}{2\pi\gamma}} e^{-\frac{\gamma}{2}} \quad \gamma \gg 1$$

(5.31)

Thus using approximation (5.31), for $\gamma > 10$ dB, the quantities $Q\left(\sqrt{\gamma}\right)$ and $Q^2\left(\sqrt{\gamma}\right)$ can be ignored. Now, equation (5.30) becomes

$$P_b = -\frac{nL-2}{2n(L-1)} \sum_{l=0}^{L-1} (-1)^l \left( L \atop l+1 \right) e^{-\gamma(l+1)}$$

(5.32)

The SER for LFSK is given as [Lathi: 98]

$$P_{e,FSK} = \frac{1}{L} \sum_{l=0}^{L-1} (-1)^l \left( L-1 \atop l \right) e^{-\gamma(l+1)}$$

(5.33)

$$= -\frac{1}{L} \sum_{l=0}^{L-1} (-1)^l \left( L \atop l+1 \right) e^{-\gamma(l+1)}$$
Using the relationship of BER with SER (equation (5.18)), the BER of LFSK becomes

\[ P_{b_{,FSK}} = \frac{1}{2(L-1)} \sum_{i=1}^{L} (-1)^i \left( \frac{L}{i+1} \right) e^{-2\gamma/(i+1)} \]  \hspace{1cm} (5.34)

So equation (5.32) becomes

\[ P_b = \frac{nL-2}{nL} P_{b_{,FSK}} < P_{b_{,FSK}} \] \hspace{1cm} (5.35)

Also, 4QAM always performs better than BFSK \((L = 2)\) and increasing the number of orthogonal frequencies, \(L\), improves the performance [Proakis: 89] so any combination of HQFM with \(M = 4\) is expected to perform better than their respective FSK/ QAM counterparts. Equation (5.30) is not true for QAM with large constellation size \((M > 4)\) because the role of \(Q\) function cannot now be ignored while evaluating equation (5.24).

The results tabulated in table 5.I are also confirmed through Monte Carlo simulations. The total samples taken are \(10^{11}\). Following assumptions are made while performing these simulations:

- There is no pulse shaping performed at transmitter i.e. the pulse shape for transmitted data is assumed to be rectangular.
- The channel assumed here is AWGN only.
- Data symbols both employed in QAM and FSK are independent and equiprobable i.e. \(Pr(x = QAM) = 1/M\) and \(Pr(y = FSK) = 1/L\).
- There is no filtering within the system, and as a result no ISI is introduced.
- The phase acquisition algorithm, described in section 5.3, is applied before the usual QAM demodulation process.
- For all cases, the function assumes the use of a Gray-coded signal constellation

Figure 5.9 and 5.10 shows the curves for BER for different formats of 6-bit and 8-bit HQFM respectively. Also the results are compared with 64- and 256- QAM (outermost curves). Both figures 5.9 and 5.10 show that the results fully agree with
equation (5.24), hence, prove the validity of the equation.

**Figure 5.9:** MC simulated BER curves for 2/32, 4/16, 8/8 and 16/4 HQFM compared with 64QAM

**Figure 5.10:** MC simulated BER curves for 2/128, 4/64, 8/32 and 16/16 HQFM compared with 256QAM
5.5 BW Efficiency vs. BER

The HQFM signaling technique discussed so far are compared with other digital modulation schemes like QAM or FSK in number of ways. For example, we compare it on the basis of required SNR $\{E_b/N_0\}$ to achieve a specified probability of error, say $P_b = 10^{-5}$. This tells us the power efficiency of the modulation system. Also, we compared HQFM with MQAM and LFSK on the basis of bandwidth occupancy or in other words, in terms of bandwidth efficiency. The most compact and meaningful comparison is the one that is based on the normalized data rate $\{R_b/B\}$ (bits per second per Hertz of bandwidth) versus SNR per bit, $\{\gamma_b\}$ required to achieve a given error rate.

Table 5.II summarizes the bandwidth efficiency $\{\eta_B = R_b/B\}$ and bit error rate probability formulas for HQFM signals compared with MQAM and LFSK. A comparison of their performance is illustrated in figure 5.11 for $P_b = 10^{-5}$ (equation (5.24)). For the purpose of comparisons, figure 5.11 also illustrates the capacity of a band limited AWGN channel having bandwidth $B$ and a SNR per bit $\gamma_b = E_b/N_0$.

According to Shannon- Hartley capacity theorem [Shannon: 48], the capacity of such a channel in bits per second is

$$C_H = B \log_2 \left(1 + \frac{C_H \cdot E_b}{B \cdot N_0} \right)$$

Hence

$$\frac{E_b}{N_0} = \frac{2^{C_H/B} - 1}{C_H / B}$$

When $C_H/B = 1$, $E_b/N_0 = 1$ (0 dB). Also, $E_b/N_0$ increases exponentially as $C_H/B \to \infty$. On the other hand as $C_H/B \to 0$, $E_b/N_0 = \ln 2 = -1.6$ dB. This value of $E_b/N_0$ is called the Shannon limit. It is not possible in practice to reach the Shannon limit, however in the case of orthogonal signaling such as LFSK, the lower limit is achieved by increasing the number of frequencies $L$ to $\infty$. 
### Table 5.11: Performance and Rate of L/M HQFM Compared with MQAM and LFSK

<table>
<thead>
<tr>
<th>Modulation</th>
<th>$\eta_R = R_b/B$</th>
<th>Error Rate Performance&lt;sup&gt;1&lt;/sup&gt;</th>
</tr>
</thead>
</table>
| L/M HQFM   | $\log_2 L + \log_2 M / L$ | \[\frac{nL - \log_2 M}{2n(L-1)} - \left(\frac{nL - \log_2 M}{2nL(L-1)}\right) \]
+ \[\frac{4}{nL}\left(1 - \frac{C}{M}\right)Q\left(\sqrt{2gnR_b}\right)\]
+ \[\frac{4}{nL}\left(1 - \frac{1}{CM}\right)^{2c}Q^2\left(\sqrt{2gnR_b}\right)\]
\[
\times \sum_{q=0}^{L-1} (-1)^q \left(\frac{L}{q+1}\right)e^{-\gamma q(q+1)}
\]
| MQAM       | $\log_2 M$       | \[\frac{4}{n}\left(1 - \frac{C}{M}\right)Q\left(\sqrt{2gnR_b}\right)\]
\[
- \left(1 - \frac{1}{CM}\right)^{2c}Q^2\left(\sqrt{2gnR_b}\right)
\]
| LFSK       | $\log_2 L / L$   | \[\frac{1}{2(L-1)} \sum_{l=1}^{L-1} (-1)^{l+1} \left(\frac{L}{l+1}\right)e^{-\gamma_l(l+1)}
\]

---
<sup>1</sup> $n$ is the total number of bits per symbol
CHAPTER 5  PERFORMANCE IN AWGN

Bandwidth Efficiency

$\eta_B (\text{bps/Hz})$

$0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \quad 30 \quad 35$

$10 \quad -1 \quad 10 \quad 0 \quad 10 \quad 1 \quad 10 \quad 2$

$2F \quad 4F \quad 8F \quad 16F \quad 32F \quad 64F$

$512Q \quad 1024Q$

$\text{Shannon's Channel Capacity Limit}$

$\text{Bandwidth Limited } R_b/B > 1$

$\text{Power Limited } R_b/B < 1$

FW 5.11: Bandwidth Efficiency Plane for HQFM, QAM and FSK

BW efficiency vs. required $E_b/N_0$ for MQAM, illustrated in figure 5.11, clearly states that the BW efficiency increases by increasing $M$ but this is achieved against the required $E_b/N_0$. Consequently, MQAM is appropriate for channels in which SNR is large enough to support a normalized data rate $R_b/B \geq 1$. On the other hand, orthogonal LFSK make inefficient use of channel bandwidth in the sense that $R_b/B \leq 0.5$ i.e. LFSK trade bandwidth for reduction in $E_b/N_0$ required to achieve a given error rate. So, it is appropriate for channels in which SNR is small and there is sufficient bandwidth to allow $R_b/B < 1$.

HQFM signals lie between the two limits i.e. they are simultaneously bandwidth and power efficient. As the number of keying frequencies increases, bandwidth efficiency decreases, which decreases the required $E_b/N_0$. On the other side, when QAM size grows, the bandwidth efficiency increases which increases the required $E_b/N_0$. There are many possible combination of $L/M$ HQFM for which $\eta_B > 1$ e.g. $2/M$, $4/M$ and $8/M$ ($M > 4$). All $L/4$ HQFM requires $E_b/N_0 \leq 10\text{dB}$ for reliable transmission at $P_b = 10^{-5}$.

Comparing the data in Table 3.I (bandwidth efficiency) and Table 5.I (power

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efficiency); we observe that the same bandwidth efficiency can be achieved by using different combinations of HQFM schemes. For example, \( R_b/B = 2.0 \) bps/Hz can be obtained by conventional 4QAM, or by HQFM with a combination \( 2/8 \) \( \{L = 2, M = 8\} \) or \( 4/64 \) \( \{L = 4, M = 64\} \) while requiring a \( E_b/N_0 \) of 12.6, 13.07 and 19.41 dB, respectively, to achieve a bit error rate of \( 10^{-5} \). As another example, 16/16HQFM provides approximately the same power efficiency at a given bandwidth efficiency, \( R_b/B = 0.5 \). A point to be noted is that \( L/4 \) HQFM provides better power and bandwidth efficiency than the conventional non-coherent LFSK. Finally, HQFM with all frequency and QAM combinations provides better power efficiency than the conventional ML QAM system.

A further advantage of using HQFM appears when the comparisons are based on the power and bandwidth that contains 90\% and 99\% of the total power. In general, \( L/M \) HQFM requires less power than either \( M \) or \( ML \)-QAM. A specific example illustrating this point is obtained by observing 4/4 HQFM and the conventional 4QAM systems; these modulation schemes have approximately the same bandwidth efficiency in terms of the 90\%bandwidth, but 4/4 HQFM outperforms 4QAM by 3.33 dB at a BER = \( 10^{-5} \). From the data given in Tables 3.III and 5.I, the 99\% bandwidth efficiency of any combination of \( L/M \) HQFM is almost equal to that of the \( ML \)-QAM, but \( L/M \) HQFM requires less power than \( ML \)-QAM. As an example, 2/32, 4/16 and 8/8 HQFM have approximately the same bandwidth efficiency (\( = 0.3 \)) in terms 99\% power containment bandwidth as that of conventional 64-QAM, but these HQFMs outperform 64-QAM by 3.11, 6.31 and 9.7 dB respectively at a BER = \( 10^{-5} \).

### 5.6 BER Performance of HQFM-OFDM in AWGN

It has been shown in chapter 2 (section 2.4) that in an OFDM system, the received data symbol transmitted in \( p^{th} \) signaling interval on the \( q^{th} \) subcarrier is given by the corresponding transmitted symbol, multiplied with the channel frequency response sampled at the \( q^{th} \) subcarrier frequency plus noise i.e.

\[
y_{p,q} = h_{p,q} x_{p,q} + n_{p,q}
\]  

(5.38)

If there is an ideal linear time invariant frequency non-dispersive AWGN
channel, this translates to a parallel set of AWGN channels, with equal SNR. As a consequence, under perfect time and frequency synchronization, OFDM is completely transparent and its performance is identical to single-carrier modulation over AWGN, except for the SNR loss due to the cyclic prefix [Hara, Prasad: 03]. Thus, the expression for $P_b$ in the case of HQFM-OFDM is same as single-carrier HQFM (equation(5.24)) except for the loss in SNR {$\gamma'_b$}. No apparent advantage of OFDM over single-carrier systems can be seen here. However, OFDM is very flexible and has a structure well suited for wireless channels, which are far from ideal.

The transmitted energy per subcarriers is $\int |\phi_k(t)|^2 = T_r / (T_r - T_{CP})$ and the SNR loss because of discarded cyclic prefix in the receiver, becomes $\gamma'_b = -10 \log \left(1 - \frac{T_{CP}}{T_r}\right)$, where $\gamma_{CP} = T_{CP}/T_r$ is the relative length of the cyclic prefix. Longer the cyclic prefix, larger the SNR loss. Typically $\gamma_{CP}$ is small and the ICI- and ISI-free transmission motivates $\gamma'_b$ (less then 1dB for $\gamma_{CP} < 0.2$) [Edfors, Sandell: 96].

REFERENCES


Chapter 5 has described the design and performance of HQFM operating over classical AWGN channel. In this chapter, we consider the problems of signal designs, receiver structure and performance for more complex channels, namely channels having random time-variant impulse responses. These channels are termed as fading channels.

6.1 Preliminary Discussion: Characterization of Fading Multipath Channels

Detailed discussions on characteristics of multipath fading channels can be found in [Proakis: 89] [Rappaport: 01] [Sklar: 97]. This section shows the essence of the literature.

Multipath fading is due to multipath reflections of a transmitted wave by local dispersers such as houses, buildings, and other man-made structures, or natural objects such as forest surrounding a mobile unit. Through the multipath fading channel with $P$
paths, the received signal is written as

$$y(t) = \sum_{p=1}^{P} \alpha_p(t)e^{-j2\pi f_p \tau_p(t)} = \sum_{p=1}^{P} \beta_p(t)$$  (6.1)

where $\beta_p(t)$ is a complex-valued stochastic process. Also, $\alpha_p(t)$ and $\tau_p(t)$ are the complex-valued channel loss (or gain) and real-valued time delay for the $p^{th}$ path, both of which can be modeled as stochastic processes. Thus, the received signal is the sum of stochastic processes, so for large number of paths, according to central limit theorem, the received signal, $y(t)$ can be modeled as a complex valued Gaussian stochastic process with its mean $\mu$ and variance $\sigma^2$.

When the impulse response is modeled as a zero mean complex-valued Gaussian process, the phase $\theta(t) = \arg \{y(t)\}$ of the received signal is uniformly distributed in the interval $[0, 2\pi]$ and the envelope $\rho(t) = |y(t)|$, at any instant $t$, is Rayleigh distributed and the channel is called Rayleigh fading channel.

On the other hand, when a direct path is also available and the channel has signal reflectors, $y(t)$ cannot be modeled as a zero mean process. In this case, the envelope has a Rice distribution, and the channel is called Ricean fading channel.

### 6.1.1 Multipath Delay Profile

Assuming that the loss and phase shift of the channel associated with path delay $\tau_1$ is uncorrelated with the loss and phase shift of the channel associated with path delay $\tau_2$ [this is called uncorrelated scattering (US)], then the autocorrelation function $\phi_h(\tau_1, \tau_2; \Delta t)$ of a wide sense stationary (WSS) impulse response of the multipath channel is given as [Proakis:89]

$$\phi_h(\tau_1, \tau_2; \Delta t) = \phi_h(\tau_1; \Delta t)\delta(\tau_1 - \tau_2)$$  (6.2)

where $\delta(t)$ is the Dirac’s delta function. When setting $\Delta t = 0$; $\phi_h(\tau) \equiv \phi_h(\tau; 0)$ is called the multipath delay profile or multipath intensity profile, describing the average power output of the channel as a function of the time delay $\tau$. Figure 6.1(a) shows the graphical relationship of multipath intensity profile, $\phi_h(\tau)$ and channel impulse response, $h(\tau, t)$. 

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CHAPTER 6 PERFORMANCE IN FADING MULTIPATH CHANNELS

FIGURE 6.1: Relationship between (a) $\phi_h(\tau)$ and $h(\tau,t)$ (b) $H(f;t) = \mathcal{S}\{h(r;t)\}$ and $D_H(\zeta) = \mathcal{S}\{\phi_h(\Delta t)\}$.

If the average received powers of multipaths with equidistant delays are exponentially decaying, then this kind of profile is called an *exponentially decaying* profile and is often encountered in indoor environments [Saleh, Valenzuela: 87] [Hashemi: 93]. But, if the average received powers of multipaths with equidistant delays, are all the same, then it is called *independent and identically distributed* (i.i.d.) profile and is used to test the system performance [Steele: 92].

There are two important multipath channel parameters, the mean excess delay,

---

1 These figures are taken from [Hara, Prasad: 03]
{\bar{\tau}}, and the root mean square (r.m.s.) delay spread, {\tau_{\text{rms}}}. These parameters can be determined from power delay profile. They are respectively defined as

\begin{align}
\bar{\tau} &= \frac{1}{\sigma^2} \int_{-\infty}^{\infty} \tau \phi_{h}(\tau) d\tau \\
\tau_{\text{rms}} &= \sqrt{\frac{1}{\sigma^2} \int_{-\infty}^{\infty} \tau^2 \phi_{h}(\tau) d\tau - \bar{\tau}^2}
\end{align}

Typical values of \tau_{\text{rms}} are on the order of \mu s in outdoor mobile radio channel and on the order of ns in indoor channel.

### 6.1.2 Frequency Selective and Frequency Nonselective (Flat) Fading Channels

When channel impulse response, \( h(\tau; t) \), is a WSSUS Gaussian stochastic process, then its transfer function \( H(f; t) = \Im \{ h(\tau; t) \} \) is also a WSSUS Gaussian stochastic process, so its autocorrelation function can be defined as

\[ \phi_{h}(\Delta f; \Delta t) = \int_{-\infty}^{\infty} \phi_{h}(\tau; \Delta t)e^{-j2\pi \Delta f \tau} d\tau \]  

(6.5)

In equation (6.5), if we set \( \Delta t = 0 \), we obtain the spaced-frequency correlation function of the channel \( \phi_{h}(\Delta f) = \phi_{h}(\Delta f; 0) \) which describes the correlation between frequency variations of the channel separated by \( \Delta f \).

The multipath channel generally has a coherence bandwidth, \{\Delta f_{C}\} where channel variations are highly correlated, i.e. \( \phi_{h}(\Delta f)/\phi_{h}(0) \approx 1 \). If \( \Delta f_{C} \) of the channel, through which the signal is transmitted, is small compared with the bandwidth of the transmitted signal, the signal is severely distorted and the channel is called to be frequency selective. On the other hand, if \( \Delta f_{C} \) is much larger compared with the bandwidth of the transmitted signal; the channel is called to be frequency nonselective or flat. The coherence bandwidth \{\Delta f_{C}\} is inversely proportional to r.m.s. delay spread \( \tau_{\text{rms}} \). A common rule of thumb is that a channel is frequency selective if the symbol period \( T_s < 10 \tau_{\text{rms}} \) and is flat fading if \( T_s \geq 10 \tau_{\text{rms}} \) [Rappaport: 01].
6.1.3 Time Selective and Time Nonselective Fading Channels

In equation (6.5), if $\Delta f = 0$, we obtain the spaced-time correlation function of the channel $\phi_H(\Delta t) = \phi_H(0; \Delta t)$, describing the correlation between time variations of the channel separated by $\Delta t$.

The multipath channel generally has a time duration, called coherence time $\{\Delta t_C\}$, where channel variations are highly correlated, i.e. $\phi_H(\Delta t)/\phi_H(0) \approx 1$. If $\Delta t_C$ of the channel, through which the signal is transmitted, is small compared with the symbol duration of the transmitted signal, the channel is called to be time selective or fast. On the other hand, if $\Delta t_C$ is much larger compared with the symbol duration of the transmitted signal; the channel is called to be time nonselective or slow.

6.1.4 Doppler Spectrum

A Doppler spectrum can be determined by taking the Fourier transform of the spaced time correlation function of the channel i.e. $D_H(\zeta) = \mathfrak{I}\{\phi_H(\Delta t)\}$. Figure 6.1(b) gives the relationship between $H(f; t) = \mathfrak{I}\{h(r; t)\}$ and $D_H(\zeta) = \mathfrak{I}\{\phi_H(\Delta t)\}$ [Hara, Prasad: 03]. Another important parameter, Doppler spread $\{B_D\}$ is a measure of the spectral broadening caused by the time rate of change of mobile radio channel and is defined as a range of frequencies over which the Doppler spectrum is essentially zero. A channel is said to be fast fading if the bandwidth of the transmitted signal, $B_s < B_D$ and is slow fading if $B_s \gg B_D$.

For Rayleigh fading with a vertical receive antenna with equal sensitivity in all directions, the Doppler Spectrum, has been shown to be [Clarke:68] [Gans: 72]

$$D_H(\zeta) = \mathfrak{I}\{\phi_H(\Delta t)\} = \frac{\sigma^2}{\pi \sqrt{f_D^2 - \zeta^2}}$$  \hspace{1cm} (6.6)

where $\zeta = f_D \cos \theta$ is the Doppler shift of the signal; $\theta$ and $f_D$ are the direction arrival of a signal from the direction of motion and the maximum Doppler shift respectively.

A Rayleigh fading channel can be modeled by generating the real and imaginary parts of a complex number according to independent normal Gaussian variables.
However, it is sometimes the case that it is simply the amplitude fluctuations that are of interest. The most popular approaches to this is Jakes classical model [Jakes: 75], which produces a signal that has the Doppler power spectrum given in equation (6.6). Jakes’ Classical fading channel model, which is actually a Clarke and Gans’ model, can be changed from Rayleigh to Ricean by making a single frequency component dominant in amplitude within $D_H(\zeta)\big|_{\zeta=0}$.

In a 3-D isotropic scattering environment, where the angles of arrival are uniformly distributed in the azimuth and elevation planes, the Doppler spectrum is found theoretically to be flat [Clarke, Khoo: 97]. A flat Doppler spectrum is also specified in some cases of the ITU-R 3G channel models reference channel models, for indoor (commercial) applications [ITU-R: 97]. The normalized flat Doppler power spectrum is given analytically by

$$D_H(\zeta) = \frac{1}{2f_D} |\zeta| \leq f_D \tag{6.7}$$

### 6.2 Performance of HQFM in Rayleigh Fading Channel

To evaluate the probability of symbol error of any modulation scheme in a slow fading channel, the probability of error of that particular scheme in AWGN channel is averaged over the possible strength due to fading [Proakis: 89] i.e.

$$P_e = \int_0^\infty P_e(\gamma) p(\gamma) d\gamma \tag{6.8}$$

where $P_e(\gamma)$ is the probability of a given modulation at a specific $\gamma = \alpha^2 E_s/N_0$ and $p(\gamma)$ is p.d.f. of $\gamma$ due to fading channel; $\alpha^2$ represent the instantaneous power values of the fading channel w.r.t. the non-fading $E_s/N_0$.

For Rayleigh fading channel, the fading amplitude $\alpha$ has Rayleigh p.d.f., so the fading power $\alpha^2$ and consequently $\gamma$ has chi square distribution with two degrees of freedom. Therefore

---

1 Proposed by [Clarke: 68] and analyzed by [Gans: 72].
where \( \overline{\gamma} = E[\alpha^2] E_s/N_0 \) is the average SNR. For unity gain fading channel, \( E[\alpha^2] = 1 \), \( \overline{\gamma} = n\overline{\gamma}_b \); \( n \) is the number of bits per symbol and \( \overline{\gamma}_b \) is average SNR per bit.

As mentioned in chapter 5, HQFM reception involves two-stage demodulation; therefore the probability of symbol error is the sum of the probabilities of symbol error at each stage i.e.

\[
P_e = P_{e,FSK}(\gamma) + (1 - P_{e,FSK}(\gamma))P_{e,QAM,FSK}(\gamma)
\]

(6.10)

It was shown in [Proakis: 89] that

\[
P_{e,FSK}(\gamma) = \sum_{q=1}^{L-1} \frac{(-1)^{q+1}}{q+1} \left( L-1 \right) e^{-\gamma q/(q+1)}
\]

(6.11)

When no diversity is applied, it can be shown that average probability of symbol error can be evaluated using equations (6.8), (6.9) and (6.11) as

\[
P_{e,FSK} = \int_0^{\infty} P_{e,FSK}(\gamma)p(\gamma)d\gamma
\]

\[
= \sum_{q=1}^{L-1} \frac{(-1)^{q+1}}{q+1} \left( L-1 \right) e^{-\gamma q/(q+1)} \int_0^{\infty} e^{-\gamma q/(q+1)}d\gamma
\]

(6.12)

At, stage 2, when the estimate on correct frequency is already made, the probability of symbol error is due to QAM demodulation which is given as

\[
P_{e,QAM,FSK}(\gamma) = 4 \left( 1 - \sqrt{\frac{C}{M}} \right) Q\left( \sqrt{2g\gamma} \right) - 4 \left( 1 - \sqrt{\frac{1}{CM}} \right)^2 Q^2 \left( \sqrt{2g\gamma} \right)
\]

(6.13)

where \( g = 1.5/(M-1) \) and \( C = 0.5 \) for cross QAM constellations otherwise 1.

[Craig: 91] and [Simon, Divsalar: 98] respectively showed that Gaussian Q-function and its square can be generalized in the form of definite integrals.
\[ Q(x) = \frac{1}{\pi} \int_0^{\pi/2} e^{-x \sin^2 \theta} d\theta; \]
\[ Q^2(x) = \frac{1}{\pi} \int_0^{\pi/4} e^{-x \sin^2 \theta} d\theta; \quad x \geq 0 \]  
(6.14)

Therefore, average probability of symbol error with no diversity can be evaluated using equations (6.8), (6.9), (6.13) and (6.14) [Alouini: 98]

\[
P_{e,\text{QAM/FSK}} = \int_0^\infty P_{e,\text{QAM/FSK}}(\gamma) p(\gamma) d\gamma 
= \frac{4}{\pi^7} \left( 1 - \sqrt{\frac{C}{M}} \right) \int_0^{\pi/2} \int_0^{\pi/4} e^{-\frac{\gamma (\sin^2 \theta)}{\sin^2 \phi}} d\phi d\gamma 
\]
\[
- \frac{4}{\pi^7} \left( 1 - \sqrt{\frac{1}{CM}} \right) \int_0^{\pi/2} \int_0^{\pi/4} e^{-\frac{\gamma (\sin^2 \theta)}{\sin^2 \phi}} d\phi d\gamma
\]
\[
= 2 \left( 1 - \sqrt{\frac{C}{M}} \right) \left( 1 - \sqrt{\frac{g\gamma}{1 + g\gamma}} \right) 
+ \left( 1 - \sqrt{\frac{1}{CM}} \right) 2C \frac{4}{\pi} \sqrt{\frac{g\gamma}{1 + g\gamma}} \cot^{-1} \left( \sqrt{\frac{g\gamma}{1 + g\gamma}} \right) - 1 \]  
(6.15)

So the equation (6.10) in terms of equation (6.12) and (6.15) becomes

\[
P_e = \sum_{q=0}^{L-1} \left( \frac{L-1}{q} \right) (-1)^{q+1} \left( \frac{L-1}{q + q\gamma} \right) \left[ \left( 1 - \sqrt{\frac{C}{M}} \right) \left( 1 - \sqrt{\frac{g\gamma}{1 + g\gamma}} \right) \right] \left( 1 - \sqrt{\frac{1}{CM}} \right) 2C \frac{4}{\pi} \sqrt{\frac{g\gamma}{1 + g\gamma}} \cot^{-1} \left( \sqrt{\frac{g\gamma}{1 + g\gamma}} \right) - 1 \]  
(6.16)

The relationship of \( P_b \) and \( P_e \) is evaluated in chapter 5 and will be restated here as

\[
P_b = \frac{nL-k}{2n(L-1)} P_{b,\text{FSK}} + \frac{1}{n} \left( 1 - P_{e,\text{FSK}} \right) P_{e,\text{QAM/FSK}} \]  
(6.17)

where \( n = \log_2(ML) \) and \( k = \log_2(M) \). Substituting \( \gamma = n\gamma_e \) and replacing values from equations (6.12) and (6.15), equation (6.17) becomes
\[ P_b = \frac{nL - k}{2n(L-1)} \sum_{q=0}^{L-1} \left( \frac{L-1}{q+1} \right) (-1)^{q+1} \left( 1 + q + qn\gamma_b \right) \]

\[ + \frac{1}{n} \sum_{q=0}^{L-1} \left( \frac{L-1}{q+1} \right) (-1)^q \left( 1 - \frac{C}{\sqrt{M}} \right) \left( 1 - \frac{gn\gamma_b}{1 + gn\gamma_b} \right) \]

\[ + \left( 1 - \frac{1}{CM} \right) ^{2C} \left[ \frac{4}{\pi} \left( \frac{gn\gamma_b}{1 + gn\gamma_b} \right) \cot^{-1} \left( \frac{\sqrt{gn\gamma_b}}{1 + gn\gamma_b} \right) - 1 \right] \] (6.18)

It can be proved easily that the whole HQFM system reduces to pure QAM when \( L = 1 \) and pure FSK when \( M = 1 \). From (6.18), the observation made is that the error rates decreases only inversely with SNR. In contrast, the decrease in error rate on a non-fading (AWGN) channel is exponential with SNR. This means that, on a fading channel, the transmitter transmits a large amount of power in order to achieve the required BER. This poor performance is due to non-zero probability of very deep fades, when the instantaneous BER is as low as 0.5. Significant improvement can be achieved by using diversity techniques or error control coding totally avoids the probability of deep fades [Rappaport: 01].

Figure 6.2 and 6.3 are plotted for different \( \frac{L}{M} \) HQFM formats showing the BER performance in Rayleigh slow fading channel using equation (6.18).

Figure 6.2 show that the performance is degraded by either increasing the QAM size \( M \) or decreasing the number of frequencies \( L \) while keeping the number of bits per symbol \( n \) constant. This figure also compare the BER performance of \( n \)-bit QAM in slow Rayleigh fading channel using equation (6.18) by setting \( L = 1 \). It is evident, that all the HQFM formats performs better than the pure QAM symbol.

However, there is no performance improvement by increasing the number of frequencies in \( \frac{L}{M} \) HQFM while keeping \( M \) constant\(^1\), as shown in figure 6.3. Thus one has to choose the right format for transmission for the required QAM size \( M \). However the performance is degraded by increasing the QAM size for the same number of frequencies \( L \).

\(^1\) Number of bits per HQFM symbols changes accordingly.
Figure 6.2: BER Performance of 64QAM compared with different $L/M$ HQFM \(\{ML = 64\}\) formats in Slow Rayleigh Fading Channel

Figure 6.3: BER Performance of $L/M$ HQFM formats with $M = \{4, 16, \text{and } 64\}$ each having $L = \{2, 4, 8, \text{and } 16\}$ frequencies
6.3 Frequency Selective Mobile Channels

Frequency selective fading caused by time delay spread causes ISI, which results in an irreducible BER floor for mobile systems. However, even if the mobile systems are frequency non-selective, the motion of the mobile system causes time varying Doppler spread which results in irreducible error floor. The irreducible error floor in frequency selective channel occurs when [Rappaport: 01]:

(a) The main signal component is removed through multipath cancellation
(b) Non zero value of delay spread $\{\tau_{rms}/T_s\}$ causes ISI
(c) Sampling time is shifted as a result of delay spread.

Equalization can mitigate the effect of channel induced ISI brought due to by frequency selectivity of the channel. Usually zero-forcing or maximum-likelihood sequence estimation (MLSE) equalizer is used to combat these effects. Due to the time varying channel impulse response, therefore an adaptive feedback equalizer is needed which constantly tracks the changes induced by channel.

To avoid the use of complex equalizer, OFDM is used which combats frequency selective effects of the channel by prefixing cyclically some symbols whose length is greater than $\tau_{rms}$. But OFDM, besides it good performance in frequency selective channels, has high PAPR. To suggest some modification to OFDM symbols so that PAPR is reduced, HQFM was proposed. The PAPR properties of HQFM in OFDM systems were discussed in Chapter 4. The remaining chapter 6 will discuss its BER performance in frequency selective channels.

6.4 Channel Estimation Techniques in HQFM-OFDM

A dynamic estimation of channel is necessary before the demodulation of OFDM signals since the radio channel is frequency selective and time-varying for wideband mobile communication systems. For HQFM-OFDM we have used the pilot-based channel estimation method described in [Coleri, Ergen: 02]. A brief description of
this method is described below:

The pilot tones, used for channel estimation, are inserted after signal mapping (any constellation like MPSK or MQAM or CPM; in our case HQFM), prior to IFFT and CP insertion operations. The received signal, after passing through a time varying fading channel with additive noise is given by [Zhao, Huang: 97] [Hsieh, Wei: 98]

\[ Y(k) = H(k)X(k) + N(k) + I(k); \quad k = 0,1,...,N-1 \]  \hfill (6.19)

where \( Y(k) = \mathcal{F}\{y(j)\} \) is the received signal, \( X(k) = \mathcal{F}\{x(j)\} \) is the transmitted signal, \( H(k) = \mathcal{F}\{h(j)\} \) is the channel impulse response, \( I(k) \) is ICI because of Doppler frequency and \( N(k) = \mathcal{F}\{n(j)\} \) is AWGN. After FFT operation at receiver, the pilot signals are extracted and the estimated channel \( H_c(k) \) for the data sub-channels is obtained. The transmitted data is then estimated by

\[ X_c = \frac{Y(k)}{H_c(k)} \quad k = 0,1...N-1 \]  \hfill (6.20)

The pilot-based channel estimation in OFDM systems can be either block-typed (insertion of pilot tones into all OFDM subcarriers with a specific period) or comb-typed (insertion of pilot tones into each OFDM symbol).

In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all subcarriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error since the pilots are sent at all subcarriers. The estimation can be performed by using either least square (LS) or minimum mean-square estimate (MMSE) [Beek, Edfors: 95] [Edfors, Sandell: 98]. If ISI is eliminated by CP, we write (6.19) in matrix notation

\[ Y = XFh + n \]  \hfill (6.21)

where \( Y_{Nx1} = \) received matrix; \( X = \text{diag}(X_{Nx1}) = \) transmitted matrix; \( F_{NxN} = \) Fourier matrix [Proakis, Manolakis: 95]; \( h_{Nx1} = \) channel impulse response matrix and \( n_{Nx1} = \) AWGN matrix. If the time domain channel vector is Gaussian and uncorrelated with the channel noise, the frequency domain MMSE of is given by [Edfors, Sandell: 98]

\[ H_{MMSE} = FR_h R_Y^{-1} Y \]  \hfill (6.22)
where $R_{hY}$ is the cross covariance matrix between $h$ and $Y$ and $R_{YY}$ is auto-covariance matrix of $Y$. The LS estimate is represented by

$$H_{LS} = X^H Y$$

(6.23)

which minimizes $(Y - XFh)^H (Y - XFh)$; $(.)^H$ denote the conjugate transpose. When the channel is slow fading, the channel estimation inside the block can be updated using the decision feedback equalizer at each subcarrier. This type of channel estimation is usually done in the physical layer design for HIPERLAN/2 [-Jush, Schramm: 99].

In comb-type pilot based channel estimation, the pilot signals are uniformly inserted. If $\{H_p(k); k = 0, 1 \ldots N_p - 1\}$is the frequency response of the channel at pilot subcarriers, the estimate of the channel at pilot subcarriers based on LS estimation is then given by

$$H_p = \frac{Y_p}{X_p}$$

(6.24)

where $Y_p(k)$and $X_p(k)$ are output and input at the $k^{th}$ pilot subcarrier respectively. Since LS estimate is susceptible to noise and ICI, MMSE was proposed. But MMSE includes the matrix inversion at each iteration, the simplified linear MMSE estimator was suggested [Reimers: 98] which needs the inversion only once. The complexity is further reduced with a low-rank approximation using singular value decomposition [Hsieh, Wei: 98].

### 6.5 HQFM-OFDM over Rayleigh Slow Fading Channel

Here the OFDM system, employing HQFM modulation format [Latif, Gohar: 06], assumes perfect subcarrier synchronization between the transmitter and receiver i.e. no presence of CFO. Therefore, the insertion of comb-type pilot symbols is eliminated in the simulations. However, to attain a distinct correlation peak and a reasonable SNR, block-type pilots known as special training symbol (STS) is used for which the data content is known to the receiver. It consists of 52 subcarriers plus a zero value at DC, which are then modulated by the elements of the PN sequence of length 53 and are time-domain transformed using 64 point-FFT. Each OFDM symbol is preceded by this
time-domain transformed STS, which is used, to estimate the channel and to improve the system performance.

The channel model simulated for the results in this section uses 3 taps WSSUS channel, having Rayleigh distribution. The channel delays are specified to be [0, 20, 40] ns with a gain vector of [0, -3, -6] dB. Therefore r.m.s. delay spread is 0.632 ns. Other simulation parameters are listed in Table 6.I.

**Table 6.I: Simulation Parameters for HQFM-OFDM in Rayleigh Slow Fading Channel**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$ - OFDM Symbol size</td>
<td>64</td>
</tr>
<tr>
<td>$N_{\text{used}}$ - Number of Active subcarriers</td>
<td>52</td>
</tr>
<tr>
<td>$f_D$ - Doppler's Frequency (Hz)</td>
<td>50, 100, 200</td>
</tr>
<tr>
<td>$n$ - total number of bits/subcarrier</td>
<td>8</td>
</tr>
<tr>
<td>$M$ - QAM levels</td>
<td>4,8,16</td>
</tr>
<tr>
<td>$L$ - Number of FSK Frequencies</td>
<td>4,8,16</td>
</tr>
<tr>
<td>$h$ - Modulation Index for FSK</td>
<td>1</td>
</tr>
<tr>
<td>Encoder Used</td>
<td>RS (13,7)</td>
</tr>
<tr>
<td>$f_c$ - Centre Frequency</td>
<td>450 MHz</td>
</tr>
<tr>
<td>Data Rate Supported</td>
<td>50 Mbps</td>
</tr>
<tr>
<td>OFDM Symbol Period</td>
<td>8.32 $\mu$s</td>
</tr>
</tbody>
</table>

The simulation results shown in this section employs Reed-Solomon (RS) encoding and decoding algorithms, because of their good distance properties. These codes are selected because they are the most popularly used block codes, particularly useful for correcting bursty channels [Sklar: 02]. The OFDM link in presence of fading multipath is a very good application for this code. The BER performance of HQFM-OFDM compared with conventional QAM-OFDM, shown in figures 6.4 - 6.7 in this section, was produced using Monte-Carlo simulations instead of analysis.

The BER for the different HQFM-OFDM formats each comprising 8 bits/subcarrier, in Rayleigh fading channels, compared with 256QAM-OFDM is shown in figure 6.4. With $f_D T = 0.0064$, using non-coherent detection of FSK frequencies, this figure shows that, the system performs better than 256QAM-OFDM when RS coding is
applied. The required $E_b/N_0$ for the 256QAM-OFDM to be detected correctly at $P_e = 10^{-3}$ is ~38 dB (~32 dB with coding) while for 16/16 HQFM-OFDM, it is ~36 dB and it can be reduced to ~23 dB when RS coding is employed. Other HQFM formats i.e. 4/64 and 8/32 requires ~44 and ~40 dB respectively at $P_e \approx 10^{-4}$ without coding and it can reduced to 26 and 24 dB respectively with RS coding. So, HQFM can replace with 256QAM when proper coding is employed to attain comparable power efficiency.

![Figure 6.4: BER of Different HQFM formats (n = 8 bits/subcarrier)-OFDM in Rayleigh Slow Fading Channel with $f_{DT} = 0.0064$. Legend: Solid Line: With RS Coding; Dotted line: Without Coding](image)

Figure 6.5 compares the BER performances of $16/M$ HQFM - OFDM \{L = 16, different $M = 4, 8, 16$\} with conventional 256QAM-OFDM and it shows that BER performances is degraded by increasing the QAM size while keeping $L$ constant. Also, one can infer from this figure that required $E_b/N_0$ for HQFM-OFDM can be reduced significantly when coding is employed. At $P_e = 10^{-4}$ and RS(13,7), the required $E_b/N_0$ for $M = 4, 8, 16QAM$ with 16 number of frequencies, is 25 (extrapolated value), 28 and 26.5 dB respectively while it is 32.5 dB (extrapolated value) for 16QAM. The coding gain, for instant, for $16/16$ \{L = 16, $M = 16$\} HQFM is 13 dB while it is only 6.5 dB for 256QAM.
CHAPTER 6  PERFORMANCE IN FADING MULTIPATH CHANNELS

FIGURE 6.5: BER Curves for $16/M$ HQFM OFDM $M = \{4, 8, 16\}$ ($f_dT = 0.0064$).
Legend: Solid Line: With RS Coding; Dotted line: Without Coding

FIGURE 6.6: BER Curves with $L/16$ HQFM–OFDM $L = \{4, 8, 16\}$ ($f_dT = 0.0064$)
Figure 6.6 shows the BER performance of different HQFM formats when same QAM size i.e. $M = 16$ are employed with different number of frequencies $L = \{4, 8, \text{ and } 16\}$. This also compares the performance with conventional 16QAM-OFDM system. The figure clearly shows that there is no significant improvement by increasing the number of orthogonal frequencies in Rayleigh fading channel. These results were also deduced from equation (6.18) and figure 6.3. In AWGN, HQFM performance is improved as number of frequencies are increased while keeping QAM size constant. This can be achieved when proper coding is done as shown in this figure. It shows that BER performance is improved but this improvement is not significant when $L$ increases beyond 8. For instance, the coding gain for L/16 {L = 16 and 8} HQFM-OFDM is about 9.5 dB (this gain is about 11.5 dB for 4/16 HQFM-OFDM) while it is 6 dB for 16QAM at $P_b = 10^{-4}$.

In mobile systems, due to Doppler spread, even if there is no time dispersion, the BER does not decrease below a certain irreducible floor. Figure 6.7 clearly illustrates the effect of Doppler-induced fading for 16/16 HQFM-OFDM. As velocity or normalized Doppler Shift {$f_DT$} of arbitrary mobile increases, the error floor increases, despite an increase in required $\mathcal{E}_b/\mathcal{N}_0$. Thus, there is no improvement in link performance, once a certain $\mathcal{E}_b/\mathcal{N}_0$ is achieved.

![Figure 6.7: BER Curves with 16QAM/16FSK with different $f_DT$](image-url)
6.6 HQFM-OFDM over Frequency Selective Channel

HiperLAN/2 and IEEE 802.11a WLAN systems are deployed in a wide range of environments such as offices, industrial buildings, exhibition halls, and residential environments. Different channel models have been produced to represent these different environments [Medbo, Schramm: 98] [Medbo, Hallenberg: 99]. Channel model A (for Office NLOS condition, Table 6.II) and B (for large open space and office environments, NLOS condition, Table 6.III), used in simulations for this section, are wideband, with Rayleigh modeled tapped delay lines, with a mean corresponding to an exponentially decaying average power delay profile.

The supported data rate is $R_b = 30$ Mbps operating at 5GHz. The mobile speed assumed is 10 km/hr giving rise to a Doppler frequency of 46.3 Hz. For simulation purpose, we assumed $f_D = 50$ Hz. Other system parameters to perform simulations are described in Table 6.IV. Figure 6.8 and 6.9 shows the BER performance of 4/16 HQFM OFDM with frequency separation of $1/NT_s$ where $T_s$ is the HQFM symbol period in Channel A and B respectively. The performance of 64 MC-QAM, only, with same data rate is also shown for comparison.

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\tau_\kappa$ (ns)</th>
<th>$P(\tau_\kappa)$ (dB)</th>
<th>$\kappa$</th>
<th>$\tau_\kappa$ (ns)</th>
<th>$P(\tau_\kappa)$ (dB)</th>
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<td>-7.3</td>
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<td>170</td>
<td>-9.9</td>
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<td>80</td>
<td>-6.9</td>
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<td>-26.7</td>
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TABLE 6.III: HIPERLAN/2 CHANNEL MODEL B

<table>
<thead>
<tr>
<th>κ</th>
<th>τ_κ (ns)</th>
<th>P(τ_κ) (dB)</th>
<th>κ</th>
<th>τ_κ (ns)</th>
<th>P(τ_κ) (dB)</th>
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<td>-9.6</td>
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<td>18</td>
<td>730</td>
<td>-24.6</td>
</tr>
</tbody>
</table>

TABLE 6.IV: SIMULATION PARAMETERS FOR HQFM-OFDM IN FREQUENCY SELECTIVE FADING CHANNEL

| Number of Data + Pilot Carriers, N_{used} | 52 |
| Number of OFDM subcarriers, N | 64 |
| Symbol duration, T = NT_s | 3.2μsec |
| Cyclic Prefix, T_{CP} = \frac{1}{4} T | 800ns |
| Total Symbol Duration, T_t = T + T_{CP} | 4μs |
| Subcarrier Spacing, Δf = \frac{1}{NT_s} | 312.5kHz |
| Sampling Rate, F = NΔf | 20MHz |

Figure 6.8 shows the BER performance of 4/16 HQFM \{L = 4, M = 16\} compared with 64QAM, both employed in 64-carrier OFDM using the tapped delay line model for Channel A. This model is listed in Table 6.II. The r.m.s. delay spread, τ_{rms}, for this channel is 50 ns. The max delay \tau_{max} = 390 ns which is less than the cyclic prefix, \( T_{CP} = 800 \) ns. The performance of 4/16HQFM is 29 dB without coding which is 7 dB lower than 64QAM at \( P_b = 10^{-4} \). This performance is improved when RS (13, 7) is used. The achieved \( E_b/N_0 \) for 4/16 HQFM with this code is 17.5 dB (approx.) which is 18.5 dB lower than 64QAM at \( P_b = 10^{-4} \). The coding gain for 4/16 HQFM is 11.5 dB, while it is 8.5 dB for 64QAM OFDM.
CHAPTER 6  PERFORMANCE IN FADING MULTIPATH CHANNELS

![Figure 6.8: BER Performance of 4/16 HQFM with 64QAM employed in OFDM for Channel A NLOS Conditions](image)

![Figure 6.9: BER Performance of 4/16 HQFM with 64QAM employed in OFDM for Channel B NLOS Conditions](image)
Figure 6.9 shows the BER performance of 4/16 HQFM \( \{L = 4, M = 16\} \) compared with 64QAM, both employed in 64-carrier OFDM using tapped delay line model for Channel B. This channel is designed for large offices and open areas with r.m.s. delay spread, \( \tau_{\text{rms}} = 100 \text{ ns} \). This channel is listed in Table 6.III. The max delay \( \tau_{\text{max}} = 730 \text{ ns} \), therefore, again the cyclic prefix of 16subcarriers \( (T_{\text{CP}} = 800 \text{ ns}) \) is enough. The performance of 4/16HQFM is 24dB without coding which is 10 dB lower than 64QAM at \( P_{b} = 10^{-4} \). This performance is improved when RS \((13, 7)\) is used. The achieved \( E_b/N_0 \) for 4/16 HQFM with this code is 15 dB (approx.) which is 10 dB lower than 64QAM at \( P_{b} = 10^{-4} \). The coding gain for 4/16HQFM is 9 dB while it is 10dB for 64QAM OFDM.

Therefore, the performance is improved for 4/16 HQFM as compared to 64QAM, in both channel conditions. However the performance of HQFM is better in Channel A than channel B in terms of coding gain.

### 6.7 Performance of HQFM-OFDM, HQFM-I and HQFM-II

The OFDM system parameters [Latif, Gohar: 07], used in simulation, for this section are listed in table 6.V.

<table>
<thead>
<tr>
<th>Table 6.V: OFDM Parameters Defined for Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Modulation Type</strong></td>
</tr>
<tr>
<td><strong>Number of Subcarrier, N</strong></td>
</tr>
<tr>
<td><strong>Cyclic Prefix, ( N_{\text{cp}} )</strong></td>
</tr>
<tr>
<td><strong>Number of Active Subcarriers, ( N_{\text{used}} )</strong></td>
</tr>
<tr>
<td><strong>OFDM symbol duration, ( T_t )</strong></td>
</tr>
<tr>
<td><strong>Cyclic Prefix Duration, ( T_{\text{CP}} )</strong></td>
</tr>
<tr>
<td><strong>Symbol Rate, ( I/T )</strong></td>
</tr>
<tr>
<td><strong>Subcarrier Spacing, ( \Delta f )</strong></td>
</tr>
<tr>
<td><strong>Data Rate, ( R_b )</strong></td>
</tr>
<tr>
<td><strong>Channel Bandwidth, ( B )</strong></td>
</tr>
</tbody>
</table>
The modulation index, $h = 1$. The side-information for PTS-OFDM $\in [0, \pi/2, \pi, 3\pi/2]$ is transmitted along with each OFDM symbol while the rotation vector for HQFM-II $\in [0, \pi]$ which is detected iteratively at the receiver before proper HQFM demodulation. The maximum Doppler shift assumed for this simulation is $f_D = 200$ Hz. This Doppler shift is induced when a mobile with maximum speed of 240 km/hr is operating at a centre frequency of 900 MHz.

For Rayleigh slow fading channel, the channel is assumed as exponential decay profile with channel delay $= [0, 60, 120]$ x 3 ns and gains $= [0, -3, -6]$ dB. The maximum delay, $\tau_{\text{max}} = 360$ ns, therefore a cyclic prefix of 128-carriers (>360*2 ns) is sufficient to combat the frequency selectivity of such a channel induced.

![Figure 6.10: BER of 4/16 HQFM, OFDM in Rayleigh Fading Channel, compared with HQFM-I and HQFM-II](image)

Figure 6.10 shows the BER comparison of HQFM-OFDM, HQFM-I and HQFM-II, after PAPR reduction, both in AWGN (dotted line) and fading channel (solid lines). The figure shows that HQFM-I performs better than ordinary HQFM and HQFM-II. This is because of the use of multi-FSK utilization in HQFM II. Comparing it with QAM-OFDM, its performance shows robustness against channel impairments with an increasing number of FSK tones. Slight performance degradation of HQFM-II,
compared to conventional HQFM and HQFMI, can be stated as an extra overhead paid to decode the iterative phase vector before the conventional HQFM demodulation.

6.8 Performance in Frequency Selective Channels

The second set of experiments was performed using a more realistic channel model. The system simulated here is tested over different channel models suggested by ITU-R M.1225 which are recommended for third generation IMT 2000 systems [ITU-R: 97]. The operating frequency, $f_c$, is assumed to be 2000MHz, which is the suggested centre frequency for ITU-R channels. The total symbol period for OFDM is $T_i = T + T_{CP} = 39 \mu s$ where $T_{CP} = \frac{1}{4}T$. The number of subcarriers ($N$) = 512 with 208 active subcarriers. The modulation index, $h$, is 1 (non-coherent case). Five channel models for ITU-R are suggested for the simulations in this section are:

(i. Indoor office with Doppler frequency $f_D = 0-5.55$ Hz (Channel A and B). Maximum mobile speed is 0-3 km/hr.

(ii. Outdoor-to-indoor and pedestrian, with $f_D = 5.55 - 55.6$ Hz (Channel A and B). Maximum mobile speed is 3-30 km/hr

(iii. Vehicular - high antenna with $f_D = 55.6 - 463$ Hz (Channel A). Maximum mobile speed is 30 – 250 km/hr.

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\tau_\kappa$ (ns)</th>
<th>$P(\tau_\kappa)$ (dB)</th>
<th>$\tau_\kappa$ (ns)</th>
<th>$P(\tau_\kappa)$ (dB)</th>
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<tbody>
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<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>100</td>
<td>-3.6</td>
</tr>
<tr>
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<td>200</td>
<td>-7.2</td>
</tr>
<tr>
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<td>-18.0</td>
<td>300</td>
<td>-10.8</td>
</tr>
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<tr>
<td>6</td>
<td>310</td>
<td>-32.0</td>
<td>700</td>
<td>-25.2</td>
</tr>
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</table>
The tapped-delay line impulse response parameters for the indoor test environment are given by Table 6.VI. The Doppler spectrum for each tap is specified as flat given by equation (6.7). The A model has 6 rays with maximum delay $\tau_{\text{max}} = 310$ ns and r.m.s. delay spread of $\tau_{\text{rms}} = 35$ ns. The B model has 6 rays with $\tau_{\text{max}} = 700$ ns and a r.m.s. delay spread of $\tau_{\text{rms}} = 100$ ns. Therefore, model A in this test environment refers to indoor small office or residential building, while model B refers to indoor large office [Rappaport: 01]. The cyclic prefix duration of $39 \mu\text{sec}/5 = 7.8 \mu\text{s}$ is sufficient to combat the frequency selectivity of both channels in this case. Here we assumed a maximum Doppler shift of 5.5 Hz.

### Table 6.VII: Tapped Delay Line Impulse Response Specification for Outdoor to Indoor and Pedestrian Test Environment

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>Channel-A</th>
<th>Channel-B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_{\kappa}$ (ns)</td>
<td>$P(\tau_{\kappa})$ (dB)</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>110</td>
<td>-9.7</td>
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<tr>
<td>3</td>
<td>190</td>
<td>-19.2</td>
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<tr>
<td>4</td>
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<td>-22.8</td>
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<td></td>
</tr>
<tr>
<td>6</td>
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</tr>
</tbody>
</table>

The tapped-delay line impulse response parameters for the pedestrian test environment are given by Table 6.VII. The Doppler spectrum is specified as classic Jakes’ model suggested in equation (6.6). The A model has 4 rays with maximum delay spread of $\tau_{\text{max}} = 410$ ns and r.m.s. delay spread $\tau_{\text{rms}} = 45$ ns. The B model has 6 rays with $\tau_{\text{max}} = 3.7 \mu\text{s}$ and r.m.s. delay spread $\tau_{\text{rms}} = 750$ ns. Channel model A refers to an indoor small office or residential building while channel B refers to very large offices or factory or high rise apartments. Again the cyclic prefix of duration $7.8 \mu\text{s}$ is sufficient for both channel conditions. Here we assume a maximum Doppler shift of 50 Hz.

The tapped-delay line impulse response parameters for the vehicular test environment are given by Table 6.VIII. The Doppler spectrum is specified as classic Jakes’ model. The A model has 6 rays with maximum delay $\tau_{\text{max}} = 2.51 \mu\text{s}$ and r.m.s.
delay spread $\tau_{rms} = 370$ ns. Again the cyclic prefix of duration 7.8 $\mu$s is sufficient for this channel condition and a maximum Doppler shift of 220Hz (medium case scenario) is assumed.

**Table 6.VIII: Tapped Delay Line Impulse Response Specification for Vehicular Test Environment**

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>Channel-A</th>
<th>$\tau_\kappa$ (ns)</th>
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Figure 6.11, 6.12 and 6.13 shows the BER comparison of 4/16HQFM \{L = 4, M = 16\}, HQFM-I and HQFM-II compared with 64QAM OFDM in all channel conditions described above. All these figures show that any HQFM type (simple $L/M$ HQFM, HQFM-I and HQFM-II) performs better than 64QAM OFDM. Thus, QAM with same number of bits/ subcarrier can be replaced with any HQFM type supporting the same data rate. Comparing the performance of HQFM-I with simple $L/M$ HQFM and HQFM-II, the former performs best in all channel conditions. This is because of a multi stage FSK and QAM modulator and demodulator employed at transmitter and receiver. Its performance in indoor test environment and indoor-to-outdoor channel A is almost same i.e. $E_b/N_0$ is around 20 dB at $P_b = 10^{-3}$ and typical r.m.s. delay spread is between 35 ns and 100 ns. When r.m.s. delay spread exceeds 100ns, as in case of indoor-to-outdoor channel B and vehicular channel A, the performance is degraded by about 2.5 dB i.e. $E_b/N_0 = 22.5$ dB (approx.) at $P_b = 10^{-3}$. 

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Figure 6.11: BER Performance of HQFM, HQFM-I and II compared with 64QAM OFDM in Indoor Test Environment

Figure 6.12: BER Performance of HQFM, HQFM-I and II compared with 64QAM OFDM in Pedestrian Test Environment
The iterative detection of rotation vector in case of HQFM-II accounts for performance degradation as compared to simple $L/M$ HQFM and HQFM-I. Like HQFM-I, its performance in indoor office environment (channel A with r.m.s. delay spread = 35 ns) is around 27 dB at $P_b = 10^{-5}$. When r.m.s delay spread is 45 ns (indoorto-outdoor channel A), the performance is improved by about 2 dB at $P_b = 10^{-5}$. However, the performance in vehicular environment is almost same as that of indoor office. It is around 28 dB at $P_b = 10^{-5}$. In case of channel B, the performance in indoor office environment is 3 dB better than that of pedestrian environment.

Similar results can be drawn when 4/16 HQFM is compared in different channel condition. For channel A, the performance in indoor and vehicular environment is same i.e. $E_b/N_0 = 26$ dB (approx.) at $P_b = 10^{-5}$. The performance is improved in pedestrian environment (channel A) i.e. $E_b/N_0 = 22.5$ dB (approx.) at $P_b = 10^{-5}$. In case of channel condition B, the performances in indoor office and pedestrian environment are almost same.
REFERENCES


CHAPTER 6  PERFORMANCE IN FADING MULTIPATH CHANNELS


Orthogonal Frequency Division Multiplexing (OFDM) is a modulation scheme which is proved to be robust against multipath fading and has found its application in many high speed transmission systems like HDTV, DAB and DVB and is adopted as standard in PHY layer of IEEE 802.11.a and HIPERLAN/2. It has the potential to surpass the capacity of CDMA systems and can provide the wireless access method for 4G systems.

Despite its robustness against multipath fading, it suffers from two main drawbacks; one is its sensitivity to carrier frequency offset (CFO) and the other is its high peak to average power ratio (PAPR). In this dissertation, the PAPR problem and how to reduce it at baseband is discussed.

Several researchers have tried to find a solution to this problem. These techniques achieve PAPR reduction at the expense of transmit signal power increase, bit error rate (BER) increase, data rate loss, computational complexity increase, and so on. PAPR reduction schemes can be classified depending on the approach taken by them.
The PAPR can be reduced either prior to OFDM modulation i.e. before IFFT operation or after OFDM modulation. The second type includes clipping and amplifier linearization techniques. The PAPR reduction schemes that are applied prior to OFDM modulation include all those techniques which deploy coding and signal transformation. Some researchers tried to solve this problem by simultaneously applying coding prior to IFFT and clipping and filtering after IFFT. But this scheme is not suitable for large number of subcarriers.

The goal of reduction schemes based on signal transformation is to transform the signal (mapping bits to a set of complex numbers) prior to IFFT so that PAPR is reduced after OFDM modulation. But all these schemes are either application specific or they need an extra overhead in the form of side information to be transmitted along the OFDM symbol to retranslate the signal at the receiver. In this dissertation, we proposed a modulator that replaces the conventional modulations like QAM or PSK which reduces PAPR at the output of IFFT without transforming the signal. Thus, the need of side information is eliminated.

Conventionally OFDM uses either QAM or PSK as a baseband modulator. Recently CPM (of which FSK is a special case) is proposed in place of QAM/PSK as a baseband modulator. But, QAM/PSK is bandwidth efficient while FSK (if orthogonality is maintained) is bandwidth inefficient. If compared in terms of BER performance, QAM/PSK is power inefficient and FSK is power efficient. Comparing QAM and PSK, we know that QAM is more power efficient than PSK. In this dissertation a hybrid of bandwidth efficient QAM and power efficient FSK is used and found that when applied in OFDM, PAPR is reduced and is referred here as hybrid QAM-FSK modulator (HQFM).

The power spectral density (PSD) of the proposed modulator is evaluated using conventional statistical tools. Null-to-null bandwidth efficiency is then computed from the PSD obtained and found that the bandwidth efficiency lies between the bandwidth efficiencies of QAM and FSK. For practical point of view, 90% or more precisely 99% (FCC authorized bandwidth) of the power bandwidth is needed; therefore, fractional-out-of band power (FOBP) is evaluated. It is found that the bandwidth efficiency of HQFM in terms of $B_{99\%}$ is the same as that of QAM for the same number of bits per
subcarrier. The PSD and consequently bandwidth efficiency of OFDM is not affected by replacing QAM with HQFM as OFDM’s PSD is independent of the mapping scheme used and is merely a function of subcarrier spacing, \(\Delta f\).

It has been found that many factors control the PAPR statistics of HQFM OFDM. Regardless of the fact that PAPR depends on number of subcarriers, \(N\), it also depends on number of FSK frequencies, \(L\) and the modulation index, \(h\) (or in other words FSK tone spacing \(f_\Delta\)). PAPR is found to be reduced if number of signaling frequencies is increased. HQFM-OFDM performs the best when orthogonality of the FSK tones is not disturbed and this happen when \(h\) is odd multiple of either 0.5 (for coherent FSK) or 1 (non-coherent FSK). The PAPR reduction capability of the proposed modulator is found to be not as good as Partial Transmit Sequences (PTS), therefore a modification is suggested which makes use of multi-stage HQFM modulator (HQFM-I). The performance of HQFM-I is found to be comparable PTS. Another modification, HQFM-II is used which employs PTS type algorithm without transmitting the side information and reduces PAPR as compared to PTS.

Comparing \(B_{99\%}\) power efficiency and PAPR reduction capability of different HQFM formats, it is found that there are many combinations of HQFM formats (number of frequencies or QAM size) for which same bandwidth and improved power efficiency and PAPR reduction capability can be achieved, which gives the system designer a wider class of possibilities in the choice of an optimum system under given specification and practical limitations on design techniques.

To complete the analysis, the power efficiency of HQFM (probability of bit error) is evaluated mathematically both in AWGN and in Rayleigh fading channel. A two-stage demodulator is suggested which detects the FSK frequencies first and then QAM symbols are extracted, which are then demodulated using conventional minimum distance criterion. Prior to QAM demodulation, an algorithm is proposed which corrects any phase discrepancy occurred due to FSK part in first stage. Standard available channel models are used to evaluate the BER performance in fading channels. It is found using Monte Carlo techniques that HQFM and its modification is better than QAM in any channel condition.
7.1 **Future Extension**

Research is a never ending process so in this section, we are proposing some suggestions for future extension of our work:

1. In this dissertation, we compared our system with PTS-OFDM to observe the PAPR reduction capability and proposed two modifications; one based on the concept of varying simultaneously $L$ and $M$ where $Q=ML$ is constant and choosing the least PAPR signal, the other is based on PTS algorithm. Our systems can be compared with other available PAPR reduction algorithm like Tone Injection or Tone reservation. Both these algorithms are devised for QAM based OFDM systems.

2. In order to avoid ISI among HQFM signals, the upsampling factor $J = 8$ or greater is considered. This demands for large IFFT size and increases PAPR also. So efforts can be made to redesign the HQFM signals so that $J$ can be reduced to a factor of 4 or 2.

3. BER expressions are evaluated for AWGN and Rayleigh fading channels. For cross QAM (odd bit mapping), the BER expressions are approximated. Efforts can be made to evaluate the exact expressions if cross QAMs are hybrid with FSK in HQFM. Also, BER expression for other fading channels like Rician and Nakagami-m Channels can be evaluated and confirmed through simulations.

4. In this dissertation we have not applied diversity i.e. our system is based on SISO transmission. Performance evaluation of our modulation can be done using frequency, time or space diversity.

5. CDMA is applied in conjunction with OFDM to get the advantages of both OFDM and CDMA. Our system has
potential to be applied in MC-CDMA.

6. In this dissertation, we have considered only the non-coherent orthogonal FSK. Research can be directed to use non-coherent/non-orthogonal and coherent, orthogonal/non-orthogonal FSK and apply all the possible research directions mentioned in above paragraphs. Possibility of using GFSK (Gaussian shaped FSK) can also be explored in this context.

7. QAM is usually detected coherently. The most popular non-coherent QAM is Star-QAM, so, Hybrid Star-QAM FSK Modulation (HsQFM) can be employed to draw results.

8. Other research challenges concerning OFDM i.e. sensitivity to CFO, phase noise and strict synchronization are still unexplored in this context. Research can be done in this direction and new or existing algorithms can be applied to draw the results.
APPENDIX

POWER SPECTRAL DENSITY OF MFSK

The spectral analysis of nonlinear LFSK signals is much more complicated than linear modulated signals like MQAM or MPSK. Also LFSK signals are correlated to each other and have memory. Recall that a low pass FSK signal is represented as $u_i(t) = e^{j \phi_i} g(t)$ where $f_i(t) = 2\pi f_i \Delta t = 2\pi h \sum_{\tau=-\infty}^\infty I_i q(t); \ q(t) = \int_0^\infty g(\tau) d\tau$ is zero outside interval $[0,T_s]$. FSK is considered as full response CP M with modulation index either 0.5 or 1. The average autocorrelation function of this low pass signal is given as

$$
\phi_{uu}(\tau) = \frac{1}{2T_s} \int_0^{T_s} \mathbb{E} \left\{ u^*_i(t) u_i(t+\tau) \right\} dt
$$

$$
= \frac{1}{2T_s} \int_0^{T_s} \mathbb{E} \left[ \exp \left( j2\pi h \sum_{i=-\infty}^\infty I_i \left[ q(t+\tau-iT_s) - q(t-iT_s) \right] \right) \right] dt
$$

$$
= \frac{1}{2T_s} \int_0^{T_s} \mathbb{E} \left[ \prod_{i=-\infty}^\infty \exp \left( j2\pi h I_i \left[ q(t+\tau-iT_s) - q(t-iT_s) \right] \right) \right] dt
$$

$$
= \frac{1}{2T_s} \int_0^{T_s} \prod_{i=-\infty}^\infty \left\{ \sum_{l=1}^L \Pr \left\{ I_i = l \right\} e^{j2\pi hl \left[ q(t+\tau-iT_s) - q(t-iT_s) \right]} \right\} dt
$$

(A.1)
where $\Pr\{I_i\}$ is the probability of occurrence of information sequence $I_i$. These symbols are assumed statistically independent and identically distributed. For equal probable symbols, $\Pr\{I_i\}=1/L$. Taking the Fourier transform of equation (A.1) yields the power spectral density (PSD) of LFSK i.e.

$$\Phi(f) = \mathfrak{I}(\phi_{aw}(\tau)) = \int_{-\infty}^{\infty} \phi_{aw}(\tau) e^{-j2\pi f \tau} d\tau$$

$$= 2\Re \left[ \int_{0}^{r_c} \phi_{aw}(\tau) e^{-j2\pi f \tau} d\tau \right]$$

$$= 2\Re \left[ \int_{0}^{r_c} \phi_{aw}(\tau) e^{-j2\pi f \tau} d\tau + \int_{r_c}^{r} \phi_{aw}(\tau) e^{-j2\pi f \tau} d\tau \right]$$

$$= 2\Re \left[ \int_{0}^{r_c} \phi_{aw}(\tau) e^{-j2\pi f \tau} d\tau + \frac{1}{1-\psi e^{-j2\pi f r_c}} \int_{r_c}^{r} \phi_{aw}(\tau) e^{-j2\pi f \tau} d\tau \right]$$

$$= 2 \left[ \int_{0}^{r_c} \phi_{aw}(\tau) \cos 2\pi f \tau d\tau \right]$$

$$+ \frac{1-\psi \cos 2\pi f r_c}{1+\psi^2 - 2\psi \cos 2\pi f r_c} \int_{r_c}^{r} \phi_{aw}(\tau) \cos 2\pi f \tau d\tau$$

(A.2)

where

$$\psi = \psi(jh) = \frac{\sin L\pi h}{L \sin \pi h}$$

(A.3)

is the characteristic function.

If rectangular pulse shapes of unit amplitude are used then $|G_i(f)|=T_s \sin \pi(fT_s - \lambda_i h / 2)$ where $\lambda_i = 2i-1-L$. In this case $q(t) = \int_{0}^{T_s} g(\tau)d\tau$ becomes linear in interval $[0,T_s]$. The PSD in equation (A.2) then can be simplified as

$$\Phi(f) = \frac{1}{T_s} \left\{ \frac{1}{L} \sum_{i=1}^{L} |G_i(f)|^2 + \frac{2}{L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} B_{ij}(f)|G_i(f)||G_j(f)| \right\}$$

$$= \frac{1}{T_s L^2} \left\{ \sum_{i=1}^{L} \left( \lambda_i + 2B_{ii}(f) \right)|G_i(f)|^2 + 4 \sum_{i=1}^{L} \sum_{j=1}^{L} B_{ij}(f)|G_i(f)||G_j(f)| \right\}$$

(A.4)

where
\[ B_y(f) = \frac{\cos \pi \left( 2f T_s - \left( \lambda_i + \lambda_j \right) \frac{h}{2} \right) - \psi \cos \pi \left( \lambda_i + \lambda_j \right) \frac{h}{2}}{1 + \psi^2 - 2\psi \cos 2\pi f T_s} \]  

(A.5)

Putting \( G_i(f) = T_c A_i(f) = T_c \log_2 L A_i(f) \) where \( A_i(f) = \frac{\sin \pi (f T_s - \lambda_i h / 2)}{\pi (f T_s - \lambda_i h / 2)} \), the normalized PSD \( \Phi'(f) = \Phi(f) / T_b \) becomes

\[ \Phi'(f) = \frac{\log_2 L}{L} \left( \sum_{i=1}^{L} (L + 2B_{ii})A_i^2 + 4 \sum_{i=1}^{L} \sum_{j \neq i} B_{ij} A_i A_j \right) \]  

(A.6)

For coherent FSK, \( h = f_s T_c = 0.5 \), hence the characteristic function, \( \psi = 0 \). For non-coherent FSK, \( \psi = -1 \), therefore \( B_{ij} \) reduces to

\[ B_{ij} = \begin{cases} \cos \left( 2\pi f T_s - \alpha_{ij} \right) & \psi = 0 \text{(Coherent FSK)} \\ -\frac{1}{2} (-1)^{i+j} & \psi = -1 \text{(Non-coherent FSK)} \\ -\frac{1}{2} & i = j \end{cases} \]  

(A.7)

where \( \alpha_{ij} = \pi \left( \lambda_i + \lambda_j \right) \frac{h}{2} = \pi (i + j -1 - L) h \).

Using Identity \( \frac{\sin a \sin b}{ab} = \frac{1}{2ab} (\cos(a-b) - \cos(a+b)) \)  

(A.8)

It can be shown easily that

\[ A_i A_j = \begin{cases} \cos \pi h (j-i) - \cos (2\pi f T_s - \alpha_{ij}) & i \neq j \\ 2\pi^2 (f T_s - \lambda_i h / 2)(f T_s - \lambda_j h / 2) & i = j \end{cases} \]  

(A.9)

Now equation (A.6) becomes

\[ \Phi'(f) = \frac{\log_2 L}{\pi^2 L^2} \left( \sum_{i=1}^{L} (L + 2B_{ii}) \sin^2 \pi (f T_s - \lambda_i h / 2) / (f T_s - \lambda_i h / 2)^2 \\ -2 \sum_{i=1}^{L} \sum_{j \neq i} B_{ij} \frac{\cos (2\pi f T_s - \alpha_{ij}) - \cos \pi h (i-j)}{(f T_s - \lambda_i h / 2)(f T_s - \lambda_j h / 2)} \right) \]  

(A.10)

For coherent FSK, when \( h = 0.5 \) (\( \psi = 0 \)) and by substituting \( B_{ij} \) from equation(A.7), equation (A.10) becomes
\[
\Phi'(f) = \frac{\log_2 L}{\pi^2 L^2} \left\{ \sum_{i=1}^{L} \left( \frac{\sin^2 \pi (fT_s - \frac{\lambda_i}{4})}{(fT_s - \frac{\lambda_i}{4})^2} \right) \frac{\sin^2 \pi (fT_s - \frac{\lambda_j}{4})}{(fT_s - \frac{\lambda_j}{4})^2} \right\} (A.11)
\]

Noting \( \alpha_{ij} = \frac{1}{2} \pi \lambda_i \) and using identity (A.8) \((a = b)\), equation (A.11) becomes

\[
\Phi'(f) = \frac{\log_2 L}{\pi^2 L^2} \left\{ \sum_{i=1}^{L} \left( \frac{\frac{1}{2} \cos(2\pi fT_s - \alpha_{ij}) - \cos \frac{\pi}{2} (i - j) \cos(2\pi fT_s - \alpha_{ij})}{(fT_s - \frac{\lambda_i}{4})(fT_s - \frac{\lambda_j}{4})} \right) \right\} (A.12)
\]

For the simplest case when \( L = 2 \)

\[
\Phi'(f) = \frac{1}{4 \pi^2} \left\{ \sum_{i=1}^{2} \frac{1 - \cos^2 (2\pi fT_b - \alpha_{ij})}{(fT_b - \frac{\lambda_i}{4})^2} \right\} - 2 \frac{\cos^2 (2\pi fT_b - \alpha_{12})}{(fT_b - \lambda_1/4)(fT_b - \lambda_2/4)} \right\} (A.13)
\]

Since \( \alpha_{12} = 0 \) and after substituting \( \alpha_{ab} = \frac{\pi}{2} \lambda_i \) and knowing \( 1 - \cos^2 a = \sin^2 a \), equation (A.13) can be rewritten as

\[
\Phi'(f) = \frac{1}{4 \pi^2} \left\{ \sum_{i=1}^{2} \frac{\sin^2 (2\pi fT_b - \frac{\pi}{2} \lambda_i)}{(fT_b - \frac{\lambda_i}{4})^2} \right\} - 2 \frac{\cos^2 2\pi fT_b}{(fT_b - \lambda_1/4)(fT_b - \lambda_2/4)} \right\} (A.14)
\]

Since \( \lambda_i \) is always odd integers, so \( \sin^2 (2\pi fT_b - \frac{\pi}{2} \lambda_i) = \cos^2 2\pi fT_b \). Equation (A.14), after simplification and putting the values of \( \lambda_1 \) and \( \lambda_2 \), becomes

\[
\Phi'(f) = \frac{4 \cos 2\pi fT_b}{\pi \left( (4 fT_b)^2 - 1 \right)} \right\} (A.15)
\]

This is the normalized PSD of coherent BFSK. It is also obvious from equation (A.15) that the sidelobes in BFSK fall off with the fourth power of frequency offset from the centre frequency \( f_c \).

Figure A.1 shows the power spectral density of BFSK as a function of
normalized frequency $fT_s$. The PSD for higher order FSK ($L > 2$) is also plotted using equation (A.12). For $L > 2$, the normalized PSD in equation (A.12) is difficult to evaluate. However as it is evident from figure A.1, the rate of fall off LFSK with $L > 2$ is same as that of BFSK i.e. fourth power of frequency offset from centre frequency. Note that only the right half of the bandwidth occupancy is shown in graphs. The origin corresponds to the centre frequency $f_c$. These graphs show that the spectrum of LFSK is relatively smooth and well confined. Also the rate of fall of spectral sidelobes is approximately the same as that of BFSK i.e. fourth power of frequency offset from centre frequency $f_c$ (origin in this case).

![Normalized Power Spectral Densities, $\Phi(f)/T_s$ for Coherent LFSK with $L = 2, 4, 8, 16$](image)

**Figure A.1:** Normalized Power Spectral Densities, $\Phi(f)/T_s$ for Coherent LFSK with $L = 2, 4, 8, 16$

Now consider the case of non-coherent LFSK i.e. when $h = f_\lambda T_s = 1$. Substituting the value of $B_{ij}$ from equation (A.7) and noting $j-i$ always as integer, equation (A.10) can be simplified as

$$
\Phi(f) = \frac{\log_2 L}{\pi^2 L^2} \left\{ (L-1) \sum_{i=1}^{L} \frac{\sin \frac{\pi (fT_s - \lambda_i/2)}{fT_s - \lambda_i/2}}{fT_s - \lambda_i/2} 
- \sum_{i=1}^{L} \sum_{j>i} \frac{1 - (-1)^{j-i} \cos \frac{\pi (2fT_s - \alpha_{ij})}{fT_s - \lambda_j/2}}{(fT_s - \lambda_j/2)(fT_s - \lambda_j/2)} \right\} 
$$

(A.16)
Since $\lambda_i$ is always odd integer so $\sin(\pi f T_s - \pi \lambda_i / 2) = \pm \cos \pi f T_s$ depending on the value of $\lambda_i$. Also $\cos(2\pi f T_s - \alpha_y) = -(-1)^{i+j} \cos 2\pi f T_s$. Equation (A.16) can be now written as

$$
\Phi'(f) = \frac{\log_2 L}{\pi^2 L^2} \left\{ \left( L - 1 \right) \sum_{i=1}^{L} \frac{\cos^2 \pi f T_s}{\pi^2 (f T_s - \lambda_i / 2)^2} \right. \\
\left. - \sum_{i=1}^{L} \sum_{j=1}^{L} \frac{1 + \cos 2\pi f T_s}{(f T_s - \lambda_i / 2)(f T_s - \lambda_j / 2)} \right\} 
$$

(A.17)

Using another identity $1 + \cos 2a = 2 \cos^2 a$

(A.18)

Equation (A.17) becomes

$$
\Phi'(f) = \frac{\log_2 L \cos \pi f T_s}{\pi^2 L^2} \left\{ \left( L - 1 \right) \sum_{i=1}^{L} \frac{1}{(f T_s - \lambda_i / 2)^2} \right. \\
\left. - 2 \sum_{i=1}^{L} \sum_{j=1}^{L} \frac{1}{(f T_s - \lambda_i / 2)(f T_s - \lambda_j / 2)} \right\} \\
= \frac{\log_2 L \cos \pi f T_s}{\pi^2 L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} \left\{ \frac{1}{(f T_s - \lambda_j / 2)} - \frac{1}{(f T_s - \lambda_i / 2)} \right\}^2 \\
= \log_2 L \sum_{i=1}^{L} \sum_{j=1}^{L} \left( \frac{4|i-j| \cos \pi f T_s}{\pi L (2f T_s - \lambda_i)(2f T_s - \lambda_j)} \right)^2 
$$

(A.19)

Equation (A.19) shows that the sidelobes in non-coherent orthogonal LFSK also fall off with the fourth power of frequency offset from the centre frequency $f_c$. This equation is plotted in figure A.2 with different values of $L$. Comparing figure A.2 with figure A.1, it is obvious that the main lobe of non-coherent LFSK is 50% wider than coherent LFSK, hence the bandwidth occupancy of non-coherent LFSK is twice than the bandwidth occupancy of coherent FSK. This is the price paid for the unknown carrier phase reference required for demodulation. This section is concluded by observing figure A.1 and A.2 that when number of keying frequencies i.e. $L$ increases, the main lobe of power spectral density of both coherent and non-coherent LFSK widens, which means that spectral occupancy of LFSK increases with increase in $L$ which decreases the bandwidth efficiency.
Normalized Power Spectral Density, $\Phi(f)/T_b$, dB

*Figure A.2*: Normalized Power Spectral Densities, $\Phi(f)/T_b$ for Non-coherent LFSK with $L = 2, 4, 8, 16$
function alpha = phaseOpt(Q,M)

% Possible QAM Points
refA = complex([1,3,3,5,5,5,7,7,7],[1,1,3,1,3,5,3,5,7]);

Q = Q/(modnorm(qammod([0:M-1],M),'avpow',1)); % Apply Normalization
phOff = angle(qammod(M-1,M))-angle(Q(1,1));
alpha(1,1) = Q(1,1)*exp(complex(0,-phOff));

for j = 2:length(Q)

% Calculate the Possible QAM Point on I-Q Plane from the received % QAM Symbol
[y,idx] = min((abs(refA)-abs(Q(j,1))).^2);
a = refA(1,idx)*[1 complex(0,1) -1 complex(0,-1)];

% Compute the closest neighboring points from Rx-QAM.

% Point1
[y idx] = min(abs(a-Q(j,1)));
REF1 = a(1,idx);

% Point2
a = conj(a);
[y idx] = min(abs(a-Q(j,1)));
REF2 = a(1,idx);

if REF1 == REF2
    phOff = angle(REF1)-angle(Q(j,1));
    alpha(j,1) = REF1;
else
    temp1 = REF1*exp(complex(0,-phOff));
    temp2 = REF2*exp(complex(0,-phOff));
    if abs(temp1-Q(j,1)) >= abs(temp2-Q(j,1))
        alpha(j,1) = REF2;
    else
        alpha(j,1) = REF1;
end
end

alpha = abs(Q).*exp(complex(0,angle(alpha)));
alpha = alpha*(modnorm(qammod([0:M-1],M),'avpow',1));

**FIGURE B.1:** Flow Chart Diagram of Phase Offset Acquisition Algorithm
This Dissertation is written in Microsoft ® Office WORD 2003.

Equations are produced using MathType™ 6.0.

Figures and block diagrams are generated using Microsoft ® Office Visio ® Professional 2003.

HQFM-OFDM Signal Models are developed under Simulink® Environment and results for PSD, FOBP, PAPR and BER analysis are drawn using Matlab® R2007b.