# Contents

1 **Introduction** 4

1.1 Literature review 6

1.1.1 Vibrations of cylindrical shells 6

1.1.2 Vibrations of fluid-filled cylindrical shells 10

1.1.3 Vibrations of FGM cylindrical shells 12

1.1.4 Vibrations of cylindrical shells on elastic foundations 16

1.2 Motivation and objectives of the present work 18

2 **Preliminaries** 21

3 **Vibrations of FGM thin cylindrical shells with exponential volume fraction law** 30

3.1 Introduction 30

3.2 Mathematical formulation 32

3.3 Strain-displacement and curvature-displacement relations 33

3.4 Lagrangian energy functional 38

3.5 Rayleigh-Ritz method 38

3.6 Modal displacement form 39

3.7 Implementation of Rayleigh-Ritz method and derivation of frequency equation 41

3.8 Axial modal dependence and boundary conditions 44

3.9 Functionally graded materials 45

3.10 Modified exponential volume fraction law 47

3.11 Numerical results and discussion 47

3.11.1 Variation of volume fraction 47

3.11.2 Validity and accuracy of present method 48

3.11.3 Variation of natural frequency (Hz) 49

3.12 Conclusion 52

Tables 53

Figures 60
4  Vibrations of functionally graded cylindrical shells
   on elastic foundations 65
4.1 Introduction 65
4.2 Lagrangian energy functional 66
4.3 Derivation of dynamical equations for a FGM
   cylindrical shell 67
4.4 Wave propagation approach 69
4.5 Results and discussion 73
   4.5.1 Validation of wave propagation approach 73
   4.5.2 Vibration frequency analysis based on elastic
   foundations 73
4.6 Conclusion 76
Tables 77
Figures 80

5  Vibration characteristics of fluid-filled cylindrical
   shells based on elastic foundations 87
5.1 Introduction 87
5.2 Derivation of dynamical equations for a fluid-filled
   cylindrical shells based on elastic foundation 88
5.3 Induction of the acoustic pressure field 89
5.4 Numerical procedure 90
5.5 Results and discussion 92
   5.5.1 Validation of the present approach 92
   5.5.2 Vibration frequency analysis of empty as well
   as fluid-filled cylindrical shells based on Winkler
   and Pasternak foundations 93
5.6 Conclusion 95
Tables 96
Figures 100