Symbol Detection Techniques in a Spatial Multiplexing System

Author
Adnan Ahmed Khan
05-UET/PhD-CASE-CP-16

Supervisor
Dr. Syed Ismail Shah
Professor, Department of Electrical and Computer Engineering

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
CENTRE FOR ADVANCED STUDIES IN ENGINEERING
UNIVERSITY OF ENGINEERING AND TECHNOLOGY
TAXILA

DECEMBER 2008
Symbol Detection Techniques in a Spatial Multiplexing System

A dissertation submitted in partial fulfillment of the degree of Doctor of Philosophy (PhD) in Electrical and Computer Engineering

Author

Adnan Ahmed Khan
(05-UET/PhD-CASE-CP-16)

Checked and Recommended by the Foreign Experts:

Dr. Zaheer Ahmad
Centre for Communications Systems Research
University of Surrey, Guildford, Surrey, GU2 7XH
United Kingdom, Email: Zaheer.ahmad@surrey.ac.uk

Dr. Irfan Awan
University of Bradford, West Yorkshire, BD7 1DP, UK, United Kingdom
Email: I.U.Awan@Bradford.ac.uk

Approved by:

____________________
Dr. Syed Ismail Shah
Thesis Supervisor

____________________
____________________
Dr. Shoab A. Khan  Dr. Jamil Ahmed  Dr. Mudassar Farooq
Member Research Committee  External Member  Member Research Committee
ECE Department, CASE  Dean, Iqra University, Islamabad  ECE Department, CASE
Islamabad  Islamabad  Islamabad

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
CENTRE FOR ADVANCED STUDIES IN ENGINEERING
UNIVERSITY OF ENGINEERING AND TECHNOLOGY TAXILA

DECEMBER 2008
DECLARATION

The substance of this thesis is the original work of the author and due references and acknowledgements has been made, where necessary, to the work of others. No part of this thesis has been already accepted for any degree, and it is not being currently submitted in candidature of any degree.

Adnan Ahmed Khan
SP-03-036
Thesis Scholar

Countersigned:

Dr. Syed Ismail Shah
Thesis Supervisor
Dedicated to
my family
Acknowledgement

In the name of ALLAH, the most gracious, the most merciful

Alhamdulillah, all praises be to Allah who gave me the strength and patience to complete this thesis. The research work took a long and strenuous effort which was not easy to complete, without the help of my PhD supervisor Prof. Dr. Syed Ismail Shah. I am deeply thankful for his able guidance and useful advices throughout my research. Without his consistent support I might not have reached this milestone. I am also grateful to Prof. Dr. Asrar U. H. Sheikh of King Fahd University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia and Dr. Xiadong Li of RMIT University, Australia. Their useful comments enabled me to produce some useful research work, which was published in renowned journals. I am impressed by the dynamic personality of Dr. Shoab Khan who guided and bucked me up during course work and research. I am also thankful to Dr. Jamil Ahmed for providing me with his able guidance that helped me to meet the international standards of research. I am thankful to the whole faculty of Computer Engineering department and my colleagues especially Sajid Bashir, Muhammad Naeem and Khawaja Tauseef for their time to time discussion/advices on my work. I appreciate the patience, cooperation and understanding of my family members during this research work.

The financial support of Higher Education Commission of Pakistan through HEC merit Scholarship Scheme (200 Scholarships) is greatly acknowledged. I am thankful to my institute Centre for Advanced Studies in Engineering for providing me opportunity to study/work and providing conducive environment. I acknowledge my parent department (Pakistan Army) for allowing me to avail this opportunity and spare me for the study.
Abstract

Significant performance gains are achievable in wireless communication systems using a Multi-Input Multi-Output (MIMO) communications system employing multiple antennas. This architecture is suitable for higher data rate multimedia communications. One of the challenges in building a MIMO system is the tremendous processing power required at the receiver side. MIMO Symbol detection involves detecting symbol from a complex signal at the receiver. The existing MIMO detection techniques can be broadly divided into linear, non-linear and exact detection methods. Linear methods like Zero-Forcing offer low complexity with degraded Bit Error Rate (BER) performance as compared to non-linear methods like VBLAST. Non-linear detectors are computationally not very expansive with acceptable performance. Exact solutions like Sphere Decoder provide optimal performance however it suffers from exponential complexity under certain conditions. The focus in the early part of this thesis is on non-linear approximate MIMO detectors and an effort has been made to develop a low complexity near-optimal MIMO detector. Computational Swarm Intelligence based Meta-heuristics are applied for Symbol detection in a MIMO system. This approach is particularly attractive as Swarm Intelligence (SI) is well suited for physically realizable, real-time applications, where low complexity and fast convergence is of absolute importance. Application of Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO) algorithms is studied. While an optimal Maximum Likelihood (ML) detection using an exhaustive search method is prohibitively complex, we show that the Swarm Intelligence optimized MIMO detection algorithms gives near-optimal Bit Error Rate (BER) performance in fewer iterations, thereby reducing the ML computational complexity significantly. In this thesis novel non-conventional MIMO detection approaches based on Swarm-Intelligence techniques have been presented.

An effective and practical way to enhance the capacity of MIMO wireless channels is to employ space-time (ST) coding. Space-time block coding (STBC) is a transmit diversity technique in which the data stream to be transmitted is encoded in blocks, which are distributed among multiple antennas and across time. Alamouti’s simple STBC scheme for
wireless communication systems uses two transmit antennas and linear maximum-likelihood (ML) decoder. This system was generalized by Tarokh to an arbitrary number of transmit antennas by applying the theory of orthogonal designs. In the later part of this thesis a simple multi-step constellation reduction technique based decoding algorithm that further simplifies the linear ML detection in Orthogonal Space-Time Block Coded systems is proposed. This approach reduces the computational complexity of these schemes while presenting the ML performance.

In addition, Spatial Multiplexing systems using Orthogonal Walsh codes are also studied. This approach has a potential to reduce the search space to allow efficient symbols detection in Spatial Multiplexing systems.
## Contents

**Acknowledgements** vi

**Abstract** vii

**List of Figures** xv

**List of Tables** xviii

**List of Abbreviations** xix

**Glossary** xx

### 1. Introduction

<table>
<thead>
<tr>
<th>1.1 Challenges in uncoded Multi-Antenna Systems</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 Approach to Optimize MIMO Detection</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Exact Solution in a Coded Multi-Antenna System</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Orthogonal Coded Spatial Multiplexing System</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td>1.5 Organization of Thesis</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>6</td>
</tr>
</tbody>
</table>

### 2. Multipath Fading Channels, Modulation Techniques and Antenna Diversity

<table>
<thead>
<tr>
<th>2.1 Introduction</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2 Channel Characterizations</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
<td>2.2.1 Large Scale Propagation Models</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>2.2.1.1 Deterministic Approach</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>(a) Free Space Propagation Model</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>(b) Long-Distance path loss Model</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>2.2.1.2 Stochastic Approach</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>(a) Lognormal Shadowing Model</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10</td>
</tr>
</tbody>
</table>
### 2.2.2 Small Scale Propagation Models

#### 2.2.2.1 Mobile Multipath channels parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fading</td>
<td>12</td>
</tr>
<tr>
<td>Doppler Shift</td>
<td>12</td>
</tr>
<tr>
<td>Power Delay Profile $\Phi_c(\tau)$</td>
<td>12</td>
</tr>
<tr>
<td>Excess Delay</td>
<td>13</td>
</tr>
<tr>
<td>Delay Spread ($T_m$)</td>
<td>13</td>
</tr>
<tr>
<td>Coherent Bandwidth ($BW_c$)</td>
<td>13</td>
</tr>
<tr>
<td>Doppler Spread ($B_D$)</td>
<td>13</td>
</tr>
<tr>
<td>Coherent Time ($T_c$)</td>
<td>14</td>
</tr>
</tbody>
</table>

#### 2.2.2.2 Types of Small-Scale Fading

<table>
<thead>
<tr>
<th>Types of Fading</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Fading</td>
<td>14</td>
</tr>
<tr>
<td>Frequency-Selective Fading</td>
<td>14</td>
</tr>
<tr>
<td>Fast Fading</td>
<td>15</td>
</tr>
<tr>
<td>Slow Fading</td>
<td>15</td>
</tr>
</tbody>
</table>

#### 2.2.2.3 Statistical Models of Small-Scale Propagation Channels

<table>
<thead>
<tr>
<th>Models of Channels</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh Fading Channel</td>
<td>15</td>
</tr>
<tr>
<td>Ricean Fading Channel</td>
<td>16</td>
</tr>
<tr>
<td>Nakagami Fading Channel</td>
<td>16</td>
</tr>
</tbody>
</table>

#### 2.2.2.4 Statistical Models for Multipath Fading Channels

<table>
<thead>
<tr>
<th>Models of Channels</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TwoRay Model</td>
<td>17</td>
</tr>
<tr>
<td>Markov Channel Modeling</td>
<td>17</td>
</tr>
<tr>
<td>Finite State Markov Channel</td>
<td>17</td>
</tr>
<tr>
<td>Hidden Markov Models</td>
<td>19</td>
</tr>
</tbody>
</table>

#### 2.2.2.5 Multi-Input Multi-Output Channel Model

<table>
<thead>
<tr>
<th>Models of Channels</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Channel Model</td>
<td>20</td>
</tr>
<tr>
<td>Physical Scattering Model</td>
<td>22</td>
</tr>
</tbody>
</table>

### 2.3 Modulation Techniques

#### 2.3.1 Classification of Modulation Schemes

<table>
<thead>
<tr>
<th>Classification</th>
<th>Page</th>
</tr>
</thead>
</table>

#### 2.3.2 Selection of Digital Modulation Schemes

<table>
<thead>
<tr>
<th>Classification</th>
<th>Page</th>
</tr>
</thead>
</table>

#### 2.3.3 Different Modulation Schemes

<table>
<thead>
<tr>
<th>Classification</th>
<th>Page</th>
</tr>
</thead>
</table>

#### 2.3.3.1 Linear Modulation Schemes

<table>
<thead>
<tr>
<th>Classification</th>
<th>Page</th>
</tr>
</thead>
</table>
2.3.3.2 Constant Envelope Modulation Schemes - 24
2.3.3.3 Combining Linear and Constant Envelope Modulation - 24
   (a) M-ary Phase Shift Keying (MPSK) - 25
   (b) Quadrature Amplitude Modulation (MQAM) 26
2.3.3.4 Spread Spectrum Modulation Techniques - 27
2.3.3.5 Orthogonal Frequency Division Multiplexing - 29

2.4 Diversity - 32
2.4.1 Diversity Techniques - 32
   2.4.1.1 Spatial Diversity - 32
   2.4.1.2 Polarization Diversity - 33
   2.4.1.3 Frequency Diversity - 33
   2.4.1.4 Time Diversity - 33
2.4.2 Diversity Combining Methods - 34
   2.4.2.1 Selection Combining - 34
   2.4.2.2 Maximum Ratio Combining - 34
   2.4.2.3 Equal Gain Combining - 35

2.5 Summary - 36

3. Multiple Antennas Communication Systems - 37
3.1 Introduction - 37
3.2 Narrowband MIMO - 37
3.3 MIMO Channel Decomposition - 39
3.4 MIMO Channel Capacity - 41
   3.4.1 Static Channels - 41
      3.4.1.1 Perfect Channel Knowledge at Transmitter - 42
      3.4.1.2 Channel unknown at Transmitter - 43
   3.4.2 Fading Channels - 44
      3.4.2.1 Perfect Channel Knowledge at Transmitter - 44
      3.4.2.2 Channel unknown at Transmitter - 44
5.4.2.3 Channel unknown at Transmitter and Receiver - 46
3.5 Space-Time Coding - 46
   3.5.1 General Matrix for Orthogonal Space-Time Codes - 46
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5.2</td>
<td>A Simple Space-Time Block Coded System</td>
<td>48</td>
</tr>
<tr>
<td>3.5.2.1</td>
<td>The Transmission Model</td>
<td>48</td>
</tr>
<tr>
<td>3.5.2.2</td>
<td>ML Decoding of O-STBC over Quasi-Static Channel</td>
<td>48</td>
</tr>
<tr>
<td>3.5.2.3</td>
<td>Quasi-Static Channel</td>
<td>50</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Other Space-Time Block Codes</td>
<td>52</td>
</tr>
<tr>
<td>3.5.4</td>
<td>Spatial Multiplexing and BLAST Architecture</td>
<td>53</td>
</tr>
<tr>
<td>3.6</td>
<td>Frequency-Selective MIMO Channels</td>
<td>55</td>
</tr>
<tr>
<td>3.7</td>
<td>Summary</td>
<td>56</td>
</tr>
<tr>
<td>4.</td>
<td>Symbol Detection in MIMO System</td>
<td>57</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>57</td>
</tr>
<tr>
<td>4.2</td>
<td>MIMO System Model</td>
<td>57</td>
</tr>
<tr>
<td>4.3</td>
<td>MIMO Detection Problem Formulation</td>
<td>58</td>
</tr>
<tr>
<td>4.4</td>
<td>Existing MIMO Detection Algorithms</td>
<td>59</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Sphere Decoder</td>
<td>60</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Linear Detection</td>
<td>62</td>
</tr>
<tr>
<td>4.4.2.1</td>
<td>Zero-forcing Detection</td>
<td>62</td>
</tr>
<tr>
<td>4.4.2.2</td>
<td>Minimum Mean Square Error Detection</td>
<td>63</td>
</tr>
<tr>
<td>4.4.3</td>
<td>Non-linear (V-BLAST) Detection</td>
<td>64</td>
</tr>
<tr>
<td>4.4.4</td>
<td>ZF-ML Detection</td>
<td>66</td>
</tr>
<tr>
<td>4.4.5</td>
<td>SDP Detection</td>
<td>67</td>
</tr>
<tr>
<td>4.5</td>
<td>Summary</td>
<td>68</td>
</tr>
<tr>
<td>5.</td>
<td>Meta-heuristic Techniques</td>
<td>69</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>69</td>
</tr>
<tr>
<td>5.2</td>
<td>Meta-heuristics and heuristics</td>
<td>69</td>
</tr>
<tr>
<td>5.3</td>
<td>Natural Optimization by Ants</td>
<td>71</td>
</tr>
<tr>
<td>5.4</td>
<td>Ant Colony Optimization Algorithm</td>
<td>72</td>
</tr>
<tr>
<td>5.5</td>
<td>Binary Ant System (BAS)</td>
<td>73</td>
</tr>
<tr>
<td>5.6</td>
<td>Natural Optimization by Swarm</td>
<td>74</td>
</tr>
<tr>
<td>5.7</td>
<td>Particle Swarm Optimization Algorithm</td>
<td>75</td>
</tr>
<tr>
<td>5.8</td>
<td>Summary</td>
<td>80</td>
</tr>
</tbody>
</table>
## 6. Swarm Intelligence Meta-heuristics for Symbol Detection in MIMO System

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>81</td>
</tr>
<tr>
<td>6.2</td>
<td>BA-MIMO Detection Algorithm</td>
<td>81</td>
</tr>
<tr>
<td>6.3</td>
<td>Performance analysis of BA-MIMO Detection</td>
<td>82</td>
</tr>
<tr>
<td>6.4</td>
<td>Computational Complexity Comparison</td>
<td>85</td>
</tr>
<tr>
<td>6.5</td>
<td>Performance-Complexity trade-off</td>
<td>86</td>
</tr>
<tr>
<td>6.6</td>
<td>Discussion</td>
<td>87</td>
</tr>
<tr>
<td>6.7</td>
<td>PSO-MIMO Detection Algorithm</td>
<td>87</td>
</tr>
<tr>
<td>6.8</td>
<td>SPSO-MIMO Detection Algorithm</td>
<td>88</td>
</tr>
<tr>
<td>6.9</td>
<td>MPSO-MIMO Detection Algorithm</td>
<td>89</td>
</tr>
<tr>
<td>6.10</td>
<td>BPSO-MIMO Detection Algorithm</td>
<td>90</td>
</tr>
<tr>
<td>6.11</td>
<td>PSO Parameter Control</td>
<td>91</td>
</tr>
<tr>
<td>6.12</td>
<td>PSO-MIMO Detection Algorithm’s Relationship</td>
<td>92</td>
</tr>
<tr>
<td>6.13</td>
<td>Simulation Results and Performance Analysis</td>
<td>92</td>
</tr>
<tr>
<td>6.13.1</td>
<td>Experimental Setup</td>
<td>92</td>
</tr>
<tr>
<td>6.13.2</td>
<td>BER versus SNR Performance</td>
<td>93</td>
</tr>
<tr>
<td>6.13.3</td>
<td>Computational Complexity Theoretical Evaluation</td>
<td>95</td>
</tr>
<tr>
<td>6.13.4</td>
<td>BER Performance-Computational Complexity Trade-off</td>
<td>98</td>
</tr>
<tr>
<td>6.13.5</td>
<td>Effects of Change in Algorithms Parameters and Iterations</td>
<td>99</td>
</tr>
<tr>
<td>6.13.6</td>
<td>Effect of initial guess on the performance</td>
<td>100</td>
</tr>
<tr>
<td>6.13.7</td>
<td>Comparison of different PSO-MIMO Detection Techniques</td>
<td>101</td>
</tr>
<tr>
<td>6.13.8</td>
<td>An Analysis of PSO Algorithm as a MIMO Detection Technique</td>
<td>101</td>
</tr>
<tr>
<td>6.13.9</td>
<td>Discussion</td>
<td>101</td>
</tr>
<tr>
<td>6.14</td>
<td>Fitness Landscape Analysis of MIMO Detection Problem</td>
<td>102</td>
</tr>
<tr>
<td>6.15</td>
<td>Conclusions</td>
<td>105</td>
</tr>
</tbody>
</table>

## 7. Symbol Detection in Coded Multi-antenna Systems

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>106</td>
</tr>
<tr>
<td>7.2</td>
<td>Space-Time Block Codes</td>
<td>107</td>
</tr>
<tr>
<td>7.2.1</td>
<td>STBC with two transmit antennas</td>
<td>107</td>
</tr>
<tr>
<td>7.2.1.1</td>
<td>Conventional Maximum Likelihood Detection</td>
<td>108</td>
</tr>
<tr>
<td>7.2.1.2</td>
<td>Simplified Maximum Likelihood Detection</td>
<td>109</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>9.2 Future Work</td>
<td>137</td>
<td></td>
</tr>
<tr>
<td>A. List of Papers from the Research Work Conducted</td>
<td>138</td>
<td></td>
</tr>
<tr>
<td>References</td>
<td>140</td>
<td></td>
</tr>
</tbody>
</table>
# List of Figures

1.1 Spatial Multiplexing system - - - - - - - 2
2.1 Small-scale and large-scale fading manifestation - - - - - - 9
2.2 Partitioning of Received SNR and assigning each interval to a state of the FSMC 18
2.3 The Finite State Markov channel model representation- - - - - - 18
2.4 MIMO system model - - - - - - - - 21
2.5 QPSK constellation - - - - - - - - 27
2.6 16-QAM constellation diagram - - - - - - - - 27
2.7 Orthogonal sub-carriers spectra - - - - - - - - 30
2.8 Block diagram of OFDM transmitter - - - - - - - - 31
2.9 Block diagram of OFDM receiver - - - - - - - - 31
2.10 Selection Combining Method - - - - - - - - 34
2.11 Maximal Ratio Combining Method - - - - - - - - 35
3.1 $N_t 	imes N_r$ MIMO Communications system model - - - - - 38
3.2 Transmit Precoding and Receiver Shaping - - - - - - - - 40
3.3 Ergodic capacity of a $N_t 	imes N_r$ MIMO systems with CSIR and no CSIT- - - - - 45
3.4 Space-Time Coded System Block Diagram - - - - - - - - 48
3.5 Alamouti’s System Model - - - - - - - - 49
3.6 BER performance of STBC with 1,2 and 3 transmit antennas - - - - 51
3.7 Spatial Multiplexing with serial encoding technique - - - - - - 54
3.8 VBLAST: Spatial Multiplexing with Parallel encoding technique - - - - 54
3.9 VBLAST Receiver - - - - - - - - 55
4.1 A simplified linear MIMO communication system - - - - - - 58
4.2 Sphere Decoder algorithm illustration - - - - - - 60
4.3 Sphere Decoder algorithms tree structure illustration- - - - - 61
4.4 Performance of linear and non-linear MIMO detectors- - - - - 65
4.5 ZF-ML reduced constellation search - - - - - - - - 66
4.6 Performance of ZFML MIMO detectors - - - - - - - - 67
5.1 Experimental setup to observe ants behavior - - - - - - 71
5.2 Situation at the beginning of search – foraging behavior - - - - 71
5.3 Ants foraging behavior after certain time - - - - - - 72
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4</td>
<td>Routing Diagram for Ants in BAS</td>
</tr>
<tr>
<td>5.5</td>
<td>PSO flow diagram</td>
</tr>
<tr>
<td>5.6</td>
<td>Particles are pulled towards their g\textsubscript{best} and p\textsubscript{best}</td>
</tr>
<tr>
<td>6.1</td>
<td>BA-MIMO detection BER versus SNR performance for a 3x3 system</td>
</tr>
<tr>
<td>6.2</td>
<td>BA-MIMO Detection BER versus SNR performance for a 4x4 system</td>
</tr>
<tr>
<td>6.3</td>
<td>BA-MIMO Detection BER versus SNR performance for a 6x6 system</td>
</tr>
<tr>
<td>6.4</td>
<td>PSO-MIMO Detector BER versus Eb/No 2x4 system</td>
</tr>
<tr>
<td>6.5</td>
<td>SPSO and MPSO algorithms convergence with iterations</td>
</tr>
<tr>
<td>6.6</td>
<td>PSO-MIMO Detector BER versus Eb/No\ for 4-QAM 4x4 MIMO system</td>
</tr>
<tr>
<td>6.7</td>
<td>PSO-MIMO Detector BER versus Eb/No\ for 4-QAM 8x8 MIMO system</td>
</tr>
<tr>
<td>6.8</td>
<td>BPSO-MIMO Detectors Convergence with iterations at 15-dB</td>
</tr>
<tr>
<td>6.9</td>
<td>Effect on BER performance with change in Social and Cognitive components</td>
</tr>
<tr>
<td>6.10</td>
<td>3-D View of Fitness Landscape for 2x2 64 QAM MIMO system</td>
</tr>
<tr>
<td>6.11</td>
<td>3-D View of Fitness Landscape for 2x2 16 QAM MIMO system</td>
</tr>
<tr>
<td>7.1</td>
<td>SML algorithm employing 16-QAM Alamouti system</td>
</tr>
<tr>
<td>7.2</td>
<td>SML algorithm employing 16-PSK Alamouti system</td>
</tr>
<tr>
<td>7.3</td>
<td>Performance comparison of the proposed SML detector with ML using 16-QAM Alamouti System</td>
</tr>
<tr>
<td>7.4</td>
<td>Performance comparison of the proposed SML detector with ML using 16-PSK Alamouti System</td>
</tr>
<tr>
<td>7.5</td>
<td>Performance of proposed SML detection with conventional H4 OSTBC ML 16-PSK</td>
</tr>
<tr>
<td>7.6</td>
<td>Performance of proposed SML detection with conventional ML in 16-QAM H4 OSTBC system</td>
</tr>
<tr>
<td>7.7</td>
<td>Neighbor list selection method</td>
</tr>
<tr>
<td>7.8</td>
<td>Application of the proposed method on 16-QAM Alamouti system</td>
</tr>
<tr>
<td>7.9</td>
<td>BER performance of proposed detection algorithm in 16-QAM Alamouti system</td>
</tr>
<tr>
<td>7.10</td>
<td>BER performance of proposed detection algorithm in 16-QAM Alamouti system</td>
</tr>
<tr>
<td>8.1</td>
<td>Orthogonal coded MIMO system transmitter and receiver model</td>
</tr>
<tr>
<td>8.2</td>
<td>BER performance of the proposed 4x4 OCM system</td>
</tr>
<tr>
<td>8.3</td>
<td>BER performance of ML detection in the proposed 4x4-OCM and 4x4 4 QAM un-coded MIMO system</td>
</tr>
<tr>
<td>8.4</td>
<td>BER performance comparison for Linear detectors</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>8.5</td>
<td>BER performance comparison for non-linear detectors</td>
</tr>
<tr>
<td>8.6</td>
<td>BER comparison of the proposed 4-QAM 4x4-OCM system and 4x4-BPSK</td>
</tr>
<tr>
<td></td>
<td>un-coded MIMO system</td>
</tr>
</tbody>
</table>
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Pheromone Update Strategy for BA-MIMO system</td>
<td>82</td>
</tr>
<tr>
<td>6.2</td>
<td>4-QAM $N_t \times N_r$ systems - performance complexity trade-off</td>
<td>86</td>
</tr>
<tr>
<td>6.3</td>
<td>MPSO-MIMO algorithm’s pseudo-code</td>
<td>90</td>
</tr>
<tr>
<td>6.4</td>
<td>Computational Complexity Comparison- MQAM 2x4-MIMO system</td>
<td>97</td>
</tr>
<tr>
<td>6.5</td>
<td>Computational Complexity Comparison – 4QAM $N_t \times N_r$, MIMO system</td>
<td>97</td>
</tr>
<tr>
<td>6.6</td>
<td>Performance Complexity trade-off</td>
<td>98</td>
</tr>
</tbody>
</table>
List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIMO</td>
<td>Multiple-input–multiple output</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>V-BLAST</td>
<td>Vertical Bell-laboratories LAyered Space-Time</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum-Likelihood</td>
</tr>
<tr>
<td>SI</td>
<td>Swarm Intelligence</td>
</tr>
<tr>
<td>PSO</td>
<td>Particle Swarm Optimization</td>
</tr>
<tr>
<td>ACO</td>
<td>Ant Colony Optimization</td>
</tr>
<tr>
<td>Bin PSO</td>
<td>Binary Particle Swarm Optimization</td>
</tr>
<tr>
<td>BAS</td>
<td>Binary Ant System</td>
</tr>
<tr>
<td>STBC</td>
<td>Space-time block coding</td>
</tr>
<tr>
<td>STTC</td>
<td>Space-time Trellis coding</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>MUD</td>
<td>Multi-user Detection</td>
</tr>
<tr>
<td>HMM</td>
<td>Hidden Markov models</td>
</tr>
<tr>
<td>FSMC</td>
<td>Finite State Markov Channel</td>
</tr>
<tr>
<td>PAM</td>
<td>Pulse Amplitude Modulation</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Keying</td>
</tr>
<tr>
<td>CPM</td>
<td>Continuous Phase Modulation</td>
</tr>
<tr>
<td>FSK</td>
<td>Frequency Shift Keying</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>GMSK</td>
<td>Gaussian Minimum Shift Keying</td>
</tr>
<tr>
<td>DS-SS</td>
<td>Direct sequence spread spectrum</td>
</tr>
<tr>
<td>MAI</td>
<td>Multiple Access Interference</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximal Ratio Combining</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>SD</td>
<td>Sphere Decoder</td>
</tr>
<tr>
<td>SDP</td>
<td>Semi-Definite Programming</td>
</tr>
<tr>
<td>CO</td>
<td>Combinatorial Optimization</td>
</tr>
</tbody>
</table>
Glossary

\( N_r \) \quad Number of transmit antennas
\( N_r \) \quad Number of receive antennas
\( y(t) \) \quad Received signal
\( H \) \quad \( N_r \times N_t \) channel matrix
\( x(t) \) \quad Transmit signals
\( R_y \) \quad Covariance matrix
\( P_{outage} \) \quad Outage probability
\( Q_z \) \quad Quantizer/Slicer
Wireless communication systems have developed considerably since Marconi first demonstrated radio’s ability to provide contact with ships sailing the English Channel back in 1897. Since then people throughout the world have enthusiastically adopted new wireless communications methods and services. Currently, when the telecommunications industries are deploying third-generation (3G) systems worldwide and researchers are presenting many new ideas for the next generation wireless systems (termed 4G), several challenges are yet to be fulfilled.

Demands for high capacity in wireless communications, driven by cellular mobile, Internet and multimedia services have been rapidly increasing. On the other hand, the available radio spectrum is limited and the communication capacity needs cannot be met without a significant increase in the spectral efficiency of the newly proposed communication systems. Advances in coding, such as turbo [1] and low density parity check codes [2][3] made it feasible to approach the Shannon capacity limit [4] in systems with a single antenna link. Significant gain in spectral efficiency can be achieved by increasing the number of antennas at both the transmitter and the receiver [5][6] resulting in a Multiple-Input Multiple-Output (MIMO) system.

MIMO has recently emerged as one of the most significant breakthroughs in modern wireless communication systems. The technology figures prominently on the list of recent technical advances with a chance of resolving the bottleneck of capacity in future data-intensive networks. Perhaps even more surprising is that just a few years after its invention the technology seems poised to penetrate large-scale standards-driven commercial wireless products and networks such as broadband wireless access systems, wireless local area networks (WLAN), third-generation (3G) networks, WiMAX and beyond.

The idea behind MIMO such as the one shown in Fig. 1.1 is that the signals on the transmit (Tx) antennas at one end and the receive (Rx) antennas at the other end are “combined” in such a way that the bit-error rate or BER performance or the data-rate of the communication for each MIMO
user will be improved. Such an advantage can be used to increase both the network’s quality of service and the operator’s revenues significantly.

A core idea in MIMO a system is space–time signal processing in which time the natural dimension of digital communication, data is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas. The relevant information–theoretic analysis reveals that significant performance gains are achievable in wireless communication systems using a MIMO architecture employing multiple antennas [7]. This architecture is suitable for higher data rate multimedia communication [8]. Efficient exploitation of spatial diversity available in MIMO channel enables higher system capacity. A key feature of MIMO systems is the ability to turn multipath propagation, traditionally a pitfall of wireless transmission, into a benefit for the user. MIMO effectively takes advantage of random fading [9]–[11] and when available, multipath delay spread [12][13] for multiplying data transfer rates. The prospect of many orders of magnitude improvement in performance at no cost of extra spectrum (only hardware and complexity are added) is largely responsible for the success of MIMO as a topic for new research. Orthogonal Frequency Division Multiplexing (OFDM) employed in conjunction with MIMO architecture constitutes an attractive solution for modern wireless communications systems [14].

An effective and practical way to approaching the capacity of MIMO wireless channels is to employ space-time (ST) coding [15]. Space-time coding is a technique designed for use with multiple transmits antennas. Coding is performed in both spatial and temporal domains to introduce correlation between signals transmitted from various antennas at various time periods. The spatial-temporal correlation is used to exploit the MIMO channel fading and minimize transmission errors at the receiver. Space-time coding can achieve transmit diversity and power gain over spatially uncoded systems without sacrificing the bandwidth. There are various
approaches in coding structures, including space-time block codes (STBC), space-time trellis codes (STTC), space-time turbo trellis codes and layered space-time (LST) codes.

1.1 Challenges in Uncoded Multi-Antenna Systems

One of the challenges in building wide-band MIMO systems is the tremendous processing power required at the receiver. While coded MIMO schemes offer better performance than separate channel coding and modulation scheme by fully exploring the tradeoff between multiplexing and diversity [16], its hardware complexity can be significant, especially for wideband system with more than four antennas both at the transmitter and the receiver sides. On the other hand, it is easier to implement traditional channel coding schemes such as convolution or turbo for data rates of hundreds of Mbps. For this reason we discuss uncoded MIMO system also called spatial multiplexing (as shown in Fig 1.1) in the earlier part of this thesis.

MIMO symbol detection involves detecting symbol from a complex signal at the receiver. This detection process is considerably complex as compared to single antenna system. A number of architectures have been developed for MIMO systems. The Vertical Bell-laboratories LAyered Space-Time (V-BLAST) system is one successful implementation of MIMO systems [5][9]. However, the performance of the V-BLAST detection scheme [17][18], is limited due to imperfect interference cancellation and insufficient receive diversity. The Maximum Likelihood (ML) detection scheme performs the best, but its complexity increases exponentially with transmit antennas and with higher modulation schemes. Besides V-BLAST, quite a few approaches have been proposed in [19]-[27]. These MIMO detection techniques can be broadly divided into linear and non-linear detection methods [28]. Linear methods offer low complexity with degraded BER performance as compared to non-linear methods. This thesis focuses on non-linear detectors and makes an effort to improve BER performance at the cost of complexity and vice versa. ML and V-BLAST detectors [5],[9] are well known non-linear MIMO detection methods. ML outperforms V-BLAST in BER performance, while V-BLAST has a lower computational complexity than ML. Both of these non-linear detectors have their own advantages and disadvantages. ML outperforms VBLAST in terms of BER, while V-BLAST possesses low complexity than ML. In [29] and [30] the researchers have proposed techniques that take advantage of these two methods and the results show a performance complexity trade off between the two methods.
1.2 Approach to Optimize MIMO Detection

Being NP-hard [28] computational complexity of optimum ML technique is generically exponential. Therefore, in order to solve these problems for any non-trivial problem size, exact, approximate or un-conventional techniques such as meta-heuristics based optimization approach can be used. The exact method exploits the structure of the lattice and generally obtains the solution faster than a straightforward exhaustive search [28]. Approximation algorithm provides approximate but easy to implement low-complexity solutions to the integer least-squares problem. Whereas, nonconventional methods like meta-heuristics based algorithm can also work reasonably well to optimize such NP-hard problems.

Real-life optimization problems (like MIMO detection) are often so complex that finding the best solution becomes computationally unfeasible. Therefore, an intelligent approach is to search for a good approximate solution consuming lesser computational resources. Several engineering problems contain multiple objectives that need to be addressed simultaneously such as aircraft design, the oil and gas industry, automobile design, or wherever optimal decisions is to be taken in with a trade-offs between two conflicting objectives. Many techniques have been proposed that imitate nature’s own ingenious ways to explore optimal solutions for both single and multi-objective optimization problems. Earliest of the nature inspired techniques are genetic and other evolutionary heuristics that evoke Darwinian evolution principles.

Swarm Intelligence (SI) [31][32] is a novel distributed intelligent paradigm for solving optimization problems that originally took its inspiration from the biological processes like swarming, flocking phenomena in vertebrates and the cooperative foraging strategy of real ants. Particle Swarm Optimization (PSO) meta-heuristics is a population-based Swarm Intelligence (SI) technique inspired by the coordinated movements of birds flocking introduced by Kennedy and Eberhart in 1995 [33]. Standard PSO uses a real-valued multidimensional solution space, whereas in binary PSO particle positions are binary rather than real valued [34]. The combination of the pure heuristics like PSO with local search (LS) is termed as “memetic algorithms” (MAs) [35]. MAs are extensions that apply additional procedure to further refine the search results efficiently. This hybridization improves search efficiency [36].

ACO meta-heuristics is another SI technique that is based on the cooperative foraging strategy of real ants [37],[38]. In this approach, several artificial ants perform a sequence of operations
Chapter 1 Introduction

iteratively. Ants are guided by a greedy heuristic algorithm which is problem dependent, that aid their search for better solutions iteratively [39]. Ants seek solutions using information gathered previously to perform their search in the vicinity of good solutions. Its binary version known as Binary Ant System (BAS) is well suited for constrained optimization problems with binary solution structure [40].

Simple mathematical model of these algorithms, their resistance to being trapped in local minima and convergence to near optimal solution in fewer iterations makes them a suitable candidates for real-time NP-hard communication problems [41], in addition to other wide range of applications like traveling salesman problem [42].

In our work SI is applied on a NP-hard symbol detection problem in the area of wireless communication. The problem is to detect symbols from a composite signal, received at multiple receivers, transmitted from multiple transmitters.

1.3 Exact Solution in a Coded Multi-Antenna Systems

Depending on whether multiple antennas are used at transmitter or receiver, space diversity can be classified into receive diversity and transmit diversity. In the former case, multiple antennas are used at the receiver side to pick up independent copies of the transmitted signals whereas, in the later, multiple antennas are deployed at the transmitter side. In both the cases, multiple copies of the transmitted signals are combined to mitigate the multipath fading effects. Space-Time Block Coding (STBC) is a transmit diversity technique in which the data stream to be transmitted is encoded in blocks, which are distributed among multiple antennas and across time. Alamouti [43] proposed a simple STBC scheme for wireless communication systems using two transmit antennas and a linear maximum-likelihood (ML) decoder which was generalized by Tarokh et. al. in [15] to an arbitrary number of transmit antennas by applying the theory of orthogonal designs. The main advantage of these schemes is a decoder based on linear processing.

A constellation reduction technique based decoding algorithm that simplifies the ML detection in Orthogonal Space-Time Block Coded systems is proposed in this thesis. This approach reduces the computational complexity of these schemes while presenting ML performance. Throughout this thesis, the STBC schemes developed by Alamouti and Tarokh are discussed however, the idea presented can be generalized to other STBC schemes.
Chapter 1 Introduction

1.4 Orthogonal Coded Spatial Multiplexing System

An Orthogonal Coded MIMO system is also presented at the end in chapter-8. Walsh coded BPSK modulated user-data bits are added and transmitted. The receiver de-spreads the received signal and applies conventional detection techniques to jointly decode the transmitted symbols. The significance of this approach is relative enhancement in BER performance and reduction in the symbol detection complexity in comparison to the conventional MIMO system.

1.5 Organization of thesis

The Thesis deals with un-coded and coded MIMO systems. Initial Chapters present the basic concepts of wireless communications and Multi-antenna communication systems. Subsequently existing un-coded MIMO detection algorithms are described followed by explanation of Meta-heuristics optimization techniques. Proposed approaches to Optimize symbol detection in both un-coded and coded MIMO systems are presented next.

Chapter 2 discusses multipath fading channels and modulation techniques. Small scale and large scale fading manifestations are elaborated. Concepts of flat, frequency selective and time selective fading have are explained. These concepts will be used in the subsequent chapters.

Multi-antenna communication systems are discussed in Chapter 3. Both the un-coded and coded MIMO systems are described and their system models and capacity limits are also discussed. The Chapter also explains different un-coded MIMO architectures like V-BLAST and coded systems like Alamouti and Taurok [15].

Symbol detection techniques in MIMO systems and MIMO detection problem formulation is given in Chapter-4. This Chapter also presents a survey of existing MIMO detection techniques like linear, non-linear and exact techniques. ZF, MMSE are common linear MIMO detectors that offer very low complexity as well as BER performance, compared to non-linear methods like VBLAST. Non-linear methods are computationally expansive than linear techniques, however they have an enhanced BER performance as compare to linear detectors.

Chapter 5 describes the meta-heuristic techniques that are used to solve the MIMO detection problem discussed in the thesis. Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO); two well known swarm intelligence based optimization algorithms which have been used to solve real-time NP-hard problem such CDMA Multi-user Detection (MUD) are explained.
Chapter 1 Introduction
These nature inspired optimization techniques provide an intelligent way to search the ML space thus reducing the ML computational complexity; while keeping the BER performance near-optimal.

Chapter 6 presents the proposed algorithms for MIMO detection using ACO and PSO algorithms. Different PSO algorithms like Standard PSO and Binary PSO are used to solve the MIMO detection problem under discussion. Similarly, Binary ACO is also applied.

Chapter 7 explains the proposed symbol detection in coded multi-antenna systems. Conventional ML detection in these systems offers linear complexity which is further reduced by the proposed multi-step constellation reduction algorithm. Results for the proposed reduced search approach are used to improve the complexity of coded multi-antenna systems in a Quasi-static channel. The proposed techniques are simple but result in substantial reduction in decoding complexity while keeping the BER performance at the optimal level. Similar reduced complexity ML detection is applied to OSTBC time selective channels and the suggested approach and results are in same chapter 7. Once the channel becomes time-selective the performance of conventional ML detection degrades as the transmitted symbols no more remain decoupled. Here the proposed modified reduced search detection algorithm gives an enhanced performance.

Another approach to reduce the detection complexity using orthogonal codes is discussed in Chapter 8. Orthogonal Multiplexed MIMO systems are proposed that employ orthogonal Walsh codes for antenna separation. The system reduces the complexity but also effects systems bandwidth. Finally, Chapter 9 concludes the thesis and also mentions the future work that can be perused based on this work.
Chapter 2

Multipath Fading Channels, Modulation Techniques and Antenna Diversity

2.1 Introduction

This chapter builds a background for discussing communication systems employing multiple antennas. Basic information on the channel characterizations and an overview of the different modulation techniques and the concept of diversity is presented.

2.2 Channel Characterizations

Channel characterization becomes a significant issue due to the time varying nature of the wireless channel. The analysis of mobile channel is essential in order to fully exploit its characteristics. In mobile wireless communication scenario, the channel characteristics vary with time due to relative motion between the transmitter and the receiver, time variation in the medium structure, etc. This makes the wireless channel random in nature. Its characterization becomes correspondingly tedious. The strength of the received signal depends on the channel characteristics and the transmitter-receiver separation. A defining characteristic of the mobile wireless channel is the variations of the channel strength over time and over frequency. In a broad sense, the channel can be modeled in two different ways, large-scale propagation model and small-scale propagation model. Propagation models that predict the mean signal strength for any transmitter receiver separation become useful in estimating the transmitter coverage area. This is generally termed as large-scale propagation models. Propagation models that characterize rapid fluctuations of the received signal within few wavelengths (short distances) are called small-scale propagation models. Fig 2.1 depicts a gradual large scale variations and more
frequently varying small scale variations for an indoor radio communications system [44]. These models are discussed in more details in the following subsections.

![Graph showing small-scale and large-scale fading manifestation.](image)

**Fig 2.1:** Small-scale and large-scale fading manifestation.

### 2.2.1 Large Scale Propagation Models

Large scale propagation occurs due to path loss of signal as the mobile moves through a distance of the order of cell size and shadowing caused by large objects such as buildings, hills, etc. It is frequency independent typically. Large scale propagation is generally classified into deterministic and non-deterministic categories. Both deterministic and non-deterministic or stochastic approaches can be used to describe a time-varying channel.

#### 2.2.1.1 Deterministic Approach

(a) **Free Space Propagation Model**

The received signal power decays as a function of the transmitter-receiver distance when a clear, unobstructed line-of-sight path exists. Typically, satellite communication systems and microwave radio links experience free space propagation. In this case, the free space signal power $P_r(d)$, received by a receiver antenna at a distance $d$ meters from the radiating transmitter, is given by Friis free space equation [44].

$$P_r(d) = \frac{PGG_r \lambda^2}{(4\pi)^2 d^2 L} \quad d \geq d_0(\lambda) \geq d_f$$  \hspace{1cm} (2.1)

where $P_t$ is the transmitted signal power, $G_t$ and $G_r$ are the transmitter and receiver antenna gains, respectively, $L \geq 1$ is the system loss factor not related to propagation. $\lambda$ in meters is the
wavelength. \( d_f \) is the far-field distance or Fraunhofer distance and \( d_0 \) is the received-power reference distance. The gain of the antenna is related to its effective aperture, \( A_e \) as

\[
G = \frac{4\pi A_e}{\lambda^2}
\]  

(2.2)

The effective aperture is related to antennas physical size, and \( \lambda \) is related to carrier frequency \( f \). Far field distance \( d_f \) is represented by as

\[
d_f = \frac{2D^2}{\lambda} \sqrt{d_0 D} \tag{2.3}
\]

\( D \) is the largest physical linear dimension of the antenna. The free-space received signal power at a distance \( d > d_0 \) can be represented as

\[
P_r(d) = P_r(d_0) \left( \frac{d_0}{d} \right)^\gamma \quad d \geq d_0 \geq d_f \tag{2.4}
\]

(b) Long-Distance path loss Model

Regardless of being an indoor or outdoor radio channel, the propagation models indicate that the average received power decrease logarithmically with transmitter and receiver distance, which is given by

\[
\overline{PL}(d) = \overline{PL}(d_0) + 10n \log\left( \frac{d}{d_0} \right) \tag{2.5}
\]

where \( n \) the path loss exponent shows the relationship between path loss and distance. The path loss exponent \( n \) is environment specific [44].

2.2.1.2 Stochastic Approach

(a) Lognormal Shadowing Model

The scenario in (2.5) does not cater for the fact that the surrounding environments can be considerably different at different locations with same transmitter-receiver distance. The phenomenon that describes the random shadowing effects occurring over a large number of locations with the same T-R separation with different environmental clutter on the propagation path is termed as lognormal shadowing. Experimental results show that at any value of \( d \), the
path loss $PL(d)$ is random at a particular location. Lognormal shadowing model [45] [46] states that the path loss $PL(d)$ at a particular location is lognormally (normal in dB) distributed about the mean distance-dependent value. This model is given by

$$PL(d)[dB] = PL(d_o) + 10n\log\left(\frac{d}{d_o}\right) + X_o$$  \hspace{1cm} (2.6)$$

where $X_o$ (dB) is a zero-mean Gaussian distributed random variable with a standard deviation $\sigma$ in dB.

In addition to the general large-scale propagation models described above, there are some outdoor and indoor environments specific models also. These channel models are developed based on the particular area profile. Some outdoor propagation models include the Longley–Rice model [47], Okumura Model, Hata Model and Durkin’s model [48]. Few indoor models are the Ericsson multiple breakpoint model [49] and the attenuation factor model [50]. Ray tracing and site-specific modeling techniques are also used for both outdoor and indoor environments [44].

### 2.2.2 Small Scale Propagation Models

Small-scale fading describes rapid fluctuations in the amplitudes and phases of the signal over a short distance that is typically $5\lambda$ to $40\lambda$, $\lambda$ being the wavelength of the signal. In this case, the instantaneous received signal fluctuates rapidly and results in small-scale fading. Fading occurs when two or more versions of the transmitted signals arrive at slightly different time after having traversed different paths. These waves, generally called multipath waves, add at the receiver to give a resultant signal which may vary in amplitude and phase. Different small-scale propagation models are described below

#### 2.2.2.1 Mobile Multipath channels Parameters

A multipath channel can be characterized using some useful parameters like delay spread and coherence bandwidth that describe the time-dispersive nature of the channel. Where as, Doppler spread and coherence bandwidth explain the time-varying nature of the channel in a small-scale location [52].
Chapter 2 Multipath Fading Channels and Modulation Techniques

(a) Fading
Fading or small-scale fading result when two or more attenuated versions of the transmitted signal arriving at the receiver interference in such a way that these signals are added destructively. Time dispersion due to multipath causes the signals to undergo the fading phenomenon. The speed of the mobile and the transmission bandwidth of the signal play a vital role in fading.

(b) Doppler Shift
The apparent variation in the transmitted signal frequency due to the relative motion of the mobile is termed as the Doppler shift. When a mobile moves with velocity $v$, and $\theta$ is the spatial angle between the direction of motion of the mobile and the direction of arrival of the wave with $\lambda$ wavelength, Doppler shift $f_d$ is expressed as

$$f_d = \frac{v}{\lambda} \cos \theta$$  \hspace{1cm} (2.7)

(c) Power Delay Profile $\Phi_c(\tau)$
The average output signal power of the channel as a function of excess time delay $\tau$ is termed as power delay profile $\Phi_c(\tau)$. It is usually measured by transmitting narrow pulses and cross-correlating the received signal with its delayed version. $\Phi_c(\tau)$ is also referred as delay power spectrum and multipath intensity profile. It is termed as delay power spectrum because of its frequency domain component, which gives the power spectrum density. Power delay profile can used to determine multipath channel parameters such as mean excess delay, root mean squared (rms) delay spread, and excess delay spread ($\text{XdB}$). The mean excess delay ($\tau_{\text{mean}}$) is the first moment of the power delay profile. The rms delay spread ($\sigma_\tau$) is the square root of the second central moment of the power delay profile, and the maximum excess delay ($\text{XdB}$) is termed as the time delay during which multipath energy falls to $X$ dB below the maximum value. $\tau_{\text{mean}}$ and $\sigma_\tau$ are expressed as

$$\tau_{\text{mean}} = \frac{\sum P(\tau_j)\tau_j}{\sum P(\tau_j)}$$  \hspace{1cm} (2.8)

$$\sigma_\tau = \sqrt{\text{mean}(\tau^2) - \tau_{\text{mean}}^2}$$  \hspace{1cm} (2.9)
when 

\[
\text{mean}[(\tau_j^2)] = \frac{\sum P(\tau_j)\tau_j^2}{\sum P(\tau_j)}
\]  \hspace{1cm} (2.10)

\(d\) Excess Delay

The relative delay of the \(j\)th multipath signal component from the first arriving component is termed as excess delay \(\tau_j\).

\(e\) Delay Spread \((T_m)\)

Delay spread or multipath spread of the channel is the range of values of excess time delay \(\tau\), over which power delay profile \(\Phi_c(\tau)\) is not zero.

\(f\) Coherence Bandwidth \((B_W)\)

Coherence bandwidth is a measure of range of frequencies over which the channel may be considered flat. That is it is a frequency band in which all the spectral components of the transmitted signal pass with equal gain and linear phase through a channel. The channel remains invariant over this bandwidth. Coherence Bandwidth \(B_Wc\) can be expressed in terms of rms delay spread. According to Lee [44], with a frequency correlation of approximately 90\%, \(B_Wc\) can be approximated as

\[
B_Wc \approx \frac{1}{5\sigma_c}
\]  \hspace{1cm} (2.11)

If the frequency correlation is above 50\%, the \(B_Wc\) becomes

\[
B_Wc \approx \frac{1}{5\sigma_c}
\]  \hspace{1cm} (2.12)

\(g\) Doppler Spread \((B_D)\)

Doppler spread is a measure of spectral spreading caused due to time rate of change of a mobile radio channel. It is the range of frequencies over which the Doppler spectrum received in non zero. With the transmitted a pure sinusoidal signal with frequency \(f_c\), the resultant Doppler spectrum will contain the components in the range between \((f_c - f_m)\) and \((f_c + f_m)\), where \(f_m\) is the maximum Doppler frequency shift.
Coherent Time ($T_c$)

It is the time domain dual of Doppler spread. Coherence time is a statistical measure of the time duration during which the channel's impulse response remains invariant. $T_c$ is inversely proportional to the Doppler spread, and with the maximum Doppler frequency shift, $f_m$.

$$T_c \approx \frac{1}{f_m} \quad (2.13)$$

2.2.2.2 Types of Small-Scale Fading

Small-scale fading effects are divided into two broad categories based on the Doppler spread and time delay spread. These effects are manifested depending upon the nature of transmitted signal, the channel, and the velocity of mobile. The Doppler spread based class is categorized as fast and slow fading, whereas the time delay spread based class is divided into flat fading and frequency selective fading. Fast and slow fading is independent of propagation path loss models and deal with the time rate of change of the channel and the transmitted signal relationship. These are discussed in detail below.

(a) Flat Fading

Flat fading is experienced in a mobile radio environment, if the channel response remains constant gain with linear phase over a bandwidth that is greater than the transmitted signal’s bandwidth. In flat fading the transmitted signal’s symbol period is (at least 10 times) greater than the channels delay spread. The bandwidth of the transmitted signal is smaller than the bandwidth of the channel. The Flat fading channels cause deep fades, and therefore may require 20 to 30 dB additional transmitter power to achieve better bit error rates (BERs) during the deep fades period.

(b) Frequency-Selective Fading

Frequency selective fading is experienced if the bandwidth of the transmitted signal is greater than the bandwidth over which. The transmitted signal’s symbol period is (at least 10 times) smaller than the delay spread of the channel. The frequency-selective fading channels are also known as wideband channels, since the bandwidth of the transmitted signal is more than the channel bandwidth. Frequency-selective channel cause intersymbol interference (ISI). Characterizing frequency-selective channels is tougher than the flat-fading channels is difficult since each multipath signal has to be modeled individually.
(c) Fast Fading

Fast fading is a result of rapidly changing channel impulse response within the symbol duration in a mobile radio environment. The symbol period of the transmitted signal is greater than the coherence time of the channel, also termed as *time-selective fading*. Transmitted signal bandwidth is smaller than the Doppler spread. In **fast-flat fading** channels the amplitude of the received signal varies faster than the transmitted baseband signal’s rate of change. However, in **fast-frequency-selective** channels the amplitudes, phases, and time delays of the multipath components vary faster than the transmitted signal’s rate of change.

(d) Slow Fading

Slow fading is experienced by the received signal in the mobile radio environment when the channel impulse response is varying slowly, within the symbol duration. Transmitted signal’s symbol period is smaller than the channels coherence time. The channel can be considered as static over one or more symbol durations. The transmitted signal is greater than the Doppler spread.

2.2.2.3 Statistical Models of Small-Scale Propagation Channel

The mobile wireless fading channels can be modeled statistically in a variety of ways. The most common statistical models are the Rayleigh, Ricean, and Nakagami fading channel models, described briefly in following sub-section [44].

(a) Rayleigh Fading Channel

Rayleigh distribution is generally used to describe the statistical time varying nature of the flat fading signals received envelope. When the channel impulse response $C(\tau, t)$ is modeled as a zero-mean complex Gaussian process at a time instant $t$ and delay $\tau$, the envelope $|C(\tau, t)|$ at that time instant $t$ is known as Rayleigh distributed. In this scenario, the channel is termed as a Rayleigh fading channel. Envelope of the sum of two quadrature Gaussian noise follow Rayleigh distribution. The Rayleigh distribution has the probability density function (PDF) given by.
Chapter 2: Multipath Fading Channels and Modulation Techniques

\[ p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) & (0 \leq r \leq \infty) \\ 0 & (r < 0) \end{cases} \]  \hspace{1cm} (2.14)

where \( r \) is the rms value of the received signal envelope and \( \sigma^2 \) is the time average power of the received signal before envelope detection.

(b) Ricean Fading Channel

If the mobile wireless channel has reflectors or fixed scatterers along with the randomly moving scatterers, the channel impulse response \( C(\tau, t) \) can not be modeled as a zero-mean complex Gaussian process. In this scenario the envelope has a Ricean distribution. Such a channel is termed as a Ricean fading channel. In Ricean fading, a dominant non-fading signal component is present such as line of sight propagation path. The random multipath components arriving at different angles superimposed on the stationary dominant signal. The Ricean distribution has the PDF.

\[ p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + A^2)}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right) & \text{where } (A \geq 0, r \geq 0) \\ 0 & (r < 0) \end{cases} \]  \hspace{1cm} (2.15)

parameter \( A \) denotes the peak amplitude of the dominant received signal arriving at the receiver either through a line of sight path or from a fixed scatterer. \( I_0(\cdot) \) is the modified Bessel function of first kind and zero-order. Ricean factor \( K \) is sometimes used to describe Ricean distribution, which is defined as the ratio between the dominant signal power and the variance of the scattered power as \( K = \frac{A^2}{2\sigma^2} \). If \( A \) approaches 0, the amplitude of the dominant path decreases, the Ricean fading converts to Rayleigh fading channel.

(c) Nakagami Fading Channel

Long-distance rapid fading channels are characterizes as Nakagami fading channels [52]. Nakagami distribution is used for fading channel characterization since it is closer to the experimental results of channel modeling as compared to the Rayleigh or Ricean distributions discussed above. The Ricean distribution contains a Bessel function, while the Nakagami
distribution does not [52], therefore the Nakagami distribution often leads to convenient closed-form expressions.

### 2.2.2.4 Statistical Models for Multipath Fading Channels

Many statistical channel models have been proposed to describe the observed statistical nature of the terrestrial and satellite channel environments. Examples include Clarke’s model [53], the Saleh and Valenzuela model [54], and the two-ray fading channel model. In this section we will discuss only the two-ray fading channel model since it gives a clear idea about the channel’s fading effect. Besides, we will discuss some recently researched channel models, which are based on different types of fading channel environments.

(a) **Two Ray Model**

This model considers both direct path and ground reflected propagation path between the transmitter and the receiver. This model is quite accurate for predicting large scale signal strength over longer distances for a mobile communication system. The received power at the distance $d$ from transmitter the two ray ground reflected model can be represented as:

$$ P_r = P_i G_t G_r \frac{h_t^2 h_r^2}{d^4} $$

(2.16)

Where, $h_t$ and $h_r$ are the transmitter and receiver antenna heights. Once the the distance $d$ is much larger the received power degrades at the rate of 40 dB/decade [44]. The two-ray models path loss (dB) can be represented as:

$$ PL(dB) = 40 \log d - (10 \log G_t + 10 \log G_r + 10 \log h_t + 10 \log h_r) $$

(2.17)

(b) **Markov Channel Modeling**

Finite State Markov Channel model and Hidden Markov Model which falls under this category are explained below:-

(i) **Finite State Markov Channel**

Wang and Moayeri [55] proposed the modeling of a Rayleigh fading narrowband channel using a Markov process with a finite number of states referred to as the Finite State Markov Channel
(FSMC) model. The FSMC model originated as an extension of a simpler model proposed earlier, and known as the Gilbert-Elliot channel. In the FSMC, the fading process is related to the received signal to noise ratio (SNR). Such models are applicable *primarily to flat fading channels*. The SNR is used since it is a common parameter that represents the quality of the channel [55]. For instance, the variations in the SNR can also affect the performance of other layers, like the link layer. At high average SNR the average number of lost frames due to transmission errors is expected to be low, the opposite occurs at low average SNR values. Therefore an accurate modeling of the received SNR can result in accurate channel models at the bit or frame level. Fig. 2.2 illustrates how the received SNR can be used in a FSMC model. First the SNR is partitioned into ‘n’ intervals or *levels*. Then each interval is associated with a state of a Markov process.

Fig 2.2: Partitioning of Received SNR and assigning each interval to a state of the FSMC [55]

Fig. 2.3 shows the FSMC represented by a chain of ‘n’ states. As seen in the figure, only transitions to the same state or to adjacent ones are allowed in the model. In the figure the in-state transition probabilities \((p_{ii})\) and the adjacent state transition probabilities \((p_{ij})\) are shown next to each arrow in Fig 2.3.

Fig 2.3: The Finite State Markov channel model representation [55]
Chapter 2 Multipath Fading Channels and Modulation Techniques

The goal of the model is to relate the varying nature of the channel with a loss process. For this, each of the ‘n’ states is associated with a different binary symmetric channel (BSC). The ‘n’ BSCs are shown in the lower part of Fig. 2.3. In each state the associated BSC determines how a symbol being transmitted, for example a zero or a one, could be received in error. The individual probabilities of receiving a symbol in error are called crossover probabilities and are shown in the Fig 2.3 as $1-p_i$. FSMC models are based on the theory of constant Markov processes. Constant Markov processes have the property that the state transition probabilities are independent of the time at which they occur. These processes can be defined by a finite number of possible states that are usually represented by a set $S = \{s_0, s_1, \ldots, s_{n-1}\}$ and a sequence of states $\{s_k\}$, $k=0,1,2,\ldots,n$.

The elements of the state transition probability matrix $P$ should be between 0 and 1; the rows of $P$ should add up to one. A $k \times 1$ crossover probability vector $e$ of the Binary Symmetric Channel associated with state $k$. $k \times 1$ steady state vector $\pi$ [55].

It has been shown that a FSMC is accurate under a wide range of simple modulation and error correction schemes. Nevertheless no study has taken into consideration complex schemes such as those used in IEEE 802.11b and 802.11a wireless local area network technologies. The basic assumption made by all the FSMC model studies is that the underlying signal to noise ratio process follows the Markovian property. This property indicates that the probability of transition at a time ‘n’ to a new state only depends on the state at time ‘n-1’.

(ii) Hidden Markov Models

The application of FSMC is adequate under very slowly fading applications, that is, for short durations of time. Whenever there is a need to include the effect of very long channel memory the FSMC model is no longer appropriate. This is for example in the case of the study of fade duration distributions in fading channels [56]. Here, there is a need for Markov chains with larger memory, however since the number of states grows exponentially with the process memory, the approach is no longer practical [56]. In such cases other methods such as those that use hidden Markov models can be used.

Hidden Markov models (HMM) [57] are probabilistic functions of Markov chains. These models can be used to study the fading process of a Rayleigh fading channel. A common discrete Markov process, like the one used in FSMC, is a stochastic process in which the outputs are observable. The outputs in this case are the set of states at each instant of time. Additionally, each state
corresponds to some physical and observable event. These observable models can be extended to include the case where the ‘observation is a probabilistic function of the state’. This results in a ‘doubly embedded stochastic process’ where one of the stochastic processes is not observable and hence the name hidden Markov model. A HMM is characterized with the following elements.

a. A set of the Markov chain states represented by \( S = \{1, 2, \ldots, n\} \). The number of states in the model is ‘\( n \)’. Even though these states are called ‘hidden’, in practical applications they are associated with some physical event.

b. The set \( H \) of the observable output symbols in any state represented as \( H = \{h_1, h_2, \ldots, h_m\} \) with ‘\( m \)’ elements. ‘\( m \)’ is also called the alphabet size.

c. The state transition probability distribution matrix \( P = \{p_{ij}\} \), where \( p_{ij} = P_r[\text{current state} = j | \text{previous state} = i] = P_r[s_j | s_i] \).

d. The observed output symbol probability distribution matrices \( B \). \( B \) are diagonal matrices whose elements \( b_j \) represent the probability \( p\{h | s_j\} \) where \( h \in H \) (if \( H \) is discrete).

e. The initial state probability vector \( \pi \).

2.2.2.5 Multi-Input Multi-Output (MIMO) Channel Model

(a) Matrix Channel Model

Multiple-input multiple-output (MIMO) channel model [58], is shown in Fig. 2.4. \( N_t \) transmit antennas transmit the set of signals vector \( x(t) \). Received signal \( y(t) \) arrives at \( N_r \) receive antennas after propagating through a matrix channel \( H(\tau;t) \).

The two different time components \( t \) and \( \tau \) in the channel transfer function show that these channels may be a function of time \( t \) to model a time-varying channel and a function of delay \( \tau \) to model the dispersion incurred by wideband transmission. In general a vector/matrix notation is used to keep track of all the transmitted and received signals in a MIMO system. Grouping all the transmitted and received signals into vectors, the system can be viewed as transmitting a \( N_t \times 1 \) vector signals through an \( N_t \times N_r \) matrix channel, the relationship between the output and input vectors can be established as:

\[
y(t) = \int_{-\infty}^{\infty} H(\tau;t)x(\tau) \, d\tau
\]

where \( x(t) = [x_1(t) \ x_2(t) \ldots \ x_{N_t}(t)]^T \); \( y(t) = [y_1(t) \ y_2(t) \ldots \ y_{N_r}(t)]^T \) and
In the above representation, the $(n_r, n_t)^{th}$ element of $H$, $h_{n_r, n_t}$ is the complex channel response from the $n_t^{th}$ transmit antenna to the $n_r^{th}$ receive antenna. For the narrowband, time-invariant
Chapter 2 Multipath Fading Channels and Modulation Techniques

MIMO channel model, the channel transfer matrix becomes a constant (H) that simplifies (2.18) as

\[ y = Hx \]  

(2.19)

where

\[
H = \begin{bmatrix}
h_{11} & h_{12} & \cdots & h_{1N_t} \\
h_{21} & h_{22} & \cdots & h_{2N_t} \\
\vdots & \vdots & \ddots & \vdots \\
h_{N_r1} & h_{N_r2} & \cdots & h_{N_rN_t}
\end{bmatrix}
\]

(b) Physical Scattering Model

Physical Scattering Model [59] has successfully characterized the MIMO channel. The results of this model are close to the experimental observations. This method assumes a predefined power delay profile valid under specific settings such as range, system bandwidth, and antenna beam widths. A distribution of scatterers to tune to the predefined power delay profile is calculated that is used to characterize the MIMO channel. The propagation environment is constructed by the scattering coefficient and the location and the location of each scatterer. Ray based (two ray model) approach is used to determine the channel matrix. The physical scattering model is valid for all Ricean factor, including the Rayleigh fading case.

2.3 Modulation Techniques

Modulation is the process in which the message source is encoded in a manner that is suitable for its transmission on to a medium. It pertains to translate a baseband message signal to passband signal at higher frequencies. Modulation can be accomplished by varying the amplitude, phase and frequency of the high frequency carrier. Demodulation is the process in which the baseband message signal is extracted from the carrier for decoding the originally transmitted message at the receiver. Digital modulation translates digital symbols into waveforms accustomed with the
channel characteristics. Different digital modulation techniques that are in use in various communication systems are discussed below.

### 2.3.1 Classification of Modulation Schemes

Different modulation schemes can be classified into two categories: memoryless modulation and memory modulation techniques. In memory modulation a modulator maps a digital sequence into its corresponding analog signal with the condition that an analog signal waveform at any time depends on the previously transmitted signals like differential PSK (DPSK). Where as, in case of memoryless modulation techniques bit to symbol mapping is accomplished without the need of previous waveforms. Examples of memoryless modulation include pulse amplitude modulation (PAM) and phase shift keying (PSK) [44].

Digital modulation schemes are also classified as linear and nonlinear modulation techniques. In a linear modulation scheme, a modulator uses the principle of superposition to maps a digital bits sequence into an analog counterpart, examples of linear techniques are PAM, PSK, etc. While, in nonlinear modulation scheme superposition principle is not followed. Examples of nonlinear counterpart include continuous-phase modulation (CPM), frequency shift keying (FSK), etc.

### 2.3.2 Selection of Digital Modulation Schemes

Various factors should be considered for choosing a particular modulation scheme to be used in a communications system. A modulation scheme should be spectral efficient and possess low bit-error-rate (BER) especially at lower signal-to-noise ratios (SNR). A modulation scheme should also perform well in multipath and fading environments. Cost effective and simple implementations are also key desirable features. Existing modulation techniques do not offer all these features simultaneously. Some modulations are spectral efficient, however their BER performance is not exceptional. Bandwidth efficiency and energy efficiency are the two parameters used to quantify the performance of modulation techniques. Bandwidth or spectral efficiency is the number of bits transmitted per second per hertz of bandwidth or data-rate per hertz (bits/s/Hz). Energy efficiency is the ratio of signal energy per bit to noise power spectral density (Eb/N0) required for certain BER at the receiver.
2.3.3 **Different Modulation Schemes**

2.3.3.1  **Linear Modulation Techniques**

In linear modulation schemes the amplitude of the transmitted signal varies linearly with the modulating signal. Linear techniques are well suited for wireless communication system because of their spectral efficiency. A transmitted signal $S(t)$ can be represented as follows in a linear modulation scheme [61].

$$S(t) = A[m_g(t)\cos(w_c t) - m_g(t)\sin(w_c t)]$$  \hspace{1cm} (2.20)

where $A$ is the amplitude of the modulated signal $m(t)$ in general complex form, $f_c$ is the carrier frequency. It is obvious from the above equation that the amplitude of the carrier depends linearly on the modulating signal. These linear techniques are bandwidth efficient however they need RF amplifiers for transmission which are not power efficient. When non-linear amplifiers are used these result in degrading the spectral efficiency of linear modulation techniques. Some of the linear modulation schemes are Binary Phase Shift Keying (BPSK), Differential Phase Shift Keying (DPSK), Quadrature Phase Shift Keying (QPSK) and Offset PSK (OPSK) [62].

2.3.3.2  **Constant Envelope Modulation Techniques**

Constant envelope modulation techniques have significance as these modulations use Class C power efficient amplifiers which eliminate the presence of the side lobes and help reduce the spectral widening. Many practical radio systems use non-linear modulation methods in which the amplitude of the carrier remains same regardless of the changes in the modulating waveform. However, constant envelope techniques are less spectral efficient than linear modulation methods but are more power efficient [44]. Few examples of these modulation techniques are Binary Frequency Shift Keying, Minimum Shift Keying and Gaussian Minimum Shift Keying [63], etc

2.3.3.3  **Combining Linear and Constant Envelope Modulation Techniques**

Communication systems using modulations techniques which vary the amplitude and phase of the carriers provide more room to take data bits and lesser bandwidth. Variation of signal envelope and phase provides more degree of freedom to map the baseband data in more possible
carriers. Modulation techniques using this two dimensional freedom are termed as M-ary modulation. Some of these techniques are discussed below.

(a) M-ary Phase Shift Keying (MPSK)

In MPSK the carrier phase can take a phase shift of M possible values to represent the user data bits. The MPSK modulated signal can be represented as

\[ S_j(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[\omega_c t + (j - 1)\frac{2\pi}{M}\right] \quad 0 \leq t \leq T_s \]  

(2.29)

where the symbol period \( T_s = bT_b \) and energy per symbol \( E_s = bE_b \), \( b = \log_2 M \) bits per symbol and \( j=1 \) to \( M \). Symbols are equally spaced at \( \sqrt{E_s} \) centered at origin. 8-PSK constellation diagram shown in Fig 2.6 illustrates that MPSK is a constant envelope modulation technique.

Average error probability for MPSK modulation can be represented as

\[ P_{(MPSK)_{err}} \leq 2Q\left(\sqrt{\frac{2bE_b}{N_0}} \sin\left(\frac{\pi}{M}\right)\right) \]  

(2.30)
(b) Quadrature Amplitude Modulation (MQAM)

For MQAM, the information bits are encoded in both the amplitude and phase of the transmitted signal. Thus, whereas both MPAM and MPSK have one degree of freedom in which to encode the information bits (amplitude or phase), MQAM has two degrees of freedom. As a result, MQAM is more spectrally efficient than MPAM and MPSK, in that it can encode the most number of bits per symbol for a given average energy. Fig 2.7 shows 16-QAM constellation. It’s a square lattice representation of symbols. M-QAM signal can be expressed as

\[ S_i(t) = \sqrt{\frac{2E_{\text{min}}}{T_s}} a_i \cos \omega_c t + \sqrt{\frac{2E_{\text{min}}}{T_s}} b_i \cos \omega_c t \quad 0 \leq t \leq T_s \]  

(2.31)

where \( i = 1 \) to \( M \), \( E_{\text{min}} \) is the minimum symbols energy with lowest amplitude. \( a_i \) and \( b_i \) represent the values of a particular signal point along \( x \)-axis and \( y \)-axis such that \( S = a + jb \), representing the symbols location in the constellation. In an AWGN channel with optimum detection techniques, the SER (symbol error rate) of an M-QAM system with a rectangular constellation is given by [65]

\[ P_{(MQAM),e} = 1 - \left[ 1 - 2 \left( 1 - \frac{1}{M} \right) Q \left( \sqrt{\frac{3\gamma}{M-1}} \right) \right]^2 \]  

(2.32)

where \( \gamma \) is the average SNR per symbol.

MQAM is power efficient than MPSK [60]. QAM is used in different applications, such as microwave digital radios, Digital Video Broadcasting Cable (DVB-C) systems, 3G cellular data-only systems (IS856), satellite communications etc.
2.3.3.4 Spread Spectrum Modulation Techniques

Spread-spectrum is a means of transmission in which the signal occupies a bandwidth in excess of the minimum necessary to send the information; the band spread is accomplished by means of a code that is independent of the data, and a synchronized reception with the code at the receiver is used for deseeding and subsequent data recovery.

Direct sequence spread spectrum (DS-SS) that is employed in the IS-95 standard is discussed in more details below. There exists another form of spread spectrum is called frequency-hopping spread spectrum (FH-SS) where the carrier frequency of the signal is moved (hopped) around in the band in a pseudorandom fashion.

In direct sequence code division multiple access (DS-CDMA) spreading codes like Walsh codes and Pseudo-Noise (PN) sequence are used for channelization of users in forward and reverse link. In CDMA 2000 orthogonal Walsh codes are used in the forward synchronous link. The reverse asynchronous link utilizes semi-orthogonal partial long PN codes with variable offset and spreading gain. The reason for this is that when the DS-CDMA system can be guaranteed to be synchronous it is preferable to use orthogonal sequences (Walsh codes) for spreading. However these codes have larger cross correlation values when used in asynchronous environment where users are not time aligned [65],[66]. This is really why Walsh codes are used in forward link while PN codes are used in the reverse link. In CDMA-2000 the long PN code used in reverse link has a
length of $2^{12}-1=4.4 \times 10^{12}$ chips long and repeats after 41 days [67][68]. Each user generates its spreading code from a particular offset. The user data bits get spread with the segments of the running long PN sequence. The spreading of user bits with the PN code segments and the asynchronous channel leads to Multiple Access Interference (MAI) in the reverse link.

User data is multiplied with the spreading codes after encoding and interleaving. The spectrally expanded user’s data is added and transmitted on to the channel. The channel induces noise, interferences in the transmitted signal. The received signal is despread using the corresponding codes to retrieve the original transmitted user data.

Consider a DS-CDMA system (reverse link) consisting of $N$ users transmitting BPSK signals to a single receiver. The individual user data is spread using a segment of the long PN code generated with a particular offset. The data signals $d_1(t), d_2(t)$ up to $d_n(t)$ include the data symbols spread by the random sequences $P_1(t), P_2(t)$ up to $P_n(t)$. The spreading sequences are periodic extension of single periods as shown below.

$$P_i(t) = \sum_{M=1}^{N} C_{im} P(t - (m-1)T_c)$$  \hspace{1cm} (2.32)

Where $C_{im}$ is the $m^{th}$ chip of the spreading sequence, $N$ is the spreading gain, $P(t)$ is the chip pulse shape that is assumed to be rectangular for our simulations and $T_c$ is the chip duration.

The channel introduces delays $\tau_1, \tau_2$ to $\tau_n$ to signals from different users. The signals also undergo fading before they are summed up and received by the system receiver. The received signal $r(t)$ can be written as.

$$r(t) = \sum A_k (t-\tau) P_k (t-\tau) d_k (t-\tau) + n(t)$$  \hspace{1cm} (2.33)

Where $A_k$, $P_k$ and $d_k$ are the delayed amplitude, signature code waveform and data signal of the $k^{th}$ user respectively, $n(t)$ is the Additive White Gaussian Noise (AWGN).

The advantage is CDMA is frequency reuse and its ability to mitigate multipath distortion. It employs Rake receiver to takes advantage of multipath effects in wireless channels. However needs careful power control is needed to avoid near-far problem.
2.3.3.5  Orthogonal Frequency Division Multiplexing

Frequency division multiplexing (FDM) extends the concept of single carrier modulation by using multiple subcarriers within the same single channel. The total data rate to be sent in the channel is divided between the various subcarriers. But guard bands are generally introduced in between, so that the spectrum of subcarriers do not interfere with each other. OFDM further extends the same concept and in this case subcarriers are kept orthogonal to each other. The use of orthogonal subcarriers would allow the subcarriers’ spectra to overlap, thus increasing the spectral efficiency. As long as orthogonality is maintained, it is still possible to recover the individual subcarriers’ signals despite their overlapping spectrums. Due to its robustness in environments affected by high interference and multipath and to its spectral efficiency, orthogonal frequency-division multiplexing (OFDM) is considered as an effective modulation technique for high-speed digital transmissions.

OFDM is a multi-carrier modulation scheme where the carriers are orthogonal to each other. A single stream of digital data is split into several parallel streams of low data rate, each of the parallel stream then rides a carrier frequency within the original bandwidth such that all the carriers remain orthogonal to each other. The carriers are multiplexed to form a single OFDM carrier. It is also known as Multi-carrier modulation (MCM) or Discrete Multi Tone modulation (DMT).

The main concept of OFDM is the orthogonality of sub-carriers. Since the carriers are all sine/cosine waves, we know that the area under one period of a sine or a cosine is zero. If we take a sine wave of frequency \( m \) and multiply it by a sinusoid (sine or cosine) of a frequency \( n \), where both \( m \) and \( n \) are integers, the area under the product remains zero.

In general for all integers \( n \) and \( m \), \( \sin mx, \sin nx, \cos nx \) and \( \cos mx \) are all orthogonal to each other. These frequencies are called harmonics. This is a key idea in the concept of OFDM. The orthogonality allows simultaneous transmission on a lot of sub-carriers in a tight frequency space without interference from each other.

The binary data stream to be transmitted is first forward error correction (FEC) encoded. Its purpose is to detect and correct any error in the data at the receiver end, caused by channel impairment. OFDM generally employs convolutional encoding for FEC. The coded version of OFDM is also called Coded OFDM (COFDM). The data stream is then demultiplexed into \( N \) parallel data streams. Each of the parallel data stream has a bit rate which is \( 1/N \) of the bit rate of the initial data stream. Each of the parallel data stream modulates a sub-carrier using QPSK, QAM
or any other digital modulation scheme. All the (N) sub-carriers are orthogonal to each other as discussed above. In OFDM, sinc-shaped pulses are used as subcarrier spectra. Zero crossings of sinc-pulses, are located at the multiples of 1/T (T is the symbol interval) as shown in Fig 2.8. Subcarrier orthogonality is maintained by selecting these frequencies according to equation (2.34).

\[ f_i = i/T \quad i = 0,1,\ldots,N \]  

(2.34)

![Fig 2.7 Orthogonal sub-carriers spectra](image)

The modulated orthogonal sub-carriers are combined together to form a composite OFDM signal. This is done by taking Inverse Discrete Fourier Transform (IDFT) of the samples. The IDFT transforms a set of samples in the frequency domain into an equivalent set of samples in the time domain. The DFT performs the reverse operation at the receiver end. This transform pair could be expressed mathematically as:

\[ DFT : b_n = \sum_{k=0}^{N-1} B_k e^{-j2\pi kn/N} \quad n = 0,1,\ldots,N-1 \]  

(2.35)

\[ IDFT : B_k = \frac{1}{N} \sum_{n=0}^{N-1} b_n e^{j2\pi kn/N} \quad k = 0,1,\ldots,N-1 \]  

(2.36)

where the sequences \( b_n \) and \( B_k \) are the frequency domain and time domain samples respectively. In practical implementation the above process is performed by an efficient (digital signal processing) computational tool called the Fast Fourier Transform / Inverse Fast Fourier Transform (FFT/IFFT) pair. For FFT/IFFT implementation N should be a power of 2. Direct computation of N point DFT requires \( N^2 \) iterations, whereas a typical FFT/IFFT algorithm can do the same calculation in \( (3/2 \times N) \log N \) iterations (Cooley and Tukey algorithm). The basic strategy that is
used in the FFT algorithm is one of “divide and conquer,” which involves decomposing an N-point DFT into successively smaller DFT’s. Forward FFT takes a random signal, multiplies it successively by complex exponentials over the range of frequencies, sums each product and plots the results as a coefficient of that frequency. The coefficients are called a spectrum and represent how much of that frequency is present in the composite signal. IFFT quickly computes the time domain signal instead of having to do it one carrier at a time and then adding. In OFDM the input of the IFFT is also a time domain signal instead of a frequency domain signal. But IFFT being a mathematical concept treats the incoming block of bits as a spectrum, and then produces the correct time domain result. The OFDM signal can be represented mathematically as in (2.37) which is basically an equation for IFFT.

\[
s(k) = \sum_{k=0}^{N-1} b_n e^{j2\pi kn/N} \quad k = 0, 1, ..., N - 1
\]

The output of the IFFT consists of, N time samples of a complex envelope for each period T, which are parallel to serial converted and then finally digital to analog converted to facilitate transmission of the incoming data stream over the wireless channel. The basic block diagram of OFDM transmitter and Receiver module is given in Fig 2.9 and Fig 2.10 below.

**Fig 2.8** Block diagram of OFDM transmitter.

**Fig 2.9** Block diagram of OFDM receiver.
Chapter 2 Multipath Fading Channels and Modulation Techniques

2.4 Diversity

Multipath fading in a wireless channel can cause severe degradation in the performance of a communication system. In order to mitigate the effects of fading, diversity techniques are used to improve the reliability of the transmission without increasing the transmitted power or sacrificing the bandwidth [44]. The motivation behind the diversity is that, if two or more independent samples of the signal samples are taken, these samples will fade in an uncorrelated manner [15] i.e. some samples are severely faded while others are less attenuated so if one radio path undergoes deep fade, another independent path may have a strong signal. These techniques are commonly classified as spatial diversity, polarization diversity, time diversity, frequency diversity. Each of the diversity techniques is discussed briefly.

2.4.1 Diversity Techniques

Diversity techniques such as Spatial, Time, Polarization and Frequency diversity techniques are elaborated below:-

2.4.1.1 Spatial Diversity

Performance in multipath fading environment can be improved if the transmission and reception of information is carried out through multiple antennas placed at appropriate distances in space and suitable signal processing done that combines the received signals from the various antennas. Typically a distance of few wavelengths between the antennas is sufficient. Spatial diversity is also called antenna diversity. The spatial diversity can be achieved by using multiple antennas at the transmitter and receiver. If multiple antennas are used at the receiver to collect independently faded copies of the transmitted signal then it is called receive diversity. If multiple antennas are employed at the transmitter then it is called transmit diversity. The spatial diversity does not introduce loss in the bandwidth efficiency which is very striking feature for high data rate wireless communication systems.

2.4.1.2 Polarization Diversity

Orthogonally polarized waves with independent fading characteristics can be used as a source of diversity. Such techniques are called polarization diversity techniques. In urban environments where space is limited, polarization diversity is particularly advantageous as we can place antennas
together. It however provides only two diversity branches. In environments with a number of reflections, the polarization may ultimately be lost and this technique may no longer be useful. Line of sight communications can use polarization diversity to better its performance.

2.4.1.3 Frequency Diversity
If data is sent on carriers sufficiently spaced apart in frequency, fading can be considered to be independent. We define coherence bandwidth of a channel as [44]
“Coherence bandwidth is the range of frequencies over which two frequency components have strong amplitude correlation”
Based on this definition we can have different measures of coherence bandwidth depending on how high a value of correlation we may want to use. Thus we may say that if data is sent on carriers that are spaced by more than the coherence bandwidth of the channel we can utilize frequency diversity.

2.4.1.4 Time Diversity
If the transmitted signal is repeated in time and the interval of time between repetitions is large enough for the channel characteristics to change, the received signals can undergo independent fading. We define coherence time of the channel as [44]
“Coherence time is the statistical measure of time during which the two received signals have strong amplitude correlation”
Based on this definition we can have different measures of coherence time depending on how high a value of correlation we may want to use. Thus we may say that if data is repeated in time with time duration greater than the coherence time of the channel we can utilize time diversity. If the data rate is fixed, higher Doppler will provide greater time diversity though not necessarily better signal as higher Doppler means deeper fades.

In some practical wireless communication systems, two or more of the above mentioned diversity techniques are combined to provide multidimensional diversity in order to meet the specified system performance requirements. For example, in GSM cellular systems multiple receive antennas at the base stations are used in conjunction with interleaving and error control coding to simultaneously exploit space and time diversity.
2.4.2 Diversity Combining Methods

The main characteristic of all the diversity techniques is an improvement in the error rate performance, or stated in other words, there is a very low probability of uneven deep fades. Generally, the performance of communication system employing diversity techniques highly depends upon how the different copies of same signal are combined at the receiver to maximize the overall received signal-to-noise ratio (SNR). The three most common space diversity techniques employed at the receiver for combining the received signal replicas are selection combining, maximal ratio combining, and equal gain combining. A short summary of each one of the diversity combining methods is presented below.

2.4.2.1 Selection Combining

In selection combining (SC) multiple receive antennas are placed at large enough distances so that fading is independent in each receive antenna as shown in Fig. 2.11. The best signal from all the receive antennas is selected for detection.

![Selection Combining Method](image)

Fig 2.10: Selection Combining Method

2.4.2.2 Maximal Ratio Combining

In maximal ratio combining (MRC), the channel is estimated and the signals received by each receive antenna are weighed according to the estimates so as to maximize the SNR. Fig 2.12 shows a block diagram of a maximum ratio combining diversity. The output signal from the combiner is a linear combination of a weighted replica of all the received signals. It is given by

\[
\text{Output} = \sum_{i=1}^{n_R} w_i r_i
\]

where \( w_i \) is the weight assigned to the \( i \)-th receive signal, and \( r_i \) is the received signal at the \( i \)-th antenna.
where \( r_i \) is the received signal at receive antenna \( i \), and \( \alpha_i \) is the weighing factor for receive antenna \( i \) and is dependent upon the channel estimates in maximum ratio combining.

This method requires perfect channel state information (CSI). Hence, it can be used in conjunction with coherent detection only.

### 2.4.2.3 Equal Gain Combining

In equal gain combining (EGC), the weights do not depend on the channel estimates. Signals from each receive antenna are multiplied by the same weight so as to give a lower SNR performance when compared to MRC. Even though the performance for EGC is lower than for MRC, no channel estimation needs to be done in EGC.
2.5 Summary

The chapter built a foundation for the subsequent chapters and discusses relatively basic concepts regarding the channel characterization, modulation methods and diversity techniques. Small scale and large scale manifestation are explained and multipath effect is discussed. Introduction to CDMA and OFDM is presented. Antenna diversity techniques are explained. Using diversity techniques to combat degradation caused by multipath fading is highlighted. The next chapter focuses on coded and un-coded multi-antenna communication systems.
Multiple Antenna Communication Systems

3.1 Introduction

Systems with multiple antennas at the transmitter and receiver referred as multiple- input multiple-output (MIMO) systems are used to increase data rates through multiplexing or to improve BER performance through diversity. In MIMO systems transmit and receive antennas can be employed simultaneously to achieve spatial diversity gains. Multiplexing is obtained by exploiting the structure of the channel gain matrix to obtain independent transmission paths that can be used to send independent data. MIMO systems were proposed by Winters [69], Foschini [5], Gans [9], later Telatar [6][11] worked out the spectral efficiencies and performance for wireless systems with multiple transmit and receive antennas. These spectral efficiency gains often require channel state information (CSI) at the receiver, and at the transmitter. In addition to spectral efficiency gains, ISI and interference from other users can be reduced using smart antenna techniques. The cost of the performance enhancements obtained through MIMO techniques is the added cost of deploying multiple antennas, the space and power requirements of these extra antennas, and the added complexity required for multi-dimensional signal processing. In this chapter we examine the different uses for multiple antennas and find their performance advantages.

3.2 Narrowband MIMO

Consider a narrowband MIMO channel. A narrowband point-to-point communication system of $N_t$ transmit and $N_r$ receive antennas is shown in Fig 3.1 This system can be represented by the following discrete time model:
The MIMO system in (3.1) can be represented simply as:

\[ \mathbf{y} = \mathbf{Hx} + \mathbf{v} \quad (3.2) \]

\[
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_{Nr}
\end{bmatrix} =
\begin{bmatrix}
  h_{11} & h_{12} & \cdots & h_{1Nr} \\
  h_{21} & h_{22} & \cdots & h_{2Nr} \\
  \vdots & \vdots & \ddots & \vdots \\
  h_{Nr1} & h_{Nr2} & \cdots & h_{NrNr}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_{Nr}
\end{bmatrix} +
\begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_{Nr}
\end{bmatrix}
\]

(3.1)

Fig 3.1 \( N_t \times N_r \) MIMO Communications system model.

Here \( \mathbf{x} \) represents the \( N_t \)-dimensional transmitted symbol, \( \mathbf{v} \) is the \( N_r \)-dimensional noise vector. \( \mathbf{H} \) being the \( N_r \times N_t \) matrix of channel gains \( h_{ij} \), representing the gain from \( j^\text{th} \) transmit antenna to \( i^\text{th} \) receive antenna Complex Gaussian noise with zero mean and covariance matrix \( \sigma_n^2 \mathbf{I}_{N_r} \). \( \sigma_n^2 \) is \( N_0 B \), where \( B \) is the channel bandwidth. For a transmit power constraint \( P \) assume an equivalent model with a noise power of unity and transmit power \( P/\sigma_n^2 = \rho \). \( \rho \) is the average SNR per receive antenna under unity channel gain. The transmitted symbols must satisfy:

\[ \sum_{i=1}^{N_t} \mathbb{E}[x_i x_i^*] = \rho \quad (3.3) \]
Channel gain matrix $H$ can be assumed to be known at both transmitter and receiver; it is represented as channel side information at the transmitter (CSIT) and channel side information at the receiver (CSIR), respectively. For a static channel CSIR is typically assumed, since the channel gains can be obtained fairly easily by sending a pilot sequence for channel estimation. When the feedback path is present then CSIR from the receiver can be transferred to the transmitter to have CSIT. When the channel is not known some distribution on the channel gain matrix must be assumed. The most common model for this distribution is a zero-mean spatially white model, where the entries of $H$ are assumed as i.i.d. unit variance, zero mean, complex circularly symmetric Gaussian random variables.

Optimal detection of the transmitted symbol requires ML decoding, due to the cross-coupling between the possible transmitted symbols at the receivers. In a general MIMO matrix channel, when the transmitter do have a perfect CSI the exhaustive ML detection is prohibitive even for small number of transmitters [70]. Where as, the detection complexity can be reduced meaningfully if under perfect CSIT conditions which is discussed next section.

### 3.3 MIMO Channel Decomposition

Diversity gain can be achieved by using multiple antennas at the transmitter or receiver. Utilizing multiple antennas both at the transmitter and receiver end, results in multiplexing gain. The characteristics of the MIMO matrix channel to be decomposed into R parallel independent channels, enables multiplexing gain. The increase in the data rate due to this MIMO channel gain matrix is called multiplexing gain.

Consider a MIMO matrix channel with $N_r \times N_t$, channel gain matrix $H$ assumed to known perfectly at both transmitters and receivers. Singular Value Decomposition (SVD) can be done on $H$ as

$$H = U \Sigma V^H$$

(3.4)

When $R_H$ is the rank of $H$, $U$ and $V$ are unitary matrices with dimensions $N_r \times N_r$ and $N_t \times N_t$ respectively. $\Sigma$ is a $N_r \times N_t$ singular valued diagonal matrix of values $\sigma_i$ of $H$. For $i^{th}$ eigenvalue $\lambda_i$ of $HH^H$ these singular values have $\sigma_i = \sqrt{\lambda_i}$ and $R_H$ is non zero, $R_H \leq \min(N_r, N_t)$. In a
scenario once $\mathbf{H}$ is full ranked, $R_H = \min(N_t, N_r)$ the environment is sometimes referred to as rich scattering environment.

Parallel decomposition of MIMO channel is achieved by transmit precoding and receiver shaping. The input vector $\mathbf{x}$ is passed through a linear transformation to get $\mathbf{x} = \mathbf{V}^H \mathbf{\tilde{x}}$. The channel output $\mathbf{y}$ is multiplied with $\mathbf{U}^H$ to perform the receiver shaping. The transmitter precoding and receiver shaping operations are shown in Fig 3.2.

\[ \mathbf{y} = \mathbf{U}^H (\mathbf{H} \mathbf{x} + \mathbf{v}) \]

where $\mathbf{\tilde{y}} = \mathbf{U}^H \mathbf{v}$ and $\sum$ is a diagonal matrix of singular values of $\mathbf{H}$. The operation of transmit precoding and receiver shaping as shown above transforms the MIMO matrix channel into $\mathbf{R}_H$ parallel independent channels which do not interfere with each other, therefore we can term their gains as independent. As the parallel channels do not interfere the optimal ML detection complexity is linear in $\mathbf{R}_H$. In addition transmission of independent data through parallel channels, allow the MIMO channel to support data rate of SISO channel multiplied with the $\mathbf{R}_H$ times. Hence the multiplexing gain is $R_H$. 

\[ \mathbf{\hat{y}} = \mathbf{U}^H \mathbf{y} \]
3.4 MIMO Channel Capacity

Shannon capacity gives the maximum data rate possible that can be transmitted through the channel with a very small probability of bit error, is discussed for a MIMO matrix channel. Capacity versus outage defines the maximum rate which can be transmitted through the channel with some nonzero outage probability. Channel capacity is related to the knowledge of the channel gain matrix at the transmitter and/or receiver. Throughout this section it is assumed that the channel matrix $H$ is known at the receiver, since for static channels a good estimate of $H$ can be obtained fairly easily. First the static channel capacity will be explained, which forms the basis for the subsequent section on capacity of fading channels [70].

3.4.1 Static Channels

The capacity of a MIMO channel is an extension of the mutual information formula for a SISO channel [70] to a matrix channel. Specifically, the capacity is given in terms of the mutual information between $x$ the input vector and $y$ the output vector, represented as

$$C = \max_{p(x)} I(X;Y) = \max_{p(x)} [H(Y) - H(Y|X)]$$

(3.5)

for $H(Y)$ and $H(Y|X)$ is the entropy in $y$ and $y|x$ [70]. By the definition of entropy $H(Y|X) = H(V)$ which is the noise entropy. Maximizing the mutual information is similar to maximizing the entropy since the fixed entropy of noise $v$ is not dependent on channel.

The mutual information of $y$ depends on the covariance $\mathbf{R}_y = E[\mathbf{y}\mathbf{y}^H]$. 

$$\mathbf{R}_y = \mathbf{H}\mathbf{R}_x\mathbf{H}^H + \mathbf{I}_N$$

(3.6)

It can be seen that for all random vectors with covariance matrix $\mathbf{R}_y$, the entropy of $y$ is maximum once it is a zero-mean circularly-symmetric complex Gaussian (ZMCSG) random vector [5], this is true one the input $x$ is also ZMCSG. Therefore, the mutual information becomes [3,5,6,7].

$$I(X;Y) = B \log_2 \det[I_N + \mathbf{H}\mathbf{R}_x\mathbf{H}^H]$$

(3.7)

Maximizing the mutual information (3.7) over $\mathbf{R}_x$ which satisfy the power constraint yields the capacity of the MIMO channel as:

$$C = \max_{\mathbf{R}_x} B \log_2 \det[I_N + \mathbf{H}\mathbf{R}_x\mathbf{H}^H]$$

(3.8)
This optimization depends upon whether not \( \mathbf{H} \) is known at transmitter.

### 3.4.1.1 Perfect Channel Knowledge at Transmitter

Parallel decomposition of MIMO channel as discussed in section 3.2 assumes perfect channel knowledge at the transmitter and receiver. It is observed that the capacity is equal to the accumulated capacity of the parallel channels. Using (3.8) and assuming perfect channel knowledge MIMO channel capacity results as

\[
C = \max_{\rho_i \geq \rho, \sum_i \rho_i < \rho} \sum_i B \log_2 (1 + \sigma_i^2 \rho_i) \tag{3.9}
\]

Where \( \rho = P / \sigma_n^2 \). Above capacity can also be expressed in power allocation \( P_i \) of the \( i^{th} \) parallel channel as

\[
C = \max_{\rho_i \geq \rho, \sum_i \rho_i < \rho} \sum_i B \log_2 \left( 1 + \frac{P_i \gamma_i}{P} \right) \tag{3.10}
\]

\( \rho_i = P_i / \sigma_a^2 \) and \( \gamma_i = \sigma_i^2 P / \sigma_a^2 \) is SNR of the \( i^{th} \) parallel channel with full power. This results into a water filling power allocation MIMO channel

\[
\frac{P_i}{P} = \begin{cases} 
\frac{1}{\gamma_0} - \frac{1}{\gamma_i} & \gamma_i \geq \gamma_0 \\
0 & \gamma_i < \gamma_0
\end{cases} \tag{3.11}
\]

where \( \gamma_0 \) is cutoff value, The resulting capacity becomes

\[
C = \sum_{i: \gamma_i \geq \gamma_0} B \log \left( \frac{\gamma_i}{\gamma_0} \right) \tag{3.12}
\]

The MIMO capacity with channel matrix \( \mathbf{H} \) can also be expressed in terms if vector \( \mathbf{h} \) consisting of channel gains, and an optimal weight vector \( \mathbf{c} \) as

\[
C = B \log_2 (1 + \rho \mathbf{h} \mathbf{c}) \tag{3.13}
\]
where \( c = \frac{h^*}{\|b\|} \) and \( \rho = \frac{P}{\sigma_n^2} \).

3.4.1.2 Channel unknown at transmitter

In a case when the receiver only has channel knowledge the transmitter will not be able to optimize its covariance structure across the antennas at the input. When \( H \) follows a ZMSW gain the best strategy can be allocating equal power to each transmitter without any bias in terms of mean of \( H \), this results input covariance matrix as \( R_x = (\rho/M_t)I_{M_t} \). The mutual information of channel is maximized by the input covariance matrix [11]. The mutual information for \( N_t \) transmit antennas and \( N_r \) receive antennas is represented as

\[
I = B \log_2 \det[I_{M_t} + \frac{\rho}{M_t} HH^H] \tag{3.14}
\]

Above expression can also be rewritten using SVD as

\[
I = \sum_{i=1}^{R_H} B \log_2 \left( 1 + \frac{\gamma_i}{N_t} \right) \tag{3.15}
\]

where \( \gamma_i = \sigma_i^2 \rho \) and \( R_H \) is nonzero singular values of \( H \).

The singular values of channel \( H \) influence the mutual information in (3.15). The probability distribution of the singular values of channel \( H \) effect the average mutual information of a random matrix \( H \) [9], [11]. For static channels if the transmitter is ignorant of the channel’s average mutual information it cannot control data transmit rate to enable accurate reception of data. Under such scenarios channel capacity is more appropriately defined as outage capacity. In outage capacity the transmitter keeps the data transmission rate \( C_T \) as constant, the outage probability of \( C_T \) is the probability that channel \( H \) possess mutual information lesser than \( C_T \) expressed below

\[
p_{\text{outage}} = p \left( H : B \log_2 \det \left[ I_{N_r} + \frac{\rho}{N_t} HH^H \right] < C_T \right) \tag{3.16}
\]

When \( N_t \) and \( N_r \) increase singular values of \( H \) distribution can be defined using central limit theorem as a constant mutual information for all possible channel coefficients [14]. This constant
mutual information was used to derive MIMO channel capacity with uncorrelated fading channels in [71-74]. Similarly for correlated fading channels MIMO channel capacity is discussed in [75-77].

3.4.2 Fading Channels

Under flat-fading MIMO matrix channel gains $h_{ij}$ vary with time. The channel capacity is dependent on the knowledge of $H$ at the transmitter and receiver. With no CSI available at the transmitter outage capacity is used to describe the MIMO channel capacity under any arbitrary realization of $H$. A detailed account on channel capacity is given below.

3.4.2.1 Perfect Channel Knowledge at Transmitter

The average of the channel capacities related with each channel realization under perfect CSIT and CSIR is as represented in (3.8) with optimal power allocation and is termed as ergodic capacity. When the power of each channel realization is equal to the average power constraint $P$ the ergodic capacity is represented as

$$C = E_H \left[ \max_{\rho \sum_i \rho_i \leq \rho} \sum_i B \log_2 \left( 1 + \frac{P_i \rho_i}{\rho} \right) \right]$$

(3.17)

When there are different powers for different channel realizations under the condition that the average power constraint over all the channel realizations, the ergodic capacity now becomes

$$C = \max_{\rho_i} E_H \left[ \max_{\rho, \|H\| = \rho} B \log_2 \det[I_N + HH^H] \right]$$

(3.18)

3.4.2.2 Channel unknown at Transmitter

In the scenario when the channel is time-varying and the channel $H$ is not known at the transmitter but known at the receiver, a ZMSW distribution of $H$ is assumed at the transmitter. Here the capacity can be defined as ergodic capacity as and the outage capacity. Maximum rate transmission, averaged over all channel realizations based only on distribution of $H$ is regarded as ergodic capacity. It is quantified as

$$C = \max_{\rho_i} E_H \left[ B \log_2 \det[I_N + HH^H] \right]$$

(3.19)
For ZMSW model the transmit power is divided among $N_t$ and symbols are transmitted independently the ergodic capacity becomes

$$C = E_{ll} \left[ B \log_2 \det[I_{N_t} + \frac{P}{N_t} HH^H] \right] \text{bps / Hz}$$

(3.20)

Once the channel is not ZMSW the channel capacity is dependent on the singular values distribution for the random channel matrix which is analyzed in [13]. The ergodic capacity of a $N_t \times N_r$ MIMO communications system is represented in Fig. 3.3. Here the i.i.d complex Gaussian channels gains with CSIT and CSIR are assumed in these results. Fig 3.3 shows significant capacity gains with multiple antennas in comparison to single antenna system.

Fig 3.3 Ergodic capacity of a $N_t \times N_r$ MIMO systems with CSIR and no CSIT

It can be shown that when both transmitter and receivers increase, the capacity grows linearly with $\min(N_t, N_r)$. If receivers are increased and transmitters are kept fixed, the capacity increases logarithmically with receivers. However, if the receivers are fixed while the number of transmitters increases, the capacity saturates at some fixed value. Ergodic capacity has the multiplexing gain of $\min(N_t, N_r)$, this means that each 3-dB SNR leads to an increase of $\min(N_t, N_r)$ bps/Hz in spectral
efficiency. Plots of the Ergodic capacity in Figure 3.3 for 1x1, 2x2, 4x4, etc cases are shown in Figure 3.3. A linear increase of capacity with respect to SNR is observed till 5-dB. 1x1 system has a slope of 1 bit/3-dB, while 4x4 system has a slope of 4bits/3-dB.

3.4.2.3 Channel unknown at Transmitter and Receiver

When the CSI is neither at transmitter nor at receiver the capacity increase with number of transmit, receive antennas is no more observed. In [79] it is discussed that in fading channels when channel is not known both at transmitters and receivers, data rate is not increased with the increase in the number of transmit antennas. Whereas, under correlated fading conditions the increase in transmitters does affect the channel capacity positively [80].

3.5 Space-Time Coding

The MIMO system is constructed with multiple element array antennas at both ends of the wireless link. Space-time coding is related to practical signal design approaches that aim at reaching MIMO channels information theoretic capacity limits. The fundamentals of space-time block coding have been established by Tarokh et al. in 1998 [15]. Space-time coding and related MIMO signal processing has evolved into a most vibrant research topic in wireless communications.

Space time coding introduces joint correlation in transmitted signals in both the temporal and spatial domains. The spatial-temporal correlation is used to exploit the MIMO channel fading and minimize the transmission errors at the receiver, as well as to gain high spectral efficiency.

This chapter starts with defining a generalized structure of the matrix for O-STBC. Then a simple space-time coded system is discussed and simulation results for its BER performance are provided. Finally some other high rate space-time block codes are presented.

3.5.1 General Matrix for Orthogonal Space-Time Block Codes

We have $k$ symbols $s_1, s_2, \ldots, s_k$ and their conjugates. These symbols are chosen from a constellation having $2^b$ constellation points where $b$ is the number of bits in each symbol. The amount of information that is transmitted in the space time block encoder is $k \times b$.

Let us suppose that number of transmitter antennas is $p$ and the number of time slots is $n$, then the general form of the matrix for space time block code will be
Following are the notable properties of the above matrix

a. Any entry $g_{ij}$ in the matrix above is a linear combination of symbols $s_1, s_2, \ldots, s_k$ and their conjugates.

b. Entries in a row are transmitted by transmit antennas $T_{x_1}, T_{x_2}, \ldots, T_{x_p}$ simultaneously.

c. Entries in a column are transmitted by the same transmit antenna in different time slots.

d. The columns of the matrix $G_p$ are orthogonal to each other.

Thus encoding has been done both in space and time, where space is running horizontally and time is running vertically in the above matrix. Since $k$ symbols are transmitted over $n$ time slots, the space-time block code rate is $R = k / n$. If $B$ is the available bandwidth for transmission, then the bandwidth efficiency is defined as $\rho = R / B$.

Let’s take an example of Alamouti’s space-time block code given by the matrix (i.e. $p=2$)

$$
G_2 = \begin{bmatrix}
  s_1 & s_2 \\
  -s_2^* & s_1^*
\end{bmatrix}
$$

(Note that the columns in the above matrix are orthogonal to each other)

Number of symbols to be transmitted = $k = 2$ (i.e. $[s_1 \; s_2]$). Number of time slots = $n = 2$ and code rate is, $R = k / n$.

Block diagram of a space-time coded system is shown in Fig 3.4. Information bits are first modulated by the chosen modulation scheme. The resulting symbols are then sent to different antennas for transmission over the channel. Finally, the transmitted signal propagates through the channel and reaches at the receiver.
3.5.2 A Simple Space-Time Block Coded System

In this section, the simplest of all Alamouti’s space-time block code scheme is presented. The system model is developed first and then the maximum likelihood decoding of O-STBC over quasi-static fading channel is presented.

3.5.2.1 The Transmission Model

We start with a wireless communication system having with two transmit antennas and one receive antenna as shown in Fig 3.5. The information bit sequence \( \{b(n)\} \) is first modulated, by a signal constellation \( \Omega \) of size \( |\Omega|=2^b \), into a complex symbol sequence \( \{s(n)\} \) having symbol duration \( T_s \) and symbol energy \( E_s \). The complex sequence \( \{s(n)\} \) is then parsed into code vectors \( s = [s_1 \ s_2] \) which is then transmitted according to the Alamouti’s O-STBC scheme with the following code matrix

\[
C = \begin{bmatrix}
  s_1 & s_2 \\
  -s_2^* & s_1^*
\end{bmatrix}
\]

The first row of \( C \) is transmitted by the transmit antennas \( T_1 \) an \( T_2 \) at time slot ‘t’ and the second row is transmitted at time ‘\( t+T_s \)’.
The received signal from transmitter ‘j’ at time ‘i’ is

\[ r_i = \sum_{j=1}^{2} h_j(i)c_{ij} + n_i \]  

(3.24)

The channel gains \( h_j(i) \) are modeled as independent, identically distributed (i.i.d.) complex Gaussian random variables having zero mean and variance \( \sigma_h^2 = 1 \). From, the following signals received at two time instants can be retrieved.

At time \( i=1 \), the received signal is given by

\[ r_1 = h_1(1)s_1 + h_2(1)s_2 + n_1 \]  

(3.25)

At time \( i=2 \), the received signal is given by

\[ r_2 = -h_1(2)s_1^* + h_2(2)s_2^* + n_2 \]  

(3.26)

By taking conjugate of \( r \) we can again write the above two equations as

\[ r_1 = h_1(1)s_1 + h_2(1)s_2 + n_1 \]  

(3.27)

\[ r_2^* = h_2^*(2)s_1 - h_1^*(2)s_2 + n_2^* \]  

(3.28)

or written in matrix form as
\[ \bar{\mathbf{r}} = \mathbf{H} \bar{s} + \bar{n} \]  

where

\[ \bar{r} = \begin{bmatrix} r_1 & r_2^* \end{bmatrix}^T \] is the received vector, \( \bar{s} = \begin{bmatrix} s_1 & s_2 \end{bmatrix}^T \) is the transmitted code vector and

\[ \mathbf{H} = \begin{bmatrix} h_1(1) & h_2(1) \\ h_2^*(2) & -h_1^*(2) \end{bmatrix} \] is the channel matrix.

### 3.5.2.2 ML Decoding of O-STBC over Quasi-Static Channel

The optimum decoder for the received signals given in (3.27) and (3.28) is the ML decoder that selects code vector \( \hat{s} = \begin{bmatrix} \hat{s}_1 & \hat{s}_2 \end{bmatrix}^T \) according to

\[ \hat{s} = \arg \min_s \| \bar{r} - \mathbf{H}s \|^2 \]  

where the search in (3.30) is performed over all possible code vectors \( s = \begin{bmatrix} s_1 & s_2 \end{bmatrix}^T \) with \( s_1, s_2 \in \Omega \).

Now we will consider two separate scenarios for the decoding of the received signals using the decoding metric given in (3.30).

### 3.5.2.3 Quasi-Static Channel

Let us first assume that the channel is quasi-static. In such a scenario the channel gains \( h_i(i) \) remains constant over entire codeword length i.e. \( 2T_s \)

Or

\[ h_1(1) = h_1(2) \\
 h_2(1) = h_2(2) \]  

Under these conditions the channel matrix \( \mathbf{H} \) given above can be simplified to

\[ \mathbf{H} = \begin{bmatrix} h_1(1) & h_2(1) \\ h_2^*(1) & -h_1^*(1) \end{bmatrix} \]  

Now the following property of \( \mathbf{H} \) is notable
Chapter 3 Multi Antenna Communication Systems

\[ H^H H = \begin{bmatrix} h_1^*(1) & h_2^*(1) \\ h_2^*(1) & -h_1^*(1) \end{bmatrix} \begin{bmatrix} h_1(1) & h_2(1) \\ h_2^*(1) & -h_1^*(1) \end{bmatrix} \]

\[ = \left( |h_1(1)|^2 + |h_2(1)|^2 \right) I_2 \]

(3.33)

which depicts the orthogonality of the channel matrix. Based on this fact we can use (3.30) and (3.33) to compute the decision statistic vector \( \hat{r} = \begin{bmatrix} \hat{r}_1 \\ \hat{r}_2 \end{bmatrix} \)

as

\[ \hat{r} = H^H \bar{r} = \left( |h_1(1)|^2 + |h_2(1)|^2 \right) s + \hat{n} \]

(3.34)

where \( \hat{n} \) is still a zero mean vector of Gaussian random variables. Compared to (3.29), it can be seen clearly from (3.34) that the symbols \( s_1 \) and \( s_2 \) are decoupled from each other. Using (3.34) in the ML decoding metric given in (3.30) will yield the optimum results.

Fig 3.6 illustrates the performance of space-time block coding scheme in flat Rayleigh faded, quasi-static channel for the case of one, two, and three transmit antennas. BPSK modulation scheme is used for the two transmit antenna system and QPSK modulation scheme is used for the three transmit antenna system. The receiver can perfectly estimate the set of channel coefficients. Fading is assumed to be uncorrelated. The energy of symbols from each transmit antenna is assumed to be unity.

![Fig 3.6: BER performance of STBC with 1,2 and 3 transmit antennas](image)

Fig 3.6: BER performance of STBC with 1,2 and 3 transmit antennas
3.5.3 Other Space-Time Block Codes

Invoking the theory of orthogonal codes, space-time block codes with more than two transmit antennas have been built [9]. Following are the STBC matrices for 3 and 4 transmit antennas

\[
G_3 = \begin{bmatrix}
  s_1 & s_2 & s_3 \\
  -s_2 & s_1 & -s_4 \\
  -s_3 & s_4 & s_1 \\
  -s_4 & -s_3 & s_2 \\
  s_1^* & s_2^* & s_3^* \\
  -s_2^* & s_1^* & -s_4^* \\
  -s_3^* & s_4^* & s_1^* \\
  -s_4^* & -s_3^* & s_2^* \\
\end{bmatrix}, \quad G_4 = \begin{bmatrix}
  s_1 & s_2 & s_3 & s_4 \\
  -s_2 & s_1 & -s_4 & s_3 \\
  -s_3 & s_4 & s_1 & -s_2 \\
  -s_4 & -s_3 & s_2 & s_1 \\
  s_1^* & s_2^* & s_3^* & s_4^* \\
  -s_2^* & s_1^* & -s_4^* & s_3^* \\
  -s_3^* & s_4^* & s_1^* & -s_2^* \\
  -s_4^* & -s_3^* & s_2^* & s_1^* \\
\end{bmatrix}
\]

Following are the notable properties of the above systems

a. With \(G_3\) and \(G_4\), the code rate and bandwidth efficiency is reduced to half as compared to the space time block code \(G_2\).

b. The number of time slots across which the channel gains are required to have a constant fading envelop (flat fading) is 8, which is higher than that of space time code \(G_2\) by a factor of 4.

To further enhance the bandwidth efficiency, Tarokh constructed the rate \(\frac{3}{4} (k/n)\) space-time block codes with 3 and 4 transmitter antennas [9]. The high rate STBC for three and four antennas are

\[
H_3 = \begin{bmatrix}
  s_1 & s_2 & s_3 & \frac{s_3}{\sqrt{2}} \\
  -s_2^* & s_1^* & s_3^* & \frac{s_3}{\sqrt{2}} \\
  s_3^* & s_3^* & \frac{s_3}{\sqrt{2}} & -s_3-s_1^*+s_2-s_2^* \\
  \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} & s_2+s_2^*+s_1-s_1^* & 2 \\
\end{bmatrix}
\]
Chapter 3 Multi Antenna Communication Systems

\[ \mathcal{H}_t = \begin{bmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_1}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} \\ s_2^* & s_1^* & -s_1 - s_1^* + s_2 - s_2^* & -s_2 - s_2^* + s_1 - s_1^* \\ \sqrt{2} & \sqrt{2} & 2 & 2 \\ \sqrt{2} & -\sqrt{2} & 2 & -s_1^* - s_1 + s_2 + s_2^* \end{bmatrix} \]

A brief summary of the properties of all the above codes is presented in the Table 3.1.

<table>
<thead>
<tr>
<th>STBC</th>
<th># of Transmit Antennas</th>
<th># of Input Symbols</th>
<th>Code Span</th>
<th>Code Rate $(R=k/n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_2$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$G_3$</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$G_4$</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$H_3$</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>$H_4$</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>$\frac{3}{4}$</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of Space-Time Block Codes

3.5.4 Spatial Multiplexing and BLAST Architecture

In Spatial Multiplexing $N_t$ independent symbols in each symbol period are transmitted using space and time dimensions. To achieve complete available diversity the encoded bit stream should be transmitted over all $N_t$ transmitters achieved using serial encoding shown in Fig 3.7. In serial encoding the bit stream is temporally encoded encoded over the channel blocklength $T$, interleaved, mapped to a symbol in the constellation and then demultiplexed to different transmitting antennas $N_t$. 

53
Another method to achieve spatial multiplexing introduced at Bell Laboratories is Bell Labs Layered Space Time (BLAST) architecture for MIMO channels [5]. It employs parallel encoding as illustrated in Fig 3.8.
User data bits are demultiplexed into $N_t$ transmit antennas as independent streams. Each stream is temporal encoded, interleaved, mapped to constellation alphabets and transmitted through its corresponding transmitter. This architecture of converting the user data bit streams into vertical vectors is termed as V-BLAST [5]. This system can achieve a diversity order of $N_t$ because each transmitted symbol is received by $N_r$ transmitters. V-BLAST has a linear encoding complexity in terms of number of antennas. Whereas, optimal decoding performs joint symbols detection which is exponential in complexity, it has been studied in [81] that the symbol detection complexity can be meaningfully reduced by utilizing symbol interference cancellation method as shown in Fig 3.9. The symbol interference cancellation technique firstly orders the $N_t$ transmitted symbols with decreasing SNR, estimate of the symbol with highest SNR is performed assuming it to be correctly received. This process is repeated to detect all the transmitted symbols. This algorithm is explained in more details in the next chapter. An interesting combination that utilizes the parallel encoding and diversity of serial encoding is known as D-BLAST [5].

### 3.6 Frequency-Selective MIMO Channels

When the MIMO channel bandwidth is large in comparison to the channel’s multipath delay spread, the channel experiences ISI, similar to SISO channels. Equalization can be used to mitigate the effects of ISI. However, the equalizer is much complex in MIMO channels since the channel must be equalized over both space and time. Moreover, when the equalizer is used in conjunction with a space-time code, the nonlinear and noncausal nature of the code further
complicates the equalizer design. In some cases the structure of the code can be used to convert the MIMO equalization problem to a SISO problem for which well-established SISO equalizer designs can be used [82-84].

An alternative to equalization in frequency-selective fading is multicarrier modulation or orthogonal frequency division multiplexing (OFDM). OFDM technique converts the wideband channel into a set of narrowband sub channels that only exhibit flat fading. Applying OFDM to MIMO channels results in a set of narrowband MIMO channels, and the space-time modulation and coding techniques described above for a single MIMO channel are applied to the parallel set.

### 3.7 Summary

In this chapter we discusses the theory of multiple antenna communication systems. Coded and un-coded MIMO architectures were explained. First an un-coded MIMO communication system model is formulated along with its channel capacity. Increase in system capacity using multiple antennas is highlighted. Later, Alamouti and Tarokh OSTBC systems for coded MIMO are introduced. Simplified ML detection for the coded MIMO systems having linear complexity in Quasi-static channels is also discussed. The issue of symbol detection in coded and un-coded MIMO systems which is discussed next.
Chapter 4

Symbol Detection in MIMO System

4.1 Introduction

In the previous chapter we discussed that using multiple antennas at the transmitter and receiver enable significant increase in the spectral efficiency and the reliability of a mobile radio channel without increasing the system transmission power or bandwidth. However, developing a cost effective MIMO system needs considerable amount of effort. The bottle neck is designing low computational complexity and efficient receivers that can fully exploit the benefits of the MIMO architecture without taking a long time to decode the transmitted symbols. In this chapter the Optimal Maximum-Likelihood MIMO detection problem is formulated and a survey of existing detection techniques which reduce the ML complexity for MIMO systems in the un-coded spatial multiplexing system is presented.

4.2 MIMO System Model

Consider a MIMO system as shown in Fig 3.1 where \( N_t \) different signals are transmitted and arrive at an array of \( N_r \) \((N_r \leq N_t)\) receivers via a rich-scattering flat-fading environment. Grouping all the transmitted and received signals into vectors, the system can be viewed as transmitting an \( N_r \times 1 \) vector signal \( x \) through an \( N_r \times N_r \) matrix channel \( H \), with \( N_r \times 1 \) Gaussian noise vector \( v \) added at the input of the receiver as depicted in (3.2)

\[
y = Hx + v
\]

(4.1)
where $y$ is the received $N_r \times 1$ vector. The $(n_r, n_t)^{th}$ element of $H$, $h_{n_r, n_t}$, is the complex channel response from the $n_t^{th}$ transmit antenna to the $n_r^{th}$ receive antenna. $x$ is zero mean and has covariance matrix of $R_x = E\{xx^*\} = \sigma_x^2 I$. The vector $v$ is also zero-mean and $R_v = E\{vv^*\} = \sigma_v^2 I$. The entries of channel matrix $H$ are assumed to known at the receiver but not at the transmitter. If training or pilot signals are sent to get the channel information this assumption is reasonable. Channel parameters are constant for some coherent interval.

### 4.3 MIMO Detection Problem Formulation

The task is that of detecting $N_t$ transmitted symbols from a set of $N_r$ observed symbols that have passed through a non-ideal communication channel, typically modeled as a linear system followed by an AWGN as shown in Fig 4.1.

Transmitted symbols from a known finite alphabet $\chi = \{x_1, \ldots, x_M\}$ of size $M$ are passed to the channel. The detector chooses one of the $M^{N_t}$ possible transmitted symbol vectors from the available data. Assuming that the symbol vectors $x \in \chi^{N_t}$ are equiprobable, the Maximum Likelihood (ML) detector always returns an optimal solution according to the following:

$$\hat{x} = \arg \max_{x \in \chi^{N_t}} P(y \text{ is observed} | x \text{ was sent})$$

Assuming the additive noise $v$ to be white and Gaussian, the ML detection problem of Fig 4.1 can be expressed as the minimization of the squared Euclidean distance to a target vector $y$ over $N_r$-dimensional finite discrete search set.
Chapter 4 Symbol Detection in MIMO System

\[ x_\star = \arg \min_{x \in \mathcal{X}} \| y - Hx \| \tag{4.3} \]

Optimal ML detection scheme needs to examine all \( M^{N_t} \) or \( 2^{bN_t} \) symbol combinations (\( b \) is the number of bits per symbol). The problem can be solved by enumerating over all possible \( x \) and finding the one that causes the minimum value as in (4.3).

In ML detection, optimization is performed over the space of all possible vectors \( x \). Since the search space is discrete with \( x \) having integer components, this problem is posed in the literature as an integer least-squares optimization problem [85], and it belongs to the class of nondeterministic polynomial-time hard, NP-hard, combinatorial optimization problems [86, 87].

A combinatorial optimization (CO) problem involves searching values for discrete variables in such a way an optimal solution with respect to a selected objective function is detected. A straightforward approach to the solution of a CO problem would be exhaustive search, i.e. the enumeration of all possible solutions and choosing the one that minimizes the objective function in equation (4.3). A naive implementation of this search strategy results in a prohibitive complexity, as the number of candidate solutions increases exponentially with the problem size. Therefore, for a \( N_t \times N_r \) MIMO system with symbols from M-QAM constellation alphabet the computational complexity increases exponentially with constellation size \( M \) and number of transmitters \( N_t \) as can be observed from (4.3).

This work focuses on designing MIMO detection algorithms capable of finding a near optimal solution with lesser than ML computational complexity. These will be low complexity near optimal uncoded MIMO detectors.

4.4 Existing MIMO Detection Algorithms

Two classes of algorithms are available for the solution of combinatorial optimization problems: exact and approximate algorithms. Exact algorithms find the optimal solution for every finite size of a combinatorial optimization problem; however, for NP-hard problems, exact algorithms have an exponential worst-case complexity, and they generally suffer from a strong rise in computation time when the problem size increases.

Approximate algorithms, on the other hand, trade optimality for efficiency; they exploit some problem-specific knowledge to produce reasonable solutions at a comparatively low computational complexity with no surety to produce optimal solutions.
In the following, we will briefly review some of the exact and approximate algorithms that have been used to solve the MIMO detection problem.

### 4.4.1 Sphere Decoder

Fincke and Pohst introduced the Sphere Decoder (SD) algorithm [88] as an exact MIMO detection algorithm which gives ML BER performance [89]. Transmitted symbols $\mathbf{x}$ vector are represented as points on a rectangular integer grid $N_t$-dimensional lattice. The MIMO channel $\mathbf{H}$ is assumed as a lattice generating matrix. $\mathbf{Hx}$ which is $N_r$-dimensional spans the transformed skewed lattice. Hence considering a received vector $\mathbf{y}$ at $N_r$ receivers and $\mathbf{Hx}$ the transformed lattice, ML detection problem reduces to determining the nearest lattice point to $\mathbf{y}$ in terms of minimum Euclidean distance. The basic concept of SD is to search for the lattice points within a radius $R$ around the received vector $\mathbf{y}$, instead of traversing the complete search space of the lattice. The SD searches the point that gives the minimum Euclidean distance within the hypersphere with radius $R$, as shown in Fig 4.3.

![Fig 4.2: Sphere Decoder algorithms illustration.](image)

The resultant point in hypersphere that are closest $\mathbf{y}$ is the solution returned by the SD algorithm. The issue of selecting the radius $R$ and finding the nodes that are with this radius actually determine the efficiency of SD algorithm.

SD is a type of Branch and Bound tree search algorithms [89]. Applying QR factorization on $\mathbf{H}$ matrix channel enables converting it into tree structure inherent to it. A product of unitary matrix $\mathbf{Q}$ and upper triangular matrix $\mathbf{R}$ is substituted in place of $\mathbf{H}$ in (4.3). The resultant expression is multiplied with $\mathbf{Q}^T$ to yield the following equivalent problem representation:
Chapter 4 Symbol Detection in MIMO System

$$x^*_s = \arg \min_{x \in \mathcal{X}^N} \| y' - R x \|^2$$

(4.4)

Here $y' = Q' y$. The above function can be rewritten owing to the upper triangular nature of $R$ in terms of SD algorithm.

$$\sum_{j=1}^n \left( y'_j - \sum_{l=j}^n r_{j,l} x_l \right)^2 \leq C_0$$

(4.5)

where $C_0$ is the squared radius of n-dimensional sphere with origin at $y'$. This condition needs to be justified to ensure that the lattice point $x$ falls within the hypersphere for all the components $x_j$, where $j = 1$ to $n$. This constraint is observed in a depth first method meaning that the root node is searched first till the algorithm reaches the leaf node to determine a solution that satisfies the above constraint. Similarly reverse traversing in the hunt to find more leaf nodes which fulfill the criteria is done. SD algorithm then outputs the lattice points that possess the least Euclidean distance with $y'$. A tree constructed by SD algorithm for 2x2 4-QAM MIMO system is illustrated in Fig 4.3.

![Fig 4.3: Sphere Decoder algorithms tree structure illustration.](image)

The points not fulfilling the constraints imposed are pruned from the tree, the lattice points that are within the sphere are $x_1 = (0, 1, 1, 0)$ and $x_2 = (0, 0, 1, 1)$.

The procedure for finding the radius of search $C_0$ which was not originally specified was worked out by Vikalo and Hassibi in [90], by utilizing noise variance. In [91] it was proposed that $C_0$ be initially kept as infinity till searching of first lattice point, and then $C_0$ is set equal to the Euclidean
distance from received signal. A significant research work has been done in SD algorithm suggesting numerous methods to efficiently search the lattice tree structure, like lower bound pruning [91], lattice reduction [89], incrementing search radius [90], etc.

The worst case computational complexity of SD remains exponential; however its expected complexity is regarded as cubic over certain SNR and problem dimensions [90]. An exponential lower bound on SD average complexity has been worked out by Jalden in [94], however there will definitely be a problem dimension in which an approximate algorithm with polynomial time proves to be efficient than SD especially at lower SNR and larger modulation alphabets sizes.

Remainder of this chapter discusses the approximate MIMO detection algorithms such as linear, non-linear and Semi definite programming methods.

4.4.2 Linear Detection

Linear detection is used in the class of receivers in which the symbol estimate $\hat{x}$ is taken by a linear transformation of the received symbol $y$ [28] as:

$$\hat{x} = Q_z(Wy)$$  \hspace{1cm} (4.6)

where matrix $W$ is dependent on the channel $H$ and $Q_z$ is a quantizer or slicer that maps its arguments on to the nearest constellation point.

4.4.2.1 Zero-forcing Detection

These detectors solve the integer least square problem by removing the discreteness constraint on the components of $x$. Zero-Forcing detection is low complexity linear detection algorithm that gives the estimate of $x$ as:

$$\hat{x} = Q_z(\hat{x}_{ZF})$$  \hspace{1cm} (4.7)

and

$$\hat{x}_{ZF} = H^+y$$  \hspace{1cm} (4.8)
where $H^+$ denotes the pseudo-inverse of $H$ and $\hat{x}_{ZF}$ is mapped to nearest integer in the constellation alphabet from which $x$ is derived [28]. The receiver tries to force the cross correlation between the estimation error and the transmitted vector $X$ to zero, therefore it is termed as Zero-Forcing detector.

ZF detection algorithm is a linear detection algorithm since it behaves as a linear filter separating different data streams to perform decoding independently on each stream, therefore eliminating the multi-stream interference. The drawback of ZF detection is retarded BER performance due to noise enhancement. The AWGN noise $v$ loses its whiteness property as it is enhanced and correlated across the data streams. In addition, ZF detection gives $N_r-N_t+1$ diversity order in a $N_t \times N_r$ MIMO system with $N_r$ possible diversity order.

For $n \times n$ MIMO system, ZF detector possess a polynomial complexity of cubic order $O(n^3)$ which constitutes the computational complexity of calculating the pseudo-inverse of the matrix channel $H$.

### 4.4.2.2 Minimum Mean Square Error Detection

Minimum Mean Square Error (MMSE) detector estimates the transmitted vector $x$ by applying the linear transformation to the received vector $y$. It finds out the estimate $\hat{x}$ of the transmitted symbol vector $x$ as:

$$\hat{x} = Q_z(\hat{x}_{MMSE})$$  \hspace{1cm} (4.9)

and

$$\hat{x}_{MMSE} = Wy$$  \hspace{1cm} (4.10)

where

$$W = \left( H^H H + \frac{1}{SNR} I_{N_t} \right)^{-1} H^H$$  \hspace{1cm} (4.11)

where $W$ is selected to minimize the mean square error as $E\left[\|Wy - x\|^2\right]$.
Chapter 4 Symbol Detection in MIMO System

MMSE detectors balance the noise enhancement and multi-stream interference by minimizing the total error. Its BER performance is superior to ZF detection due to mitigating the noise enhancement. Its computational complexity is dominated by the matrix inversion in (4.11), which is cubic order $O(n^3)$.

4.4.3 Non-linear (V-BLAST) Detection

V-BLAST MIMO architecture uses Successive Interference Cancellation (SIC) technique to decode the transmitted symbols. Instead of detecting the transmitted symbol vector with $N_t$ symbols jointly it decodes the symbols based on the energy of the symbols. V-BLAST decodes the first transmitted symbol by fulfilling the ZF or MMSE performance criterion and assuming the remaining symbols contribution as interference, subsequently the contribution of the earlier detected symbol is cancelled out to get a reduced order integer least square problem having $N_t - 1$ unknowns. The V-BLAST is a non-linear recursive detection procedure that extracts the components of the transmitted vector $x$ according to a certain ordering $(l_1, l_2, \ldots, l_{N_t})$ of the indices elements of $x$ based on channel realization $H$. The ZF based V-BLAST algorithm is a V-BLAST derivative based on ZF criterion. Its algorithm is given below [95].

Initialization:

$$W_i = H^+$$
$$i = 1$$

Repetition:

$$k_i = \arg \min \left\| (W_i)_{j} \right\|^2 \quad \text{when} \quad j \notin [k_1, \ldots, k_{i-1}]$$
$$y_{k_i} = (W_i)_{k_i} y_i$$
$$\hat{x}_{k_i} = Q_Z(y_{k_i})$$
$$y_{i+1} = y_i - \hat{x}_{k_i} (H)_{k_i}$$
$$W_{i+1} = H_{\bar{x}_{k_i}}^+$$
$$i = i + 1$$
Chapter 4 Symbol Detection in MIMO System

$H^+$ is the pseudoinverse [85] of $H$, $j^{th}$ of $W_i$, $(H)_k$ is the $k^{th}$ column of $H$. $H_k^+$ is the resultant matrix by zero forcing the columns of $H$. $Q_{d(.)}$ is a slicer.

The above algorithm first of all orders the symbols to be decoded, carries out nulling and gets decision statistics, decides the first symbol. The coefficients of $H$ corresponding to the detected symbol having maximum energy are cancelled out, inverse of the new Channel matrix $H$ are computed. The process is repeated to iteratively decode the transmitted symbols.

This ZF VBLAST detection algorithm uses ZF approach discussed earlier. The ordering of the symbols is based on the minimum noise variance $\left\| (H^+) \right\|_2^2 N_0$.

The VBLAST computational complexity for $n \times n$ MIMO systems is $O(n^4)$. It possesses enhanced BER performance than linear detectors like ZF and MMSE, however suffers from error propagation. VBLAST performance suffers degraded performance when the first symbol is decoded incorrectly.

![Performance Analysis of Linear and Non-linear detectors-4-QAM 4x4 MIMO System](image)

**Fig 4.4:** Performance of linear and non-linear MIMO detectors.
4.4.4 ZF-ML Detection

The quadratic form of (4.2) given as:

$$f(x) = \|y - Hx\|^2$$  \hspace{1cm} (4.12)

The function $f(x)$ in (4.12) is convex. This detection algorithm also termed as multi-step reduced-constellation (MSRC) detection performs local search of the target symbols within certain constraint specified reduced search space. In fact a ZF initial solution estimate is used to define the radius of search. Constellation points around the ZF solution are searched in steps using (4.2) to find out the minimum Euclidian distance. This particular method which starts with the ZF processing is termed as ZFML detection [90].

First $y$ is computed and then a ML search around the neighborhood of $y$ is performed as shown in Fig 4.6. Each of the $N_t$ symbol generates a neighbor list, then a joint ML search our reduced constellations is performed. For a 16-QAM 4x4 MIMO and fixed neighbor size of 4 for each antenna, there are nine entries in the lookup table, with each entry containing 4 constellation points. For simplicity pivot points can be defined like in the above case nine pivot points are defined. We determine one pivot point which lies close to the ZF solution found, now the neighbors of each pivot point are searched jointly.

![ZF-ML reduced constellation search](image.png)

Once the first-step search result is generated, we can form a second neighbor list around it and perform a second-round search, and so on. The neighbor lists for the second round can be
Chapter 4 Symbol Detection in MIMO System

generated and searched. BER performance comparison of ZF-ML, ML and linear detection methods is represented in Fig 4.7.

Compared to ML that performs a coarse search over the complete search space the ZF-ML used a reduced constellation, therefore its computational complexity is \( \left( \frac{M}{M_n} \right)^{N_t} \). Where M is the constellation size, \( M_n \) is the neighbors list and \( N_t \) is the number of transmitters.

![Performance Analysis of ZFML Detector in 4-QAM 4x4 MIMO System](image)

Fig 4.6: Performance of ZFML MIMO detectors.

4.4.5 SDP Detection

Convex relaxation technique is another approach to solve the integer least-squares problems. In these techniques the objective function to be minimized is expressed in a relaxed form while keeping the problem convex and the convexified problem is solved using mathematical programming techniques.

Semi-Definite Programming (SDP) approach has been successfully used for MIMO Detection [97],[98]. Semidefinite programming technique deals with optimization problems which can be expressed as [99].

\[
\begin{align*}
\text{minimize} \quad & T_i(CX) \\
\text{subject to} \quad & T_i(A_iX) = b_i, \quad \forall i \in 1, ..., m \\
& X \geq 0
\end{align*}
\]
where the space of real symmetric \( n \times n \) matrices is \( X \in \mathbb{S}^n \), the matrices \( A_i \in \mathbb{S}^n \) and \( C \in \mathbb{S}^n \) define the problem parameters and \( b \in \mathbb{R}^n \). Trance of \( CX \) matrix is defined by \( T_r(CX) \) and the inequality \( X \succeq 0 \) shows that \( X \) is positive semidefinite.

The integer least-squares problem of MIMO detection is represented in higher dimensions and subsequently SDP is obtained by relaxing nonconvex constraints or SDP can be derived as Lagrangian bidual \[100\]. The integer least-squares problem can also be converted into binary quadratic minimization problem with the help of different relaxation models that can be further solved by SDP \[97\]. The SDP based MIMO detectors approach near-optimal BER performance with the computational complexity for \( n \times n \) MIMO system of \( O(n^{5.5}M^{5.5}\log(1/\varepsilon)) \) with tolerance \( \varepsilon \).

### 4.5 Summary

A survey of existing linear, non-linear and exact MIMO detectors was presented in this chapter. Linear detectors like ZF, MMSE have reduced computational complexity with a lower BER performance. Non-linear techniques like VBLAST present acceptable BER performance with not very high complexity. Exact techniques such as Sphere Decoder give optimal BER performance however the complexity is still on higher side and may vary with noise variance. It is believed that under certain conditions SD complexity can become exponential.
Chapter 5

Meta-heuristic Techniques

5.1 Introduction

Real life optimization problems (like uncoded MIMO-ML detection) are often so complex that finding the best solution becomes computationally infeasible. Therefore, an intelligent approach is to search for a good with reduced computational complexity. Many techniques have been proposed that imitate nature’s own ingenious ways to explore optimal solution. Earliest of the nature inspired techniques are genetic and other evolutionary heuristics that evoke Darwinian evolution principles.

Swarm Intelligence [101-104][112] is one such novel, intelligent distributed technique for solving optimization problems that was inspired from the natural behaviors such as birds swarming and animals flocking.

A brief review on heuristics, metaheuristics and the role they play in solving hard optimization problems is discussed in this chapter. Ant Colony Optimization (ACO) [115] and Particle Swarm Optimization (PSO) [122] are the two main Swarm Intelligence techniques. ACO and PSO algorithms are discussed next.

5.2 Meta-heuristics and heuristics

Integer-least square problem introduced in the previous chapter belong to a class of NP-hard Combinatorial Optimization (CO) problems. A CO problem has three components, a set of possible candidate solutions, an objective or fitness function which assigns to each candidate solution a cost value, and a set of constraints. The candidate solution that satisfies the
Chapter 5 Meta-heuristic Techniques

constraints becomes a feasible solution. To solve a CO problem, we need to find a globally feasible solution in which fitness function can be a minimization or maximization problem. In this work we focus on minimization problem expressed in the ML detection expression given by (4.3).

$$x^* = \arg\min_{x \in \mathcal{X}} \|y - Hx\|^2$$

Most of the combinatorial optimization problems are NP-Complete [105]. Heuristics are approximate algorithms used to find near optimal solutions to hard CO problems in polynomial time.

Glover in [106] introduced the term metaheuristics in 1986. It is derived from the composition of two words: meta meaning “higher level, beyond” and heuristics meaning “to find”. It is defined as “A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem” [106]. Therefore instead of performing a coarse search over the complete fitness landscape, the metaheuristics guide the underlying heuristics to regions of better fitness according to the procedure defined by the metaheuristics. The common metaheuristics techniques are Genetic algorithm, Simulated Annealing and Tabu search [107][108]. Swarm Intelligence is a relatively newer technique, but has proved to be a powerful optimizer in many applications [101]. Many of these techniques like genetic algorithm, Tabu search, Ant Colony Optimization and Particle Swarm Optimization have been used successfully in multiuser detection (MUD) in CDMA systems [109-111]. However, these metaheuristics detection methods have not been applied to MIMO detection being a relatively newer research area. The swarm intelligence based algorithms like ACO and PSO are applied to decode the received symbols which becomes a NP-hard problem. Sub-sections below explain the ACO and PSO metaheuristics techniques.
5.3 Natural Optimization by Ants

Ants, bees and termites are social insects who exhibit an intelligent and complex group behavior [112]. The collective behavior of the insects appears from a simple and indirect interaction of members of a group called colonies. Ants being semi or completely blind, communicate with other members and their surroundings using a volatile odorous chemical, called pheromone, produced by glands in ants abdomen. Ants sense these using the receivers in their antennae. Pheromone is used by the ants to serve many purposes like threat alert, recognizing group mates, attracting mates, boundary marking for colony and making path or trails to guide others towards the food source [113]. The cooperative foraging behavior of ants was of particular interest to researchers who noticed that a colony is able to choose the shortest path between the nest and a source of food even though individual ants do not have a global vision of the path [114]. To establish this behaviour, Deneubourg et al [114] arranged a setting in which the ants colony was connected with two different routes to a food source as shown in the Fig 5.1.

When ants start exploring the space surrounding the nest in search for food, they do so in a random fashion. An ant reaches chooses either one of the two paths with equal probability at the start, as there is no initial pheromone traces placed on either paths to alert the ants to favor one over the other, Fig. 5.2.

---

Fig 5.1: Experimental setup to observe ants behavior

Fig 5.2: Situation at the beginning of search – foraging behavior.
When an ant reaches the food source, it retracts its path back to the nest, depositing pheromones along the way to alert other ants to the presence of food at the end of the path. Given the same period of time, an ant using the shorter path will be able to complete more trips going back and forth between the nest and the food source than an ant using the longer path; thus a larger amount of pheromones will accumulate on the shorter path. This increased amount of pheromones will bias the decision of ants when they reach the fork in the road once again and the shorter path will be chosen with a higher probability, which means a larger number of ants will traverse the path with the passage of time leading to an even larger amount of pheromones accumulating on the shorter path. At the end, almost all of the ants will choose the shorter path, due to this positive feedback process, Fig. 5.3.

![Ants foraging behavior after certain time.](image)

The ability of ant colonies to naturally solve optimization problems, such as selecting the shortest path, inspired Dorigo, Maniezzo and Colomi to build the first ACO algorithm mimicking ants' behavior to solve hard combinatorial optimization problems. This algorithm, known as Ant System [115], was the progenitor to a number of ant-inspired algorithms which were formally grouped within one general framework called the Ant Colony Optimization metaheuristic [116].

5.4 Ant Colony Optimization (ACO) Algorithm

ACO algorithm as discussed above is based on the foraging behavior of a colony of ants searching for food. In this approach, several artificial ants perform a sequence of operations iteratively. Within each iteration, several ants search in parallel for good solutions in the solution space. One or more ants are allowed to execute a move iteratively, leaving behind a
Chapter 5 Meta-heuristic Techniques

pheromone trail for others to follow. An ant traces out a single path, probabilistically selecting only one element at a time, until an entire solution vector is obtained. In the following iterations, the traversal of ants is guided by the pheromone trails, i.e., the greater the pheromone concentration along any particular path, the more is the chances that an ant traverses that path to its destination. The quality of produced solution is estimated via a cost function in each iteration. This estimate of a solution quality is essential in determining whether or not to deposit pheromone on the traversed path.

As the search progresses, deposited pheromone dominates ants’ selectivity, reducing the randomness of the algorithm. Therefore, ACO is an exploitive algorithm that seeks solutions using information gathered previously, and performs its search in the vicinity of good solutions. However, since the ant’s movements are stochastic, ACO is also an exploratory algorithm that samples a wide range of solutions in the solution space.

Since its introduction in 1992, several researchers have aimed to improve the performance of Ant System so that it can be applied to a wide range of problems with competitive results; the improved versions of AS became stand-alone algorithms in their own right. Among the best performing ACO algorithms, Ant Colony System (ACS) [117], Max-Min Ant System (MMAS) [118], Approximate Nondeterministic Tree Search (ANTS) [119] and Binary Ant System (BAS) [120].

ACO’s binary version BAS [120][121] which is relatively less known however, is well suited for constrained optimization problems with binary solution structure like MIMO Detection problem at hand. Therefore we will focus on BAS to optimize NP- hard MIMO symbol detection problem.

5.5 Binary Ant System (BAS)

1) Solution construction: In BAS, artificial ants construct solutions by traversing the mapping graph as shown in Fig 5.4 below.

![Routing Diagram for Ants in BAS](image)
A number of $n_a$ ants cooperate together to search in the binary solution domain per iteration. Each ant constructs its solution by walking sequentially from node 1 to node $n+1$ on the routing graph shown above. At each node $i$, ant either selects upper path $i_0$ or the lower path $i_1$ to walk to the next node $i+1$. Selecting $i_0$ means $x_i=0$ and selecting $i_1$ means $x_i=1$. The selecting probability is dependent on the pheromone distribution on the paths:

$$p_{is} = \tau_{is}(t), i = 1, ..., n, s \in \{0, 1\}$$

Here ‘t’ is the number of iterations.

2) Pheromone Update: The algorithm sets all the pheromone values as $\tau_{is}(0) = 0.5$, initially but uses a following pheromone update rule:

$$\tau_{is}(t+1) \leftarrow (1-\rho)\tau_{is}(t) + \rho \sum_{x \in S_{upd}} w_x \sum_{s \in \{0, 1\}}$$

Where $S_{upd}$ is the set of solutions to be intensified; $w_x$ are explicit weights for each solution $x \in S_{upd}$, which satisfying $0 \leq w_x \leq 1$ and $\sum_{x \in S_{upd}} w_x = 1$. The evaporation parameter $\rho$ is initially as $\rho_0$, but decreases as $\rho \leftarrow 0.9\rho$ every time the pheromone re-initialization is performed. $S_{upd}$ consists of three components: the global best solution $S_{gb}$, the iteration best solution $S_{ib}$, and the restart best solution $S_{rb}$. $w_x$ combinations are implemented according to the convergence status of the algorithm which is monitored by convergence factor $cf$, given by:

$$cf = \frac{\sum_{i=1}^{5} |\tau_{i0} - \tau_{i1}|}{n}$$

The pheromone update strategy in different values of $cf$, are given in table-1, here $w_{ib}$, $w_{rb}$ and $w_{gb}$ are the weight parameters for $S_{ib}$, $S_{rb}$ and $S_{gb}$ respectively, $cf_i, i=1,...,5$ are threshold parameters in the range of $[0,1]$. When $cf>cf_5$, the pheromone re-initialization is performed according to $S_{gb}$.

### 5.6 Natural Optimization by Swarm

This population-based search algorithm was inspired by the social behavior of animals like birds flocking, bacteria molding and fish schooling. The initial intention of the particle swarm concept was to simulate graphically the unpredictable bird flock’s choreography. The original
study of these birds patterns were aimed to discover the factors that govern the bird’s ability to synchronously fly as well as suddenly change their direction and reorganize in a formation.

PSO can be understood through an analogy. Consider a flock of birds searching for food in an area. Their goal is to find the best food spot, without any a priori knowledge. The birds start at random locations in the field with random velocities in their hunt for food. Every bird can remember the locations where it found the food and in some way know the place where the others discover food. In a state of perplexity to return to the location where a bird had personally found the food, or exploring the location reported by others, the unsure bird accelerates in both directions. Reminiscence or social pressure influence bird’s decision as it changes its trajectory to fly in the resulting direction. During travel to a newer location, a bird might find a place with more food than it had found earlier. A bird may occasionally fly over a place with more food than earlier encountered by any other bird in the flock. The whole flock would now be attracted towards that location as well as their own personal best finding. The birds explore the field in the similar fashion. Flying over locations of greatest concentration of food and then being attracted back towards them. The birds are continually checking the location they fly over against previously encountered places in their endeavor to find the absolute best food concentration. Ultimately, the birds concentrate at the best available food location in the complete field. Kennedy and Eberhart reached an optimizing heuristic in their attempt to model this natural phenomenon.

### 5.7 Particle Swarm Optimization (PSO) Algorithm

Particle Swarm Optimization meta-heuristics is a population-based Swarm Intelligence (SI) technique inspired by the coordinated movements of birds flocking introduced by Kennedy and Eberhart in 1995 [122]. Standard PSO uses a real-valued multidimensional solution space, whereas in binary PSO particle positions are binary rather than real valued [123]. The combination of the pure heuristics like PSO with local search (LS) is termed as “memetic algorithms” (MAs) [124]. MAs are extensions that apply additional procedure to further refine the search result efficiently. This hybridization improves search efficiency [125]. The main PSO terminologies are elaborated below:
1) **Particle**: One individual in the swarm (birds in our analogy). The particles are independent and move towards the personal best and global best location, continually checking its current location value.

2) **Swarm**: The entire collection of particles, like bird flock.

3) **Fitness**: It is a unique value representing the goodness of a solution in the solution space. In the example fitness function would be the best concentration of food. More is the food available better will be the fitness function. As regards the problem at hand, fitness function is the minimum value of Euclidean distance of the symbol being detected.

4) **pbest**: Location of the best fitness returned for the specific agent in the parameter space. In the analogy it is the location where the birds personally discovered the most food. This the personal best (pbest) location with the highest fitness value personally encountered. Every bird has its own pbest. In the path bird traverses it compares the fitness value of its current location with that of pbest. If the current location has higher fitness value, pbest is updated.

5) **Gbest**: This is the location of the best fitness returned for the entire swarm in parameter space. Every bird possesses a mechanism to know the best concentration of food encountered by the complete flock. The gbest is the global best location returned by the entire flock. In a flock there is one gbest, where the bird gets attracted. At every point in the flight the bird compares the fitness of the current position with that of the gbest, which is updated by the bird if it is at the location of higher fitness.

6) **Vmax**: It is Maximum allowable velocity of a particle in a particular direction.

7) **Generations/Iterations**: The maximum allowable position updates for each particle. It is the maximum number of times a particle can change its present location to reach gbest.

The PSO algorithm detailed below is also explained pictorially in a flow diagram of Fig 5.5.
1) Allocate Solution Space parameters: The initial step for implementation of the PSO is to select the parameters that need to be optimized. Allocate the parameters with a reasonable range to be searched for the optimal solution. A minimum (Xmin_n) and maximum (Xmax_n) value for each dimension in an N-dimensional optimization is specified.

2) Fitness Function definition: The definition of fitness function is crucial since it should precisely represent the goodness of the solution in a single value. The fitness function and the solution space development is optimization problem specific. The remaining implementation, however, is independent of the physical system being optimized.

3) Random Swarm Velocities and Location Initialization: Every particle starts at its own random location to begin searching for the optimal position in the solution space with a velocity that is random both in its magnitude and direction. The initial position for each particle is its respective pbest at start. Then from amongst these initial positions the first gbest is selected.

4) Particles Systematically Fly-through the Solution Space: Each particle (like a bird in the flock) must traverse through the solution space parameter. The heuristic is applied on every particle one after another, moving it by a small distance and cycling through the entire swarm. The following steps act individually on each particle.
a) **The Particle’s Fitness Evaluation:** The fitness function returns a fitness value for the present location using the coordinates of the particle in solution space. The locations are updated if the fitness value is greater (or smaller, problem dependent) than the value at the respective $p_{best}$ or the $g_{best}$.

b) **The Particle’s Velocity Update:** The particle’s velocity updating is the heart of the entire optimization algorithm. The particle’s velocity is altered according to the relative locations of $p_{best}$ and $g_{best}$. A Particle is accelerated in the directions of the location of the best fitness according to the relation (4) given below:

$$
\hat{v}_{ij}(t+1) = w \cdot v_{ij}(t) + c_1 r_1(t)[y_{ij}(t) - x_{ij}(t)] + c_2 r_2(t)[\hat{y}_j(t) - x_{ij}(t)]
$$

(5.5)

Where $y_{ij}(t)$ is the $p_{best}$ for particle $i$ in dimension $j$, $\hat{y}_j(t)$ is the $g_{best}$. $v_{ij}(t)$ is the velocity of particle $i$ in dimension $j$. $x_{ij}(t)$ is the position of particle $i$ in dimension $j$ at a particular time $t$. $c_1$ and $c_2$ are positive acceleration constants used to scale contribution of cognitive and social elements can also be termed as learning factors. $r_1$ and $r_2$ are the random number function rand() which returns a number between 0 and 1 randomly. Two independent random numbers are used to stochastically vary the relative pull of $g_{best}$ and $p_{best}$. $w$ is the inertial weight. A number chosen between 0 and 1, shows the particles resistance to the drag of $p_{best}$ and $g_{best}$. The particle motion is based on (5.4). The greatest “pull” from the respective locations is experiences by the particle farthest from $g_{best}$ or $p_{best}$. Therefore move toward them more rapidly than any nearer particle. The particles accelerate in the direction of the place of greatest fitness until they run over them. Now these will be attracted back in the reverse direction. It is considered that this over running of the local and global maxima is a to the PSOs success. The velocity consists of three components first is ‘previous velocity’ which is the memory of the previous flight direction (inertia). Secondly a ‘Cognitive component’ it is the $p_{best}$ for a particle, and last is the ‘Social component’ that quantifies the influence on particle based on $g_{best}$ and $p_{best}$. This is depicted in Fig 5.6, where $x(t)$ is the present and $x(t+1)$ is the new position of particle in the parameter space. $y(t)$ and $\hat{y}(t)$ are $p_{best}$ and $g_{best}$ respectively.
Fig. 5.6: Particles are pulled towards their gbest and pbest

A particle at its present position experiences these cognitive, inertial and social velocities and moves in the resultant direction of new velocity to jump to next location.

c) **Particles Movement:** The velocity is applied for a given time-step, and the particle moves to the next position. New coordinate are computed for each of the dimensions in the parameter space based on the following equation:

$$x_i(t+1) = x_i(t) + v_i(t+1)$$  \hspace{1cm} (5.6)

Where $x_i(t+1)$ and $v_i(t+1)$ is the new position and velocity for $i^{th}$ particle.

5) **Repeat:** This process is enacted on each particle in the swarm; the procedure is repeated starting at Step 4. In this way the particles move for discrete time intervals before being evaluated. It is as though a snapshot is taken of the entire swarm every second. At that time the positions of all the particles are evaluated, and corrections are made to the positions of pbest, and gbest before letting the particles fly around for another second. Repetition of this cycle is continued until the termination criteria met or stopping condition is reached.

A key attractive feature of the PSO approach is its simple mathematical model involving two model equations and fewer parameters to adjust.

In the binary version of PSO known as binPSO [123], velocity loses its physical meaning. It is used to determine a probability by squashing velocities to the range (0,1) by using sigmoid function. A proof of explicit PSO equations and its guaranteed convergence is given in [104].

The nature of the ML optimization problem, suggests BinPSO algorithm to be use as ML function optimizer as discussed in the next chapter.
5.8 **Summary**

This chapter introduced the nature inspired optimization algorithms. Swarm Intelligence which is an intelligent paradigm for solving NP-hard optimization problems was explained. PSO and ACO algorithm were discussed in details to get an insight on their functionality to carry out optimized search for better solution. The socio-cognitive behavior of PSO algorithm and exploratory-exploitive optimization performed by ACO algorithm make these a suitable candidate to be used for optimized traversing of MIMO-ML search space to find out better solution with lower complexity. In the next chapter we discuss the optimized uncoded-MIMO detection algorithms based on these ACO and PSO algorithms.
Chapter 6

Swarm Intelligence Meta-heuristics for
Symbol Detection in MIMO System

6.1 Introduction

The major challenge in designing BAS and BinPSO algorithm based MIMO detector is the selection of effective fitness function which is problem dependent and perhaps is the only link between the real world problem and the optimization algorithm. Fitness function is unique for each optimization problem. An important step to implement BAS-MIMO detection algorithm is to define a fitness function; this is the link between the optimization algorithm and the real world problem. Basic fitness function used by this optimization algorithm is to converge to the optimal solution is (4.3). Choice of the initial solution plays a vital role in the fast convergence of the optimization algorithm to a suitable solution. The MIMO detector assisted by BAS makes a start with the ZF or VBLAST inputs. The proposed BAS assisted MIMO detection algorithm [125] is discussed.

6.2 BA-MIMO Detection Algorithm

The proposed detection algorithm is described as follows:

1) Consider the output of ZF or VBLAST as initial input to algorithm instead of starting with random values, such that $x_i \in \{0,1\}$. The number of nodes $n$ visited by $n_a$ ants is $bxN_t$ i.e ML search space size ($x_i$). Here $x_i$ represents the bit strings of the detected symbols at the receiver and $i=1 \text{ to } n$. 
2) The probability of selecting $x_i=0$ or 1 depends on the pheromone deposited according to (5.3). Where $\tau_{is}(0) = 0.5$ for equal initial probabilities.

3) Evaluate the fitness of solution based on (5.1):

$$ f = \| y - Hx \|^2 $$

Minimum Euclidean distance for each symbol represents the fitness of solution. The Euclidean distance for $x_i$ is measured.

4) Perform pheromone updation based on (5.3). Calculate $S_{upd}$ that consists of $S^{gb}$, $S^{ib}$, and $S^{rb}$ with weights $w_i$ based on $cf$ (5.4) and Table 6.1.

5) Go to step-3 until maximum number of iterations is reached.

As $cf \to 0$, the algorithm converges, once $cf > cf_5$, the pheromone re-initialization procedure is performed according to $S^{gb}$, i.e.

$$\begin{align*}
\tau_{is} &= \tau_{\mu} \quad \text{if } is \in S^{gb} \\
\tau_{is} &= \tau_{L} \quad \text{otherwise}
\end{align*}$$

where $\tau_{\mu}$ and $\tau_{L}$ are the two parameters satisfying $0 < \tau_{L} < \tau_{\mu} < 1$ and $\tau_{L} + \tau_{\mu} = 1$.

After extensive experimentations, we choose the algorithm parameters as: $x_i = bN$, $\tau_0 = 0.5$, $\tau_1 = 0.65$ and $\rho_0 = 0.3$.

<table>
<thead>
<tr>
<th></th>
<th>$cf &lt; cf_i$</th>
<th>$cf \leq [cf_1, cf_2)$</th>
<th>$cf \leq [cf_2, cf_3)$</th>
<th>$cf \leq [cf_3, cf_4)$</th>
<th>$cf \leq [cf_4, cf_5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{ib}$</td>
<td>1</td>
<td>2/3</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$w_{rb}$</td>
<td>0</td>
<td>1/3</td>
<td>2/3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$w_{gb}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.1: Pheromone Update Strategy for BA-MIMO system.

### 6.3 Performance analysis of BA-MIMO Detection

In this section, we compare the results obtained using the proposed method with the existing detection techniques for the spatial multiplexing systems. The SNR ($E_b/N_0$) is the average Signal to noise ratio per antenna ($P/\sigma_v^2$) where $P$ is the average power per antenna and $\sigma_v^2$ is the noise variance. The simulation environment assumes Rayleigh fading channel with no correlation.
between sub-channels. The channel is assumed to be quasi-static for each symbol, but independent among different symbols. Perfect sampling and carrier frequency offset synchronization are assumed. An average of no less than 10,000 simulations is obtained for each result in order to report statistically relevant results.

A 3x3 ($N_t \times N_r$), 4x4, and 4-QAM 6x6 MIMO systems are considered. The symbols $x_i$ and number of algorithm iterations ($N_{itr}$) depends on $N_t$ and the QAM constellation size. For 3x3, 4-QAM system, $x_i$ equals 6 and it grows to 12 for 6x6, 4-QAM system. $N_{itr}$ is kept in the range of 10 to 20 in our simulations. Iterations are according to the system requirements. Larger $N_{itr}$ can result in better BER at the cost of increased complexity. However, the algorithm reaches saturation after a certain number of iterations and therefore $N_{itr}$ needs to be tuned carefully. Optimum $N_{itr}$ value is taken after a number of trials to find the best BER with the least complexity.

Fig 6.1 presents the BER versus $E_b/N_0$ performance of proposed detector compared with ML and VBLAST detectors for 3x3 ($N_{itr}=10$). At $10^{-3}$ BER, the proposed BAV algorithm (with VBLAST as initial guess) and BAZ algorithm (with ZF as initial guess), result in 3-dB and 5-dB degraded performance in comparison with ML. However, in comparison with VBLAST, both the BAV and BAZ show 7-dB and 5-dB enhanced performance, respectively.

![Fig. 6.1. BA-MIMO detection BER versus SNR performance for a 3x3 system.](image)

In a 4x4 system in Fig. 6.2 ($N_{itr}=13$), the BER gain of both BAZ and BAV detectors, in comparison with VBLAST increase by 7-dB and 8-dB, respectively. The BER performance of
BAZ and BAV detectors lower by 4-dB and 3-dB from ML but with some complexity reduction.

![Fig. 6.2. BA-MIMO Detection BER versus SNR performance for a 4x4 system.](image)

For a 6x6 ($N_{tx}=18$) system in Fig 6.3, the BER improvement in comparison to VBLAST for both the BAZ and BAV detectors is 7-dB and 8-dB, respectively. However, ML has 7-dB (approximately) superior BER performance but its complexity is also significant. Increase in system configuration ($N_t \times N_r$), results in exponential increase of search space, therefore more iterations are required to converge to near-optimal solution. A trade off between systems BER performance and iterations has to be maintained according to the system requirement and priority.

![Fig. 6.3. BA-MIMO Detection BER versus SNR performance for a 6x6 system.](image)
6.4 Computational Complexity Comparison

We now examine the computational complexity of the reported detector and compare it with ML and VBLAST detectors. As the hardware cost of each algorithm is implementation-specific, we try to provide an estimate of complexity in terms of number of complex multiplications. The computational complexity is computed in terms of the $N_b$, $N_r$ and the constellation size $M$.

For ML detector as seen from (4.2) $M^{N_t}(N_tN_r)$ multiplications are required for matrix multiplication operation and an additional $M^{N_t}N_r$ multiplications are needed for square operation. Therefore, ML complexity becomes:

$$\gamma_{ML} = N_r (N_t + 1)M^{N_t} \tag{6.3}$$

In case of ZF, the pseudo-inverse of matrix $(H^H H)^{-1} H^H$ takes $4N_t^3 + 2N_t^2N_r$ multiplications [15]. Therefore ZF complexity becomes:

$$\gamma_{ZF} = 4N_t^3 + 2N_t^2N_r \tag{6.4}$$

For VBLAST the pseudo-inverse matrix is calculated $N_t$ times with decreasing dimension. In addition, the complexity of ordering and interference canceling is $\sum \left[N_r(N_t - i) + 2N_t\right]$. Therefore, total complexity of VBLAST ($\gamma_{VBLAST}$) results in.

$$\gamma_{VBLAST} = \sum_{i=0}^{N_t-1} (4i^3 + 2N_t^2i) + \sum_{i=0}^{N_t-1} [N_r(N_t - i) + 2N_t]$$

$$= N_t^4 + (5/2 + 2/3N_r)N_t^3 + (7/2 + N_r)N_t^2 + 1/3N_tN_r \tag{6.5}$$

For BA-MIMO detector, first fitness using (6.1) in $x_t$ is calculated. Therefore, the complexity ($\gamma_{BA-MIMO}$) becomes,

$$\gamma_{BA-MIMO} = x_t (N_tN_r) \tag{6.7}$$

Pheromone update requires $\mu_p$ additional multiplications per iteration with from (5.2). Therefore $\mu_p$ becomes 2, the complexity becomes,

$$\gamma_{BA-MIMO} = x_t (N_tN_r + \mu_p) \tag{6.8}$$

This procedure is repeated $N_{itr}$ (same as $t$) times to converge to the near-optimal BER performance. Therefore,

$$\gamma_{BA-MIMO} = x_t (N_tN_r + \mu_p) N_{itr} \tag{6.9}$$
The Proposed detector takes initial solution guess as ZF or VBLAST output therefore, it is added into get the resultant complexity of BA-MIMO detector.

It is obvious that the complexity of ML is exponential with $N_t$ and $M$. ML complexity for a 4-QAM 4x4 system is 5120 and it grows to 4.7 M for 8x8 system. This increase is even significant with higher order modulation schemes.

Computational complexity of VBLAST for 4-QAM 4x4 and 6x6 systems computed from (6.9) is 712 and 3054, respectively. The complexity of proposed detector with ZF initialization for 4x4 and 6x6 configurations is 2256 and 9504. However, the complexity with VBLAST input comes out to be 2584 and 11262, respectively. This complexity estimate is only meaningful in the order of magnitude sense since it is based on the number of complex multiplications.

<table>
<thead>
<tr>
<th>Performance Comparison</th>
<th>4x4 ($n_a=8, N_{tr}=13, \mu_p=2$)</th>
<th>6x6 ($n_a=12, N_{tr}=18, \mu_p=2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Performance (10^{-3} BER)</td>
<td>Computational Complexity</td>
</tr>
<tr>
<td>BAZ and ML</td>
<td>4-dB less</td>
<td>56% less</td>
</tr>
<tr>
<td>BAV and ML</td>
<td>3-dB less</td>
<td>50% less</td>
</tr>
<tr>
<td>BAV and VBLAST</td>
<td>8-dB more</td>
<td>71% more</td>
</tr>
<tr>
<td>BAZ and VBLAST</td>
<td>7-dB more</td>
<td>68% more</td>
</tr>
</tbody>
</table>

Table 6.2. 4-QAM $N_t x N_r$ systems - performance complexity trade-off.

### 6.5 Performance-complexity trade-off

A reasonable performance-complexity trade-off exists when a comparison of the proposed detector is drawn with ML and VBLAST detectors. Table 6.2 shows a comparison of the proposed detector with optimal ML and VBLAST detectors. Compared to ML, complexity reduction of the proposed detector given by $(\gamma_{ML} - \gamma_{BA-MIMO})/\gamma_{ML}$ is significant. In comparison to
VBLAST, for a 4x4 system, the proposed detector with ZF input results in 7-dB increased BER performance at the cost of increased 68 % computational time. However, as compared to ML, the complexity is still 56 % lower with a 4-dB reduction in BER performance. This computational complexity saving increases to as high as 94% at the cost of 7-dB lesser BER performance for a 6x6 system with ZF input.

### 6.6 Discussion

Binary ant system assisted symbol detection novel algorithm for a spatial multiplexing system was presented in the previous section. The algorithms simple model, lesser implementation complexity, resistance to being trapped in local minima, convergence to reasonable solution in fewer iterations and exploratory-exploitive search approach makes it a suitable candidate for real-time wireless communications systems. This algorithm shows promising results when compared with the optimal ML and traditional VBLAST detectors. This BA-MIMO symbol detection mechanism approaches near-optimal performance with much reduced computational complexity, especially for complex systems with multiple transmitting antennas, where conventional ML detector is computationally expensive and impractical to implement. When compared to VBLAST detector the proposed unconventional detection method results in enhanced BER performance but at the cost of increase in complexity. The simulation results suggest that the proposed detector in a 6x6 spatial multiplexing system improves VBLAST BER performance by 7-dB. However, the ML complexity is reduced by 94% with a reasonable BER performance.

### 6.7 PSO-MIMO Detection Algorithm

Exploitation of parsimonious PSO algorithm’s potential to optimize symbol detection in MIMO system is now discussed. The PSO algorithm assisted MIMO detection technique used in this section uses the same fitness function as that of the earlier case i.e. reducing the Euclidean distance in (4.3). The fitness function using the coordinates of the particle returns a fitness value to be assigned to the current location. If the value is greater than the value at respective personal best (pbest) for each particle, or global best (gbest) of the swarm, then previous locations are
updated with the present locations. The velocity of the particle is changed according to the relative locations of \( pbest \) and \( gbest \) as shown in Fig 5.6. Once the velocity of the particle is determined, it simply moves to the next position. After this process is applied on each particle, it is repeated till the maximum numbers of iterations are reached.

This exploratory-exploitive optimization approach can be extended to MIMO detection optimization problem [126-128]. The major challenge in designing PSO based MIMO detectors is selection of the algorithm parameters that fit the symbol detection optimization problem. Selection of effective fitness function is also vital and problem dependent. The fitness function used by the optimization algorithm to converge to the near optimal solution is (6.1) which is minimum Euclidian distance as discussed earlier. In addition, the choice of initial solution guess plays an important role in the fast convergence to a suitable solution. Initial guess is essential for these algorithms to perform well. Therefore, the proposed PSO-MIMO detector takes the output of ZF or ZF-VBLAST as its initial solution guess. This educated guess enables the algorithm to reach a more refined solution iteratively by ensuring fast convergence. Similar initial guesses were applied to the input of BAS algorithm discussed in the previous sections. Assuming random initialization does not guarantee convergence in less iteration.

### 6.8 SPSO-MIMO Detection Algorithm

The proposed MIMO detection algorithm based on standard continuous PSO [126] is described below:

1) Initialize the particle size (swarm) by taking an initial guess. Also, initialize the algorithm parameters.

2) Calculate the fitness of each particle with is a potential candidate solution using (6.1):

\[
 f = \| y - Hx \|^2 
\]  

Minimum Euclidean distance for each symbol represents the fitness of the solution. Find the global best performance ‘\( gbest_d \)’ in the population that represents the least Euclidean distance found so far. Record the personal best ‘\( pbest_{id} \)’ for each bit along its previous values.

3) Velocity for each particle is computed using the following PSO velocity update equation:

\[
 v_{id}(k) = v_{id}(k-1) + \phi_1 \text{rand}_1[pbest_{id} - x_{id}(k-1)] + \phi_2 \text{rand}_2[gbest_d - x_{id}(k-1)] 
\]  

88
with $v_{id} \in \{-v_{max}, v_{max}\}$.

4) The particle position is updated depending on the following PSO velocity update equation:

$$x_{id}(k) = x_{id}(k-1) + v_{id}(k) \quad (6.12)$$

5) Go to step-2 until maximum number of iterations are reached. Here ‘k’ is the number of iterations. An optimum number of iterations are tuned for efficient performance.

### 6.9 MPSO-MIMO Detection Algorithm

Hybridization of standard PSO with local search is termed as ‘memetic’ PSO (MPSO). The MPSO procedure further refines the solution found out by SPSO-MIMO using Lower Significant Bit (LSB) flipping. The algorithm is explained below:

1) Initialize the algorithm parameters. Assume the initial solution guess.

2) Find fitness using (6.10). Find ‘$g_{best_d}$’ and ‘$p_{best_d}$’. Perform velocity and position for each particle using (6.11) and (6.12).

3) Apply neighborhood search, by initializing ‘$b_s$’. Evaluate the fitness of neighbors iteratively; update ‘$g_{best_d}$’ and ‘$p_{best_d}$’.

4) Go to step-2 until maximum number of iterations is reached.

The pseudo-code is shown in Table 6.3. The degree of local search ‘$b_s$’ indicate the LSBs of the best solution found so far. Keeping search degree as 2 would mean four neighbors would be searched for better fitness. In our local search algorithm ‘$b_s$’ is kept as 1 and 2 respectively.
Table 6.3. Pseudo Code of the MPSO-MIMO algorithm

<table>
<thead>
<tr>
<th>Phase-1:</th>
<th>Initialize the particle size (Np); Randomly get particle position; Set boundaries for velocities; Initialize parameters;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase-2: For i=1: Np</td>
<td>%Get velocity for next position updating</td>
</tr>
<tr>
<td></td>
<td>[ v_{id}(k) = v_{id}(k-1) + \phi_1 \text{rand}_1[</td>
</tr>
<tr>
<td></td>
<td>If velocity &lt; boundary</td>
</tr>
<tr>
<td></td>
<td>%Update particle position;</td>
</tr>
<tr>
<td></td>
<td>[ x_{id}(k) = x_{id}(k-1) + v_{id}(k) ]</td>
</tr>
<tr>
<td></td>
<td>Else</td>
</tr>
<tr>
<td></td>
<td>%Velocity = Cyclic velocity;</td>
</tr>
<tr>
<td></td>
<td>End</td>
</tr>
<tr>
<td>%Apply neighborhood search</td>
<td>Initialize lower bits to search</td>
</tr>
<tr>
<td>For J = 1: Number of neighbors</td>
<td>%Iterative search in neighborhood</td>
</tr>
<tr>
<td></td>
<td>Evaluate();</td>
</tr>
<tr>
<td></td>
<td>If Fitness(x_{id}(k)) &lt; Fitness(x_{id}(k-1))</td>
</tr>
<tr>
<td></td>
<td>%Update Local Best;</td>
</tr>
<tr>
<td></td>
<td>(p_{besti} = x_{id}(k));</td>
</tr>
<tr>
<td></td>
<td>End</td>
</tr>
<tr>
<td>%Update Global Best;</td>
<td>(G_{best} = \min(p_{best}));</td>
</tr>
<tr>
<td>End</td>
<td></td>
</tr>
</tbody>
</table>

6.10 BPSO-MIMO Detection Algorithm

The application of binary version of PSO for symbol detection in MIMO system results in a further improved performance \[127][128\] as compared to the earlier discussed SPSO-MIMO detector. Here the particles are binary rather than real valued as in earlier case of SPSO-MIMO algorithm. The proposed binary PSO based MIMO detection algorithm is explained below:

1) Take the output of ZF or ZF-VBLAST such as \(x_i \in \{0,1\}\) as initial particles (initial solution bit string) instead of using random values from the solution space.

2) The algorithm parameters are initialized. \(v_{id}\) is initialized to zero (equal probability for binary decision), \(p_{bestid}\) and \(g_{bestid}\) are initialized to maximum Euclidean distance depending upon the QAM size.
3) Evaluate the fitness of each particle (bit) using the same fitness function of (6.1). Measure the effect on the Euclidean distance due to search space bits. Find the Global best performance ‘$g_{bestd}$’ in the population and record the personal best ‘$p_{bestd}$’ for each bit along its previous values.

4) For each search space bit at $d^{th}$ side of the bit string of particle $x_i$, compute the bits velocity using the PSO velocity update equation (6.12).

5) The particle position is updated depending on the following binary decision rule:

$$\text{If } \text{rand}_3 < S(v_{id}(k)), \text{ then } x_{id}(k) = 1, \text{ else } x_{id}(k) = 0.$$ (6.13)

6) Go to step 3 until maximum number of iterations are reached.

The value ‘$\text{rand}$’ is a random number generated uniformly in [0,1] and ‘$S$’ is sigmoid transformation function.

$$S(v_{id}(k)) = \frac{1}{1 + \exp(-v_{id}(k))}$$ (6.14)

The parameter ‘$v_i$’ is the particles predisposition, used in the sigmoidal function to get output as 1 or 0. The parameter ‘$v_i$’ determines the probability threshold to make the binary choice. The individual is more likely to choose 1 for higher $v_{id}(k)$, whereas its lower values will result in the choice of 0. Such a threshold needs to stay in the range of [0,1]. The sigmoid logistic transformation function maps the value of $v_{id}(k)$ to the range [0,1].

### 6.11 PSO Parameter Control

The terms $\phi_1$ and $\phi_2$ are positive acceleration constants used to scale the contribution of cognitive and social components such that $\phi_1 + \phi_2 < 4$ [102][123]. These are used to stochastically vary the relative pull of $p_{best}$ and $g_{best}$. $v_{max}$ sets a limit to further exploration after the particles have converged. Its values are problem dependent but usually set in the range of ±4 for binary PSO and ±10 for standard PSO [102]. The particle size is assumed fixed for SPSO and MPSO however, it varies with the system in case of BPSO. These parameters are discussed next.
6.12 PSO-MIMO Detection Algorithm’s Relationship

In Standard PSO originally proposed in [122] particle positions are real valued, however in its later version binary PSO [123], the particles represent binary bits rather than real values. Therefore, the MIMO detection problem formulation using SPSO as discussed in section 6.8 is based on real valued search space. Whereas, in case of BPSO-MIMO detection algorithm explained in section 6.10, the problem formulation is different. Here the solution is converted into binary string and particles are bits in this string. The Binary PSO algorithm is applied on each bit to get the newer solution bit string for fitness analysis.

The ‘memetic’ PSO based MIMO detection algorithm finds out the solution using exactly the same procedure as that used for SPSO-MIMO algorithm; however the solution returned is further refined using local search method. Therefore, the MPSO-MIMO detection technique offers better performance with additional complexity overhead in comparison to SPSO-MIMO detection algorithm. In depth performance analysis of these proposed PSO-MIMO detection methods is presented next.

6.13 Simulation Results and Performance Analysis

This section provides simulations results and theoretical analysis to ascertain the performance of the proposed PSO-MIMO detectors.

6.13.1 Experimental Setup

We evaluate the performance of proposed detectors for a 2x4 \(N_t \times N_r\) MIMO system with 4-QAM, 16-QAM, 32-QAM and 64-QAM constellations. In addition, the proposed detectors are also tested in a 4x4 and 8x8 4-QAM MIMO configurations. BER performance is analyzed at different SNR. The \(SNR (E_b/N_0)\) is the average Signal to noise ratio per antenna \(P/\sigma^2\), where \(P\) is the average power per antenna and \(\sigma^2\) is noise variance. The simulation environment assumes Rayleigh flat-fading channel with no correlation between sub-channels. An average of no less than 30,000 simulations were taken to report statistically relevant
results. In the simulated system acceleration constants $\phi_1$ and $\phi_2$ are assumed to be unity for simplification, whereas $v_{\text{max}}=\pm4$ for BPSO and $v_{\text{max}}=\pm10$ for SPSO detection algorithms [122]. The particle size in SPSO and MPSO detection algorithms is kept as 16 for the result shown in Fig 6.4. Here random initial particle positions are assumed. For BPSO-MIMO system shown in Fig 6.6 and Fig 6.7, particle size ‘$N_p$’ dependents on the QAM size and number of transmitters used in the MIMO system. In case of BPSO, $N_p = b \times N_t$ where ‘$b$’ is bits per symbol. For a 4x4, 4-QAM system, ‘$N_p$’ equals 8 and it grows to 16 for an 8x8, 4-QAM system. Algorithm iterations ‘$N_{itr}$’ is kept according to the system requirements. As an initial estimate we use the result of VBLAST for the results obtained in Fig 6.6 and Fig 6.7. Lesser $N_{itr}$ will now be required due to the refined initial solution guess instead of random particle positions as assumed in the earlier results obtained in Fig 6.4 and Fig 6.5. For the results shown in Fig 6.6 and Fig 6.7 ‘$N_{itr}$’ is kept in the range of 10 to 20.

6.13.2 BER versus SNR Performance

Fig 6.4, depicts BER versus $E_b/N_o$ performance of SPSO-MIMO and MPSO detectors in comparison with the optimal ML. The former shows a 4-dB, while later 2-dB reduced performance at $10^{-2}$ BER as compared with the ML detector. Fig 6.5, demonstrates the convergence behavior of SPSO and MPSO algorithms with increase in algorithm iterations using a random initial solution guess. SPSO-MIMO algorithm with 16-QAM converges to optimal BER in 25 iterations, whereas 32-QAM systems takes 32 iterations to converge. Similarly, MIMO-MPSO detection requires 18 and 25 iterations for with 16-QAM and 32-QAM systems to converge to the ML performance.
Fig. 6.4: PSO-MIMO Detector BER versus Eb/No 2x4 system

Fig. 6.5: SPSO and MPSO algorithms convergence with iterations.

Fig 6.6, shows the BER versus Eb/No performance of BPSO and SPSO detectors compared with ML for 4x4 4-QAM MIMO system. Now the initial guess from VBLAST is assumed for fast convergence. \( N_{10} \) is kept at 10. At \( 10^{-3} \) BER, binary PSO and standard PSO detector results in 3-dB and 6-dB degraded BER performance with respect to ML. Similarly, for a 8x8, 4-QAM, MIMO system in Fig 6.7, at \( 10^{-3} \) BER, BPSO and SPSO algorithms result in 4-dB and 7-dB degraded BER performance in comparison with the optimal detector. However, a substantial ML complexity reduction is achieved which is discussed in the next subsection.
6.13.3 Computational Complexity Theoretical Evaluation

The computational complexity of the reported PSO-MIMO detectors is examined now. A comparison with conventional ML optimal detection method is also drawn. The computational complexity is represented in terms of $N_o$, $N_t$ and the constellation size $M$. 

Fig. 6.6 PSO-MIMO Detector BER versus $E_b/N_o$ for 4-QAM 4x4 MIMO system.

Fig. 6.7 PSO-MIMO Detector BER versus $E_b/N_o$ for 4-QAM 8x8 MIMO system.
ML, ZF and VBLAST detectors complexities were computed in the previous section in (6.3), (6.4) and (6.6).

For the proposed PSO-MIMO detector, the fitness of each particle in population \( N_p \) using (6.1) is calculated first. The multiplication complexity \( (\gamma_{PSO}) \) of the said algorithm is,

\[
\gamma_{PSO} = N_p (N_iN_r)
\]  

(6.15)

Velocity update in PSO and pheromone updates require \( \mu_{vel} \) additional multiplications per iteration from (6.11). To reduce some complexity \( w=1 \) and \( \varphi_1 = \varphi_2 = 1 \) are assumed. Therefore \( \mu_{vel} \) becomes 2, the complexity becomes,

\[
\gamma_{PSO} = N_p (N_iN_r + \mu_{vel})
\]  

(6.16)

This procedure is repeated \( N_{itr} \) times to converge to the near-optimal BER performance. Therefore,

\[
\gamma_{PSO} = N_p (N_iN_r + \mu_{vel})N_{itr}
\]  

(6.17)

The computational complexity for local search MPSO results in.

\[
\gamma_{MPSO} = N_i (N_iN_r + \mu_{vel})N_{itr} \gamma^{bs}
\]  

(6.18)

Where \( \gamma^{bs} \) represents the degree of local search in bits.

If the detectors take initial solution guess of ZF or VBLAST solution its complexity is also added to get the resultant complexity \( \gamma_{PSO-Total} \):

\[
\gamma_{PSO-Total} = \gamma_{PSO} + (\gamma_{VBLAST} or \gamma_{ZF})
\]  

(6.19)

From (6.1) it is obvious that the complexity of ML is exponential with \( N_i \) and \( M \). ML complexity for a 4-QAM 4x4 system is 5120 and it grows to 4.7 M for 8x8 system. This increase is even significant with higher order modulation schemes in MIMO systems with more transmitters.

A detailed complexity comparison is shown in Table 6.4 and Table 6.5. However, this complexity estimate is only meaningful in the order of magnitude sense since it is based on the number of complex multiplications only.
### Table 6.4. Computational Complexity Comparison- MQAM 2x4-MIMO system

<table>
<thead>
<tr>
<th>Method</th>
<th>16-QAM</th>
<th>32-QAM</th>
<th>64-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML Detector</td>
<td>3072</td>
<td>12288</td>
<td>49152</td>
</tr>
<tr>
<td>SPSO-MIMO using (15)</td>
<td>(N_p=10, N_t=25, mu_vel=2) 2500</td>
<td>(N_p=12, N_t=32, mu_vel=2) 3480</td>
<td>(N_p=16, N_t=38, mu_vel=2) 6080</td>
</tr>
<tr>
<td>ML complexity</td>
<td>19%</td>
<td>71%</td>
<td>88%</td>
</tr>
<tr>
<td>reduction (r_{ML} - r_{PSO})/r_{ML}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPSO-MIMO using (16)</td>
<td>(N_p=10, N_t=18, mu_vel=2, b_s=1) 3600</td>
<td>(N_p=12, N_t=25, mu_vel=2, b_s=1) 6000</td>
<td>(N_p=16, N_t=32, mu_vel=2, b_s=1) 10240</td>
</tr>
<tr>
<td>ML complexity</td>
<td>14% (more complex)</td>
<td>51%</td>
<td>79%</td>
</tr>
<tr>
<td>reduction (r_{ML} - r_{PSO})/r_{ML}</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 6.5. Computational Complexity Comparison – 4QAM N_t x N_r MIMO system

<table>
<thead>
<tr>
<th>Method</th>
<th>4x4</th>
<th>8x8</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML</td>
<td>5120</td>
<td>4.7 M</td>
</tr>
<tr>
<td>SPSO-MIMO using (19)</td>
<td>(N_p=10, N_t=10, mu_vel=2, ( \gamma_{BLAST} = 712 )) 2512</td>
<td>(N_p=20, N_t=20, mu_vel=2, ( \gamma_{BLAST} = 8864 )) 36064</td>
</tr>
<tr>
<td>ML complexity</td>
<td>51%</td>
<td>99%</td>
</tr>
<tr>
<td>reduction (r_{ML} - r_{PSO})/r_{ML}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BPSO-MIMO using (19)</td>
<td>(N_p=8, N_t=10, mu_vel=2, ( \gamma_{BLAST} = 712 )) 2152</td>
<td>(N_p=16, N_t=20, mu_vel=2, ( \gamma_{BLAST} = 8864 )) 29984</td>
</tr>
<tr>
<td>ML complexity</td>
<td>58%</td>
<td>99%</td>
</tr>
<tr>
<td>reduction (r_{ML} - r_{PSO})/r_{ML}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.13.4 BER Performance- Computational Complexity Trade-off

Table 6.6 suggests that a reasonable performance-complexity trade-off exists when a comparison of the proposed detectors is made with the exhaustive search ML detector. With 32-QAM, MIMO-SPSO detectors improves the computation time by 71% and this improvement reaches 88% with 64-QAM. Similarly 64-QAM MIMO-MPSO detection algorithm reduces the computation time by 79% approximately. For a 4x4, 4-QAM system, at 10^{-3} BER the performance of BPSO detector is 3-dB inferior to that of ML with 58% ML complexity reduction. Similarly, in 8x8, 4-QAM system, the BPSO algorithm achieves 10^{-3} BER at 4-dB more SNR than ML, while ML complexity reduction is 99%.

<table>
<thead>
<tr>
<th>Performance complexity comparison</th>
<th>2x4 32-QAM</th>
<th>4x4 4-QAM</th>
<th>8x8 4-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML and BPSO Detector</td>
<td>Complexity reduction</td>
<td>-</td>
<td>58%</td>
</tr>
<tr>
<td></td>
<td>Performance degradation at 10^{-3} BER</td>
<td>-</td>
<td>3-dB</td>
</tr>
<tr>
<td>ML and SPSO Detector</td>
<td>Complexity reduction</td>
<td>71%</td>
<td>51%</td>
</tr>
<tr>
<td></td>
<td>Performance degradation at 10^{-2} BER</td>
<td>4-dB</td>
<td>6-dB</td>
</tr>
<tr>
<td>ML and MPSO Detector</td>
<td>Complexity reduction</td>
<td>51%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Performance degradation at 10^{-2} BER</td>
<td>2-dB</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.6 Performance Complexity Trade-Off
6.13.5 Effects of Change in Algorithm Parameters and Iterations

These detection algorithms described in the previous sections converge to near optimal performance iteratively, however these algorithm also experience saturation after reaching a particular threshold BER. Therefore, iteration tuning is required for optimum performance. Figure 6.5 shows the convergence of SPSO and MPSO algorithms with an increase in iterations with random initialization. $N_{it}$ is kept 18 and 25, 16-QAM and 32-QAM systems SPSO and MPSO systems are assumed. Fig 6.8 presents the convergence pattern of BPSO with for ZF and VBLAST initial inputs. The algorithm gets saturated at around 10 iterations with VBLAST input and 15 iterations for ZF initialization case. Therefore, $N_{it}$ is kept at 10 for a 4x4 MIMO system. Choice of good initial guess has an effect on the detectors convergence as can be seen from Fig 6.8.

Fig 6.9 shows the effect of changing the algorithm parameters on the detectors performance. Values of the cognitive component ($c_1$) and social component ($c_2$) are changed. Results in Fig 6.9 assume $c_1 = c_2 = .5, 1.49$ and 2.

![Graph showing the convergence of BPSO-MIMO Detectors](image)

Fig. 6.8. BPSO-MIMO Detectors Convergence with iterations at 15-dB.

Larger values of social and cognitive components results in an improvement in BER performance. A possible reason is frequent fly over and coming back to better solution is achieved with higher $c_1$ and $c_2$ values.
6.13.6 Effect of Initial Guess on the Performance

Choice of an educated initial guess plays a vital role towards the performance of these proposed detection methods. Fig 6.8 suggests that the BER performance is better in case of VBLAST as initial guess to the PSO detector as compared to ZF. This appears logical as well, since a better initial solution will result into better further refined solution. However, the complexity of VBLAST is more than ZF detector. The computational complexity of ZF and VBLAST detectors is computed in (6.4) and (6.6). In the case of ACO based detection the results in Table 6.2 give a fair idea regarding the effect of initial solution guess. ZF initialized BA-MIMO detector has lesser complexity as compared to VBLAST initialized BA-MIMO detector, however at the cost of lower BER performance. It may observed be that even when VBLAST is supplied as initial solution guess, the complexities of these detectors do not increase significantly. Therefore, proposed VBLAST initialized PSO and ACO algorithm assisted MIMO detection algorithms becomes a useable approach for practical MIMO systems.
6.13.7 **Comparison of Different PSO-MIMO Detection Techniques**

Different PSO algorithms such as Standard PSO, Memetic PSO and Binary PSO were used to optimize the exhaustive-search ML detection problem in a MIMO communications system. BPSO-MIMO detection algorithm performs the best among the other PSO-MIMO techniques. The reason for efficient performance of BPSO-MIMO detection algorithm is the inherent binary nature of the MIMO detection problem. The MIMO detection fitness function is best optimized using Binary PSO algorithm. That is why the binary version of PSO has outperformed the other two MPSO-MIMO and SPSO-MIMO detection algorithms in BER performance as well as computational complexity.

6.13.8 **Analysis of PSO Algorithms as MIMO Detection Technique**

Particle Swarm Intelligence assisted detection approach shows promising results. Their simple mathematical model, reduced implementation complexity, resistance to being trapped in local minima and guaranteed convergence to a reasonable solution in lesser iterations make these nature inspired techniques suitable for real-time symbol detection in MIMO system. PSO algorithms imitate nature’s own ingenious ways to explore the search space to find out an optimal solution from a complex ML cost surface. The efficiency of these algorithms also lies in a simple computer code in the central algorithm with few parameters to tune. Exploratory-exploitive search approach which is an essence of PSO makes it an efficient ML search optimizer. The reduction in computation time with higher order modulations and more transmitting antennas makes these proposed detection algorithms particularly useful for high data rate communication systems.

6.13.9 **Discussion**

These Particle Swarm Intelligence meta-heuristics proved to be powerful ML function optimizers. Their simple model with lesser implementation complexity makes them suitable for this NP-hard MIMO detection problem. PSO optimized MIMO symbol detection methods approach near-optimal performance with significantly reduced computational complexity, especially for higher constellation systems with multiple transmitting antennas and larger constellation alphabet sizes, where conventional ML detector is computationally expensive and non practical to implement. The simulation results suggest that the proposed PSO detectors
reduces the ML computational complexity by as high as 99% with near optimal BER performance for an 8x8 MIMO system. Therefore, these proposed detection algorithms are particularly suitable for future high-speed wireless communications systems employing multiple antennas and higher order modulation schemes.

### 6.14 Fitness Landscape Analysis of MIMO Detection Problem

Fitness landscapes theory [139], [140] was developed originally to provide an analytical framework to analyze the behavior of evolutionary optimization problems. It has proved to be a useful method to understand the performance and behavior of combinatorial optimization algorithms. Now we perform the fitness landscape analysis of MIMO detection NP-hard problem to analyze the usefulness of PSO and ACO algorithms.

Generally commenting, a fitness landscape is a representation of search space and here in our case ML search space. One can visualize it as a mountainous area with hills, valleys and craters.

Formally, a fitness landscape of combinatorial optimization problem instance consists of a set of solutions similar to ML search space solutions and an objective function like the one shown in (6.1). In a minimization problem similar to (4.3), a heuristic algorithm can be considered as navigating through the landscape to find the lowest point as shown in Fig 6.10 and Fig 6.11. A number of maxima and minima can be observed. The exhaustive search traverses the complete cost surface to find out the optimal solution. Fig 6.10 presents 3-D view of MIMO detection fitness landscape. The system considered for this plot is 2x2 64-QAM system. X-axis represents the symbols transmitted from transmitter-1, these range from 0 to 63. Y-axis represents 0 to 63 symbols transmitted by transmitter-2 against each of the symbols transmitted from transmitter-1. Fig. 6.10 gives a congested plot of a number of maxima’s and minima’s, therefore, this graphical representation helps us to visualize the complexity of MIMO detection problem. This also suggests the use of heuristics methods as done in this thesis to find the near optimal solution from this complex landscape. It will be difficult to plot higher configuration system
using more than two transmitter and receivers, since the dimensions will be large enough for representation.

Fig 6.10. 3-D View of Fitness Landscape for 2x2 64 QAM MIMO system

In order to see a less congested plot we can reduce the constellation size to 16 QAM. Fig 6.11 shows the fitness landscape of MIMO ML detection for a 2x2 16 QAM system. A 16 X 16 matrix showing the numerical values of the fitness function plotted in Fig. 6.11 is also shown on next page. The entries of the matrix show the ML fitness values for complete ML search space. The exhaustive search method finds the smallest value in the entire fitness landscape. The proposed meta-heuristics assisted MIMO detection approach traverses the fitness landscape in search of global minima.

Fig 6.11. 3-D View of Fitness Landscape for 2x2 16 QAM MIMO system
6.15 Conclusions

ACO and PSO algorithm’s assisted un-coded MIMO optimized detection algorithms were presented. In case of BA-MIMO detection method a considerable reduction in un-coded MIMO ML computational complexity with near optimal BER performance is observed. In case of PSO algorithms different PSO variants like standard PSO, BinPSO and memetic PSO are applied. Binary versions of these algorithms present better performance owing to the binary nature of the problem.
7.1 Introduction

Spatial diversity is one of the well-known techniques used to mitigate the adverse effects of a multipath fading channel as already highlighted in the Chapter 3. It is achieved by deploying multiple antennas either at the transmitter or receiver, or at both the sides. Depending on whether multiple antennas are used at the transmitter or receiver, space diversity can be classified into receive diversity and transmit diversity. In the former case, multiple antennas are used at the receiver side to pick up independent copies of the transmitted signals whereas, in the later case, multiple antennas are deployed at the transmitter side. In both the cases, multiple copies of the transmitted signals are combined to mitigate the multipath fading effects. Space-time block coding (STBC) is a transmit diversity technique in which the data stream to be transmitted is encoded in blocks, that are distributed among multiple antennas and across time. Alamouti [130] proposed a simple STBC scheme for wireless communication systems using two transmit antennas and a linear maximum-likelihood (ML) decoder which was generalized by Tarokh et. al. in [15] to an arbitrary number of transmit antennas by applying the theory of orthogonal designs. The main advantage of these schemes is a decoder based on linear processing.

A constellation reduction technique based decoding algorithm that simplifies the ML detection in Orthogonal Space-Time Block Coded systems has been introduced. The proposed approach reduces the computational complexity of these schemes while preserving the ML performance.
In this chapter STBC scheme developed by Alamouti and Tarokh are used but the idea presented here can easily be generalized to other STBC schemes such as $G_6$ systems.

### 7.2 Space-Time Block Codes in Quasi-Static Channel

#### 7.2.1 STBC with two transmit antennas

Consider an Alamouti coded wireless communication system [130] employing two transmit antennas and one receive antenna. At each signaling interval, two complex symbols $s_1$ and $s_2$ are passed through an STBC encoder which transmits the symbols according to the following space-time matrix

$$
C = \begin{bmatrix}
c_{ij}
c_{ij}
\end{bmatrix} = \begin{bmatrix}
s_1 & s_2 \\
s_2^* & s_1^*
\end{bmatrix}
$$

(7.1)

In the space-time matrix defined in (7.1), the entry $c_{ij}$ is transmitted by transmitter $j$ at time $i$. We assume flat frequency, quasi-static fading. The received signal at time $i$ is

$$
r_{i} = \sum_{j=1}^{2} h_j c_{ij} + n_i
$$

(7.2)

Here, $n_i$ is complex additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_n^2$. The channel is assumed to be Rayleigh fading with zero mean and variance $\sigma_h^2 = 1$. Channel gain from transmitter $j$ to receiver $i$ is $h_j$, which is constant for two symbol periods.

From (7.2), the received signal in two symbol intervals can be written as

$$
r_1 = h_1 s_1 + h_2 s_2 + n_1
$$

(7.3)

$$
r_2 = -h_1 s_2^* + h_2 s_1^* + n_2
$$

(7.4)

The channel model can be written in matrix form as
where $s$ is the transmitted vector, $r$ is the received vector, $n$ is the complex noise vector and $H$ is the channel matrix given as:

$$s = [s_1 \quad s_2]^T, r = [r_1 \quad r_2]^T, n = [n_1 \quad n_2]^T \quad \text{and} \quad H = \begin{bmatrix} h_1 & h_2 \\ h_1' & -h_2' \end{bmatrix}$$

Perfect estimate of Channel State Information (CSI) is assumed to be available at the receiver.

### 7.2.1.1 Conventional Maximum Likelihood Detection

The optimum detector computes the estimate of the transmitted symbol vector $s$ from the received signal vector $r$ in (7.5) according to Maximum Likelihood rule

$$\hat{s} = \arg \min_{s \in \chi} \|r - Hs\|^2$$

(7.6)

where $\chi$ is the constellation of size $|\chi| = 2^b$ with $b$ bits per symbol. Here, we can use the orthogonality property of channel matrix $H$

$$H^H H = \left( |h_1|^2 + |h_2|^2 \right) I_2$$

(7.7)

Using the above property, the decision statistic vector can be defined as [131]

$$\tilde{r} \equiv H^H r = H^H (H s + n)$$

(7.8)

$$\tilde{r} = \left( |h_1|^2 + |h_2|^2 \right) s + \tilde{n}$$

(7.9)

where $\tilde{n} = H^H n$ is the transformed noise vector with zero mean and a conditional covariance matrix as

$$\left( |h_1|^2 + |h_2|^2 \right) \sigma_n^2 I_2$$

(7.10)
Using (7.9), the estimate of transmitted symbols $s_1$ and $s_2$ can be computed independent of each other using conventional maximum-likelihood (ML) criterion in (7.6) simplified to

$$\hat{s}_i = \arg\min_{s \in \mathcal{X}} \left| \hat{r}_i - \left( |h_1|^2 + |h_2|^2 \right) s \right|^2 \quad (7.11)$$

The search is performed over the entire signal constellation. Thus an Alamouti encoded space-time system employing 16 QAM/PSK modulation scheme will perform $16 \times 2 = 32$ searches in order to estimate the two transmitted symbols.

### 7.2.1.2 Simplified Maximum-Likelihood Detection

Alamouti encoded system with 16 QAM/PSK modulation scheme will have to look for 32 distinct combinations in order to estimate the two transmitted symbols.

The proposed technique is based on constellation reduction. The method is quite general but for the sake of simplicity we explain the idea for 16-QAM/PSK. After applying orthogonal transformation $\mathbf{H}^H$ on the received signal vector $\mathbf{r}$, we come up with the decision statistics vector $\mathbf{\hat{r}}$, which forms the soft estimate of the transmitted vector $\mathbf{s}$.

At this stage, to find the estimate of each transmitted symbol, conventional ML detector performs a search over all the sixteen constellation points per symbol and decides in favor of the one that minimizes the criteria in (7.11) in order to find the estimate of transmitted symbol.

Based on this observation, we present a simplified ML detector that uses a set of four reference constellation points in four different quadrants as shown in Fig 7.1 and Fig 7.2.

$$\kappa = \{a + jb, -a + jb, -a - jb, a - jb\} \text{ for } a, b > 0 \quad (7.12)$$

The simplified ML estimate of a symbol is obtained in two steps. In the first step, the ML criteria in (7.11) is applied to the reference points in (7.12) (instead of the entire constellation points) to reduce the search space for symbol under detection to that particular quadrant whose reference point minimizes the criteria in (7.11). During the second step, a lookup table is used in order to get the other constellation points associated with that reference constellation point in
(7.12) and then ML search in (7.11) is once again performed on the remaining points only. The symbol, which minimizes the ML criteria this time, is taken as ML estimate of the transmitted symbol. Note that there is no need to apply the ML criteria to the reference constellation point the second time.

In case of 16-QAM modulation, this method gives the estimate of the two symbols by performing only 14 ML searches.

When applied to 16-PSK modulation, this technique gives the estimate of two transmitted symbols in 16 ML searches. This is due to the fact that two of the constellation points lie exactly on quadrant boundaries that can be computed by using rotated PSK constellation

The complexity reduction is even more significant for higher modulation levels (more than 16)

Fig. 7.1 SML algorithm employing 16-QAM Alamouti system
7.2.1.3 Performance Results

Two transmit and one receive antenna Alamouti scheme under flat fading, quasi-static Rayleigh channel is assumed, where the channel gains remain constant for two signaling intervals.

In Fig 7.3 and Fig 7.4 we used Alamouti system with 16-QAM and 16-PSK modulations respectively in a quasi-static Rayleigh fading channel. BER results show that the proposed Simplified ML detection possesses optimal BER performance in both modulation schemes.

7.2.1.4 Computational Complexity

As we have already seen that, Alamouti space-time block coded system employing 16-QAM/PSK modulation performs 16x2=32 searches in order to estimate the two transmitted symbols if it uses a convention ML detector.

In contrast, for the same case of 16-ary modulations, the simplified ML detector first performs four searches to identify the quadrant in which the received symbol is located (which reduces the search space). In second step, it then further performs three searches in case of 16-QAM and four searches in case of 16-PSK to form the estimate of one symbol. Collectively, it performs only 7x2=14 searches for 16-QAM modulation and 8x2=16 searches for 16-PSK modulation in
order to compute the estimates of two transmitted symbols. Note that this complexity reduction is even more significant for higher modulation levels.

Apart from low computational complexity, the proposed simplified ML detector gives us the same performance as the conventional ML as presented in [130].

Fig. 7.3 Performance comparison of the proposed SML detector with ML using 16-QAM Alamouti System

Fig. 7.4 Performance comparison of the proposed SML detector with ML using 16-PSK Alamouti System
From simulation results, it is evident that the simplified ML detector performs analogous to the conventional ML detector and that too with less than 50% reduction in computational complexity in case of QAM and exactly 50% reduction in computational complexity for PSK modulation. Therefore, the proposed constellation reduction algorithm can be considered as a promising approach. This technique is equally applicable to the other STBC schemes.

### 7.2.2 STBC with four transmit antennas

Consider an $\mathcal{H}$–encoded wireless communication system employing four transmit antennas and one receiving antenna. At each signaling interval, three complex symbols $s_1, s_2$ and $s_3$ are passed through an $\mathcal{H}$ encoder which transmits the symbols according to the following space-time matrix

$$
C = [c_j] = \begin{bmatrix}
  s_1 & s_2 & \frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} \\
  -s_1 & s_2 & \frac{-s_3}{\sqrt{2}} & \frac{-s_3}{\sqrt{2}} \\
  \frac{s_1}{\sqrt{2}} & \frac{s_2}{\sqrt{2}} & \frac{s_3}{2} & \frac{-s_3}{2} \\
  \frac{-s_1}{\sqrt{2}} & \frac{-s_2}{\sqrt{2}} & \frac{-s_3}{2} & \frac{s_3}{2}
\end{bmatrix}
$$

(7.13)

where $c_{ij}$ is transmitted by transmitter $j$ at time $i$. Let the channel gain from transmitter $j$ to receiver at time $i$ be $h_j(i)$. The received signal at time $i$ is

$$
r_i = \sum_{j=1}^{4} h_j(i)c_{ij} + n_i
$$

(7.14)

Here, $n_i$ is zero mean, complex additive white Gaussian noise (AWGN) with variance $\sigma_n^2$. Channel is assumed to be Rayleigh fading with zero mean and variance $\sigma_h^2 = 1$. Path gain from transmitter $j$ to receiver is $h_j$, which is constant for two symbol periods. Assuming perfect estimate of Channel State Information (CSI) available at the receiver, the manipulated received signal (MRS) vectors are defined as follows $[15][131]$.

$$
g_i = \begin{bmatrix} r_i & r_2 & \frac{r_3 - r_1}{\sqrt{2}} & \frac{(r_3 + r_1)^*}{\sqrt{2}} \end{bmatrix}^T
$$

(7.15)
To obtain an expression of the channel matrix in relation to the above MRS vectors, the real and imaginary parts of the transmitted symbols are considered separately [131]. Let, 
\[ s_j = x_i + j y_i \] 
then 
\[ s = [s_1, s_2, s_3]^T = x + j y \] 
where 
\[ x = [x_1, x_2, x_3]^T, \quad y = [y_1, y_2, y_3]^T \]

Now using (7.13)-(7.17) the expression for the channel matrix in relation to the MRS vectors is found to be [131]

\[
g_i = H_i^g x + j^* H_i^y y + v_i \quad i = 1, 2, 3
\] 

where \( v_i \) is corresponding AWGN noise vector with zero mean and variance \( \sigma_n^2 \). The channel matrices are

\[
H_i^g = \\
\begin{bmatrix}
  h_i(1) & h_i(1) & \frac{h_i(1)+h_i(1)}{\sqrt{2}} \\
  h_i^*(2) & -h_i^*(2) & \frac{h_i(2)\!-\!h_i(2)}{\sqrt{2}} \\
  \frac{h_i(3)-h_i(4)}{\sqrt{2}} & \frac{h_i(4)+h_i(3)}{\sqrt{2}} & \frac{h_i(3)-h_i(4)+h_i(3)-h_i(4)}{2} \\
  \frac{h_i(3)+h_i(4)}{\sqrt{2}} & \frac{h_i(4)-h_i(3)}{\sqrt{2}} & \frac{h_i(3)+h_i(3)+h_i(4)-h_i(4)}{2} \\
\end{bmatrix}
\]

\[
H_i^y = \\
\begin{bmatrix}
  h_i(1) & h_i(1) & \frac{h_i(1)+h_i(1)}{\sqrt{2}} \\
  h_i^*(2) & -h_i^*(2) & \frac{h_i(2)\!-\!h_i(2)}{\sqrt{2}} \\
  \frac{h_i(4)-h_i(3)}{\sqrt{2}} & \frac{h_i(4)+h_i(3)}{\sqrt{2}} & \frac{h_i(3)+h_i(4)+h_i(3)+h_i(4)}{2} \\
  \frac{h_i(4)+h_i(3)}{\sqrt{2}} & \frac{h_i(4)-h_i(3)}{\sqrt{2}} & \frac{h_i(3)+h_i(3)+h_i(4)-h_i(4)}{2} \\
\end{bmatrix}
\]

\[
H_i^v = \\
\begin{bmatrix}
  h_i(1) & h_i(1) & \frac{h_i(1)+h_i(1)}{\sqrt{2}} \\
  h_i^*(2) & -h_i^*(2) & \frac{h_i(2)\!-\!h_i(2)}{\sqrt{2}} \\
  \frac{h_i(3)-h_i(4)}{\sqrt{2}} & \frac{h_i(4)+h_i(3)}{\sqrt{2}} & \frac{h_i(3)+h_i(4)+h_i(3)+h_i(4)}{2} \\
  \frac{h_i(3)+h_i(4)}{\sqrt{2}} & \frac{h_i(4)-h_i(3)}{\sqrt{2}} & \frac{h_i(3)+h_i(3)+h_i(4)-h_i(4)}{2} \\
\end{bmatrix}
\]
Chapter 7 Symbol Detection in Coded Multi-antenna Systems

7.2.2.1 Conventional ML Detector

As we know that the conventional detector always assumes the channel to be quasi-static. This implies constant channel gains for $4T_s$ i.e. $h_j(i) = h_j$. Substituting this condition for quasi-static channel into (7.18) we get

$$\mathbf{R}_i = \mathbf{H}^n_i \mathbf{x} + j^* \mathbf{H}_i^t \mathbf{y} + \mathbf{v}_i \quad i = 1, 2, 3$$  \hspace{1cm} (7.19)

where the expressions for the perceived channel matrices $\mathbf{R}_i$, $\mathbf{H}_i$ can be found by substituting the assumed channel condition into the $\mathbf{H}^n_i$, $\mathbf{H}_i$ expressions above for $i = 1, 2, 3$.

An important property of the channel matrices $\mathbf{H}^n_i, \mathbf{H}_i$ for a quasi-static channel given in [132] is “there exists a solution to the following zero-forcing equation with respect to $\theta_i$”

$$\theta_i \mathbf{H}^n_i = \lambda_i \left[ \delta_i, \delta_{i + 1}, \delta_{i + 2} \right]$$  \hspace{1cm} (7.20)

$$\theta \mathbf{H}_i = \lambda_i \left[ \delta_i, \delta_{i + 1}, \delta_{i + 2} \right]$$  \hspace{1cm} (7.21)
where $\lambda_i = \sum_{j=1}^{4} |h_{ij}|^2$ and $\delta_{ij}$ is Kronecker delta.

Such a solution is given as

$$\theta_i = [\tilde{H}_i^T(c,i)]^H = [\tilde{H}_i(c,i)]^H$$ (7.22)

where $A(:,i)$ means the $i^{th}$ column of matrix $A$.

Now from above property, the conventional detector in [15] is defined using a two-step procedure shown below [131].

To detect symbol $s_i = x_i + j y_i$, $i = 1, 2, 3$

**Step-I**

Apply the linear transform, $\theta_i$, in (7.22) to $g_i$ in (7.16)

$$g_i' = \theta_i g_i = \theta_i \tilde{H}_i^T x + j \theta_i \tilde{H}_i^T y + \theta_i v_i = \lambda_i s_i + v_i'$$ (7.23)

**Step-II**

Perform minimum distance search criteria as follows

$$\hat{s}_i = \arg \left\{ \min_{s \in S} |g_i' - \lambda_i s|^2 \right\}$$ (7.24)

where $S$ is the symbol alphabet. It can be shown that $E[v_i'] = 0$ for $j \neq i$.

### 7.2.2.2 Simplified Maximum-Likelihood Detection

In all the simulation of the $\mathcal{H}_4$ system presented assume Gray encoded 16-PSK and 16-QAM system. In this $\mathcal{H}_4$ system will have to look for 48 distinct combinations in order to estimate the four transmitted symbols.

The approach applied is similar to the constellation reduction technique used in previous case of STBC system using two transmit antennas. The method is quite general but for the sake of
simplicity we explain the idea for 16-QAM/PSK. After applying orthogonal transformation $H^T$ on the received signal vector $r$, we come up with the decision statistics vector $\hat{r}$, which forms the soft estimate of the transmitted vector $s$.

At this stage, to find the estimate of each transmitted symbol, conventional ML detector performs a search over all the sixty four constellation points per symbol and decides in favor of one that minimizes the criteria in (7.24) in order to find the estimate of transmitted symbol.

Based on this observation, we present a simplified ML detector that uses a set of four reference constellation points in four different quadrants as shown in Fig. 7.1, Fig.7. 2 and (7.12).

The simplified ML estimate of a symbol is obtained in two steps. In first step, ML criteria in (7.24) is applied to the reference points in (7.12) (instead of the entire constellation points) to reduce the search space for symbol under detection to that particular quadrant whose reference point minimizes the criteria in (7.24). During the second step, a lookup table is first consulted in order to get the other constellation points associated with that reference constellation point in (7.24) and then ML search in (7.12) is once again performed with only these remaining points. The symbol, that minimizes the ML criteria this time, is taken as ML estimate of the transmitted symbol. Note that there is no need to apply the ML criteria to the reference constellation point the second time.

In case of 16-QAM $H_4$ system, this method gives the estimate of the four symbols by performing only 21 ML searches. Similarly with 16-PSK modulation, this technique estimates the four transmitted symbols in 24 ML searches. This is due to the fact that two of the constellation points lie exactly on quadrant boundaries, which can be addressed by using rotated PSK constellation. The complexity reduction is even more significant for higher modulation levels (more than 16).
7.2.2.3 Simulation Results

We assume four transmit and one receive antenna SML constellation Reduction algorithm 16-QAM $H_4$ system under flat fading, quasi-static Rayleigh channel where the channel gains remain constant for three signaling intervals.

In Fig.7.5 and Fig.7.5 we used $H_4$ system with 16-PSK and 16-QAM modulations respectively in a quasi-static Rayleigh fading channel. BER results show that the proposed simplified ML detection possesses optimal BER performance in both modulation schemes.

7.2.2.4 Computational Complexity

As we have already seen, $H_4$ space-time block coded system employing 16-QAM/PSK modulation performs $16 \times 3 = 48$ searches in order to estimate four transmitted symbols if it uses a convention ML detector.

In contrast, for the same case of 16-ary modulations, the simplified ML detector first performs four searches to identify the quadrant in which the received symbol is located (which reduces the search space). In second step, it then further performs three searches in case of 16-QAM and four searches in case of 16-PSK to form the estimate of one symbol. Collectively, it performs only $7 \times 3 = 21$ searches for 16-QAM modulation and $8 \times 3 = 24$ searches for 16-PSK modulation in order to compute the estimates of four transmitted symbols. Note that this complexity reduction is even more significant for higher modulation levels.

Apart from low computational complexity, the proposed simplified ML detector results in the same performance as the conventional ML as presented in [15].
Fig. 7.5 Performance of proposed SML detection with conventional 16-PSK H4 OSTBC ML

Fig. 7.6 Performance of proposed SML detection with conventional ML in 16-QAM H4 OSTBC system
7.3 Space-Time Block Codes in Time-Selective Channel

The relative motion between the transmitter and the receiver makes the channel time-selective and hence the transmitted symbols can no more be decoupled from the received signal. This phenomenon is illustrated below:

\[ \tilde{r} \equiv H^H r = H^H (H s + n) = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} s + \tilde{n} \quad (7.25) \]

The performance of conventional ML detector given in (7.11) is degraded severely under such scenarios. The following proposed method explained below suggests an approach to decode the transmitted symbols with enhanced BER performance and reduced complexity as compared to the conventional ML detection.

7.3.1 Proposed Multi-Step Constellation Reduction Method for Fast Fading Channels

An Alamouti encoded system with M-QAM modulation scheme will have to look for 2M searches as shown in (9) in order to estimate the two transmitted symbols. However, due to the time selective nature of the channel the BER performance degrades.

The proposed technique is based on joint ML detection with reduced search space. The method is quite general but for the sake of simplicity we explain the idea for 16-QAM and 64-QAM constellations. The proposed multi-step reduced space (MSRS) detection algorithm is outlined below:
(a) Obtain an initial zero-forcing guess \( \hat{\mathbf{r}}_{ZF} \) as follows:

\[
\hat{\mathbf{r}}_{ZF} = \mathbf{H}^{-1} \mathbf{r}
\]  

(7.26)

(b) A local search around \( \hat{\mathbf{r}}_{ZF} \) is performed. From each element \( r_i^{ZF} \) in \( \hat{\mathbf{r}}_{ZF} \), a neighbor list is generated as shown in Fig 7.7.

(c) Joint ML detection according to (6) is then performed in the reduced search space generated from the neighbor lists, to decode the transmitted symbols.

The MSRS scheme generates the reduced search space based on soft estimate \( \hat{\mathbf{r}}_{ZF} \). The proposed idea is to first choose four reference points in the constellation one in each quadrant. Keeping the neighbor size as four we generate neighbor list using a lookup table with \( r_i^{ZF} \) as input. For example, in 16-QAM, their will be four entries in the lookup table, with each containing four constellation points as shown in Fig-7.8.

Joint ML detection (7.6) with search space generated from only these two neighbor lists is performed to find the two transmitted symbols. Similarly, in 64-QAM, the algorithm will be applied in multiple steps. This proposed method is termed as Multi-step Reduced Space detection algorithm for OSTBC under time-selective fading.
7.3.2 Simulation Results

We assume two transmit and one receive antenna Alamouti scheme under Time-Selective Rayleigh channel where the channel gains do not remain constant for two signaling intervals.

In Fig 7.9 and Fig 7.10 we used Alamouti system with 16-QAM and 64-QAM modulations respectively. The BER results show that the proposed multi-step reduced constellation ML detection approach presents significantly enhanced BER performance in both modulation schemes in comparison to the conventional ML detection. The performance of the proposed method is also reasonable in comparison to the approach presented in [138].
Fig 7.9: BER performance of proposed detection algorithm in a 16-QAM Alamouti system

Fig 7.10: BER performance of proposed detection algorithm in a 64-QAM Alamouti system
7.3.3 **Computational Complexity Analysis**

As we have already seen that, Alamouti space-time block coded system employing 16-QAM/PSK modulation performs $16 \times 2 = 32$ searches in order to estimate the two transmitted symbols if it uses a conventional ML detector.

In contrast, for the same case of 16-ary modulations, the simplified ML detector first performs four searches to identify the quadrant in which the received symbol is located (which reduces the search space). In second step, it further performs joint detection to decode the two transmitted symbols using each of the neighbor lists. ML detection on the reduced constellations with four symbols will require 16 searches. Therefore, the proposed technique requires 20 searches in comparison to 32 searches for the conventional ML detectors and the detector presented in [138].

Note that this complexity reduction is even more significant for higher modulation levels. As in case of 64-QAM the complexity of the proposed multi-step constellation reduction method reduces to half in comparison to the other two detectors.

Apart from low computational complexity, the proposed detector presents a slightly enhanced BER performance as compared to Tran et. al detector [138] proposed for Space-Time Block Codes in Time-Selective Channel.

### 7.4 Conclusions

In this chapter we discussed some simple multiple step constellation reduction techniques that further reduce the already linear computational complexity of STBC detection. Similar approaches have been employed in MIMO and few other systems however these have not been used in STBC systems. Simple ML detection is first applied on Alamouti Systems which result in optimal BER performance with almost 50% reduction in its conventional ML complexity. The concept is further extended to Tarokh systems using three transmit antennas. The performance of the proposed simple ML detection is optimal with 50% reduction in computational complexity.

A multi-step reduced search space detection algorithm for OSTBC under time-selective
fading is also discussed. The algorithm has been applied on 16-QAM and 64-QAM Alamouti encoded system. The simulation results suggest that the proposed MSRS algorithm presents significantly enhanced BER performance with reasonably reduced computational complexity as compared to the conventional ML detection method. The proposed detector also shows competitive performance as compared to Tran et. al. detector for time selective channel. Therefore, the proposed constellation reduction algorithm can be considered as a promising approach for fast fading OSTBC communication systems. Such coded Multi-Input Multi-Output (MIMO) systems will form the core for wireless communications standards like IEEE 802.11n, IEEE 802.16e and LTE. The particular application of this approach is high speed trains. This technique is equally applicable to the other modulation as well as STBC schemes.
Chapter 8

Orthogonal Multiplexed MIMO Systems

8.1 Introduction

MIMO systems are well known for their unprecedented performance gains in terms of system capacity and diversity as already discussed in Chapter 3. The design of MIMO systems for achieving spatial diversity gains has been an active research topic. A number of MIMO architectures have been developed for in the last couple of years. The Vertical Bell-laboratories LAYered Space-Time (V-BLAST) system is one promising implementation of MIMO systems. However, the performance of the V-BLAST detection scheme, is limited due to imperfect interference cancellation and insufficient receive diversity. Maximum Likelihood (ML) detection scheme performs the best, but its complexity increases exponentially when numbers of transmit antennas increase and complex modulation schemes are used. Combining Code Division Multiple Access (CDMA) with MIMO enable complete exploitation of spatial and temporal diversity of mobile radio channels [133-135].

We propose an Orthogonal Coded MIMO (OCM) system [136] in which Walsh coded BPSK modulated user-data bits are added together (multiplexed) before transmission. The receiver de-spreads the received signal and applies conventional detection techniques to jointly decode the transmitted symbols. The significance of this approach is relative enhancement in Bit Error Rate (BER) performance and reduction in the symbol detection complexity in comparison to the conventional MIMO system. The proposed system enables the codes reuse for different transmitters. This approach results in reduction in the systems bandwidth in comparison with MIMO-CDMA systems since the spreaded bits are added together before transmission in the MIMO channel.
8.2 System Model

Consider an OCM system in which user-data bits are Walsh coded, BPSK modulated and then added together before transmission from \( N_t \) transmit antennas, as shown in Fig. 8.1. Where \( N_t \) is kept equal to the Walsh code length \( N \) to reduce the effects of bandwidth expansion. These signals arrive at receive antenna having \( N_r \) elements (\( N_t \leq N_r \)), after propagating through a flat Rayleigh fading channel. The de-multiplexer in the transmitter splits the input data \( d_i \) into \( N \) sub-streams.

The transmitted signal \( x_i \) can be represented as

\[
 x_i = \sum_{j=1}^{N} d_{i,j} w_{j-1} \quad i = 1, 2, \ldots, N \tag{8.1}
\]

The received signal at received antenna \( i \) can be represented as \( N_r \times 1 \) vector \( y_i \)

\[
 y_i = \sum_{j=1}^{N} H_{i,j} x_j + n_i \quad i = 1, 2, \ldots, N \tag{8.2}
\]

\( H \) is the channel matrix in which \((i,j)^{th}\) element \( h_{i,j} \) is complex channel gain between \( j^{th} \) transmit and \( i^{th} \) receive antenna and \( n \) is also zero-mean Gaussian noise vector with \( R_n = E\{n n^*\} = \sigma_n^2 I \).

The received signal at antenna \( i \), de-spread by \((j-1)^{th}\) Walsh code is
Chapter 8 Orthogonal Multiplexed MIMO Systems

\[ r_i^j = y_{j-1}^T w_{i-1} \quad i, j \in 1, 2, \ldots, N \quad (8.3) \]

Using (8.2) and (8.3) we can write

\[ r_i = H d + \eta_i \quad (8.4) \]

where \( r_i = [r_i^1 \ r_i^2 \ \ldots \ r_i^N]^T \); \( \eta_i = [\eta_i^1 \ \eta_i^2 \ \ldots \ \eta_i^N]^T \); \( d_i = [d_i^1 \ d_i^2 \ \ldots \ d_i^N]^T \) and \( H \) is represented as

\[
H = \begin{bmatrix}
h_{11} & h_{12} & \cdots & h_{1N} \\
h_{21} & h_{22} & \cdots & h_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
h_{N1} & h_{N2} & \cdots & h_{NN}
\end{bmatrix} \quad (8.5)
\]

It is assumed that the entries of the channel matrix \( H \) are known at the receiver and not at the transmitter. This is a reasonable assumption if pilot or training signals are sent to learn the channel. The channel will be constant for some coherent interval of time.

### 8.3 Detection in the Proposed System

A number of detection schemes can be used for symbol detection in the proposed transmission system. We start with the Optimal Maximum Likelihood (ML) Detection method for decoding the transmitted data.

#### 8.3.1 ML Detection

Considering equiprobable symbol transmission, Maximum Aposteriori Probability (MAP) rule on (8.4) simplifies to Maximum Likelihood (ML) detection as

\[
\hat{d}_i = \arg\min_{d \in \mathcal{X}^N} \| r_i - H d \|_2^2 \quad (8.6)
\]

where the modulation alphabet size \( \mathcal{X} = 2 \) for BPSK. (8.6) is in fact minimization of squared Euclidean distance to a target vector \( r_i \).
8.3.2 Linear Detectors

8.3.2.1 Zero Forcing Detection

By assuming perfect channel state information (CSI), a low-complexity linear Zero-Forcing (ZF) detection algorithm can also be applied that outputs

\[ \hat{d}_i = \circ (H^{-1}r_i) \]  

(8.7)

where \( \circ \) is a quantizer that maps its argument to nearest signal point in the constellation.

8.3.2.2 QR decomposition based detection

Channel matrix \( H \) can be decomposed as \( H = QR \) where \( Q \) is a unitary matrix and \( R \) is an upper triangular matrix. Hence (4) can now be written as

\[ r_i = QRd_i + \eta_i \]  

(8.8)

As \( Q \) is unitary so we can apply the linear transformation \( W = Q^H \) on (8.8) as

\[ \mathbf{u}_i = W\mathbf{r}_i = R\mathbf{x} + \mathbf{w}_i \]  

(8.9)

Where \( \mathbf{w}_i = Q^H\eta_i \). Transmitted data bits can now be extracted iteratively from (8.9).

8.3.2.3 Minimum Mean Square Error

MMSE detector is a linear detector that minimizes the mean square error between the desired symbols and the output of the detector

\[ \hat{d}_i = \circ (W\mathbf{r}_i) \]  

(8.10)

Where \( W \) is chosen to minimize the mean square error as \( E\left[ \|W\mathbf{r}_i - \mathbf{d}_i\|^2 \right] \).

8.3.3 Non-Linear Detectors

The Successive Interface Cancellation (SIC) detection algorithm in [1] can be applied to (8.4) to estimate the data bits. SIC is a non-linear recursive procedure that extracts the components of the transmitted vector \( \mathbf{d} \) according to a certain ordering of the elements of \( \mathbf{r} \) based on channel realization \( H \).
8.4 Performance Results

8.4.1 Experimental Setup
We evaluate the performance of the proposed OCM system and compare it with an equivalent uncoded MIMO system using BPSK and 4-QAM modulations. The SNR (E_b/N_0) is the average signal to noise ratio per antenna (P/σ_v^2) where P is the average power per antenna and σ_v^2 is the noise variance. The constellations are normalized to have an average energy of 1-watt. The simulation environment assumes Rayleigh fading channel. The channel is assumed to be quasi-static for each symbol, but independent among different symbols. Perfect channel state information (CSI) is assumed. Walsh codes of length N are generated using Hadamard function.

8.4.2 BER Versus SNR Performance

Fig. 8.2 depicts the BER performance of the proposed 4x4 OCM system. BER comparison of the proposed 4x4 system with 4x4 4-QAM un-coded conventional MIMO system is shown in Fig. 8.3 to Fig. 8.5. Optimal ML, Linear ZF, QR, MMSE and non-linear SIC detectors are compared. While Fig 8.6, gives the performance comparison of 4x4-4-QAM proposed system with 4x4 un-coded MIMO system employing BPSK modulation.
Chapter 8 Orthogonal Multiplexed MIMO Systems

Fig. 8.3: BER performance of ML detection in the proposed 4x4-OCM and 4x4-4 QAM un-coded MIMO system

Fig. 8.4: BER performance comparison for Linear detectors

Fig. 8.5: BER performance comparison for non-linear detectors
Fig. 8.6: BER comparison of the proposed 4-QAM 4x4-OCM system and 4x4-BPSK un-coded MIMO system

The results in Fig. 8.3 to Fig. 8.5 suggest that the proposed Spatial Multiplexing technique offers 4-dB and 7-dB BER improvement for ML and SIC detectors in comparison to 4x4 4-QAM conventional un-coded MIMO system. The BER performance enhancement in case of linear detectors presented in Fig. 8.4 is also appreciable. ZF, QR and MMSE linear detectors enhance the BER performance by 6-dB, 7-dB and 5-dB. Performance of the proposed architecture remains same as that of 4x4 BPSK un-coded MIMO system as given in Fig. 8.6.

8.4.3 Computational Complexity Comparison

Here we examine the computational complexity of the optimal ML detector of the proposed Spatial Multiplexing system and compare it with conventional MIMO ML.

ML performs an exhaustive search over all possible symbol combinations \((8.6)\). Its complexity is exponential with \(N_t\) and modulation alphabet size \(\chi\). Conventional MIMO-ML has complexity \(O(\chi^N)\). Whereas, proposed OCM-ML detector offers a complexity \(O(N, 2^\chi)\). Here \(\chi = 2\) for BPSK. For a 4x4 MIMO-ML employing 4-QAM modulation scheme, the complexity is 256 and in case of OCM-ML detector it reduces to 64.
8.4.4 Performance Analysis

The proposed OCM system results in an enhanced BER performance with reduction in receiver complexity. Results propose up to 7-dB BER improvement with 75% decrease in complexity as compared to 4x4 4-QAM MIMO system. However, the above gains are at the cost of bandwidth efficiency. In case of 4x4-4QAM MIMO the bandwidth efficiency $R/W$ (bits per second per hertz of bandwidth) is 8-bits/s/Hz, whereas in the proposed system with 4x4 configuration it reduces to 4 bits/s/Hz due to a spreading factor of 4.

The effect of bandwidth expansion can be reduced by using only those spreading codes that result in low bandwidth expansion, such as $W_0$ and $W_3$. Orthogonal and quasi-orthogonal can also be used to increase the choice of codes selection [137].

Another advantage of this particular approach is the addition of orthogonal data bits before transmission unlike the concept of MIMO-CDMA systems. Thus the increase in systems bandwidth is smaller in case of the proposed system as compared to the MIMO-CDMA systems. Thus the proposed approach may be more attractive once the consideration is enhanced BER, where some compromise in spectral efficiency is acceptable.

8.5 Summary

MIMO communications system with orthogonal multiplexed data streams in proposed. The system offers acceptable results as compared to the conventional un-coded MIMO and MIMO-CDMA systems. Optimal ML, Linear and Non-linear detection techniques are compared for performance analysis. The proposed architecture reduces the detection complexity for higher modulation techniques using orthogonally BPSK codes data streams. The simulation results also suggest an improvement in the BER. The trade-off due to spreading can be overcome by employing low transition orthogonal and quasi-orthogonal codes. Proposed transmitter and receiver architecture possess capability and promise to be one of the candidates for future high-speed data intensive communication systems.
Employing multiple antennas at transmitter as well as both transmitters and receiver sides enables considerable performance enhancement in wireless communications systems. Performance analysis and optimization of coded as well as uncoded Multi-antenna systems has been discussed in this thesis.

Symbol detection in uncoded MIMO systems is an active research area. In this thesis, we considered the MIMO receiver design problem in a spatial multiplexing scheme. The MIMO detection problem is equivalent to solving the integer least-squares problem which is NP-hard. In the literature, suboptimal detection algorithms, typically of polynomial complexity, are often employed, while exact algorithms, like the sphere decoder, that solve the MIMO detection problem optimality, have an average exponential complexity. MIMO-ML detection is prohibitively complex to be practically deployed. ML detection complexity grows exponential with the Modulation alphabetic size and number of transmitters. MIMO detection complexity reduction techniques based on nature inspired algorithms like Ant Colony Optimization and Particle Swarm Optimization were proposed in this thesis.

Coded multiple antenna systems such as O-STBC systems using multiple transmit antennas were also been explored. Simplified ML detection is STBC system is proposed. The presented reduced constellation search method results in optimal performance while significantly reducing the symbol detection complexity.

A multi-step reduced search space detection algorithm for OSTBC under time-selective fading is also discussed. The simulation results suggest that the proposed MSRS algorithm presents significantly enhanced BER performance with reasonably reduced computational complexity as compared to the conventional ML detection method. The proposed detector
also shows competitive performance as compared to Tran et. al. detector for time selective channel [138]. Therefore, the proposed constellation reduction algorithm can be considered as a promising approach for fast fading OSTBC communication systems. Such coded Multi-Input Multi-Output (MIMO) systems will form the core for wireless communications standards like IEEE 802.11n, IEEE 802.16e and LTE. The particular application of this approach is high speed trains reaching 300 Km/h speed mark. This technique is equally applicable to the other modulation as well as STBC schemes.

In addition a Walsh coded MIMO system was also been discussed. This approach enables usage of simple modulation techniques in spite of large constellation schemes while keeping the performance at par to the complex modulated systems.

### 9.1 Contribution to Knowledge

The work presented in this thesis has contribution toward knowledge. Following are major achievements.

a. Symbol Detection is Spatial Multiplexing Systems using an un-conventional approach has been proposed. Swarm Intelligence has been applied to optimize search in a coarse ML cost surface. The main contribution of this thesis is designing BA-MIMO and PS-MIMO detection approximate algorithms that perform optimized search of MIMO detection problems fitness landscape. The Swarm Intelligence algorithms simple model, lesser implementation complexity, resistance to being trapped in local minima, convergence to reasonable solution in fewer iterations and exploratory-exploitive search approach makes it a suitable candidate for optimizing symbol detection in MIMO systems. The proposed algorithms show promising results when compared with the optimal ML and traditional VBLAST detectors. The BA-MIMO symbol detection mechanism approaches near-optimal performance with much reduced computational complexity, especially for complex systems with multiple transmitting antennas, where conventional ML detector is computationally expensive and impractical to implement. When compared to VBLAST detector the proposed unconventional detection method results in enhanced BER performance but at the cost of increase in complexity. The simulation results suggest that the proposed detector in a 6x6 spatial multiplexing system improves VBLAST
Chapter 9 Conclusion

BER performance by 7-dB. However, the ML complexity is reduced by 94% with a reasonable BER performance. Similarly, the proposed PSO detectors reduces the ML computational complexity by as high as 99% with near optimal BER performance for an 8x8 MIMO-OFDM system. Therefore, these proposed detection algorithms are particularly suitable for future high-speed wireless communications system employing multiple antennas and higher order modulation schemes.

b. The BA-MIMO and PS-MIMO algorithms presented in Chapter 6 have been published in International conferences of repute [126][127]. The work has also been published in internationally recognized journals [125], [128].

c. The work on STBC systems presented in chapter-7 pertains to Simple ML detection in O-STBC systems with multiple transmit antennas. A low complexity optimal ML detector for Alamouti and $H_4$ encoded system based on multi-step constellation reduction technique is presented. From simulation results, it is evident that the simplified ML detector performs analogous to the conventional ML detection, however it offers more than 50% reduction in computational complexity in case of QAM and exactly 50% reduction in computational complexity for PSK modulation. Using rotated PSK constellation in which the optimum rotation angle is selected such that there is no constellation point is on the quadrant boundary can further reduce the complexity in case of PSK signaling. Therefore, the proposed constellation reduction algorithm can be considered as a promising approach. This technique is equally applicable to the other STBC schemes like $G_4$ encoded systems. This work is currently under review. Similarly, STBC systems for time-selective channels are also discussed. Reduced search space detection algorithm for fast fading environment has been proposed. The proposed approach improves performance as compared to the conventional ML detector and Tran et. al detector.

d. Orthogonally Coded MIMO systems discussed in Chapter 8 shows a way to improve the performance in uncoded MIMO systems with simple orthogonal Walsh coded transmitted data streams. The resultant Walsh coded MIMO system shows meaningful BER performance with a check on detection complexity. Some of the work on Orthogonal Coded MIMO systems was published in [136].
9.2 Future Work

Ant Colony Optimization a Particle Swarm Optimization algorithms are applied to reduce MIMO-ML detection complexity. These approaches have worked reasonably well to produce an approximate solution with lesser complexity. However, it may be interesting to apply other algorithms like Angle Modulated PSO, Beam ACO, etc to solve this NP-hard problem.

Distributed, Cooperative and Virtual MIMO communication techniques are relatively newer concepts, the receiver complexity in these cases will also be a research challenge. Similar, meta-heuristics assisted optimization techniques can be applied in these relatively newer MIMO scenarios.

Multi-step constellation reduction detection methods for STBC systems was proposed. An adaptive approach to dynamically select the reduced search space according to the noise variance can be an interesting addition to the work already done in this thesis.

Using Orthogonal or Quasi-orthogonal codes that result in lower bandwidth expansion can also be used in place of Walsh codes in Orthogonal Coded MIMO systems. These codes may result in better BER performance which low expansion of systems bandwidth.
References


References


References


142
References


[38] M. Dorigo, L. Gambardella, M. Middendorf, and T. Stutzle : Guest editorial: special section on ant colony optimization,2002


[40] M. Kong, and P. Tian: “Introducing a Binary Ant colony Optimization” , Fifth International Workshop on Ant Colony Optimization and Swarm Intelligence, 2006


References


References


[63] F. Xiong “Modem Techniques in Satellite Communications”, IEEE Communications Magazines, pp. 84-97, August 1994


[70] A. Goldsmith, Wireless Communications, Cambridge University Press, 2005


References


References


147
References


References


References


[135] Sana SFAR and Khaled Ben Letaief, Group ordered successive interference
References

cancellation for multiuser detection in MIMO CDMA systems”, IEEE Wireless Communications and Networking conference, WCNC 2003


Appendix A

List of Papers form the Research Work Conducted

Journals

   URL: [http://www.springerlink.com/content/5730u45822761r44/](http://www.springerlink.com/content/5730u45822761r44/)


Conferences


Appendix A


