# CHAPTER NINE

# CONCLUSION AND DISCUSSION

Ambrose Bierce, in *The Devil's Dictionary*, gave the following definition:

> MIND, *n*.— A mysterious form of matter secreted by the brain. Its chief activity consists in the endeavour to ascertain its own nature, the futility of the attempt being due to the fact that it has nothing but itself to know itself with.

This is the challenge. Despite the scale and complexity of the problem, there have been attempts to understand the function of the *brain*. Now we are at a stage where we can construct simple mathematical models and test their predictions, sometimes even in humans. These efforts may not only be able to improve health care through efficient treatment of nervous and psychiatric disorders but also lead us towards intelligent computing machines. The future of neuroscience, therefore, seems to be very promising.

In this work we have studied a linear model of global electrocortical activity. We have considered the oscillations of electrical potentials in segments of dendritic trees as a system of damped coupled harmonic oscillators. Generalized coordinates consisting of electrical potentials and their rate of change generated a linear transformation which formed a group. The identity of this group corresponded to the physiological state of *brain death*. A covariant model was constructed by writing the equations of electrical potential in the comoving frame of the signal. When these equations were transformed into the laboratory frame a magnetic vector potential appeared along with electrostatic potential. In the presence of a weak magnetic field first-order corrections to the frequencies as well as a method to calculate the ratio of components of signal velocities was given.

A generalized coupling was suggested in which the electrical potentials were coupled both to other potentials as well as their time rate of change. The effect on frequency eigenvalues in the presence of a time-varying magnetic field was studied.

In the presence of weak magnetic field a generalized potential of the form

$$\mathcal{V}_i = \varphi_i - \gamma_i A_{ext} . v_i$$

may be viewed as a gauge transformation for the electrical potential in the comoving frame of the signal. Using this potential Lagrangian and Hamiltonian densities could be written in the nearest neighbour interaction approximation. By iterative procedure the various coefficients in the potential expansion can be calculated. This may be a first step towards the construction of a quantum field theory for the electro-cortical activity. This field theory would describe the behavior of the system at absolute zero. To construct a field theory at room temperature we have to assume a temperature bath.

Since the system is bounded we expect discrete energy states. Nunez (1981) has already considered the possibility of viewing electroencephalogram as a system of standing waves. Similar ideas may be applicable to magnetoencephalogram, and simulation may predict stochastic properties of the system.

We are trying to simulate EEG signals as a system of harmonic oscillators (Siddiqui, Kamal, Khan 1988). The results will be compared with the digitized electroencephalograms provided by local neurologists.

Since the spinal cord plays an important role in human locomotion, a three-dimensional model of the spinal column was developed (Kamal 1983; 1987). This model generates the spinal outline from the projections in frontal and sagittal planes. The outline could be obtained from roentgenograms or moiré topographs. Degree of correction of trunk deformity could be estimated by measurements performed on moiré topographs of scoliosis patients in standing and hanging positions. The model is described fully in appendix C. Comparison of moiré topographs with X rays in Malmö, Sweden on a few patients suggested that this model gives the best results in the absence of rotation. This model may, therefore, be useful in the study of normal subjects. Outline of spine may be constructed from moiré topographs in various activities. We are in the process of developing special averaging algorithms which could be used to study the spinal columns in persons capable of performing highly coordinated motor skills (e.g. gymnasts), normals and patients having neurological impairment effecting their motor system (e.g. cerebral palsy patients).

The next step could be the unification of the spinal model with the model of brain to obtain a complete picture of the motor system and motor function in humans. This would, however, require experimental support at every stage.

Since human brain is a delicate system, we cannot use invasive means for its study. Therefore, there is a need of noninvasive techniques which do not pose any risk to the subject under study. At present we are concentrating on moiré and raster techniques to study noninvasively the spinal column, and hence the motor system. We are planning dynamic studies concentrating on gaits of normals and epileptics during the *washable memory period* using FM-80 projection type moiré camera.

Further theoretical work may involve developing a space-time metric tensor formalism (Pellionisz, Llinás 1982) for our covariant model (Kamal, Siddiqui, Husain 1989a;b). The metric tensor may, in turn, generate a scalar potential. Mathematical formulation to construct pseudo-Newtonian potentials in geometrodynamics already exists (Qadir, Quamar 1986).

We are, therefore, beginning to explore the frontiers of a kingdom always seemed mysterious to us. One wonders how many more mysteries of Nature have evaded human exploration. The seekers of *truth* will, perhaps, someday uncover them!

# BIBLIOGRAPHY

Adair IV, van Wijk MC, Armstrong GWD (1977) Moiré topography in scoliosis screening. *Clin. Orthop.* 129:165-171

Berkhout J, Walter O, Adey WR (1969) Alterations of the human electropencephalogram induced by stressful verbal activity. *Electroencephalogr. Clin. Neurophysiol.* 27:55-94

Bukhari N, Kamal SA (1987) How to cope with different systems of units. *Proc. 3rd Workshop on Teaching of Physics*, edited by AF Hasnanin, Karachi, pp. 36-50

Cowan JD (1974) Mathematical models of large scale nervous activity. *Lectures on Mathematics in the Life Sciences* 6: 101

*Doreland's Pocket Medical Dictionary* (1982) 23rd edn., Saunders, New York, p. 187

El-Sayyad MM, Kamal SA (1981) Cobb's angle measurement by moiré topographs. *Proc. 34th Ann. Conf. Eng. Med. Biol.* 23: 311

Elul R (1972) The genesis of the EEG. *Int. Rev. Neurobiol.* 15: 227-272

Feynman RP, Leighton RB, Sands M (1963) *The Feynman Lectures on Physics*, vol. 1, Addison-Wesley, Reading, Mass

Gevins AS, Zeillin GM, Yingling CD, Schaffer RE, Callaway E, Yeager CL (1979) Electroencephalogram correlates of higher cortical functions.*Science* 203(66):665-667

Griffith JS (1971) *Mathematical Neurobiology*, Academic Press, London

Halliday D, Resnick R (1988) *Fundamentals of Physics*, 3rd edn., John Wiley, New York, pp. 406-410

Hodgkin AL (1965) *The Conduction of the Nerve Impulse*, University Press, Liverpool

Hodgkin AL, Huxley AF (1952) A quantitative description of membrane current and its application to conduction and excitation in nerve. *J. Physiol.* 117:500-544

Kamal SA (1981) Determinant of general tensor. *The Matrix & Tensor Quarterly* 31:64-66

Kamal SA (1983) Determination of degree of correction of spinal deformity by moiré topographs. In:*Moiré Fringe Topography and Spinal Deformity*, Proceedings of the 2nd International Symposium, Münster, FRG, edited by B Drerup, W Frobin, E Hierholzer, Gustav Fischer, Stuttgart and New York, pp. 117-126.

Kamal SA (1987) Moiré topography for the study of multiple curves of spine. In:*Surface Topography and Spinal Deformity*, Proceedings of the 4th International Symposium, Mont Sainte Marie, Québec, Canada, edited by IAF Stokes, JR Pekelsky, MS Moreland, Gustav Fischer, Stuttgart and New York, pp. 43-50.

Kamal SA (1988) Reproducibility of moiré topographs. In: *Surface Topography and Body Deformity*, Proceedings of the 5th International Symposium, Vienna, Austria, edited by G Windischbauer, H Neugebauer, Gustav Fischer, Stuttgart and New York (in press)

Kamal SA, Akram M, Bukhari N (1988) Moiré topography for the study of neurological disorders. *Second National Symposium on Frontiers in Physics*, Quaid-e-Azam Univ., Islamabad (Pakistan), Dec. 1988 (to be published in the Proceedings)

Kamal SA, Benoni G, Willner S (1988) a study to test the reproducibility of moiré topographs (preprint)

Kamal A, Bukhari N, Akram M (1988) A comparison of back moiré topographs of children in the sitting and standing positions. In:*Surface Topography and Body Deformity*, Proc. 5th Int. Symp., loc. cit. (in press)

Kamal SA, El-Sayyad MM (1981) The use of moiré topographs for the detection of orthopedic defects in children of age group four to seven years. *Med. Phys.* 8:549

Kamal SA, Lindseth RE (1980) Moiré topography for the detection of orthopedic defects. Periodic Structures, Gratings, Moiré Patterns and Diffraction Phenomena, *Proc. Soc. Photo-Opt. Instr. Eng.* 240:293-295

Kamal SA, Siddiqui KA, Husain SA (1989a) Spacetime representation of global electrocortical activity. *Biol. Cybern.* 60:307-309

Kamal SA, Siddiqui KA, Husain SA (1989b) Effects of weak magnetic fields on global electrocortical activity. *Int. J. Neuroscience* (submitted)

Kuo BC (1982) *Automatic Control Systems*, 4th edn., Prentice-Hall, Englewood Cliffs, NJ

Lane (1983) Photogrammetry in medicine. *Photogramm. Eng.* 49: 1453-1456

Lewis CG, Moreland MS, Pope MH (1981) Moiré fringe topography in gait analysis: a methodologic evaluation. In:*Moiré Fringe Topography and Spinal Deformity*, Proceedings of the 1st International Symposium, Burlington, Vt., USA, edited by MS Moreland, MH Pope, GWD Armstrong, Pergamon, New York, pp. 81-86

Ljungberg T, Ungerstedt U (1976) Sensory inattention produced by 6-hydroxydopamine-induced degeneration of ascending dopaminergic neurons in the brain. *Exp. Neurol.* 53:585-600

Marshall JF, Turner BM, Teitelbaum P (1971) Sensory neglect produced by lateral hypothalamic damage. *Science* 174:523-525

Narici L, Romani L, Salustri C, Pizzella V, Modena I, Papanicolaou AC (1987) Neuromagnetic evidence of synchronized spontaneous activity in the brain following repetitive sensory stimulation. *Intern. J. Neuroscience* 32:831-836

Narici L, Romani GL, Salustri C, Pizzella V, Torrioli G, Modena I (1987) Neuromagnetic characterization of the cortical response to median nerve stimulation in the steady state paradigm. *Intern. J. Neuroscience* 32:837-843

Nauta WJH, Karten HJ (1970) A general profile of the vertebrate brain, with sidelights on the ancestry of cerebral cortex. In: *The Neurosciences· Second Study Program*, ed. by FO Schmitt, Rockefeller Univ. Press, New York. pp. 7-26

Nunez PL (1974) avelike properties of the alpha rhythm. *IEEE Trans. Bio-Med. Eng.* **21**:473-482

Nunez PL (1981) *Electric Fields of the Brain: The Neurophysics of the EEG*, Oxford Univ. Press, New York

Nuñez PL, Allen B, Bickford RG (1978) Computer methods to study the traveling wave properties of EEG. *Proc. San Diego Bio-Med. Symp.* **17**:191-196

Olds J (1956a) A preliminary mapping of electrical reinforcing effects in the rat brain. *J. Comp. Physiol. Psychol.* **49**:281-285

Olds J (1956b) Runaway and maze behaviour controlled by basomedical forebrain stimulation in the rat. *J. Comp. Physiol. Psychol.* **49**:507-512

Olds J, Peretz B (1960) A motivational analysis of the reticular activating system. *Electroencephalog. Clin. Neurophysiol.* **12**:445-454

Pellionisz A, Llinás R (1982) Space-time representation in brain. The cerebellum as a predictive space-time metric tensor. *Neuroscience* **7**:2949-2970

Qadir A, Quamar J (1986) Pseudo-Newtonian potentials. *Europhys. Lett.* **2**: 423-425

Ricci GB, Romani GL, Salustri C, Pizzella V, Torrioli G, Buonomo S, Peresson M, Modena I (1987) Study of focal epilepsy by multichannel neuromagnetic measurements. *Electroencephalogr. Clin. Neurophysiol.* **66**:358-368

Rolls ET (1971) Involvement of brain-stem units in medical forebrain bundle of self-stimulation. *Physiol. Behav.* **7**:297-310

Rolls ET (1974) The neural basis of brain-stimulation reward.

In: *Progress in Neurobiology*, Vol. III, ed. by GA Kerkut, JW Phillis, Pergamon, Oxford and New York

Romani GL (1987) The inverse problem in MEG studies: an instrumental and analytical perspective. *Phys. Med. Biol.* 32:23-31

Rose DF, Smith PD, Sato S (1987) Magnetoencephalography and epilepsy research. *Science* 238:329-335

Rybicki GB, Lightman AP (1979) *Radiative Processes in Astrophysics*, John Wiley, New York

Schwarz JH (1986) Lectures on superstring theory. In:*Physics in Higher Dimensions*, vol. 2, Jersusalem Winter School for Theoretical Physics, edited by T Piran and S Weinberg, World Scientific, Singapore and Philadelphia, pp. 93-158

Shochat SJ, Csongradi JJ (1983) Moiré phototopography in the evaluation of anterior chest wall deformity. In:*Moiré Fringe Topography and Spinal Deformity*, Proc. 2nd Int. Symp., loc. cit., pp. 249-256

Siddiqui KA, Kamal SA (1989) Group structure of a covariant model of global electrocortical activity. *Phys. Rev. Lett.* (submitted)

Siddiqui KA, Kamal SA, Khan NU (1988) Neurophysics: A beginner's view point. *Second National Symposium on Frontiers in Physics*, Quaid-e-Azam Univ., Islamabad (Pakistan), Dec. 1988 (to be published in the Proceedings)

Suzuki N, Yamashita Y, Yamaguchi Y, Armstrong GWD (1981) Measurement of posture using moiré topography In:*Moiré Fringe Topography and Spinal Deformity*, Proc. 1st Int. Symp., loc. cit., pp. 121-131

Teyler TJ (1977) An introduction to the neurosciences. In: *The Human Brian*, ed. by MC Wittrock, Prentice-Hall, Englewood Cliffs (NJ)

Turner BH (1973) Sensorimotor syndrome produced by lesions of the amygdala and lateral hypothalamus. *J. Comp. Physiol.*

*Psychol.* 82:37-47

Walter DO, Kado RT, Rhodes JJ, Adey WR (1967) Electroencephalographic baselines in astronaut candidates estimated by computation and pattern-recognition techniques. *Aerosp. Med.* 38:371-379

Wauquine A, Rolls ET (1976) *Brain Stimulation Reward*, Elsevier/North-Holland, Amsterdam

Weinberg S (1972) *Gravitation and Cosmology*, John Wiley, New York, p. 25

Willner S (1979) Moiré topography for the diagnosis and documentation of scoliosis. *Acta Orthop. Sand.* 50:295-302

Windischbauer G (1983) Moiré topography as an optical tool in anthropometry. In:*Moiré Fringe Topography and Spinal Deformity*, Proc. 2nd Int. Symp., loc. cit., pp. 201-205

Wise RA (1978) Catecholamine theories of reward: a critical review. *Brain Res.* 152:215-247

Wise RA, Bozarth MA (1981) Brain substrates for reinforcement and drug self-administration. *Prog. Neuro-Psycho Pharmacol.* 5:467-474

Wright JJ, Craggs MD (1979) Intracranial self-stimulation, cortical arousal, and the sensorimotor neglect syndrome. *Exp. Neurol.* 65:42-52

Wright JJ, Kydd RR (1984) A linear theory of global electrocortical activity and its control by the lateral hypothalamus. *Biol. Cybern.* 50:75-82

Wright JJ, Kydd RR, Lees GJ (1984) Amplitude and phase relations of electrocortical waves generated by transhypothalamic dopaminergic neurons: a test for a linear theory. *Biol. Cybern.* 50:273-283

# APPENDICES

## APPENDIX A: THEOREMS ABOUT SIMILARITY TRANSFORMATIONS

*Theorem A*1: A similarity transformation leaves the trace of a matrix invariant.

*Proof*: Let $A$ be a matrix which transforms as $B$ under a similarity transformation $S$ i.e.

$$B = SAS^{-1} \tag{A-1}$$

The trace of $A$ is $\sum_j A_{jj}$, whereas the trace of $B$ can be written as

$$\sum_i B_{ii} = \sum_{i,j,k} S_{ij} A_{jk} (S^{-1})_{ki} = \sum_{i,j,k} (S^{-1})_{ki} S_{ij} A_{jk} \tag{A-2}$$

But $\sum_i (S^{-1})_{ki} S_{ij} = \delta_{kj}$ and so

$$\sum_i B_{ii} = \sum_{j,k} \delta_{kj} A_{jk} = \sum_j \left[ \sum_k \delta_{kj} A_{jk} \right] = \sum_j A_{jj} \tag{A-3}$$

which completes the proof.

Since trace is the sum of eigenvalues, the above theorem may also be stated as: *Under a similarity transformation the sum of eigenvalues remains invariant*. In fact a stronger result holds good for similarity transformations.

*Theorem A*2: Under a similarity transformation the eigenvalues remain invariant.

*Proof*: If $\lambda$ is an eigenvalue of matrix $A$ the secular equation can be written as

$$\det(A - \lambda\mathbb{1}) = 0 \tag{A-4}$$

where $\mathbb{1}$ is a unit matrix having determinant 1. If S is a nonsingular matrix we have $\det(S) \neq 0$ and so $\det(S^{-1}) = 1/\det(S)$. Multiplying the left hand side of (A-4) by $\det(S)$ and $\det(S^{-1})$ we have

(A-5) $$\det(S)\det(A - \lambda\mathbb{I})\det(S^{-1}) = 0$$

But $\det(A)\det(B)\det(C) = \det(ABC)$ and so

(A-6) $$\det(SAS^{-1} - \lambda S\mathbb{I}S^{-1}) = 0$$

Since $S\mathbb{I}S^{-1} = \mathbb{I}$ we have

(A-7) $$\det(B - \lambda\mathbb{I}) = 0$$

which proves the required result.

## APPENDIX B: THEOREMS ABOUT DETERMINANTS OF GENERAL TENSORS

*Theorem B1*: The determinant of an inner product is equal to the determinant of the outer product.

*Proof*: For a second rank tensor

(B-1) $$\det(A^{ab}) = \epsilon_{a_1 a_2 \ldots\ldots a_n} A^{a_1 b_1} A^{a_2 b_2} \ldots\ldots A^{a_n b_n}$$

Now

(B-2) $$\det(A^{ab})\det(B^{cd}) = (\epsilon_{a_1 \ldots a_n} \epsilon_{b_1 \ldots b_n} \epsilon_{c_1 \ldots c_n} \epsilon_{d_1 \ldots d_n}) \times (A^{a_1 b_1} B^{c_1 d_1} \ldots\ldots A^{a_n b_n} B^{c_n d_n})$$

If we replace $c$ by $b$ and sum over $b$ we have

(B-3) $$\det(A^{ab})\det(B^{bd}) = [\epsilon_{a_1 \ldots a_n} (\epsilon_{b_1 \ldots b_n})^2 \epsilon_{d_1 \ldots d_n}] \times (A^{a_1 b_1} B^{b_1 d_1} \ldots\ldots A^{a_n b_n} B^{b_n d_n})$$

$(\epsilon_{b_1 \ldots b_n})^2 = +1$ when it is non-vanishing. In those cases we may define

(B-4) $$A^{a_i b_i} B^{b_i d_i} = (AB)^{a_i d_i}$$

as the inner product of $A$ and $B$. Therefore

(B-5) $$\det(A^{ab})\det(B^{bd}) = \det(AB^{ad})$$

Note that we can change contravariant tensors to covariant (for their contraction) by multiplication with $g_{\mu\nu}$ having a determinant -1. If we define

(B-6) $$A^{a_i b_i} B^{c_i d_i} = (AB)^{a_i b_i c_i d_i}$$

as the outer product of tensors $A$ and $B$, we notice that

(B-7) $$\det(A^{ab})\det(B^{bd}) = \det(AB^{abcd})$$

We note that determinant is a number which does not depend on the choice of indices and so from equations (B-5) and (B-7) we have

(B-9) $$\det(A^{ab})\det(B^{bd}) = \det(A^{ab})\det(B^{cd})$$

and so we have

(B-10) $$\det(AB^{abcd}) = \det(A^{ad})$$

Now we prove this theorem for a determinant of a general tensor of rank $n$. Suppose it is true for a tensor of rank $k$,

(B-11) $$\det(T^{a_1a_2\cdots a_ka_{k+1}a_{k+2}}) = \det(T^{a_1a_2\cdots a_k})$$

We take a new tensor whose components are given by $C^{a_o}$ which is of first rank. Therefore according to eq. (B-11)

$$\det(C^{a_o})\det(T^{a_1a_2\cdots a_ka_{k+1}a_{k+2}}) = \det(C^{a_c})\ \det(T^{a_1a_2\cdots a_k})$$

By definition of a general tensor given in eq. (2.28) we see that the two sides are the determinants of tensors $(CT)^{a_1a_2\cdots a_ka_{k+1}a_{k+2}}$ and $(CT)^{a_1a_2\cdots a_k}$ and so

(B-12) $$\det(CT^{a_1a_2\cdots a_ka_{k+1}a_{k+2}}) = \det(CT^{a_1a_2\cdots a_k})$$

Therefore the assumption that the theorem is true for $k$ implies that it is true for $k + 1$ because on the right-hand side we have a tensor of rank $k + 1$. We already proved that it is true when $k = 2$. Hence it is true for all $n > 2$. For $n = 1$ the result can be easily proved.

*Theorem B2*: The determinants of a general tensor form an abelian group under multiplication.

*Proof*: Let $\det(A^{a_1a_2\dots a_m})$ and $\det(B^{b_1b_2\dots b_n})$ be two elements in set $D$.

(*i*) Using theorem *B*1 we can show that

$$\det(A^{a_1a_2\dots a_m})\det(B^{b_1b_2\dots b_n}) = \det(AB^{a_1a_2\dots a_m b_1 b_2\dots b_n}) \in D$$

(*ii*) For all $\det(A^{a_j})$, $\det(B^{b_k})$ and $\det(C^{c_l})$ in $D$, $j = 1,2,\dots,m$; $k = 1,2,\dots,n$; $l = 1,2,\dots,p$. Therefore

$$\begin{aligned}\det(A^{a_j})[\det(B^{b_k})\det(C^{c_l})] &= \det(ABC^{a_jb_kc_l}) \\ &= [\det(A^{a_j})\det(B^{b_k})]\det(C^{c_l})\end{aligned} \tag{B-13}$$

by theorem *B*1.

(*iii*) There exists 1 in $D$ such that

$$1.\det(A^{a_j}) = \det(A^{a_j}) = \det(A^{a_j}).1 \tag{B-14}$$

We now show that 1 is in $D$. Let us define a tensor of $m$th rank $E$ such that

$$E^{a_1a_2\dots a_m}; \qquad \alpha_j = 1,2,\dots,n \tag{B-15}$$

are given by

$$\begin{aligned}F^{a_1a_2\dots a_m} &= +1 \text{ if } \alpha_1 = \alpha_2 = \dots = a_m \\ &= 0 \text{ otherwise}\end{aligned} \tag{B-16}$$

We note that $\det(E^{a_j}) = 1$ and so 1 is in $D$.

(*iv*) For $\det(A^{a_j})$ in $D$, there exists $\det(F^{a_j})$ in $D$, such that

$$\det(A^{a_j})\det(F^{a_j}) = 1 = \det(F^{a_j})\det(A^{a_j}) \tag{B-17}$$

For this very condition we need that $\det(A^{a_j}) \neq 0$. Therefore $A^{a_j}$ must be nonsingular.

(*v*) Also we have

(B-18) $$\det(A^{a}{}_{j})\det(B^{b}{}_{k}) = \det(B^{b}{}_{k})\det(A^{a}{}_{j})$$

Therefore, all nonzero determinants of a general tensor form an abelian group under multiplication.

## APPENDIX C: THREE-DIMENSIONAL MODEL OF THE SPINAL COLUMN

To draw the outline of the spine from the moiré topograph of back consider Fig. C-1. The midpoint of the neck is joined to the midpoint of the pelvis Q. To find the position of the spine at a given point draw a line perpendicular to PQ. Let this line intersect PQ at C and a particular moiré fringe at H and E such that E is always on the right side of H. The midpoint O of the line segment HE is assumed to give the position of the spine, provided the positioning during the X-ray and moiré examinations is identical. Several such points may be found along the line segment PQ, preferably 33 to correspond to the 33 vertebrae of the spinal column, and a spinal outline in the frontal plane could be drawn as the best curve fitting these points. Similarly an outline in the sagittal plane (Fig. C-2) may be drawn from moiré topograph of the side (arm raised).

Fig. C-3 shows projection of a point O on the spine in yz and xz planes. The back lies in the yz plane (frontal plane) and the side lies in the xz plane (sagittal plane). The point B (=$B_0$) is chosen as origin. The back and side photographs may not have the same scale. To adjust the scales, photograph a stick on both back and side photographs. If $\mu$ is the ratio of the length on the back photograph to that on the side photograph, the corrected length for the side may be written as

$$z_{corrected}(\text{side}) = \mu z_{measured}(\text{side}) \qquad \text{(C-1)}$$

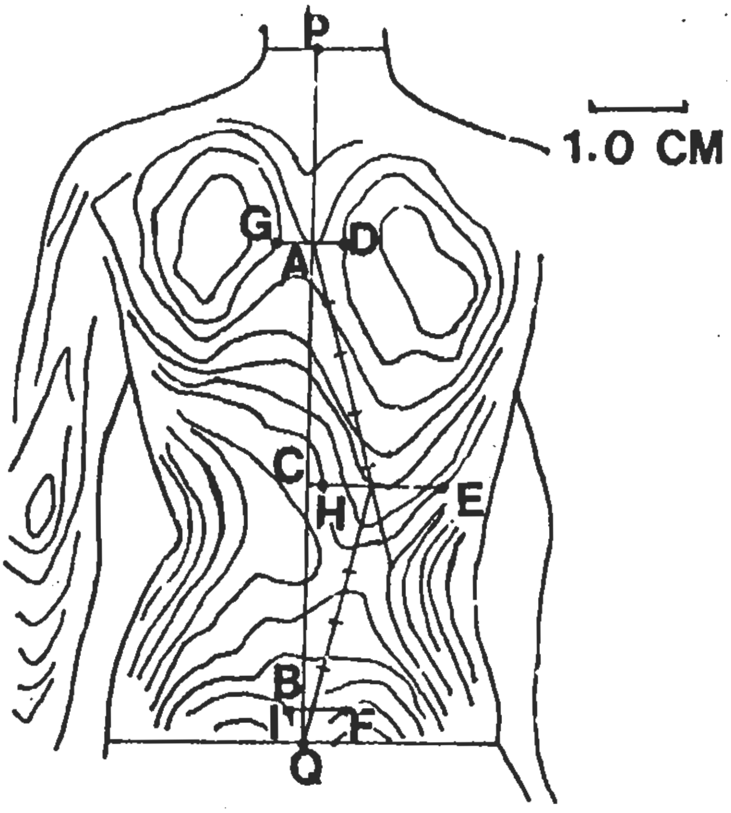

Fig. C-1: *Moiré topograph of back*

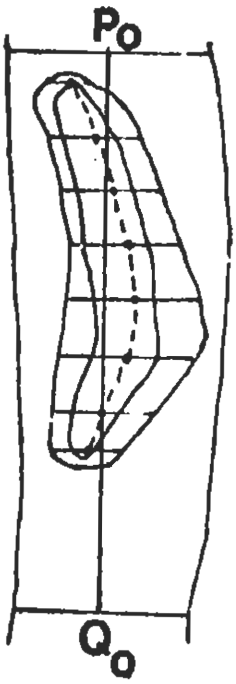

Fig. C-2: *Moiré topograph of the side*

From the measurements performed on moiré topographs of back and side, we generate a curve $x = x(\xi)$, $y = y(\xi)$, $z = z(\xi)$ which is a best fit to discrete measurements performed at various locations represented by the parameters $\xi_1, \xi_2, \ldots, \xi_n$; $n = 33$ corresponding to the 33 vertebrae of the backbone. The parameter $\xi_i$ may be taken as the distance on a line segment joining the midpoint of neck to the midpoint of the pelvis. Let us write

(C-2) $$x = f(y,z) = \tfrac{1}{2}ay^2 + byz + \tfrac{1}{2}cz^2$$

in the neighborhood of any point on the spine. The coordinate $x$ represents the deviation of the curve from the $yz$-plane. Rotating the $y$, $z$ axes by an angle $\alpha$ clockwise about the x-axis

(C-3a) $$y = y_{rot} \cos \alpha + x_{rot} \sin \alpha$$

(C-3b) $$z = - y_{rot} \sin \alpha + x_{rot} \cos \alpha$$

where

(C-4) $$\alpha = \tfrac{1}{2} \tan^{-1} [2b/(c - a)]$$

Eq. (C-3) may, therefore, be written as

(C-5) $$x = \tfrac{1}{2} \varkappa_{1i} y_{rot}{}^2 + \tfrac{1}{2} \varkappa_{2i} z_{rot}{}^2$$

where

(C-6a) $$\varkappa_{1i} = a + c - 2b^2/[4b^2 + (c - a)^2]^{1/2}$$

(C-6a) $$\varkappa_{2i} = a + c + 2b^2/[4b^2 + (c - a)^2]^{1/2}$$

The patient is then asked to hang freely from a bar and the improvement in the deformity is noted. The curvatures are again measured after guarded graduated passive correction as

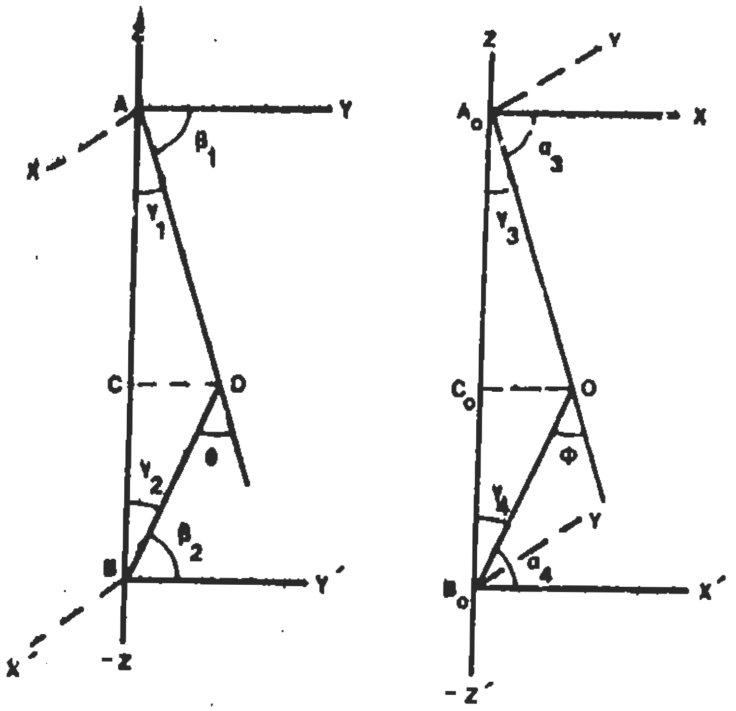

**Fig. C-3a,b:** *Projections of a point O on the spine in yz (frontal) and xz (sagittal) planes.*

$x'_{1i}$, $x'_{2i}$; $i = 1,2,\ldots,n$. The degree of correction of trunk deformity is defined as

(C-7) $$D = (50/n) \Sigma [(1 - x'_{1i}/x_{1i})^2 + (1 - x'_{2i}/x_{2i})^2]$$

The trunk deformity is classified as *severe*, *intermediate* or *mild* if D lies between 0 - 33.33, 33.34-66.66, 66.67-100 respectively. Geometrically if $x'_{1i} = x_{1i}$, $x'_{2i} = x_{2i}$; $i = 1,2,\ldots,n$, there is no correction and D = 0. On the other hand, if $x'_{1i} = x'_{2i} = 0$; $i = 1,2,\ldots,n$, the deformity is completely corrected and D = 100.

The analysis presented here may also be applied to clinical radiographs of back and side to determine the shape and curvature of the spine.

# VITA

The author was born on May 23, 1956 in Hyderabad, Pakistan. He obtained his B.Sc.(Hons.), Physics, and M.Sc., Mathematical Physics, both *magna cum laude* from the University of Karachi. He holds an M.S. in Biomedical Physics from Indiana University, Bloomington, USA where he worked on medical applications of moiré fringe topography. He was awarded Quaid-e-Azam Scholarship to attend the Johns Hopkins University, Baltimore, USA from where he graduated in 1986.

He has more than forty papers to his credit in mathematical physics and biomedical physics. He has attended more than a dozen conferences, symposia and workshops. Besides research he is actively involved in uplifting teaching standards in the country. An active participant in local and national workshops in physics teaching, Kamal has developed undergraduate and graduate physics curricula suitable for Pakistani institutions. At present he is working as Assistant Professor in the Department of Physics, University of Karachi.