

CHAPTER SEVEN

GENERALIZED COUPLING IN THE COVARIANT MODEL

7.1 NEED FOR GENERALIZED COUPLING

In the linear model of Wright and Kydd (1984) the electrical potentials ϕ_i 's are coupled to ϕ_j 's through coupling parameters $K_i^j(t)$. However, we note that a change in potential ϕ_j induces a magnetic field because of flow of current. A magnetic field will, in turn, exert Lorentz force on a charged particle and hence, in general, ϕ_j , will influence ϕ_i .

In the covariant model (Kamal, Siddiqui, Husain 1989a) the dependence of A_i 's on A_j 's was also suggested when we considered the group structure. Upon multiplication of two state transition matrices we came up with nonzero coefficients for A_j 's.

We are generalizing Wright and Kydd's damped coupled harmonic oscillator equations to include a generalized coupling which also depends on ϕ_j 's. A covariant model is constructed using this coupling and first-order corrections to EEG frequencies based on this model are calculated in the presence of a time-varying magnetic field.

7.2 MATHEMATICAL DESCRIPTION

In the comoving frame of the signal passing through a segment of the dendritic tree, the electrical potential for a mass of unit sources coupled to each other may be represented by

$$(7.1) \quad \ddot{\phi}_i + \mathcal{D}_i(\tau) \dot{\phi}_i + \mathcal{K}_i^2(\tau) \phi_i = \Sigma [A_i^j(\tau) \phi_j + \mathcal{M}_i^j(\tau) \dot{\phi}_j]$$

which forms a group.

7.3 PREDICTIONS

For a model to be considered seriously, it should have predictions which could be tested experimentally.

Consider the effects of a weak magnetic field B_{ext} ($\ll 1$ tesla) which is assumed to be uniform throughout the region concerned. The magnetic field, not varying with time, is generated by a vector potential such that $B_{\text{ext}} = \nabla \times A_{\text{ext}}$. The 4-potential is A_{ext} as given in (5.1). In the presence of this field the natural frequencies $N_i(\tau)$ will be modified to $N_i'(\tau)$. However, the damping coefficients and couplings of the individual A 's and \dot{A} 's will remain unchanged. In the lab frame eq. (7.3) now takes the form

$$(7.5) \quad \ddot{A}_i' + \Delta_i(\tau) \dot{A}_i' + \eta_i'^2(\tau) A_i' = \sum [\mathcal{X}_i^j(\tau) A_j' + \mathcal{M}_i^j(\tau) \dot{A}_j']$$

where $A_i' = A_i + A_{\text{ext}}$. Since the signal velocities are very small as compared to the velocity of light, we take $t \approx \tau$. The external field $A_{\text{ext}}(x,y,z)$ does not depend on time. We, therefore, have

$$\ddot{A}_i' = \ddot{A}_i, \quad \dot{A}_i' = \dot{A}_i$$

Subtracting eq. (7.3) from eq. (7.5) and introducing first-order correction in frequencies $\eta_i'^2 = \eta_i^2 + \delta\eta_i^2$, we have

$$\delta\eta_i^2 A_i' = -(\eta_i^2 - \sum_j \mathcal{X}_i^j) A_{\text{ext}}$$

This is same as equation (6.4). We note that all the factors related to generalized coupling with \dot{A} have cancelled out.

Therefore, we conclude that a weak, uniform, stationary magnetic field will give the same first-order shift in frequencies in the presence of generalized coupling.

Consider a magnetic field $B_{\text{ext}}(t)$ which is generated by a vector potential $A_{\text{ext}}(t)$ varying linearly with time. Defining a 4-potential A_{ext} as in (6.1), we may write eq. (7.5) in the presence of weak magnetic field. Since $A'_i = A_i + A_{\text{ext}}$, and $A_{\text{ext}}(x,y,z,t)$ has a linear time dependence, we have

$$\ddot{A}'_i = \ddot{A}_i, \quad \dot{A}'_i = \dot{A}_i$$

Subtracting (7.3) from (7.5) and introducing $\delta\eta_i$ as the first-order correction to η_i , we have

$$(7.6) \quad \delta\eta_i^2 A'_i = -(\eta_i^2 - \sum_j \mathcal{K}_i^j) A_{\text{ext}} - (\Delta_i^2 - \sum_j \mathcal{M}_i^j) \dot{A}_{\text{ext}}$$

If all the matrices can be diagonalized simultaneously, (7.6) can be written in the comoving frame of the signal as

$$(7.7) \quad \delta\mathcal{K}_i^2 \tilde{A}'_i = -(\mathcal{K}_i^2 - \sum_j \mathcal{K}_i^j) (\tilde{A}_{\text{ext}})_i - (\mathcal{K}_i^2 - \sum_j \mathcal{K}_i^j) (\dot{\tilde{A}}_{\text{ext}})_i$$

where $(\tilde{A}_{\text{ext}})_i = \lambda_i \dot{A}_{\text{ext}}$. All other quantities are defined in (6.5) and (7.2). The matrices $\delta\mathcal{K}_i$, \mathcal{K}_i , \mathcal{K}_i^j and \mathcal{M}_i^j are already diagonalized. Eq. (7.7) will yield four equations which can be solved to determine eigenvalues δ_i^μ . The results are

$$(7.8) \quad (\delta_i^\mu)^2 = [\sum_j \mathcal{K}_i^{j\mu} - (N_i^\mu)^2] \Theta_i^\mu + [\sum_j \mathcal{M}_i^{j\mu} - D_i^\mu] \Xi_i^\mu$$

where

$$(7.9a) \quad \Xi_i^0 = -\gamma_i \dot{A}_{\text{ext}} \cdot \mathbf{v}_i / (\phi_i - \gamma_i \dot{A}_{\text{ext}} \cdot \mathbf{v}_i)$$

$$(7.9b) \quad \Xi_i^r = [\delta_r^2 + v_r v_s (\gamma - 1) / v^2] \dot{A}_{\text{ext}}^s / A_{\text{ext}}^r; \quad r = 1, 2, 3$$

All the other quantities are defined after eq. (6.6).

Recall that the term $\gamma_i \dot{A}_{\text{ext}} \cdot v_i$ is very small as compared to ϕ_i . Consider a time-varying external field which is applied in the x-direction. If its magnitude is changed from $-\epsilon$ to $+\epsilon$ in a period of time Δt , i.e. on the average $|\dot{A}_{\text{ext}}| = 0$, we expect that $\delta_i^0 \propto |\dot{A}_{\text{ext}}|^{1/2}$. Using Cramer's Central Limit Theorem to replace the individual frequencies by their average, we get

$$(7.10) \quad \langle \delta^0 \rangle \propto |\dot{A}_{\text{ext}}|^{1/2}$$

This prediction may be checked experimentally.

Because of the presence of the factor $\dot{A}_{\text{ext}}/A_{\text{ext}}$ in (7.9b), the frequencies will become unbounded when $|\dot{A}_{\text{ext}}| = 0$. This is not possible physically. We, therefore, conclude that

$$(7.11) \quad \Sigma M_i^{jr} = D_i^r; \quad r = 1, 2, 3$$

This may be considered as a condition of *impedance matching*.