CHAPTER SIX

EFFECTS OF WEAK MAGNETIC FIELDS

6.1 INTRODUCTION

Magnetic fields of the brain are studied experimentally by many groups especially for localization of epileptic foci (e.g. Romani 1987; Ricci, Romani, Salustri, Pizzella, Torrioli, Buonomo, Peresson, Modena 1987; Narici, Romani, Salustri, Pizzella, Modena, Papanicolaou 1987; Narici, Romani, Salustri, Pizzella, Torrioli, Modena 1987). It would be of interest to study the effects of external weak magnetic fields on global electrocortical activity. These type of studies may be of interest in epileptic patients just after the siezure (cf. chapter 9). We expect to obtain information about the strength of coupling as well as ratio of signal velocities in various directions.

If the magnetic field is weak, it not will effect the damping coefficients and coupling of the individual vector potentials. The natural frequencies will, however, be modified. In the context of our generalized model, a first-order estimate of the shift in frequencies is given.

6.2 WEAK MAGNETIC FIELDS

Let us apply a weak magnetic field B_{ext} ($\ll$ 1 tesla) which is assumed to be uniform throughout the region concerned. The magnetic field, not varying with time, is generated by a vector potential $A_{ext}(x,y,z)$ such that $B_{ext} = \nabla \times A_{ext}$. Let us write a 4-potential as

$$(6.1) \qquad A_{ext} = \begin{bmatrix} 0 \\ \mathbf{A}_{ext} \end{bmatrix}$$

The components of this 4-potential are denoted by A^{μ}_{ext}; μ = 0, 1,2,3.

The presence of weak magnetic field will not affect the damping coefficients and coupling of the individual A's. In the presence of this weak magnetic field the natural frequencies $N_i(\tau)$ will be modified to, say, $N_i'(\tau)$. In the lab frame eq. (4 10) now takes the form

$$(6.2) \qquad \ddot{A}_i' + \Delta_i(\tau)\dot{A}_i' + \eta_i'^2(\tau)A_i' = \Sigma\, \mathcal{K}_i^{\,j}(\tau)A_j'$$

where $A_i' = A_i + A_{ext}$. Since the signal velocities are very small as compared to the velocity of light, we take $t \simeq \tau$. The external field $A_{ext}(x,y,z)$ does not depend on time. We, therefore, have

$$(6.3) \qquad \ddot{A}_i' = \ddot{A}_i,\ \dot{A}_i' = \dot{A}_i$$

Subtracting (3.10) from (5.2) and introducing $\eta_i'^2 = \eta_i^2 + \delta\eta_i^2$, ($\delta\eta_i$'s are 4 x 4 matrices having eigenvalues δ_i^{μ}; μ = 0,1,2,3 representing the shifts in frequencies) we have

$$(6.4) \qquad \delta\eta_i^2 A_i' = -(\eta_i^2 - \sum_j \mathcal{K}_i^{\,j})A_{ext}$$

We assume that $\delta\eta_i$, $\mathcal{K}_i^{\,j}$ and η_i can all be simultaneously diagonalized. In other words $[\delta\eta_i, \mathcal{K}_i^{\,j}] = 0$, $[\delta\eta_i, \eta_i] = 0$ etc. Applying the transformation $\delta\eta_i \longrightarrow \tilde{\lambda}_i\delta\eta_i\lambda_i = \delta\mathcal{N}_i$ etc. to write (5.4) in the comoving frame of the signal, we have

$$(6.5) \qquad \delta\mathcal{N}_i^2 \tilde{A}_i' = -(\mathcal{N}_i^2 - \sum_j A_i^{\,j})(\tilde{A}_{ext})_i$$

where $\tilde{A}_i' = \phi_i + (\tilde{A}_{ext})_i$; $(\tilde{A}_{ext})_i = \tilde{\lambda}_i A_{ext}$. The matrices $\delta\mathcal{N}_i$, $\mathcal{N}_i$ and $\dot{A}_i^{\,j}$ are already diagonalized. Eq. (6.5) will yield four equations each one sufficient to determine an eigenvalue δ_i^{μ}. The results are

$$(6.6) \qquad (\delta_i^{\mu})^2 = [\ \sum_j K_i^{j\mu} - (N_i^{\mu})^2\ ]\ \Theta_i^{\mu}$$

where $\Theta_i^{o} = -\gamma_i A_{ext}.v_i/(\phi_i - \gamma_i A_{ext}.v_i)$, $\Theta_i^{r} = 1$; $r = 1,2,3$.

6.3 EFFECTS ON FREQUENCIES

The shift in frequencies is a measure of the strength of coupling through the factor $[\ \Sigma\ K_i^{j\mu} - (N_i^{\mu})^2]$.

(a) If the coupling is strong i.e. $\Sigma\ K_i^{j\mu} > (N_i^{\mu})^2$, we will observe a slight increase in frequencies.

(b) If $\Sigma\ K_i^{j\mu} = (N_i^{\mu})^2$, there will be no shift in frequencies.

(c) If $\Sigma\ K_i^{j\mu} < (N_i^{\mu})^2$, there will be a decaying exponential introduced in the EEG spectrum. Therefore, the frequencies will be modulated.

Examination of (6.6) also indicates that the term $\gamma_i A_{ext}.v_i$ is very small as compared to ϕ_i. Therefore, we expect that

$$(6.7) \qquad \delta_i^{o} \propto |A_{ext}|^{1/2}$$

Taking the averages we expect that $<\delta^{o}> \propto |A_{ext}|^{1/2}$. This conclusion may be checked experimentally. In a magnetically shielded room a weak external magnetic field may be applied to the brain in the form of a horseshoe magnet. Its magnitude is varied and when the steady state is reached the EEG is taken. The dominant frequency of the EEG is expected to vary as the square root he external magnetic vector potential. The other comp ents δ_i^r; $r = 1,2,3$ are independent of the applied field.

If we apply magnetic field of same magnitude in the x-, y- and z-directions and observe δ_i^o in each case, we can measure the ratios of velocities of signals in different directions

(6.8) $$(\delta_i^o)_x : (\delta_i^o)_y : (\delta_i^o)_z = \sqrt{(v_i)_x} : \sqrt{(v_i)_y} : \sqrt{(v_i)_z}$$

If we apply Central Limit Theorem of Cramer to replace each individual frequency and velocity by its average, the above equation may provide a way to evaluate the ratio of different components of signal velocity.

6.4 GENERALIZED POTENTIAL

Eq. (6.6) also suggests that we can define a generalized potential of the form

(6.8) $$\Psi_i = \phi_i - \gamma_i A_{ext} . v_i$$

This could be viewed as a sort of gauge transformation for the electrical potential in the comoving frame of the signal. It may help in writing the Lagrangian and Hamiltonian densities for our system. The next step could be the construction of a quantum field theory for the electrocortical activity.