

UNIVERSITY OF KARACHI

SPACE-TIME REPRESENTATION IN THE BRAIN

by

SYED ARIEF KAMAL

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Department of Physics
University of Karachi
Karachi-75270

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CERTIFICATE

This is to certify that Syed Arif Kamal has successfully completed his dissertation entitled SPACE-TIME REPRESENTATION IN THE BRAIN under my supervision and guidance.

Dr. S. A. Husain
*Professor and Chairman,
Department of Physics,
University of Karachi,
Karachi-75270 (Pakistan)*

Dated _____

UNIVERSITY OF KARACHI

The undersigned certify that they have read and accepted the dissertation entitled SPACETIME REPRESENTATION IN THE BRAIN submitted by Syed Arif Kamal in conformity with the requirements for the degree of Doctor of Philosophy.

Supervisor

External Examiner

Dated _____

ABSTRACT

Wright and Kydd have developed a linear model of global electrocortical activity and its control by lateral hypothalamus. Their model rests on drastic simplifications and bypasses issues of cell-to-cell coupling, details of anatomy etc. Moreover, it does not take into account of the magnetic fields which are generated when there is a motion of charges. In the covariant description the equations for the time variation of potentials of segments of dendritic trees are written in the comoving frames of the signals. When these equations are transformed into the laboratory frame a magnetic vector potential appears along with the electrostatic potential. This model, therefore, offers a possible explanation of magnetoencephalogram (MEG). Essential theoretical features of this covariant model may be summarized as:

- (a) Electrocortical recordings reflect the transformed spatial average of cortical potentials.
- (b) The telencephalon is assumed to be a linear wave medium with regard to the gross wave potentials although the underlying microscopic interactions may be extremely non-linear.
- (c) Closed and constant boundary conditions lead the linear waves to generate activity at a large number of resonant modes, each associated with a constant natural frequency.
- (d) The values for the natural modes of the resonant frequencies are clustered about certain central values (Cramer's Central Limit Theorem).

ABSTRACT (CONTD.)

(e) Ascending inhibitory systems act partly to damp resonant activity and partly as a source of noise like driving signals.

(f) An electrical potential in a comoving frame of the signal transforms as four-potential in the laboratory frame.

The group structure of this model is also explored. By block diagonalization a nonsingular matrix is constructed from the state transition matrix. This matrix forms a group whose identity corresponds to the physiological state commonly known as *brain death*. Further, the effects of weak magnetic fields on this covariant model are considered. A method to calculate the ratio of components of signal velocities is proposed and a gauge transformation is suggested for the electrical potential. In the presence of weak magnetic field frequencies are modified, but damping coefficients and coupling constants remain essentially unchanged. A generalized coupling is suggested in which potentials are also effected by rate of change of neighboring potentials. In the presence of weak magnetic fields the effect of generalized coupling on the frequencies is calculated. In the end use of moiré fringe topography for the study of neurological disorders is discussed.

NOTATION

- Greek indices $\alpha, \beta, \gamma, \delta$ and so on generally run over the four space-time inertial coordinate labels 0, 1, 2, 3 or t, x, y, z .
- Repeated Greek indices are summed unless otherwise indicated.
- Repeated Latin indices are not summed.
- The metric $g_{\mu\nu}$ in an inertial coordinate system has elements $g_{00} = -1, g_{ij} = \delta_{ij}; i, j = 1, 2, 3$.
- A dot over any quantity denotes the time derivative of that quantity.
- Cartesian three-vectors are indicated by boldface type.
- Four-vectors are indicated by underlining them.
- The speed of light is taken to be unity except when c.g.s. units are indicated.
- Second-rank tensors are indicated by double underlining.

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TABLE OF CONTENTS

Abstract		
Notation		
List of Preprints		
Acknowledgements		
1	PHYSIOLOGY OF THE BRAIN	1
	1.1 Introduction	2
	1.2 The Human Brain	3
	1.3 Regions of Cortical Surface	3
	1.4 The Neuron	8
	1.5 Electrical Activity of the Brain	15
	1.6 Electrocortical Activity	19
2	BRAIN WAVES AS RESONANCE	20
	2.1 The Phenomenon of Standing Waves	21
	2.2 Standing Waves in Physical Systems	24
	2.3 EEG as a System of Standing Waves	27
3	MATHEMATICAL PRELIMINARIES	29
	3.1 Transformations	30
	3.2 Lorentz Transformations	31
	3.3 Four-vectors	34
	3.4 Similarity Transformations	36
	3.5 Tensors and Determinants	36
4	GLOBAL ELECTROCORTICAL ACTIVITY	39
	4.1 Linear Model	40
	4.2 Covariant Model	44
	4.3 Generalization of the Covariant Model	47

TABLE OF CONTENTS (CONTD.)

5	GROUP STRUCTURE OF THE COVARIANT MODEL	49
	5.1 The State Transition Matrix	50
	5.2 Determinant of Transition Matrix	51
	5.3 Group Structure	53
	5.4 Brain Death as Identity of the Group	54
6	EFFECTS OF WEAK MAGNETIC FIELDS	56
	6.1 Introduction	57
	6.2 Weak Magnetic Fields	57
	6.3 Effects on Frequencies	59
	6.4 Generalized Potential	60
7	GENERALIZED COUPLING IN THE COVARIANT MODEL	61
	7.1 Need for Generalized Coupling	62
	7.2 Mathematical Description	62
	7.3 Predictions	64
8	MOIRÉ TECHNIQUES IN NEUROLOGY	67
	8.1 Introduction	68
	8.2 The Moiré Technique	69
	8.3 Applications	74
	8.4 Study of Neurological Disorders	74
	8.5 Scope of Moiré Techniques	75
9	CONCLUSIONS AND DISCUSSION	76
	BIBLIOGRAPHY	81
	APPENDICES	88
	Appendix A	89
	Appendix B	91
	Appendix C	95
	Vita	