Chapter V

THEORETICAL MODEL OF ISLAMIC BANKING

Islamic banking is an integral part of Islamic economic system and as such before we construct the theoretical model of Islamic banking, it is imperative to derive an axiomatic approach to the difference between the conventional banking and the one under evaluation and consideration.

DIFFERENCE BETWEEN THE SYSTEM

The ingredients of an Islamic economic system are derived from the teachings of the Holy Quran and *Sunnah* (traditions of the Prophet - PBUH). In essence the guiding principles are equity, fairness and justice.

Interest, which basically violates these guiding principles would, therefore, be replaced by profit sharing in the Islamic banking sector. The resultant replacement of interest (fixed and pre-determined return) to the lender by profit-sharing, would thus form the pivotal difference between the conventional and Islamic banking[1] . Equity participation in a venture is admissible under *Shariah*, with due rewards only when profit is earned as against interest which is regardless of the profit or loss position of the enterprise. Lending without assuming risk on interest, therefore has to be replaced by return to lender to the extent of the productivity of his financial capital in the resulting profits.[2]

Accordingly, Islamic banking can be considered as an equity based system rather than interest based system, with the depositor, depositing funds in the bank as shareholder, entitled to a share of profits made by the bank and conversely to share the loss in the event of the bank incurring loss. On the other hand the bank can also not charge a fixed rate of interest on loans and as such would engage in profit and loss sharing agreement with the borrower.[3]

Consequent upon the resurgence of Islamic economics, to date several attempts have been made to study the behavior of Islamic banking based

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[1] Chapters II and III of this thesis have elaborated on the subject matter of *riba* and permissible modes of Financing under *Shariah*.

[2] For a synoptic view on the theoretical considerations of the Islamic Banking, please see Iqbal, Zubair and Mirakhor Abbas (1987), p.1-8.

[3] Please see Khan, Mohsin. S. (1986), p.6.

on Islamic financial principles.[4] Analysis and evaluation of the different models depict that they lack a sound micro foundation except Siddiqui and Zaman (1989). The basic purpose of this study is therefore to build a comprehensive macro economic model based on micro economic foundations.

THE MODEL: (Micro Economic Foundation for a Macro Economic System)

A mathematical economic model is basically a configuration of set of equations which are based upon analytical assumptions so adopted to describe the structure of the model.[5] Like any other theoretical work, we have simplified the economic scenario by abstracting from real world complications, and as such in order to understand the subject matter we have made the following simplified assumptions.

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[4] Studies conducted by Khan (1985), Khan (1986), Haque and Mirakhor (1987). Anwar (1987), Sattar (1991), Metwally (1992), Siddiqui and Zaman (1989) are helpful in understanding the different aspects of Islamic economic and banking system.

[5] Please see, Chiang C. Alpha (1984), p.7.

ASSUMPTIONS OF THE MODEL

i) Savers invest their surplus funds in the bank as shareholders without any guarantee of fixed return or even the nominal value of the shares, in case of loss.

ii) The bank, called the lender, lends these funds to borrower for investment on equity participation (*Musharakah*) basis and is only eligible for a return from resultant profits on a pre-agreed ratio. In the event of a loss, it shall be shared by the bank accordingly.

iii) All the individuals are assumed to have identical utility and production functions.

iv) The model is based upon two periods; in the first period the borrower possesses income m_1 and he has to make a decision about a project and consequently the amount of borrowing for financing the same. In the second period, the borrower produces and repays his liabilities (principal and profit) in case of a profit to the lender and in case of a loss settles upon the terms with the lender who also shares the loss on pre-agreed ratios.

v) There exists perfect market for borrowing and lending.

vi) Individuals aim to maximize utility and profits subject to budget constraints.

vii) All variables are in real terms.

Following Siddiqui and Zaman (1989), we shall firstly deal with fixed interest case, then equity participation *Musharakah* case and subsequently deduce our results and findings before developing the macro economic model.

FIXED INTEREST CASE:

The utility maximization problem for any borrower can be written as:

$$Max \quad U = U(C_1) + \alpha U(C_2) \qquad 0<\alpha<1$$

Subject to:

$$C_1 = m_1 + B - Q$$
$$C_2 = f(Q) - (B + rB)$$

where

C_1	=	consumption in the first period
C_2	=	consumption in the second period
α	=	discount factor (MPC) for second period
B	=	amount of borrowing in the first period
Q	=	cost of output in the first period
f(Q)	=	revenue of output produced in the first period
r	=	fixed rate of interest on borrowing

Taking 'B' and 'Q' as choice variables, the first order necessary condition for maximization is as follows:

$$Max \quad U(C_1) + \alpha U(C_2)$$

Putting the values of C_1 and C_2, we get,

$$u(m_1 + B - Q) + \alpha U [f(Q) - (1+r)B]$$

$$\Rightarrow \frac{\partial u}{\partial B} = u'(m_1 + B - Q)(1) + \alpha U'[f(Q) - (1+r)B](1)(1 + r) = 0$$

$$\Rightarrow U'(C_1) - \alpha U'(C_2)(1 + r) = 0$$

$$\Rightarrow \frac{U'(C_1)}{U'(C_2)} = \alpha(1 + r) \qquad (1)$$

$$\Rightarrow \frac{\partial u}{\partial Q} = U'(m_1 + B - Q)(-1) + \alpha U'[f(Q) - (1 + r)B]f'(Q) = 0$$

$$\Rightarrow -U'(C_1) + \alpha U'(C_2) f'(Q) = 0$$

$$\Rightarrow \frac{U'(C_1)}{U'(C_2)} = \alpha f'(Q) \qquad (2)$$

Dividing equation (1) by (2) we get,

$$1 = \frac{(1 + r)}{f'(Q)}$$

$$\rightarrow \quad f'(Q) = (1 + r) \qquad (3)$$

Equation (3) implies that at the optimal point of consumption, marginal product must equal (1+r), which is true for any production and utility function.[6] We can now, from first order condition find optimal values of the choice variables B and Q and also the equilibrium value of r. Taking a typical log utility and CES production function.[7]

$$UC_t = \ln C_t$$

where

t = 1 and 2

$$f(Q) = [\gamma K^{-\rho} + (1-\gamma)L^{-\rho}]^{-1/\rho} \qquad (4)$$

where $0<\gamma<1, -1<\rho \neq 0$

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[6] Pl. See Chapter IV of this thesis for similar deductions provided by Bohm Bawerk and Fisher.

[7] Siddiqui and Zaman (1989) have however, used power production function to derive optimal values of choice variables.

The maximization problem can be stated as under:

$$Max\ U(C_1) + \alpha\ U(C_2)$$

$$\rightarrow\ \ln(C_1) + \alpha \ln(C_2)$$

substituting the values of C_1 and C_2

$$\ln(m_1 + B - Q) + \alpha \ln [f(Q) - (1+r)B]$$

Taking choice variables as B and K (assuming k as only variable factor) the above equation can be written as:

$$Max\ \ln(m_1+B-K) + \alpha \ln [[\gamma K^{-\rho}+(1-\gamma)L^{-\rho}]^{-1/\rho}-(1+r)B]$$

First order condition can accordingly be written as

$$\frac{\partial U}{\partial B} = \frac{1}{(m_1+B-K)}(1) + \frac{\alpha(-1)(1+r)}{[[\gamma k^{-\rho} + (1-\gamma)L^{-\rho}]^{-1/\rho} - (1+r)B]} = 0$$

$$\rightarrow \frac{1}{U(C_1)} = \frac{\alpha(1+r)}{U(C_2)} \qquad (5)$$

$$\frac{\partial U}{\partial K} = \frac{1}{(m_1+B-K)}(-1) + \frac{\alpha(-1/\rho)[\gamma K^{-\rho} + (1-\gamma)L^{-\rho}]^{(-1/\rho)-1}[-\rho\gamma K^{-\rho-1}]}{[[\gamma K^{-\rho} + (1-\gamma)L^{-\rho}]^{-1/\rho} - (1+r)B]} = 0$$

$$\frac{\partial U}{\partial K} = \frac{-1}{U(C_1)} + \frac{\alpha[\gamma K^{-\rho} + (1-\gamma)L^{-\rho}]^{-\frac{(1+\rho)}{\rho}}[\gamma K^{-(\rho+1)}]}{U(C_2)} = 0 \quad (6)$$

since

$$Q = [\gamma K^{-\rho} + (1-\gamma)L^{-\rho}]^{-1/\rho}$$

and $\quad Q_K = \dfrac{\partial Q}{\partial K} = -1/\rho[\gamma K^{-\rho} + (1-\gamma)L^{-\rho}]^{-1/\rho-1}[-\rho\gamma K^{-\rho-1}]$

$$\Rightarrow \quad Q_K = [\gamma K^{-\rho} + (1-\gamma)L^{-\rho}]^{\frac{-(1+\rho)}{\rho}}[\gamma K^{-(1+\rho)}]$$

$$= [[\gamma K^{-\rho} + (1-\gamma)L^{-\rho}]^{-(1/\rho)}]^{1+\rho}[\gamma K^{-(1+\rho)}]$$

$$= \frac{\gamma(Q)^{1+\rho}}{K^{1+\rho}}$$

$$\Rightarrow \quad Q_K = \gamma(Q/K)^{1+\rho}$$

Now equation (6) can be written as

$$\frac{1}{U(C_1)} = \frac{\alpha\gamma(Q/K)^{1+\rho}}{U(C_2)} \qquad (7)$$

Dividing equation (5) by (7) we get,

$$1 = \frac{\alpha(1+r)}{\alpha\gamma(Q/K)^{1+\rho}}$$

$$\Rightarrow \quad \gamma(Q/K)^{1+\rho} = (1+r) \qquad (8)$$

Hence equation (8) states that the marginal productivity of capital is (1 + r)

For finding the optimal value of B, we cross multiply equation (5) and get

$$U(C_2) = U(C_1)\ \alpha(1+r)$$

Substituting the values of (C_2) and (C_1) we get,

$$U[f(Q) - (1+r)B] = U[(m_1+B-Q)\alpha(1+r)]$$

$$\rightarrow f(Q) - B - rB = (\alpha + \alpha r)(m_1 + B - Q)$$

$$\rightarrow f(Q) - B - Br = \alpha m_1 + \alpha B - \alpha Q + \alpha r m_1 + \alpha r B - \alpha r Q$$

$$\rightarrow -B - Br - \alpha B - \alpha Br = \alpha m_1 - \alpha Q + \alpha m_1 r - \alpha Q r - f(Q)$$

$$\rightarrow B(-1 - r - \alpha - \alpha r) = \alpha m_1 - \alpha Q + \alpha m_1 r - \alpha Q r - f(Q)$$

$$B = \frac{\alpha m_1 - \alpha Q + \alpha m_1 r - \alpha Q r - f(Q)}{(-1 - r - \alpha - \alpha r)}$$

$$\rightarrow B = \frac{\alpha m_1(1+r) - \alpha Q(1+r) - f(Q)}{[-1(1 + r) - \alpha(1+r)]}$$

Multiplying and dividing the above equations by (-1) we get

$$B = \frac{f(Q) - (\alpha m_1 - \alpha Q)(1 + r)}{(1 + r)(1 + \alpha)}$$

$$B^* = \frac{f(Q) - \alpha(1 + r)(m_1 - Q)}{(1 + r)(1 + \alpha)} \qquad (9)$$

Similarly the values of r* and Q* can be obtained by cross multiplying the equation (9) in terms of variables.

MUSHARAKAH (SHARING) CASE

In the '*Musharakah*' case the maximization problem of lender and borrower shall be different. The borrower's maximization problem can be stated as:

$$Max\ U(C_1) + \alpha U(C_2)$$

Subject to:

$$C_1 = m_1 + B - Q$$

$$C_2 = f(Q) - S[f(Q) - Q] - B$$

where S is the share of profit to the lender in the second period and is a function of Q such that $S'(Q) > 0$.[8]

Substituting the values of (C_1) and (C_2) we get,

$$Max\ U\ (m_1 + B - Q) + \alpha U\ [f(Q) - S[f(Q) - Q] - B]$$

For first order condition,

$$\frac{\partial U}{\partial B} = U'(m_1 + B - Q)(1) + \alpha U'[f(Q) - S(f(Q)-Q)-B](-1) = 0$$

[8] Khan (1992) has also taken the profit sharing ratio as a function of investment in computing the marginal productivity of capital after stating that if the sharing ratio is only taken as fixed then $MP_K = 1$ which would imply zero opportunity cost of capital and is a naive assumption.

$$\Rightarrow \quad U'(C_1) = \alpha U'(C_2) \qquad (10)$$

$$\frac{\partial U}{\partial Q} = U'(m_1 + B - Q)(-1) + \alpha U[f(Q) - S[F(Q) - Q)] - B][f'(Q) - [S(f'(Q) - 1) + S'(f(Q) - Q)] = 0$$

$$\Rightarrow \quad U'(C_1) = \alpha U'(C_2)[f'(Q) - Sf'(Q) + S - S'f(Q) + S'(Q)] \qquad (11)$$

Dividing equation (10) by (11) we get

$$1 = \frac{1}{f'(Q) - Sf'(Q) + S - S'f(Q) + S'Q}$$

$$\Rightarrow \quad f'(Q) - Sf'(Q) + S - S'f(Q) + S'Q = 1$$

$$\Rightarrow \quad f'(Q)\,(1 - S) = 1 - S + S'f(Q) - S'Q$$

$$\Rightarrow \quad f'(Q) = 1 + \frac{S'[f(Q) - Q]}{1 - S} \qquad (12)$$

From equation (12) it is evident that the Marginal productivity in profit sharing case shall be dependent upon the profit sharing ratio S, S´ (the rate at which S increases with increase in investment) and the level of profit of the enterprise (f(Q) - Q).

Similarly, the maximization problem of the lender (bank) can be defined as:

$$Max \quad U(C_1) + \alpha U(C_2)$$

subject to:

$$C_1 = m_1 - B - Q$$

$$C_2 = f(Q) + B - S^*[f(Q) - Q)]$$

Taking B and Q as choice variables, the first order maximization condition is as follows, after putting the values of (C_1) and (C_2), we get[9]

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[9] By treating lender as a bank, S^*, shall be the depositor's rate of profit, and is different from bank rate of profit (S)

$$Max\ U(m_1-B-Q) + \alpha U[f(Q)+B-S^{*}[f(Q)-Q]]$$

$$\frac{\partial U}{\partial B} = U'(m_1-B-Q)(-1) + \alpha U'[f(Q)+B-S^{*}[f(Q)-Q]] = 0$$

$$\Rightarrow U'(C_1) = \alpha U'(C_2) \qquad (13)$$

$$\frac{\partial U}{\partial Q} = -U'(C_1) + \alpha U'(C_2)[f(Q) - [S^{*}[f(Q)-1] + \acute{S}^{*}[f(Q)-Q]] = 0$$

$$\Rightarrow U'(C_1) = \alpha U'(C_2)[f'(Q) - S^{*}f'(Q) + S^{*} - S^{*\prime}f(Q) + S^{*\prime}Q \qquad (14)$$

Dividing equation (13) by (14) we get

$$1 = \frac{1}{f'(Q) - S^{*}f'(Q) + S^{*} - S^{*\prime}f(Q) + S^{*\prime}Q}$$

$$\Rightarrow f'(Q) - S^{*}f'(Q) = 1 - S^{*} + S^{*\prime}f(Q) - S^{*\prime}Q$$

$$\rightarrow \quad f'(Q)(1 - S^{*}) = 1 - S^{*} + S^{*\prime}[f(Q) - Q]$$

$$\rightarrow \quad f'(Q) = 1 + \frac{S^{*\prime}[f(Q) - Q]}{1 - S^{*}} \qquad (15)$$

Comparing the first order condition of the borrower and lender, it is deduced that $f'(Q) = 1 + \frac{S^{*\prime}[f(Q) - Q]}{1 - S^{*}}$ is identical in nature for both.

Siddiqui and Zaman (1989) also derived the same result but with the condition of constant return to scale.

If S is not a function of level of output and is assumed to be fixed at some level S_L, then the maximization problem of borrower or lender with respect to Q and B would be

$$Max\ U(C_1) + \alpha U(C_2)$$

subject to:

$$C_1 = m_1 + B - Q$$

$$C_2 = f(Q) - S_L[f(Q) - Q] - B$$

$$\Rightarrow \quad U(m_1+B-Q) + \alpha U\left[f(Q) - S_L[f(Q) - Q] - B\right]$$

First order maximization condition would be

$$\frac{\partial U}{\partial B} = U'(C_1) + \alpha U'(C_2)(-1) = 0$$

$$\Rightarrow \quad U'(C_1) = \alpha U'(C_2) \qquad (16)$$

$$\frac{\partial U}{\partial Q} = U'(C_1)(-1) + \alpha U'(C_2)[f'(Q) - S_L(f'(Q)-1)] = 0$$

$$\Rightarrow \quad U'(C_1) = \alpha U'(C_2)[f'(Q) - S_L f'(Q) + S_L] \qquad (17)$$

Dividing equation (16) by (17), we get;

$$1 = f'(Q) - S_L f'(Q) + S_L$$

$$\Rightarrow 1 = f'(Q)(1 - S_L) + S_L$$

$$\Rightarrow f'(Q) = \frac{(1 - S_L)}{(1 - S_L)}$$

$$\Rightarrow f'(Q) = 1 \qquad (18)$$

This implies that if the share of profit paid by the borrower is independent of the borrowing and is fixed, then production would be to the point where $MP_K = 1$. Since our model deals with 'S' being function of output, the ratio is to increase with 'Q'. Subsequently, the borrower would demand funds only for productive and yielding projects, simultaneously the lender would also be willing to invest with higher expected return. It can now be inferred that since the demand for and supply of funds would be dependent upon the output and in turn on S, the possibility of wastage and idle capacity would be reduced.

We have upto now discussed two cases, one with fixed and predetermined rate of interest where $MP_K = 1 + r$ and the other with profit sharing ratio 'S' as a function of 'Q' where

$MP_K = 1 + \left[\frac{S'[f(Q) - Q]}{1 - S}\right]$. From this analysis certain things emerge.

It is not certain whether the investment level would be higher or lower in sharing case as it would be dependent upon the level of profit sharing, level of profit and the rate at which 'S' increases with an increase in investment. In this context it can also be derived that in the absence of fixity of return, this system would result in risk sharing between lenders and borrowers. Consequently the investment behaviour and the level of investment may change due to this system. Under the sharing system a fixed cost for capital is no longer required to be met as a part of a firm's profit calculations and hence there will be no constraint of meeting fixed liability in shape of fixed interest charges. Hence, the MP_K can therefore be taken upto the point where maximum profits are obtained. The sharing system can thus be considered as one which has more sustainability against market failure.[10] In the study so far conducted in this section, the other conclusion regarding the effect of adoption of profit sharing system is that, it is more adjustable to shocks and fluctuation.[11] We have observed that the absence of a fixed and predetermined rate of interest is the main feature which distinguishes the profit-sharing system from the interest based system. An important inference drawn from our study has been the close link between the real and financial sector in the equity based system.

Since there will be no debt instruments in the equity based system, and the only assets that will exist would be claims to capital stock or equity shares. The question arises as to the nature of macro-economic equilibrium in such a system.

MACRO ECONOMIC MODEL BASED ON ISLAMIC FINANCIAL PRINCIPLES

The following study will represent an aggregate macro-economic model under the assumptions of a hypothetical interest free Islamic economic

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[10] Haque, Nadeem ul and Mirakhor, Abbas (1987), p. 158.

[11] Khan (1986) also demonstrates that the Islamic System may well prove to be better suited to adjusting to shocks that result in banking crisis, p.19.

system. Using the Keynesian frame work, the model with some restrictive assumptions will be simple but shall be useful to comprehend the essence and implication of the change from interest economy to interest free economy.

Using the versatile Hicksian analytical tools with the appropriate adjustments and modifications, the modified IS-LM framework will demonstrate the macro economic analysis in an Islamic economy.

The resultant model will contain capital market, money market and goods market. Labour market exists and is assumed to be in equilibrium according to Walrasian law.

ASSUMPTIONS OF THE MODEL

i) There is one financial institution called bank.

ii) The bank replaces fixed interest rate by profit sharing ratio, which indicates that the bank shall have two ratios. One which it shall share with the depositor (S^*) and the other with the borrower (S). The former may be called the depositor's rate of profit (DRP) and the latter as Bank's rate of profit (BRP).

iii) All variables are measured in real terms.

iv) The aggregate supply is completely elastic with change in spending leading to equal change in output.

v) For simplicity, we assume closed economy.

These broad assumptions when incorporated in functions would give a distinctive character to the model in terms of Islamic banking.

CAPITAL MARKET

The basic difference between the Keynesian Model and the Islamic economic Model stems from the substitution of equity market in place of bond market. Keynesian transmission mechanism operates through the bond market i.e. people substitute bond for money as expected return on bonds increases and vice versa.

In our model the banks operate as firms, issuing their deposits certificates (other than current accounts) without fixed and predetermined return or even guarantee for value of such shares/certificates. The return would thus primarily depend upon the profitability in the real sector. The supply of investible funds from savers would, therefore, depend upon the profitability in the real sector which in essence would have positive consequential relation with BRP(S) and subsequently DRP(S*).

In this context, it is however, assumed that all investments in the economy will be undertaken by borrowing from banks.

From the foregoing it can easily be deduced that as the value of shares (Deposit Certificates) increases due to increased productivity in the real sector, the demand for speculative purposes would simultaneously decrease and resource mobilization would increase. Using the discounting formula suggested by Metzler (1951) which has been also incorporated by Khan (1986) in his analysis, we determine BRP(S) in our capital market.

The formula which relates the real value of banks' shares to the capitalized value of the future real earnings of the bank can be written as:

$$\frac{E_V}{P} = \frac{Y}{S} \tag{19}$$

where

E_V stand for nominal value of shares

P stand for general price level (given)

Y stands for real income

S stands for BRP

From equation (19), the value of S can be taken as:

$$S = \frac{Y\,P}{E_V}$$

Equation (19) thus depicts that real value of shares is the capitalized value of business profits, where capitalization is done at the prevailing BRP(S).[12]

The capital market therefore in our model determines BRP(S) which in turn will determine the demand for money in the money market. The determined value of BRP(S) in the capital market will thus equate the

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[12] Khan (1986) has also used similar relationship to evaluate his capital market equilibrium.

demand for money with supply of money in the money market.

EQUILIBRIUM IN MONEY MARKET - (LM - Curve)

Under Keynesian framework, the money is demanded for precautionary, transactive and speculative purposes. However, in an Islamic economy since there is no concept of speculative motive of holding money, the only motives left would be precautionary and transactive. Since, there is no bond market, the only other demand for money will come from the market for equity capital.[13]

The demand for money then is a function of income and S (BRP), with positive relationship with income whereas negative relationship with S. Mathematically we can write,

$$M_d = f(Y, S)$$

where $\frac{\partial M}{\partial Y} > 0$ and $\frac{\partial M}{\partial S} < 0$. In linear form thus

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[13] Please see Mahdi and Asaly (1991), p.59.

$$M_d = K_Y + Is$$

The supply of money is assumed to be determined by the monetary authorities and is exogenously given such that:

$$Ms = \bar{M}_s$$

The equilibrium in money market can thus be written as:

$$\bar{M}_s = K_Y + Is \qquad (20)$$

Fig. 1(a)

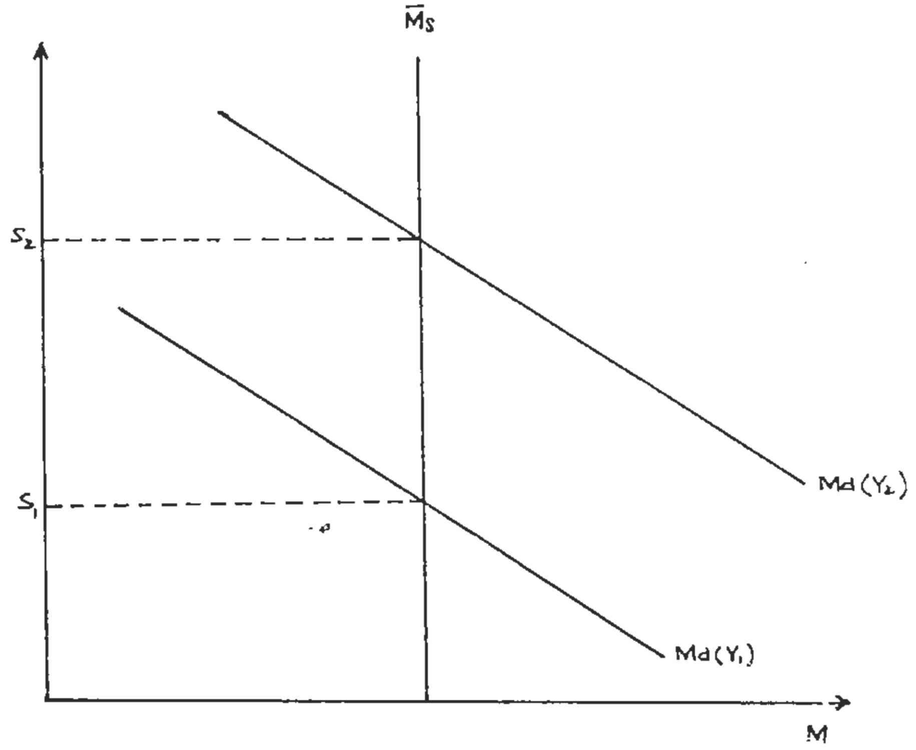
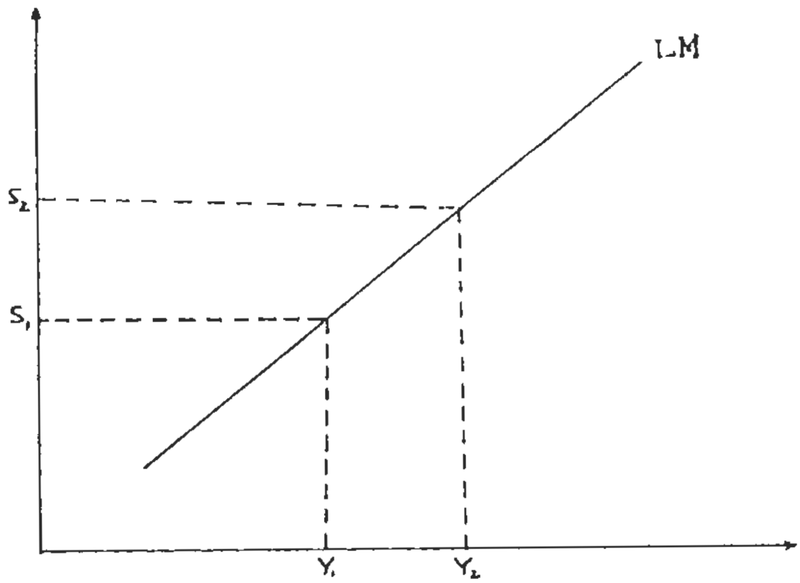


Fig. 1(b)



Equation (20) describes the money market equilibrium. Fig. 1(a) and 1(b) are the graphical representations of the derivation of LM-curve which is the locus of different combinations of Y and S that maintain equilibrium in money markets.

To find the slope of LM-curve, we take the total differentials of equation (20) which is also the equation of LM curve, we get

$$0 = \frac{\partial K}{\partial Y}dy + \frac{\partial l}{\partial S}.ds$$

$$\rightarrow \quad -\frac{\partial K}{\partial Y}dy = \frac{\partial I}{\partial S}.ds$$

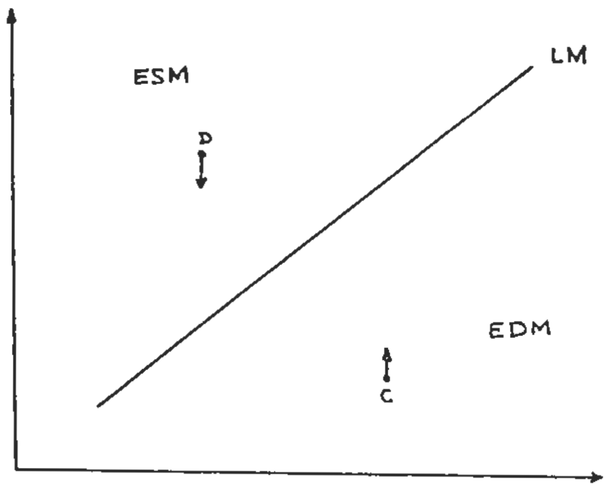
$$\rightarrow \quad \frac{\partial Y}{\partial S} = -\frac{\partial I}{\partial S}/\frac{\partial K}{\partial Y}$$

$$\rightarrow \quad \frac{dy}{ds}\Big| \bar{M}s = -\frac{I'}{K'} > 0 \qquad (21)$$

As $\frac{\partial L}{\partial S} < 0$

To the right of LM curve (Fig.2) there is an excess demand for money (EDM), and to the left of LM curve there is excess supply of money.

Fig.2



At any point to the right of LM, say at C, since there is an excess demand for money, BRP would rise to maintain equilibrium in the money market. Similarly, at point D on the left of LM curve since there is excess supply of money, BRP would fall to maintain equilibrium in money market. The changes in monetary policy therefore would shift the LM curve. The expansionary policy would shift the LM curve to the right and vice versa.

THE MARKET FOR GOODS AND SERVICES

The market for goods and services comprises of the consumption, the investment and government expenditure functions. Specifications of each function is appended below:

THE CONSUMPTION FUNCTION

Our consumption function is based upon the utility maximization behaviour with respect to the choice between present and future consumption subject to the budget constraint.

General form of consumption function can be written as:

$$C = C\,(Y_d, S^*, W)$$

where

Y_d represents real disposable income[14]

S^* represents sharing ratio of bank with depositors and

W represents wealth derived by financial assets.

The specific linear equation describing the consumption function may then be written as

$$C = C_0 + \alpha(Y - T) - \beta(S^*) + \gamma w \qquad (22)$$

subject to:

$$\frac{\partial C}{\partial Y_d} > 0, \frac{\partial C}{\partial S} < 0, \frac{\partial C}{\partial W} > 0$$

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[14] Khan (1986) has taken (r, w) and Sattar (1991) has taken (Y_d, w) as variables determining consumption

THE INVESTMENT FUNCTION

Basically, a bank in an Islamic banking frame work shall be a financial intermediary like in the conventional banking system, except that rather than charging a fixed predetermined cost on lending and fixed return paid to savers, it would be based upon profit and loss sharing system. The bank would charge a certain bank rate of profit (BRP) from the borrower and consequently pay 'depositors rate of profit' (DRP) to the savers.[15]

Although, there is no concept of a fixed cost on borrowing, however, as the appropriation of profits in company's accounts would be a net payout to lender, the behaviour of investment demand by borrower would be inversely related to BRP (S) which would be the percentage of borrowers' profit going to the bank.

In the real sector of the economy, as the investment primarily depends on the expected rate of profit.[16] This notion is also based upon our micro-foundation of the model in a sense that as BRP(S) increases,

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[15] Please see Khan (1985), p. 74 for analysis on Siddiqui's Model which postulates that Mudarabah advances are increasing function of the BRP, and on the other hand, supply of deposits by savers is a direct function of DRP.

[16] Pl. see Metzler (1951) pg.94

marginal productivity of capital decreases as depicted by equation (12). The increase in ex-post profit will simultaneously increase the BRP, in essence therefore, the increase in profitability would be coupled with increase in investment but with diminishing rate. Mathematically, we can write as follows.

$$I = f\,(S,\ e_p)$$

where $\frac{\partial I}{\partial S} \prec 0, \frac{\partial I}{\partial e_p} \succ 0$, S is BRP and e_p is expected rate of profit.

In its linear form, the above can be written as

$$I = I_0 - iS + e_p \qquad (23)$$

THE GOVERNMENT EXPENDITURE FUNCTION

An Islamic state must aim for welfare of all the people with distributive justice. The government shall therefore adjust its expenditure in such a way that efficient allocation of resources take place. The adjustment would ensure that all factors of production are working at full capacity so that the government expenditure will be dependent on the relative level of output produced in the economy. Mathematically, it can be represented as:

$$G = G_0 + f(Y_F - Y) \qquad (24)$$

Where G_0 is the exogenous government expenditure and the expression $(Y_F - Y)$ is the difference between full employment output and the equilibrium level of output produced in the economy, such that $\frac{\partial G}{\partial Y} > 0$ [17]

EQUILIBRIUM IN GOODS MARKET - (IS - CURVE)

Setting aggregate demand equal to aggregate supply, we arrive at the equilibrium condition

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[17] Sattar (1991), p. 89 has also taken the govt. expenditure a function of the difference between the full employment output level and the equilibrium level of output.

$$Y = C + I + G \qquad (25)$$

Putting the respective values of C, I and G, from equations (22), (23) and (24) in (25), we get;

$$Y = C_0 + \alpha(Y-T) - \beta(S^{*}) + \gamma W + I_0 - IS + e_p + G_0 + G_Y \qquad (26)$$

$$Y = C_0 + \alpha Y - \alpha T - \beta(S^{*}) + \gamma W + I_0 - iS + e_p + G_0 + G_Y$$

$$\Rightarrow \quad Y - \alpha Y - G_Y = C_0 + I_0 + G_0 - \beta S^{*} + \gamma w - iS + e_p - \alpha T$$

$$\Rightarrow \quad Y(1-\alpha-G) = A_0 - \beta S^{*} + \gamma w - iS + e_p - \alpha T$$

where $A_0 = C_0 + I_0 + G_0$

$$\bar{Y} = \frac{A_0 - \beta S^{*} + \gamma w - iS + ep - \alpha T}{1 - \alpha - G} \qquad (27)$$

Setting equilibrium level of National Income in generalized functional form, we can write:

$$Y = C(Y_d, S^*, W) + I(S, e_p) + G_y \qquad (28)$$

Equation (27) above is the equilibrium level of national income, which is obtained by equating aggregate demand and aggregate supply.

We now proceed to construct the IS-curve which is the locus of different combinations of Y and S (BRP) which maintains equilibrium in the goods market (Figure 3(a) and 3(b). To find the slope of IS-curve, taking total derivative of the generalized equation (28) we get,

$$dy = \frac{\partial C}{\partial Y} \cdot dy + \frac{\partial S^*}{\partial Y} \cdot dy + \frac{\partial T}{\partial S} \cdot ds + \frac{\partial I}{\partial e_p} \cdot dep + \frac{\partial G}{\partial Y} \cdot dy$$

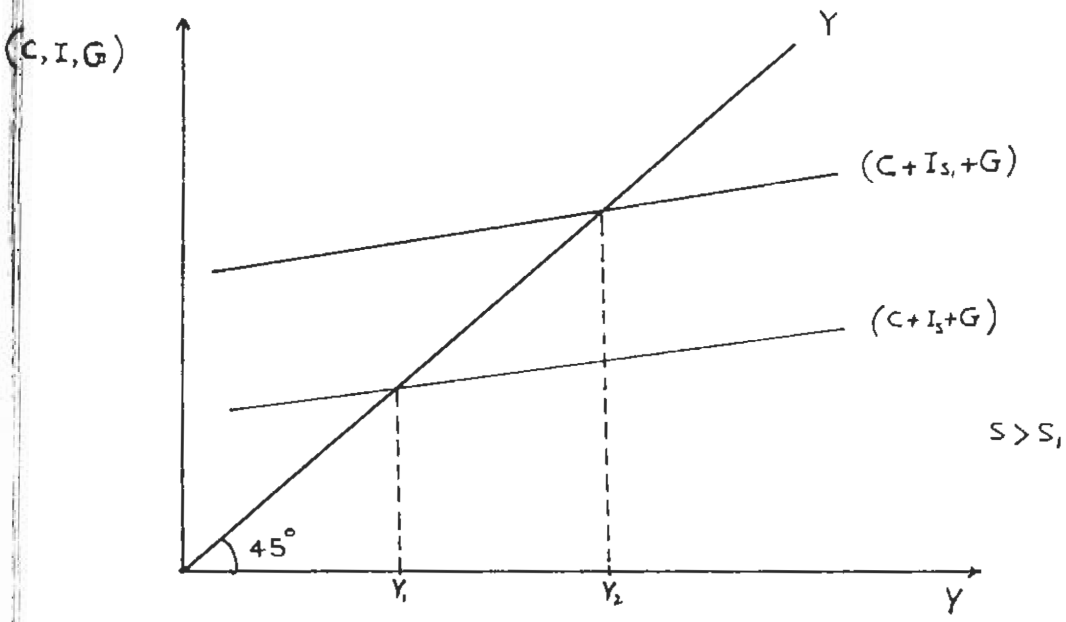
$$dy - \frac{\partial C}{\partial Y} \cdot dy - \frac{\partial S^*}{\partial Y} \cdot dy - \frac{\partial G}{\partial Y} \cdot dy = \frac{\partial I}{\partial S} \cdot ds + \frac{\partial I}{\partial e_p} \cdot dep$$

$$\Rightarrow dy\left[1 - \frac{\partial C}{\partial Y} - \frac{\partial S^*}{\partial Y} - \frac{\partial G}{\partial Y}\right] = \frac{\partial I}{\partial S} \cdot ds + \frac{\partial I}{\partial de_p} dep$$

$$\Rightarrow \quad dy = \frac{\frac{\partial I}{\partial S} \cdot ds + \frac{\partial I}{\partial ep} \cdot dep}{1 - \frac{\partial c}{\partial Y} - \frac{\partial S^{*}}{\partial Y} - \frac{\partial G}{\partial Y}}$$

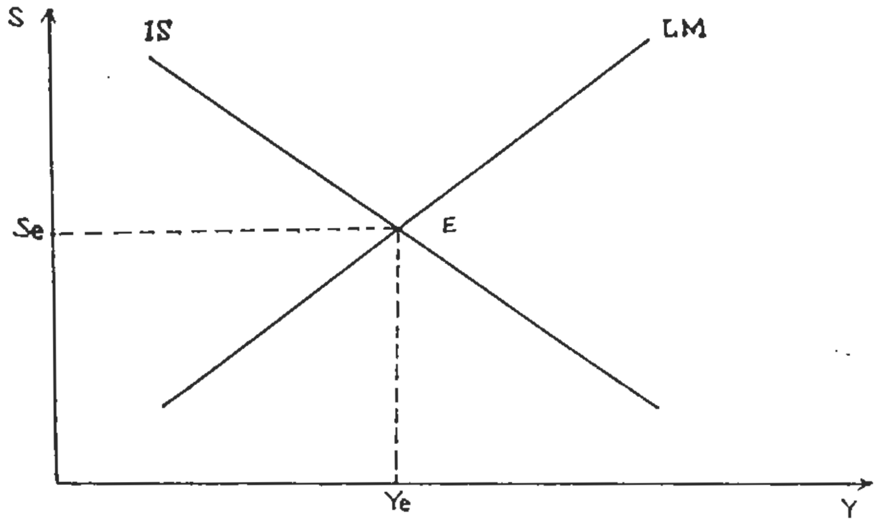
$$\frac{dy}{ds}\bigg| \; d_G = d_{ep} = dc = 0 = \frac{\partial I / \partial S}{1 - \frac{\partial c}{\partial Y} - \frac{\partial S^{*}}{\partial Y} - \frac{\partial G}{\partial Y}} < 0 \qquad (29)$$

Fig.3(a)



The equilibrium is unique at point E. As far as the stability of the equilibrium is concerned the adjustment process is indicated by arrows in Fig.6 and summarized in Table 1.

Fig. 5 & 6



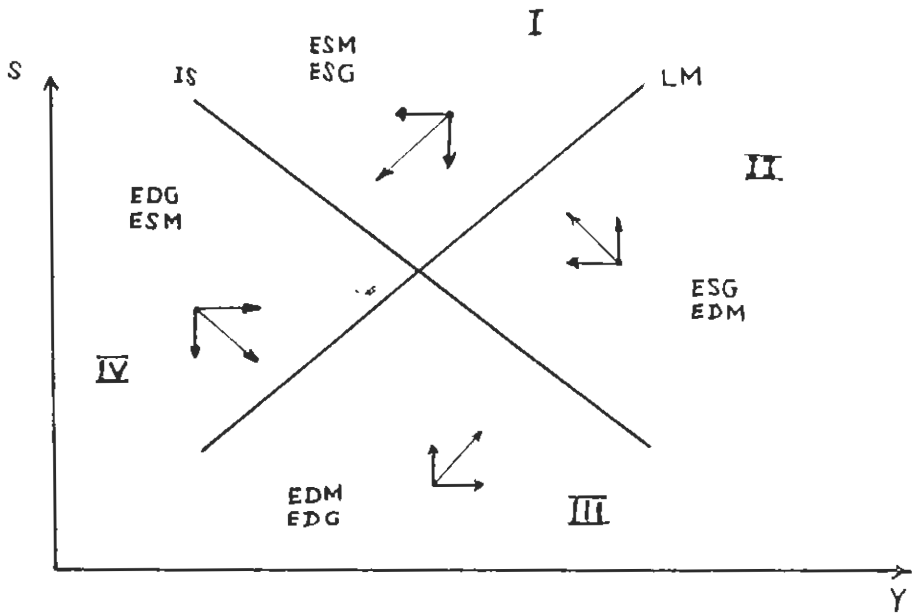


Table 1

REGION	DISEQUILIBRIUM	ADJUSTMENT		CONVERGENT/ DIVERGENT
I	ESM, ESG	S↓	Y↓	Convergent
II	EDM, ESG	S↑	Y↓	Convergent
III	EDM, EDG	S↑	Y↑	Convergent
IV	ESM, EDG	S↓	Y↑	Convergent

The analysis presented above represents the properties of comparative static analysis in depicting the convergent equilibrium for any situation of disequilibrium.

DYNAMIC ADJUSTMENTS

In order to assess the dynamic stability of our model we would check if Y tends to $\bar{Y}$ (equilibrium income level) with time. In other words the question concerning the dynamic stability is whether the time path Y(t) tends to converge to $\bar{Y}$, as t tends to ∞.

We find the time path Y(t) in the goods market by assuming that the rate of change of output with respect to time at any moment is directly proportional to excess aggregate demand (Y_d - Y_S) at that moment. Mathematically we can write:

$$\frac{dy}{dt} = j(Y_d - Y_s) \qquad (30)$$

where j > 0 and is adjustment coefficient.

We can specify (30) above in the form

$$\frac{dy}{dt} = j[C\ (Y_d, W, S^*) \ + \ I(S, e_p) \ + \ G(Y) - Y_s]$$

$$\Rightarrow \frac{dy}{dt} = j[C\ (Y_d, W, S^*) \ + \ I(S, e_p) \ + \ G(Y)] - j(Y_s)]$$

$$\Rightarrow \frac{dy}{dt} + j(Y_s) = j\ [C(Y_d, W, S^*) \ + \ I(S, e_p) \ + \ G(Y)] \qquad (31)$$

Equation (31) now is in the form of differential equation. Putting the values of c, I and G in equation (30) We get,

$$\frac{dy}{dt} = \lambda[C_0 + \alpha Y - \alpha T + \gamma W + \beta S^{*} + I_O - iS + e_p + G_O + GY - Y]$$

$$= \lambda[A_O + \alpha Y - \alpha T + \gamma W + \beta S^{*} - iS + e_p + Gy - y]$$

where $A_0 = C_0 + I_0 + G_0$

$$= \lambda[(A_O - \alpha T + \gamma W + \beta S^{*} - iS + e_p) + (\alpha + G - 1)Y]$$

$$= \lambda(A_O - \alpha T + \gamma W + \beta S^{*} - iS + e_p) - \lambda[1 - \alpha - G]Y$$

$$\frac{dy}{dt} + j[1 - \alpha - G)\ Y = \lambda[A_0 - \alpha T + \gamma W + \beta S^{*} - iS + e_p] \quad (32)$$

Equation (32) is also in the form of differential equation,

$$\frac{dy}{dt} + ay = b$$

Applying the solution formula[18]

$$y(t) = [Y(0) - b/a]e^{-at} + b/a$$

we get

$$y(t) = [y(0) - \frac{[J(A_0 - \alpha T + \gamma w + \beta S^{*} - iS^{*} + eP)]e^{-J(1-\alpha-G)t}}{j(1 - \alpha - G)}$$

$$+ \frac{j(A_0 - \alpha T + \gamma w + \beta S^{*} - iS + e_P)}{j(1-\alpha-G)}$$

where $\bar{Y} = \frac{A_0 - \alpha T + \gamma w + \beta S^{*} - iS + e_P)}{1 - \alpha - G}$ *from eq.(27)*

hence

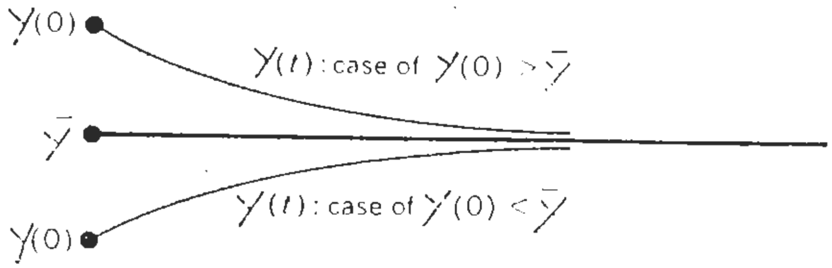
$$Y_t = [Y(0) - \bar{Y}]e^{-kt} + \bar{Y} \qquad (33)$$

[18] Please see chiang, C. Alpha Pg. 473

where $K = j(1 - \alpha - G)$

Now to check the dynamic stability of the equilibrium we see whether $Y(t) \rightarrow \bar{Y}$ as $t \rightarrow \infty$ in equation (33). As Y(0) and $\bar{Y}$ are constants, the key factor shall be the exponential expression e^{-kt}. Since k>0, and as $t \rightarrow \infty$, the expression tends to zero. Consequently the time path Y(t) will lead Y towards $\bar{Y}$, hence confirming the convergence and dynamic stability. Analyzing the concept of dynamic stability further we examine that stability dependson the relative magnitude of Y(0) and $\bar{Y}$. If $Y(0) = \bar{Y}$, then $Y(t) = \bar{Y}$, therefore the time path of Y can be drawn as the horizontal straight line. If $Y(0) > \bar{Y}$, then the first term of right side of equation (33) is positive but it will decrease as the increase in t reduces the value of e^{-kt}, hence the time path Y(t) will approach the equilibrium level from above, (as shown in figure (7) below). Similarly if $Y(0) < \bar{Y}$, then equilibrium will approach from below.[19]

Fig.7



CONCLUSION

In this chapter we have developed a model of interest free Islamic economic system using micro economic foundations for macro economic analysis. In our micro analysis the conventional utility maximization approach is adopted and it has been shown that the marginal productivity of the lender and the borrower is similar in nature and analogous to western economic theories of fixed interest case. Using the micro foundation in our macro economic model we have analyzed in

[19] Ibid, p. 477

Keynesian framework that by replacing the institution of interest by *Musharakah* finance, the system is operative on variable return on capital instead of fixed pre-determined return. The system as proven above possesses the desirable properties of uniqueness and stability of equilibrium both in static and dynamic analysis. Having developed the basic framework for micro and macro economic analysis in an Islamic economy the next chapter would deal with policy implications of such a system in terms of Islamic banking.

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