Bibliography


Harris, F. J. (1987), Exact FM detection of complex time series, in ‘Proc. ISSPA’, Brisbane, Australia, pp. 70–73.


Appendix–A

Generalized Expression of Split Radix FFT Algorithm for Zero Padded Signals

In Section 3.4.3, we have proposed an optimized flexible split radix FFT algorithm for zero padded signals. A generalized mathematical form is shown here. Consider a sequence:

\[ a_n = \{x_0, x_1, x_2, ..., x_7\} \]

\[
\sum_{n=0}^{7} a_n e^{-j\frac{2\pi}{8}nk} = \sum_{n=0}^{3} a_{2n} e^{-j\frac{2\pi}{8}2nk} + e^{-j\frac{\pi k}{4}} \sum_{n=0}^{3} a_{2n+1} e^{-j\frac{2\pi}{8}2nk} \\
= \sum_{n=0}^{3} a_{2n} e^{-j\frac{2\pi}{4}nk} + e^{-j\frac{\pi k}{4}} \sum_{n=0}^{3} a_{2n+1} e^{-j\frac{2\pi}{4}nk}
\]
Let \( a_{2n} = b_{2n} \) and \( a_{2n+1} = c_{2n} \)

\[
\sum_{n=0}^{7} a_n e^{-j\frac{2\pi}{8}nk} = \sum_{n=0}^{3} a_{2n} e^{-j\frac{2\pi}{8}nk} + e^{-j\frac{\pi}{4}} \sum_{n=0}^{3} a_{2n+1} e^{-j\frac{2\pi}{8}nk} \\
= \left[ \sum_{n=0}^{1} b_{2n} e^{-j\frac{2\pi}{4}2nk} + e^{-j\frac{\pi}{4}} \sum_{n=0}^{3} b_{2n+1} e^{-j\frac{2\pi}{8}2nk} \right] \\
+ e^{-j\frac{\pi}{4}} \left[ \sum_{n=0}^{1} c_{2n} e^{-j\frac{2\pi}{4}2nk} + e^{-j\frac{\pi}{4}} \sum_{n=0}^{3} c_{2n+1} e^{-j\frac{2\pi}{8}2nk} \right] \\
= \sum_{n=0}^{3} b_{2n} e^{-j\frac{2\pi}{8}nk} + e^{-j\frac{\pi}{8}} \sum_{n=0}^{3} b_{2n+1} e^{-j\frac{2\pi}{8}nk} \\
+ e^{-j\frac{\pi}{8}} \left[ \sum_{n=0}^{1} c_{2n} e^{-j\frac{2\pi}{8}nk} + e^{-j\frac{\pi}{8}} \sum_{n=0}^{3} c_{2n+1} e^{-j\frac{2\pi}{8}nk} \right] \tag{A.1}
\]

Now we suppose that two zeros are padded. So changing the summands with four point DFT and also changing the twiddle factors in equation (A-1),

\[
\sum_{n=0}^{1} b_{2n} e^{-j\frac{2\pi}{8}nk} + e^{-j\frac{\pi}{8}} \sum_{n=0}^{3} b_{2n+1} e^{-j\frac{2\pi}{8}nk} \\
+ e^{-j\frac{\pi}{8}} \left[ \sum_{n=0}^{1} c_{2n} e^{-j\frac{2\pi}{8}nk} + e^{-j\frac{\pi}{8}} \sum_{n=0}^{3} c_{2n+1} e^{-j\frac{2\pi}{8}nk} \right] \\
= \sum_{n=0}^{3} b_n e^{-j\frac{2\pi}{8}nk} + e^{-j\frac{\pi}{8}} \sum_{n=0}^{3} c_n e^{-j\frac{2\pi}{8}nk} \\
= \sum_{n=0}^{3} a_{2n} e^{-j\frac{2\pi}{8}nk} + e^{-j\frac{\pi}{8}} \sum_{n=0}^{3} a_{2n+1} e^{-j\frac{2\pi}{8}nk} \tag{A.2}
\]

This is DFT of eight data point DFT with eight zeros padded at the end. We generalize the results of equation (A-2) as:
\[ \sum_{n=0}^{N-1} a_n e^{-j \frac{2\pi}{Np} nk} = \sum_{n=0}^{N/2-1} a_{\text{even}} e^{-j \frac{2\pi}{Np/2} nk} + \sum_{n=0}^{N/2-1} a_{\text{odd}} e^{-j \frac{2\pi}{Np/2} nk} \]

Where \( a_n = \{x_0, x_1, x_2, ..., x_{N-1}\} \). This relationship is used recursively.
Appendix–B

DFT of an Inverted Sequence

In Section 3.2, we have derived the recursive form of Wiener-Khintchine theorem for the sliding window, which used the result in Equation (3.8) and is derived as follows. Consider a random Gaussian sequence $y[n]$ of length $N$.

$$
\text{DFT} \left[ y[N + m - 1] \right] = \sum_{m=0}^{N-1} y[N + m - 1] e^{-j \frac{2\pi}{N} nk}
$$

Let $m = N - n - 1$,

$$
\text{DFT} \left[ y[N + m - 1] \right] = \sum_{n=0}^{N-1} y[n] e^{-j \frac{2\pi}{N} (N-n-1)k}
= e^{-j \frac{2\pi}{N} (N-1)k} \sum_{n=0}^{N-1} y_2[n] e^{j \frac{2\pi}{N} nk}
= e^{j \frac{2\pi}{N} k} \left[ \sum_{n=0}^{N-1} y_2^*[n] e^{-j \frac{2\pi}{N} nk} \right]^* [B.1]
$$

For real signals, $y_2[n]^* = y_2[n]$, so

$$
\text{DFT} \left[ y[N + m - 1] \right] = e^{j \frac{2\pi}{N} k} \text{DFT}[y_2[m]]^*
= e^{j \frac{2\pi}{N} k} Y_2^*(k) \tag{B.1}
$$
Appendix–C

Recursive Padé Approximation
for ARMA(2,1)

In Chapter 4, we have developed parametric aggregate spectrogram, which works as the window slides. Recursive Padé approximation derived here, is used to efficiently update the coefficients of ARMA(2,1) model of the signal. The Padé approximation is:

\[ x[n] + \sum_{k=1}^{p} a_p(k)x(n - k) = \begin{cases} b_q(n) & n = 0, 1, \cdots, q \\ 0 & n = q + 1, \cdots, p + q \end{cases} \]  

(C.1)

In this paper, we are concerned only with the ARMA(2,1) signal model. So in this case, \( q = 1 \) and \( p = 2 \). So, the padé approximation to Equation (4.7) is:

\[ x[n] + \sum_{k=1}^{2} a_2(k)x(n - k) = \begin{cases} b_1(n) & n = 0, 1 \\ 0 & n = 2, 3 \end{cases} \]  

(C.2)

After some straightforward calculations, the AR parameters are found to be
estimated as:

\[
a_2(1) = \frac{x(1)x(2) - x(0)x(3)}{x^2(1) - x(0)x(2)} \quad (C.3)
\]

\[
a_2(2) = \frac{x(1)x(3) - x^2(2)}{x^2(1) - x(0)x(2)} \quad (C.4)
\]

The MA parameters of the model can be estimated by:

\[
b_1(0) = x(0) \quad (C.5)
\]

\[
b_1(1) = x(1) + a_2(1)x(0) \quad (C.6)
\]
Appendix–D

Recursive LSE for AR(2)

The recursive form of least square estimation, derived here, can be used for efficient updating of AR(2) model of the signal which is useful for parameter updating of AS in Chapter 4.

In LSE, we have the model:

\[ X(k) = H(k)\Theta(k) + q(k) \]  

(D.1)

where \( X(k) = \begin{bmatrix} x(k) & x(k-1) & \cdots & x(k-N+1) \end{bmatrix} \) is the window of size \( N \) at time \( t_k \), \( \Theta(k) \) is a vector of model parameters, \( q(k) \) is the noise with unknown statistics, \( x(k) = h'(k)\Theta(k) + q(k) \). \( \Theta(k) \) is chosen to minimize the error functional. The estimation model is:

\[ \hat{X}(k) = H(k)\hat{\Theta}(k) \]  

(D.2)

\( \hat{\Theta}(k) \) is the estimate of unknown parameters.

\[ \hat{\Theta}(k) = \left[H'(k)H(k)\right]^{-1}H'(k)X(k) \]  

(D.3)
We define

\[
Q(k) = H'(k)H(k) \quad (D.4)
\]

\[
R(k) = H'(k)X(k) \quad (D.5)
\]

We can find \(Q(k)\) and \(R(k)\) recursively for each window from the following equations:

\[
Q(k) = \begin{bmatrix}
\sum_{i=1}^{N} x^2(k - i) & \sum_{i=1}^{N} x(k-i)x(k-i-1) \\
\sum_{i=1}^{N} x(k-i)x(k-i-1) & \sum_{i=1}^{N} x^2(k-i-1)
\end{bmatrix} \quad (D.6)
\]

\[
R(k) = \begin{bmatrix}
\sum_{i=0}^{N-1} x(k-i)x(k-i-1) \\
\sum_{i=0}^{N-1} x(k-i)x(k-i-2)
\end{bmatrix} \quad (D.7)
\]
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